

Unsupervised Semantic Field Analysis using Community Detection Methods

Special Topic on [NETWORKS](#)

Candidate Number: [1072462](#)

Abstract

In this work we will attempt to compare methods in the domain of semantic field tagging by considering a language corpus as a large graph of neighbouring words. The network is built by performing part-of-speech tagging iteratively on the entire corpus and linking together principal words that are connected in a neighbouring sense up to a radius of r words. The resulting graph G_r is then analysed using community detection methods such as *Louvain Clustering*, *Chinese Whispers*, *Clauset Greedy Modularity* and other clustering approaches which we only discuss briefly. We derive the gain in modularity presented in the original work [Blondel et al. 2008](#) upon atomic modification of the clusters $\{C_i\}_{i=1}^{n_C}$. The Chinese Whispers algorithm is a simple graph clustering method originating from research in Natural Language Processing ([Biemann 2006](#)) and it is well-established for semantic field analysis. The abovementioned algorithms are applied to a real-world corpus of the Latin language, working with a graph of 10843 nodes and 2.4 million edges. We compare them in terms of output modularity, runtime and number of clusters on the graphs introduced, which we compare to the eigengap in the spectrum of the graph Laplacian.

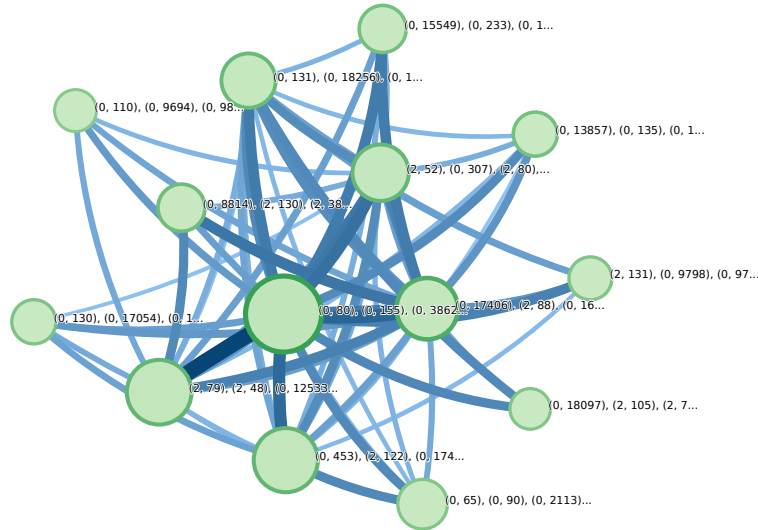


Figure 1: Visualisations of a clustering of the corpus generated semantic network G_{12} , obtained using the Map Equation approach (cf. Section 2.3.3). Visualised using the InfoMap visualisation package ([Rosvall and Bergstrom 2008](#)), where the labels indicate database references.

Contents

1	Introduction	3
2	Clustering Methods and Algorithms	6
2.1	The Louvain Method	7
2.2	Chinese Whispers	9
2.2.1	Greedy Modularity	11
2.3	Further Approaches	11
2.3.1	Embeddings	11
2.3.2	Girvan-Newman	11
2.3.3	Map Equation	12
2.3.4	Spectral Clustering Methods	12
3	Network Generation and Results	12
4	Discussion and Outlook	17

1 Introduction

Let $\mathbb{N} = \mathbb{Z}^+$ denote the positive integers and $N_0 := \{0\} \cup \mathbb{N}$ the nonnegative integers.

The methods we will discuss to identify semantic fields will be based on graph clustering algorithms applied to a text corpus word connectedness / neighbourhood network G_r . As we will discuss later, different neighbourhood radii $r \in \mathbb{N}$ give us different insight into the structure of a natural language. We will focus our attention on methods for undirected graphs, “graphs without direction” (cf. Definition 1.1).

1.1 Definition: Undirected Graph

A graph $G = (V, E)$ with vertices V and edges $E \subseteq V \times V$ is undirected if and only if $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall v_i, v_j \in V$.

