Secure Classification as a Service

Levelled Homomorphic, Post-Quantum Secure Machine Learning Inference based on the CKKS Encryption Scheme

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Outline

Privacy for Medical Applications

Post-Quantum Security

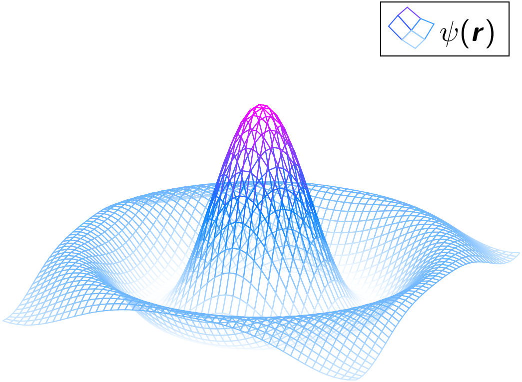


Figure: Illustration of a wave function ψ as commonly used in quantum mechanics.

The Rivest-Shamir-Adleman (RSA) Scheme

From the integers $\mathbb{Z}$, define the quotient ring $(\mathbb{Z}/q\mathbb{Z}, +, \cdot)$.

With unpadded RSA [3], some arithmetic can be performed on the ciphertext - looking at the encrypted ciphertext $\mathcal{E} : \mathbb{Z}/q\mathbb{Z} \mapsto \mathbb{Z}/q\mathbb{Z}$, $\mathcal{E}(m) := m^r \mod q$ $(r, q \in \mathbb{N})$ of the message $m_1, m_2 \in \mathbb{Z}/q\mathbb{Z}$ respectively, the following holds:

$$
\begin{aligned}
\mathcal{E}(m_1) \cdot \mathcal{E}(m_2) &\equiv (m_1)^r (m_2)^r \mod q \\
&\equiv (m_1 m_2)^r \mod q \\
&\equiv \mathcal{E}(m_1 \cdot m_2) \mod q
\end{aligned}
$$

Polynomial Rings

Definition (Cyclotomic Polynomial)

Given the n^{th} roots of unity $\{\xi_k\}$, define $\Phi_n \in \mathbb{Z}[X]$ as

$$\Phi_n(x) := \prod_{\substack{k=1 \\ \xi_k \text{primitive}}}^{n} (x - \xi_k) \,.$$

It is unique for each given $n \in \mathbb{N}$.

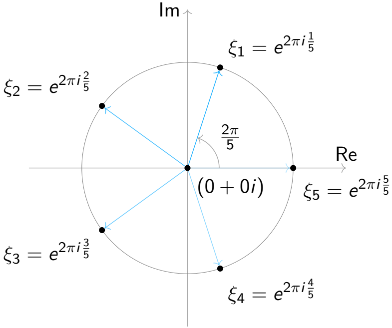


Figure: The 5^{th} roots of unity

Some Notation

- $\mathbb{Z}[X] := \{p : \mathbb{C} \mapsto \mathbb{C}\,, p(x) = \sum_{k=0}^{\infty} a_k x^k, a_k \in \mathbb{Z}\ \forall k \geq 0\}$
- $\mathbb{Z}_q[X] := (\mathbb{Z}/q\mathbb{Z})[X]$
- $\mathbb{Z}_q[X]/\Phi_M(X)$
- $\mathbb{Z}_q[X]/(X^N + 1)$ for N a power of 2.

The Learning With Errors (LWE) Problem

Definition (LWE-Distribution $A_{\boldsymbol{s},\chi_{error}}$)

Given a prime $q \in \mathbb{N}$ and $n \in \mathbb{N}$, we choose some secret $\boldsymbol{s} \in (\mathbb{Z}/q\mathbb{Z})^n$. In order to sample a value from the LWE distribution $A_{\boldsymbol{s},\chi_{error}}$:

- Draw a random vector $a \in (\mathbb{Z}/q\mathbb{Z})^n$ from the multivariate uniform distribution with its domain in the integers up to q.
- Given another probability distribution χ_{error} over the integers modulo q, sample a scalar 'error term' $\mu \in \mathbb{Z}/q\mathbb{Z}$ from it, often also referred to as noise.
- Set $b = \boldsymbol{s} \cdot \boldsymbol{a} + \mu$, with $\cdot$ denoting the standard vector product.
- Output the pair $(\boldsymbol{a}, b) \in (\mathbb{Z}/q\mathbb{Z})^n \times (\mathbb{Z}/q\mathbb{Z})$.

Search-LWE-Problem: Given m independent samples $(\boldsymbol{a}_i, b_i)_{0<i\leq m}$ from $A_{\boldsymbol{s},\chi_{error}}$, find $\boldsymbol{s}$.

