Unsupervised Semantic Field Analysis
by methods from community detection

Special Topic on Networks
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Abstract

This work will attempt to

Figure 1: Add some sort of graph plot here.

1 Motivation

For non-periodic problem settings, Chebyshev series are a fantastic choice (Biemann 2006).

In the context of Natural Language Processing, graph clustering methods can also be employed for various other tasks such as word sense induction or language separation. In this work however, we will only focus on semantic field tagging.

2 Introduction

Let $\mathbb{N} = \mathbb{Z}^+$ denote the positive integers and $N_0 := \{0\} \cup \mathbb{N}$ the nonnegative integers.

The methods we will discuss to identify semantic fields will be based on graph clustering algorithms applied to a text corpus word connectedness / neighbourhood network. As we will discuss later, different notions of connectedness can give us different insight into the structure of a natural language. We will focus our attention on methods for undirected graphs, "graphs without direction" (cf. Definition 2.1).

2.1 Definition: Undirected Graph

A graph $G = (V, E)$ with vertices V and edges $E \subseteq V \times V$ is undirected if and only if $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall\, v_i, v_j \in V$.

Vertices are also often referred to as *nodes*. Every graph G is uniquely described by its adjacency matrix $A \in \{0,1\}^{n\times n}$ (Definition 2.2), which allows us to talk about "linear algebra" of graphs.

2.2 Definition: Adjacency Matrix

Let $A \in \{0,1\}^{n\times n}$ denote the symmetric adjacency matrix of an undirected graph $G = (V, E)$. Its entries are given by $a_{ij} = \{A\}_{ij} = \mathbb{1}_{(v_i,v_j)\in E}$, so $a_{ij} = 1$ if vertex v_i is connected to v_j and 0 otherwise.

By construction, $A = A^T$ is symmetric and has all-0s in the diagonal, a definition that corresponds to the fact that you cannot be friends with yourself in a social network.

Further let $m := |E|$ and $n := |V|$ denote the number of edges and vertices, respectively. The degree d_i of a vertex $v_i \in V$ is defined by the number of edges connecting to it, so

$$d_i := \deg(v_i) = \Big|\, \{(v_j, v_k) \in E \mid v_j = v_i\} \,\Big|,$$

for an undirected graph $G = (V, E)$. The handshaking lemma (Lemma 2.1) tells us an important fact useful for normalisation.

2.1 Lemma: Handshake

For every finite, undirected graph $G = (V, E)$ the individual vertex degrees sum up to exactly twice the number of edges, so

$$\sum_{i=1}^{n} d_i = \sum_{v \in V} \deg(v) = 2m\,.$$

The individual vertex degrees can be summarised in the so-called *degree matrix* $D := \text{diag}(d_1, ..., d_n)$, $D \in \mathbb{N}_0^{n \times n}$. The graph *Laplacian* is defined by $L := D - A$.

Given a graph, we are interested in performing **graph clustering**, also referred to as **community detection** or **graph partitioning**, the goal of which is to obtain a set of mutually exclusive clusters $C_i \subseteq V$ (cf. Definition 2.3). The term *graph partitioning* is more frequently used in the context of minimal cuts, where one aims to minimise each *cut size* referring to the number of edges in between clusters.

2.3 Definition: Graph Clustering

Let $C = \{C_i \subseteq V\}_{i=1...n_C}$ denote a clustering of $G = (V, E)$ into $n_C \in \mathbb{N}$ clusters where $C_i \cap C_j = \{\}\ \forall\, i, j \in \{1, ..., n_C\}$ and $\bigcup_{i=1}^{n_C} C_i = V$. Let $s_i \in \{1, ..., n_C\}$ denote the assigned cluster of vertex $v_i \in V$.

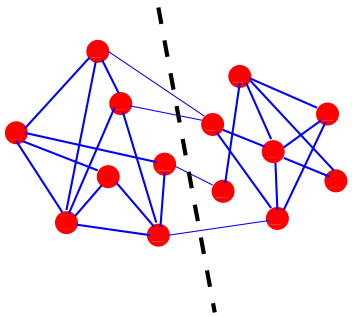

Figure 2: Partitioning of a graph using *cuts* (Fortunato and Castellano 2009).

These clusterings may be better or worse depending on the context, but a generally solid measure of "clustering goodness" is *modularity* (Definition 2.4).

2.4 Definition: Modularity

For a given undirected graph $G = (V, E)$ and clustering C, let

$$Q := \frac{1}{2m} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \delta(s_i, s_j) \,,$$

with $\delta(\cdot, \cdot)$ the Kronecker delta indicating whether two vertices v_i and v_j belong to the same cluster (Blondel et al. 2008).