Vertices are also referred to as *nodes*. Every graph G is uniquely described by its adjacency matrix $A \in \{0, 1\}^{n \times n}$ (Definition 1.2), which allows us to talk about “linear algebra” of graphs, which is especially relevant to spectral graph clustering methods (cf. Section 2.3.4).

1.2 Definition: Adjacency Matrix

Let $A \in \{0, 1\}^{n \times n}$ denote the symmetric adjacency matrix of an undirected graph $G = (V, E)$. Its entries are given by $a_{ij} = \{A\}_{ij} = \mathbb{1}_{(v_i, v_j) \in E}$, so $a_{ij} = 1$ if vertex v_i is connected to v_j and 0 otherwise.

By construction, $A = A^T$ is symmetric and has all-0s in the diagonal, a definition that corresponds to the fact that you cannot be friends with yourself in a social network.

Further let $m := |E|$ and $n := |V|$ denote the number of edges and vertices, respectively. The degree d_i of a vertex $v_i \in V$ is defined by the number of edges connecting to it, so

$$d_i := \deg(v_i) = \left| \{(v_j, v_k) \in E \mid v_j = v_i\} \right|,$$

for an undirected graph $G = (V, E)$. The handshaking lemma (Lemma 1.1) tells us an important fact useful for normalisation.

1.1 Lemma: Handshake

For every finite, undirected graph $G = (V, E)$ the individual vertex degrees sum up to exactly twice the number of edges, so

$$\sum_{i=1}^n d_i = \sum_{v \in V} \deg(v) = 2m.$$

The individual vertex degrees can be summarised in the so-called *degree matrix* $D := \text{diag}(d_1, \dots, d_n)$, $D \in \mathbb{N}_0^{n \times n}$. The graph *Laplacian* is defined by $L := D - A$.

Given a graph, we are interested in performing **graph clustering**, also referred to as **community detection** or **graph partitioning**, the goal of which is to obtain a set of mutually exclusive clusters $C_i \subseteq V$ (cf. Definition 1.3). The term *graph partitioning* is more frequently used in the context of minimal cuts, where one aims to minimise each *cut size* referring to the number of edges in between clusters.

1.3 Definition: Graph Clustering

Let $C = \{C_i \subseteq V\}_{i=1 \dots n_C}$ denote a clustering of $G = (V, E)$ into $n_C \in \mathbb{N}$ clusters where $C_i \cap C_j = \{\}$ $\forall i, j \in \{1, \dots, n_C\}$ and $\bigcup_{i=1}^{n_C} C_i = V$. Let $s_i \in \{1, \dots, n_C\}$ denote the assigned cluster of vertex $v_i \in V$.

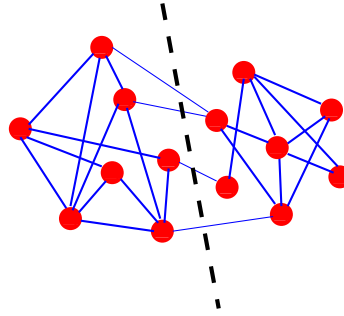


Figure 2: Partitioning of a graph using *cuts* (Fortunato and Castellano 2009).

These clusterings may be better or worse depending on the context, but a generally solid measure of “clustering goodness” is *modularity* (Definition 1.4).

1.4 Definition: Modularity

For a given undirected graph $G = (V, E)$ and clustering C , let

$$Q := \frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n \left(A_{ij} - \frac{d_i d_j}{2m} \right) \delta(s_i, s_j),$$

with $\delta(\cdot, \cdot)$ the Kronecker delta indicating whether two vertices v_i and v_j belong to the same cluster (Blondel et al. 2008).

Modularity is a measure of the quality of a clustering (also referred to as a partitioning) of G . It can also be written as the sum of individual cluster contributions

$$Q = \sum_{c=1}^{n_C} Q_c = \frac{1}{2m} \sum_{c=1}^{n_C} \left[\sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \right],$$

which might make its purpose a bit clearer.