The Learning With Errors on Rings (RLWE) Problem

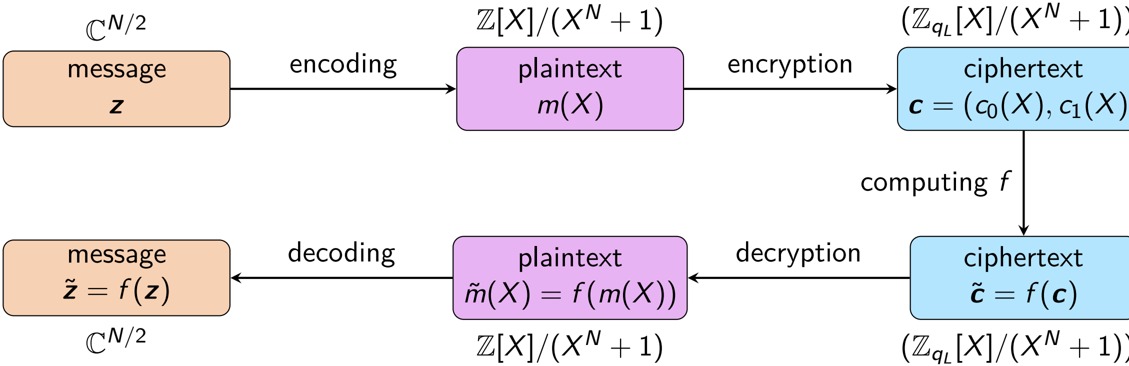
Corollary (RLWE-Distribution $B_{\mathbf{s},\chi_{error}}$)

Given a quotient $(R/qR, +, \cdot)$, we choose some secret $s \in R/qR$. In order to sample a value from the RLWE distribution $B_{s,\chi_{error}}$:

- *Uniformly randomly draw an element $a \in R/qR$*
- *Given another probability distribution χ_{error} over the ring elements, sample an 'error term' $\mu \in R/qR$ from it, also referred to as noise.*
- *Set $b = s \cdot a + \mu$, with $\cdot$ denoting the ring multiplication operation.*
- *Output the pair $(a, b) \in R/qR \times R/qR$.*

Use it to construct a cryptosystem...

Overview of Cheon-Kim-Kim-Song (CKKS)



Encoding and Decoding

CKKS.

Encode($\boldsymbol{z}$) For a given input vector $\boldsymbol{z}$, output $m = (\underline{\sigma}^{-1} \circ \underline{\rho_\delta}^{-1} \circ \underline{\pi}^{-1})(\boldsymbol{z}) = \underline{\sigma}^{-1}(\lfloor \delta \cdot \underline{\pi}^{-1}(\boldsymbol{z}) \rceil_{\underline{\sigma}(R)}) \quad \rightarrow m$

Decode(m) Decode plaintext m as $\boldsymbol{z} = (\underline{\pi} \circ \underline{\rho_\delta} \circ \underline{\sigma})(m) = (\underline{\pi} \circ \underline{\sigma})(\delta^{-1} m) \quad \rightarrow \boldsymbol{z}$

Encryption and Decryption

CKKS.

Encrypt($\boldsymbol{p}, m$) Let $(b, a) = \boldsymbol{p}$, $u \leftarrow \chi_{enc}$, $\mu_1, \mu_2 \leftarrow \chi_{error}$, then the ciphertext is
$\boldsymbol{c} = u \cdot \boldsymbol{p} + (m + \mu_1, \mu_2) = (m + bu + \mu_1, au + \mu_2) \quad \rightarrow \boldsymbol{c}$

Decrypt($s, \boldsymbol{c}$) Decrypt the ciphertext $\boldsymbol{c} = (c_0, c_1)$ as $m = [c_0 + c_1 s]_{q_L} \quad \rightarrow m$

Homomorphic Addition

CKKS.

Add$(\boldsymbol{c}_1, \boldsymbol{c}_2)$ Output $\boldsymbol{c}_3 = \boldsymbol{c}_1 + \boldsymbol{c}_2 \quad \rightarrow \boldsymbol{c}_3$

Demo: Secure Handwritten Digit Classification as a Service

Neural Networks

Matrix Multiplications

Confusion everywhere

Runtime Benchmarks

Ciphertext Visualisations

Conclusion

Crypto is good for us

Questions?

Glossary I

CKKS	Cheon-Kim-Kim-Song	10
LWE	Learning With Errors	8
RLWE	Learning With Errors on Rings	9
RSA	Rivest-Shamir-Adleman	5

Bibliography I

[1] Jung Hee Cheon, Andrey Kim, Miran Kim and Yongsoo Song. **Homomorphic Encryption for Arithmetic of Approximate Numbers**. ASIACRYPT. 2017.

[2] Daniel Huynh. **Cryptotree: fast and accurate predictions on encrypted structured data**. (2020). DOI: 10.48550/ARXIV.2006.08299. URL: https://arxiv.org/abs/2006.08299.

[3] Ronald L Rivest, Adi Shamir and Leonard M Adleman. **Cryptographic communications system and method**. US Patent 4,405,829. Sept. 1983.