Modularity is a measure of the quality of a clustering (also referred to as a partitioning) of G. It can also be written as the sum of individual cluster contributions

$$Q = \sum_{c=1}^{n_C} Q_c = \frac{1}{2m} \sum_{c=1}^{n_C} \left[\sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \right] ,$$

which might make its purpose a bit clearer.

3 Clustering Methods and Algorithms

In order to find ... Most clustering methods can be broadly categorised into spectral, partitional (such as k-means clustering), hierarchical, randomised, divisive and quality-optimisation algorithms. An interesting approach is Markov-Chain-Clustering, which may employ Simulated Annealing Fortunato 2010.

Community detection is, in principle, usually a "very hard" task given the vast number of possible system configurations as the graph grows in the number of edges or vertices, a statement which can be made more precise using complexity theory. In conventional complexity theory, problems are filed into different complexity classes when analysing their runtime and memory usage. There exist

1. NL (Nondeterministic Logarithmic space)
2. P (Polynomial time)
3. NP (Nondeterministic Polynomial time)
4. PSPACE (Polynomial space)
5. EXPTIME (Exponential time)
6. EXPSPACE (Exponential space)

computational complexity classes, sorted by the amount of problems contained in them (NL $\subseteq$ P $\subseteq$ NP $\subseteq$ PSPACE $\subseteq$ EXPTIME $\subseteq$ EXPSPACE). A particularly interesting

open problem is whether P = NP, one of the millennium prize problems and the most important open problem in computer science.

3.1 Definition: NP-Hardness

A problem is referred to as *NP-hard* if and only if it is at least as hard as the hardest problems in the complexity class NP (nondeterministic polynomial time). Formally written,

$$\mathrm{NP} := \bigcup_{k \in \mathbb{N}} \mathrm{NTIME}(n^k)$$

the union of all decision problems with runtime bounded by $\mathcal{O}(n^k)$.

Many community detection algorithms or problems relating to it are NP-hard (Fortunato 2010). It is therefore often futile to employ exact algorithms as they quickly start to become infeasible for larger system sizes.

Note that here we only consider an unweighted, undirected graph, while most clustering algorithms also allow working with a weighted graph $\tilde{G} = (V, \tilde{E})$ where $\tilde{E} \subset V \times V \times \mathbb{R}^+$ is a set of three-tuples (v_i, v_j, w_{ij}) instead of vertex pairs. Or equivalently, one can define a function $w : \tilde{E} \mapsto \mathbb{R}^+$ which maps vertex pairs (edges) to their respective weight.

3.1 Embeddings

Cosine-Similarity

$$\rho_{ij} = \arccos\left(\frac{\boldsymbol{a_i a_j}}{\|\boldsymbol{a_i}\|_2 \cdot \|\boldsymbol{a_j}\|_2}\right)$$

3.2 Girvan-Newman

The clustering method deduced in Girvan and Newman 2001 was a clear revolution at the time and brought graph clustering / community detection methods closer to practicians. Optimises centrality and betweenness.

3.3 Louvain

The popular algorithm introduced by Blondel et al. 2008 is based on modularity optimisation (cf. Definition 2.4) and has computational complexity of only $\mathcal{O}(m)$. It is referred to as the *Louvain method* named after the University of Louvain in Belgium, alma mater of the first author. The Louvain method is divided into two phases:

Phase One (local optimisation) and Phase Two (merging of vertices). Grindrod and Lambiotte 2022.