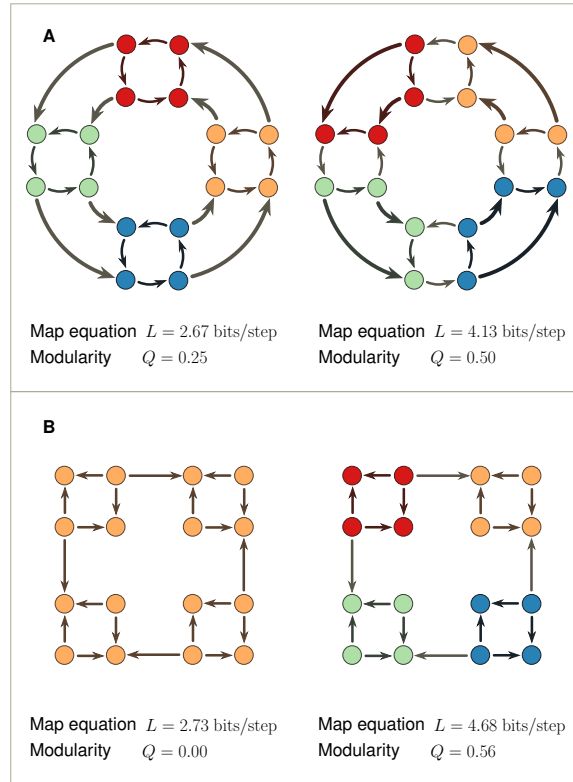


Figure 3: Four clustered graphs along with their modularity Q (and map equation value L , cf. Rosvall and Bergstrom 2008) to provide a visual illustration of modularity.

2 Clustering Methods and Algorithms

In the context of Natural Language Processing, graph clustering methods can be employed for various tasks such as word sense induction or language separation. In this work however, we will only focus on semantic field tagging.

Most clustering methods can be broadly categorised into spectral, partitional (such as k-means clustering), hierarchical, randomised, divisive and quality-optimisation algorithms (Fortunato 2010).

Community detection is, in principle, usually a “very hard” task given the vast number of possible system configurations as the graph grows in the number of edges or vertices, a statement which can be made more precise using complexity theory. In conventional complexity theory, problems are filed into different complexity classes when analysing their runtime and memory usage. There exist

1. NL (Nondeterministic Logarithmic space)
2. P (Polynomial time)
3. NP (Nondeterministic Polynomial time)
4. PSPACE (Polynomial space)
5. EXPTIME (Exponential time)
6. EXPSPACE (Exponential space)

computational complexity classes, sorted by the amount of problems contained in them ($NL \subseteq P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq EXPSPACE$). A particularly interesting open problem is whether $P = NP$, one of the millennium prize problems and the most important open problem in computer science.

2.1 Definition: NP-Hardness

A problem is referred to as *NP-hard* if and only if it is at least as hard as the hardest problems in the complexity class NP (nondeterministic polynomial time). Formally written,

$$NP := \bigcup_{k \in \mathbb{N}} NTIME(n^k)$$

the union of all decision problems with runtime bounded by $\mathcal{O}(n^k)$.

Many community detection algorithms or problems relating to it are NP-hard (Fortunato 2010). It is therefore often futile to employ exact algorithms as they quickly start to become infeasible for larger system sizes.

Note that here we only consider an unweighted, undirected graph, while most clustering algorithms also allow working with a weighted graph $\tilde{G} = (V, \tilde{E})$ where $\tilde{E} \subset V \times V \times \mathbb{R}^+$ is a set of three-tuples (v_i, v_j, w_{ij}) instead of vertex pairs. Or equivalently, one can define a function $w : \tilde{E} \mapsto \mathbb{R}^+$ which maps vertex pairs (edges) to their respective weight.

2.1 The Louvain Method

The popular algorithm introduced in Blondel et al. 2008 is based on modularity optimisation (cf. Definition 1.4) and has computational complexity of only $\mathcal{O}(m)$. It is referred to as the *Louvain method* named after the University of Louvain in Belgium, alma mater of the first author. The Louvain method is divided into two phases: Phase One (local optimisation) and Phase Two (merging of vertices) and they are repeated until modularity stops increasing (Blondel et al. 2008).

