



The Biological Setting: A Lung-Cancer Cell

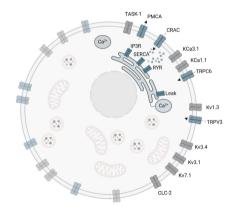


Figure: A549 Cancer Cell Model [1]

- A human cell is complex. In this project, we focus on the electrophysiological behaviour!
- Specifically, on the ion channels on the membrane of an A549 cancer cell.
- We improve simulation of the world's first electrophysiological cancer cell model [1].
- And visualise!





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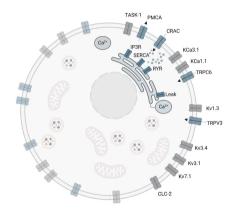


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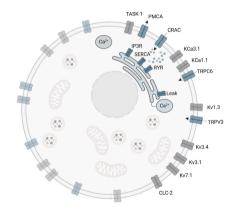


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Experiment: Patch-Clamping



Figure: Patch-Clamp System [2]

Approach in which electrophysiological behaviour of a cell can be measured in a lab. Patch-Clamping can be differentiated into two approaches:

- the Cell-attached recording method and
- the Whole-cell recording method.





Simulation of the Experiment



Hidden Markov Model

The whole cell current $I: T \to \mathbb{R}$ over time $t \in T \subset \mathbb{R}^+$ is the sum of all individual channel contributions $I_k, k \in \{1, ..., M\}$ over $M \in \mathbb{N}$ channel types

$$I(t) := \sum_{k=1}^{M} N_k I_k(t) = \sum_{k=1}^{M} N_k g_k p_{o,k} \left(V(t) - E_k \right) .$$

At each time step, the next state $\mathbf{s}_{k,n+1} \in [0,1]^{N_{s,k}}$ of the k-th channel type is obtained by

$$\boldsymbol{s}_{k,n+1} = H_k\left(V(t_n), \boldsymbol{C}(t_n), t_n\right) \boldsymbol{s}_{k,n}, \quad \text{with} \quad t_n := \sum_{i=0}^n (\Delta t)_i.$$





Solution of the Inverse Problem

• We want to find





Runtime Optimisation

The original MatLab implementation ran the entire night, so:

- Improve what we simulate
- Improve how we simulate
- Improve the inverse solution method
- Improve the solver





Quadratic Problem Formulation I

After simulation of the individual channel currents, we want to find

$$oldsymbol{N}_{ ext{opt}} = rg \min_{oldsymbol{N} \in \mathbb{N}_0^M} rac{1}{2} \|Roldsymbol{N} - oldsymbol{d}\|^2 \,.$$

the number of channels per type, with $d \in \mathbb{R}^{N_t}$ the experimentally measured current and $R \in \mathbb{R}^{N_t \times M}$ the matrix of all currents I_k per channel type. We find

$$oldsymbol{N}_{ ext{opt}} pprox rg \min_{oldsymbol{x} \in \mathbb{R}_{\perp}^{M}} f(oldsymbol{x}) = rg \min_{oldsymbol{x} \in \mathbb{R}_{\perp}^{M}} {}^{1}/{2} \|Roldsymbol{x} - oldsymbol{d}\|^{2} \,.$$





Quadratic Problem Formulation II

We manipulate the cost function $f: \mathbb{R}^M \to \mathbb{R}^+$,

$$f(\boldsymbol{x}) = \frac{1}{2}(R\boldsymbol{x} - \boldsymbol{d})^{T}(R\boldsymbol{x} - \boldsymbol{d})$$

$$= \frac{1}{2}\left(\boldsymbol{x}^{T}R^{T}R\boldsymbol{x} - \boldsymbol{x}^{T}R^{T}\boldsymbol{d} - \boldsymbol{d}^{T}R\boldsymbol{x} + \boldsymbol{d}^{T}\boldsymbol{d}\right)$$

$$= \frac{1}{2}\left(\boldsymbol{x}^{T}P\boldsymbol{x} + \boldsymbol{x}^{T}\boldsymbol{q} + \boldsymbol{q}^{T}\boldsymbol{x}\right) + \mathcal{O}(1)$$

$$= \frac{1}{2}\boldsymbol{x}^{T}P\boldsymbol{x} + \boldsymbol{q}^{T}\boldsymbol{x} + \mathcal{O}(1)$$

where we let $P := R^T R \in \mathbb{R}^{M \times M}$ and $\mathbf{q} := -R^T \mathbf{d} \in \mathbb{R}^M$ and leave out the constant $\mathbf{d}^T \mathbf{d}$ as $\mathcal{O}(1)$.





Quadratic Problem Formulation III

We can express the nonnegativity constraint $x \geq 0$ as an equality constraint using a slack variable $s \in \mathbb{R}_+^M$,

$$-x + s = 0 \Leftrightarrow Ax + s = b$$
 with $A = -1 \in \mathbb{R}^{M \times M}$ and $b = 0 \in \mathbb{R}^{M}$.

This leaves us with a constrained quadratic program,

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^M} \ ^1 / 2 \, \boldsymbol{x}^T P \boldsymbol{x} + \boldsymbol{q}^T \boldsymbol{x}, \\ s.t. \ A \boldsymbol{x} + \boldsymbol{s} &= \boldsymbol{b} \,, \ \boldsymbol{s} \in \mathbb{R}^M_+ \,. \end{aligned}$$





Quadratic Problem Formulation IV

The integer solution can then be obtained from rounding,

$$oldsymbol{N}_{ ext{opt}} = \lfloor oldsymbol{x}
ceil \in \mathbb{N}_0^M$$
 .

One obtains:

$$N_{\text{opt}} = [13, 247, 10, 1176, 38, 7, 24, 188, 15, 10, 234],$$

for the G0 cell cycle phase.





Adaptive Timestepping

In order to accelerate the simulation in areas where there is little change to the dynamics, we choose an adaptive step size $(\Delta t)_n$ based on

$$(\Delta t)_{n+1} = (\Delta t)_n \left(\frac{\Delta^{\text{tol}}}{\sum_{k=1}^M N_k \|\boldsymbol{s}_{k,n+1} - \boldsymbol{s}_{k,n}\|} \right)^{1/2}.$$





Runtime Optimisation

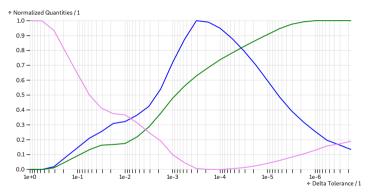


Figure: Relative change of the average timestep Δt (in blue), simulation runtime (in violet) and step acceptance rate (in green) when varying the delta tolerance Δ^{tol} .





Visualisation Dashboard



WebAssembly



Cancer Cell Simulation

Let's go!





References I

- [1] Sonja Langthaler, Theresa Rienmüller, Susanne Scheruebel, Brigitte Pelzmann, Niroj Shrestha, Klaus Zorn-Pauly, Wolfgang Schreibmayer, Andrew Koff and Christian Baumgartner. **A549 in-silico 1.0: A first computational model to simulate cell cycle dependent ion current modulation in the human lung adenocarcinoma.** *PLoS Comput. Biol.* 17.6 (June 2021), e1009091. DOI: 10.1371/journal.pcbi.1009091.
- [2] Patch-Clamp Electrophysiology. Apr. 2025. URL: https://www.criver.com/products-services/discovery-services/pharmacology-studies/neuroscience-models-assays/neuroscience-methods-endpoints/electrophysiology/patch-clamp?region=3696 (visited on 29/04/2025).