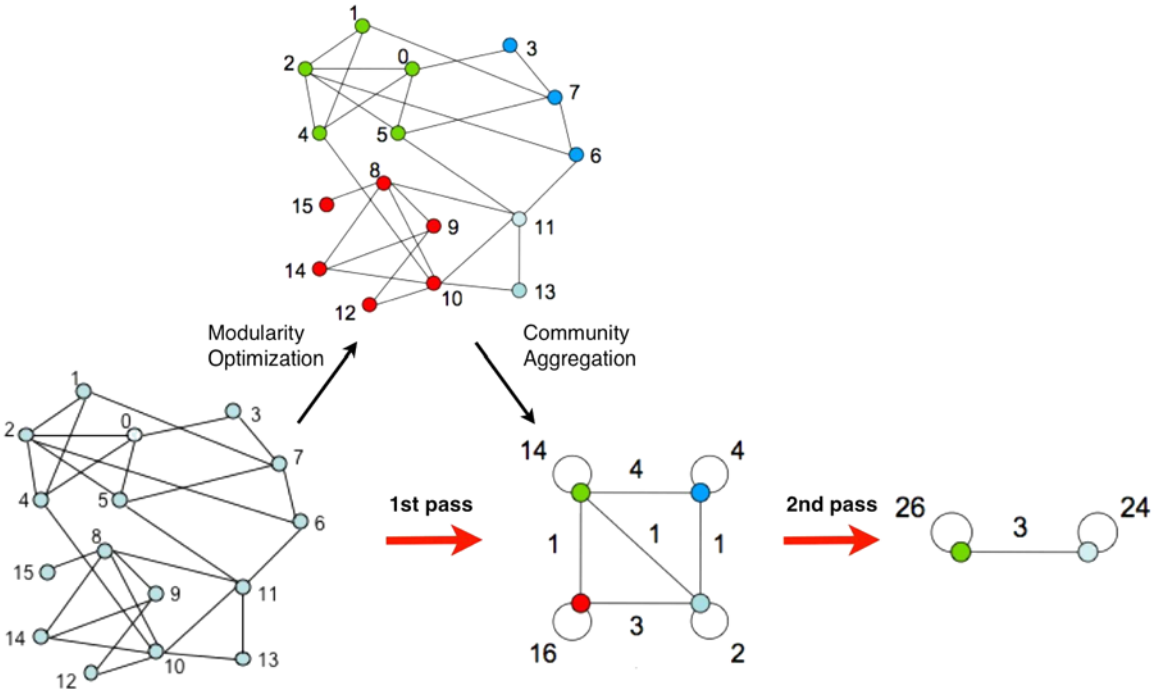

Figure 3: Louvain Graph Clustering Method Blondel et al. 2008.

Louvain is very effective, in part because a change (improvement) in modularity obtained by reassigning an **isolated** (not yet clustered) vertex v_i to a community C_j may be explicitly obtained by

$$\Delta Q = \left[\frac{\Sigma_{in}^{(j)} + k_{i,in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)} + k_i}{2m} \right)^2 \right] - \left[\frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 - \left(\frac{k_i}{2m} \right)^2 \right], \qquad (1)$$

(as given in the original paper) which is highly useful from a computational perspective (Blondel et al. 2008). Here, we have $d_{i,in}^{(j)} := \left| \left\{ (v_x, v_y) \in (C_j \times C_j) \cap E \;\middle|\; v_x = v_i \right\} \right|$ the number of in-cluster edges connected to vertex v_i and the corresponding $\Sigma_{in}^{(j)} := \sum_{v_i \in C_j} d_{i,in}^{(j)} = |(C_j \times C_j) \cap E|$ represents the total number of edges between vertices within the same cluster C_j (so neglecting edges to vertices outside C_j). Likewise, $\Sigma_{tot}^{(j)} := \sum_{v_i \in C_j} d_i$ denotes the sum of vertex degrees within the cluster C_j (so including contributions to outside nodes).

In our case, the above weightings $k_j \in \mathbb{R}^+$ reduce to the degrees d_j alone given that we operate on an unweighted, undirected graph $G = (V, E)$. Note that the above

Equation (1) simplifies to

$$\Delta Q = \cancel{\frac{\Sigma_{in}^{(j)}}{2m}} + \frac{d_{i,in}^{(j)}}{2m} - \frac{\cancel{\left(\Sigma_{tot}^{(j)}\right)^2} + 2\Sigma_{tot}^{(j)} d_i + \cancel{d_i^2}}{4m^2} - \cancel{\frac{\Sigma_{in}^{(j)}}{2m}} + \frac{\cancel{\left(\Sigma_{tot}^{(j)}\right)^2} + \cancel{d_i^2}}{4m^2} = \frac{d_{i,in}^{(j)}}{2m} - \frac{\Sigma_{tot}^{(j)} d_i}{2m^2} \,. \quad (2)$$

In more general terms, one may obtain the impact of moving a vertex $v_i \in V$ from cluster C_a to C_b by considering their individual contributions

$$Q_c = \sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) = \frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 ,$$

to the total modularity $Q = \sum_{c=1}^{n_C} Q_c^{(c)}$, which are

$$\tilde{Q}_c^{(a)} = \frac{\Sigma_{in}^{(a)} - d_{i,in}^{(a)}}{2m} - \left(\frac{\Sigma_{tot}^{(a)} - d_i}{2m} \right)^2 ,$$
$$\tilde{Q}_c^{(b)} = \frac{\Sigma_{in}^{(b)} + d_{i,in}^{(b)}}{2m} - \left(\frac{\Sigma_{tot}^{(b)} + d_i}{2m} \right)^2 ,$$

after moving vertex v_i from cluster C_a to C_b. So the change in individual cluster modularity is given by the terms

$$\Delta Q_c^{(a)} = \tilde{Q}_c^{(a)} - Q_c^{(a)} = \frac{-d_{i,in}^{(a)}}{2m} - \frac{d_i^2 - 2\Sigma_{tot}^{(a)} d_i}{4m^2} ,$$
$$\Delta Q_c^{(b)} = \tilde{Q}_c^{(b)} - Q_c^{(b)} = \frac{d_{i,in}^{(b)}}{2m} - \frac{d_i^2 + 2\Sigma_{tot}^{(b)} d_i}{4m^2} ,$$