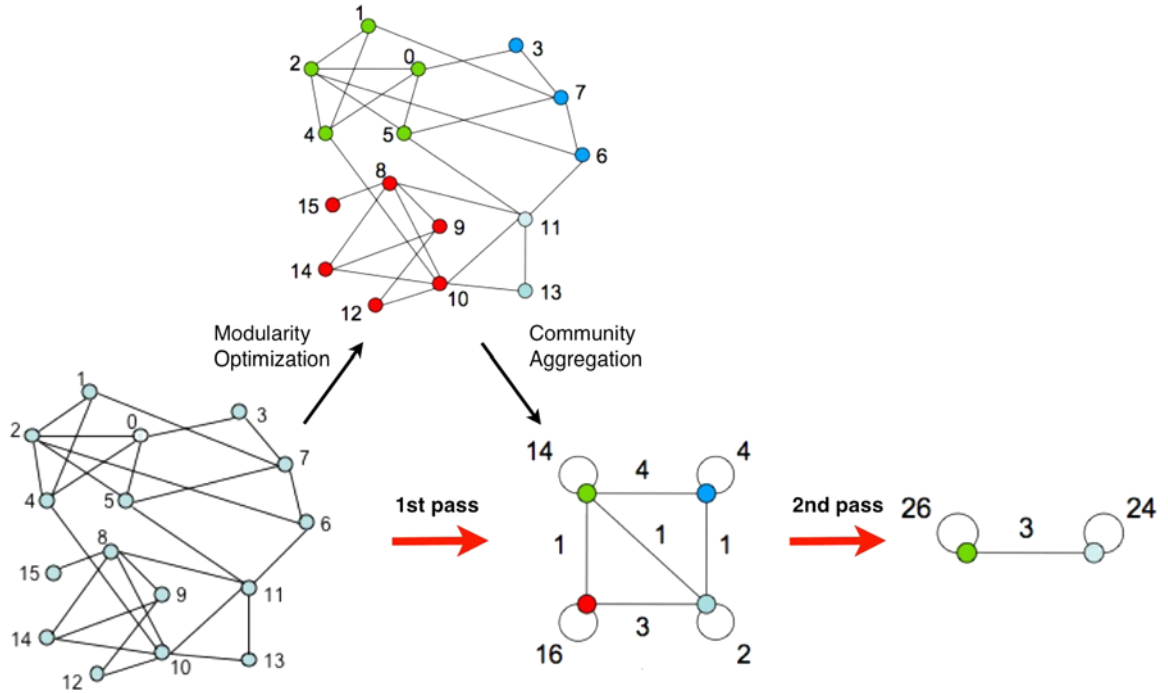


Figure 4: An illustration of the Louvain graph clustering method as described by Blondel et al. 2008.

Louvain is very effective, in part because a change (improvement) in modularity obtained by reassigning an **isolated** (not yet clustered) vertex v_i to a community C_j

may be explicitly obtained by

$$\Delta Q = \left[\frac{\Sigma_{in}^{(j)} + k_{i,in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)} + k_i}{2m} \right)^2 \right] - \left[\frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 - \left(\frac{k_i}{2m} \right)^2 \right], \quad (1)$$

(as given in the original paper) which is highly useful from a computational perspective (Blondel et al. 2008). Here, we have $d_{i,in}^{(j)} := |\{(v_x, v_y) \in (C_j \times C_j) \cap E \mid v_x = v_i\}|$ the number of in-cluster edges connected to vertex v_i and the corresponding $\Sigma_{in}^{(j)} := \sum_{v_i \in C_j} d_{i,in}^{(j)} = |(C_j \times C_j) \cap E|$ represents the total number of edges between vertices within the same cluster C_j (so neglecting edges to vertices outside C_j). Likewise, $\Sigma_{tot}^{(j)} := \sum_{v_i \in C_j} d_i$ denotes the sum of vertex degrees within the cluster C_j (so including contributions to outside nodes).