which yield the total modularity change $\Delta Q = \Delta Q_c^{(a)} + \Delta Q_c^{(b)}$ in the general case. Considering the special case when vertex $v_i \in V$ is still isolated, so when $C_a = \{v_i\}$ contains exactly that one vertex, we can further simplify to

$$\Delta Q = \cancel{\frac{d_i^2}{4m^2}} + \frac{d_{i,in}^{(b)}}{2m} - \frac{\cancel{d_i^2} + 2d_i \Sigma_{tot}^{(b)}}{4m^2} = \frac{d_{i,in}^{(b)}}{2m} - \frac{\Sigma_{tot}^{(b)} d_i}{2m^2} ,$$

because $\Sigma_{in}^{(a)} = 0$, $\Sigma_{tot}^{(a)} = d_i$ and therefore $\Delta Q_c^{(a)} = \frac{d_i^2}{4m^2}$, and we arrive at the same expression given by Blondel et al. 2008 (Equation (2)), for the special case when $C_a = \{v_i\}$. Note that the modularity contribution $Q_c = 0$ for an empty cluster.

This modularity gain of course is not restricted to the Louvain method in and of itself, it applies to all clustering methods and is relevant to those that operate on modularity.

Louvain is among the most widely used techniques for graph clustering and is available in numerous software packages.

3.4 Chinese Whispers

This method, like the Louvain method, also initialises the algorithm by assigning each vertex its own category (cluster).

The *Chinese Whispers* algorithm due to Biemann 2006

```
Input: an undirected graph G = (V, E).
Output: a graph clustering C = {C_i}_{i=1,...,n_C} into n_C classes.

Initialise with n_C = n classes, one per vertex.
while there are changes, do
  for v_i in shuffle(V), do
    Set s_i = 3
  end
end
```

Corresponds to an agent-based simulation of a social network Biemann 2006. Similar to Markov-Chain-Clustering Dongen 2000, Fortunato 2010.

The algorithm may equivalently be defined by considering the *class matrix* $\mathcal{D} \in \{0,1\}^{n \times n_C}$ of a graph $G = (V, E)$ in which rows represent nodes and columns represent clusters $C_j \in C$. So $G_{ij} = 1$ if v_i belongs to C_j and 0 otherwise. The algorithm operates iteratively and updates the class matrix $\mathcal{D}^{(k)}$ using

$$M : \{0,1\}^{n \times n_C} \mapsto \{0,1\}^{n \times n_C}, \quad \{M(\mathcal{D})\}_{ij} = \mathcal{D}_{ij}$$

$$\mathcal{D}^{(k+1)} = M(\mathcal{D}^{(k)})A, \quad k = 0, 1, 2, ..., N$$

per iteration k, where $\mathcal{D}^{(0)} = I$ is the identity, corresponding to the initialisation of individual classes for each vertex.

The paper also discusses acquisition of word classes (semantic fields), where they ran Chinese Whispers against the British National Corpus (BNC) to identify clusters of similar words. In order to improve performance, they cut off the 2000 most frequent words in the corpus which we will also employ. This set of highly frequent words is likely to contain the *core vocabulary* of the language including, but not limited to, pronouns, conjunctions, prepositions and similar parts of speech. Biemann 2006 identified 282 clusters, 26 of which contained more than 100 words. Word clustering methods such as this one may even be used to improve part-of-speech tagging (Ushioda 1996).

Table 1: Table ordered by size Biemann 2006.

Size $\|C_i\|$	Sample Words
18432	secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ...
4916	busy, grey, tiny, thin, sufficient, attractive, vital, ...
4192	filled, revealed, experienced, learned, pushed, occurred, ...
3515	White, Green, Jones, Hill, Brown, Lee, Lewis, Young, ...
2211	Ian, Alan, Martin, Tony, Prince, Chris, Brian, Harry, Andrew, Christ, Steve, ...
1855	Central, Leeds, Manchester, Australia, Yorkshire, Belfast, Glasgow, Middlesbrough, ...

3.5 Watset

A more recent work Ustalov et al. 2019 describes a meta-algorithm for graph clustering in the context of Natural Language Processing, which can be based on Chinese Whispers and also other methods.

3.6 Spectral Clustering Methods

Fortunato 2010.

3.7 Fiedler

Fortunato 2010.

4 Method

We shall also remove the 2000 most frequent words

5 Results

An itemize with some semantic field clusters we found

Plotty plot

Table 2: Results ordered by size.

Size $\lvert C_i \rvert$	Sample Words
18432	secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ...

Apply to Karate Club

Computational complexity table from Ustalov et al. 2019, also referencing complexity section above. Fortunato also has complexities.

6 Author analysis?

Authors whose works circulate around these semantic fields: bla bla, maybe not that interesting

7 Discussion and Outlook

Bla

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A Appendix Things?