In our case, the above weightings $k_j \in \mathbb{R}^+$ reduce to the degrees d_j alone given that we operate on an unweighted, undirected graph $G = (V, E)$. Note that the above Equation (1) simplifies to

$$\Delta Q = \frac{\Sigma_{in}^{(j)}}{2m} + \frac{d_{i,in}^{(j)}}{2m} - \frac{(\Sigma_{tot}^{(j)})^2 + 2\Sigma_{tot}^{(j)}d_i + d_i^2}{4m^2} - \frac{\Sigma_{in}^{(j)}}{2m} + \frac{(\Sigma_{tot}^{(j)})^2 + d_i^2}{4m^2} = \frac{d_{i,in}^{(j)}}{2m} - \frac{\Sigma_{tot}^{(j)}d_i}{2m^2}. \quad (2)$$

In more general terms, one may obtain the impact of moving a vertex $v_i \in V$ from cluster C_a to C_b by considering their individual contributions

$$Q_c = \sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) = \frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2,$$

to the total modularity $Q = \sum_{c=1}^{n_C} Q_c^{(c)}$, which are

$$\begin{aligned} \tilde{Q}_c^{(a)} &= \frac{\Sigma_{in}^{(a)} - d_{i,in}^{(a)}}{2m} - \left(\frac{\Sigma_{tot}^{(a)} - d_i}{2m} \right)^2, \\ \tilde{Q}_c^{(b)} &= \frac{\Sigma_{in}^{(b)} + d_{i,in}^{(b)}}{2m} - \left(\frac{\Sigma_{tot}^{(b)} + d_i}{2m} \right)^2, \end{aligned}$$

after moving vertex v_i from cluster C_a to C_b . So the change in individual cluster modularity is given by the terms

$$\begin{aligned} \Delta Q_c^{(a)} &= \tilde{Q}_c^{(a)} - Q_c^{(a)} = \frac{-d_{i,in}^{(a)}}{2m} - \frac{d_i^2 - 2\Sigma_{tot}^{(a)}d_i}{4m^2}, \\ \Delta Q_c^{(b)} &= \tilde{Q}_c^{(b)} - Q_c^{(b)} = \frac{d_{i,in}^{(b)}}{2m} - \frac{d_i^2 + 2\Sigma_{tot}^{(b)}d_i}{4m^2}, \end{aligned}$$

which yield the total modularity change $\Delta Q = \Delta Q_c^{(a)} + \Delta Q_c^{(b)}$ in the general case. Considering the special case when vertex $v_i \in V$ is still isolated, so when $C_a = \{v_i\}$ contains exactly that one vertex, we can further simplify to

$$\Delta Q = \frac{d_i^2}{4m^2} + \frac{d_{i,in}^{(b)}}{2m} - \frac{d_i^2 + 2d_i \Sigma_{tot}^{(b)}}{4m^2} = \frac{d_{i,in}^{(b)}}{2m} - \frac{\Sigma_{tot}^{(b)} d_i}{2m^2},$$

because $\Sigma_{in}^{(a)} = 0$, $\Sigma_{tot}^{(a)} = d_i$ and therefore $\Delta Q_c^{(a)} = \frac{d_i^2}{4m^2}$, and we arrive at the same expression given by [Blondel et al. 2008](#) (Equation (2)), for the special case when $C_a = \{v_i\}$. Note that the modularity contribution $Q_c = 0$ for an empty cluster.

This modularity gain of course is not restricted to the Louvain method in and of itself, it applies to all clustering methods and is relevant to those that operate on modularity.

The Louvain algorithm is among the most widely used techniques for graph clustering and is available in numerous software packages.

2.2 Chinese Whispers

This method, like the Louvain method, also initialises the algorithm by assigning each vertex its own category (cluster).

The *Chinese Whispers* algorithm due to [Biemann 2006](#)

```

1 Input: an undirected graph  $G = (V, E)$ .
2 Output: a graph clustering  $C = \{C_i\}_{i=1,\dots,n_C}$  into  $n_C$  classes.
3
4 Initialise with  $n_C = n$  classes, one per vertex.
5 while there are changes, do
6   for  $v_i$  in shuffled( $V$ ), do
7     Set  $s_i = \arg \max_j \left| \{(v_k, v_l) \in E \mid v_k = v_i \wedge v_l \in C_j\} \right|$ .
8   end
9 end
```

Chinese Whispers works by iterating over all vertices in random order and reassigning them to the neighbouring category with the highest number of connected edges. If there is an equal number of edges, the reassigned cluster is chosen at random. This algorithm can be thought of as an agent-based simulation of a social network ([Biemann 2006](#)) and is similar to Markov-Chain-Clustering ([Dongen 2000](#); [Fortunato 2010](#)).

The algorithm may equivalently be defined by considering the *class matrix* $\mathcal{D} \in \{0, 1\}^{n \times n_C}$ of a graph $G = (V, E)$ in which rows represent nodes and columns represent

clusters $C_j \in C$. So $G_{ij} = 1$ if v_i belongs to C_j and 0 otherwise. The algorithm operates iteratively and updates the class matrix $\mathcal{D}^{(k)}$ using

$$M : \{0, 1\}^{n \times n_C} \mapsto \{0, 1\}^{n \times n_C}, \quad \{M(\mathcal{D})\}_{ij} = \delta_{j, \arg \max_{k \in \{0 \dots n_C\}} \mathcal{D}_{ik}},$$

$$\mathcal{D}^{(k+1)} = M(\mathcal{D}^{(k)})A, \quad k = 0, 1, 2, \dots, N$$

per iteration k , where $\mathcal{D}^{(0)} = I$ is the identity, corresponding to the initialisation of individual classes for each vertex. M is a row-wise operation on its argument, which sets each entry in the row to zero except the (first appearing) largest one, which it sets to one.

The paper also discusses acquisition of word classes (semantic fields), where they ran Chinese Whispers against the British National Corpus (BNC) to identify clusters of similar words. In order to improve performance, they cut off the 2000 most frequent words in the corpus which we will also employ later. This set of highly frequent words is likely to contain the *core vocabulary* of the language including, but not limited to, pronouns, conjunctions, prepositions and similar parts of speech. [Biemann 2006](#) identified 282 clusters, 26 of which contained more than 100 words. Word clustering methods such as this one may even be used to improve part-of-speech tagging ([Ushioda 1996](#)).

Table 1: Table ordered by size [Biemann 2006](#).

Size $ C_i $	Sample Words
18432	secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ...
4916	busy, grey, tiny, thin, sufficient, attractive, vital, ...
4192	filled, revealed, experienced, learned, pushed, occurred, ...
3515	White, Green, Jones, Hill, Brown, Lee, Lewis, Young, ...
2211	Ian, Alan, Martin, Tony, Prince, Chris, Brian, Harry, Andrew, Christ, Steve, ...
1855	Central, Leeds, Manchester, Australia, Yorkshire, Belfast, Glasgow, Middlesbrough, ...

A more recent work [Ustalov et al. 2019](#) describes a meta-algorithm for graph clustering in the context of Natural Language Processing, which can be based on Chinese

Whispers and also other methods (hence the term 'meta'). Their algorithm (WATSET) is designed for *fuzzy* graph clustering which exceeds the scope of this special topic, but represents the state of the art in graph-based applications in Natural Language Processing. Fuzzy clustering algorithms are capable of producing *soft* clusterings, which relax the requirement that all clusters are strictly disjoint (so $C_i \cap C_j = \{\}$ $\forall i, j$, as introduced in Definition 1.3).

2.2.1 Greedy Modularity

This modularity-optimisation based clustering method takes a very similar route as the Louvain algorithm, which is in fact based on Clauset Greedy Modularity (Clauset, Newman and Moore 2004). The general idea is again based on maximizing modularity.

2.3 Further Approaches

2.3.1 Embeddings

Given a graph embedding, in which vertices are mapped to a point in \mathbb{R}^n , a frequently used measure of similarity is *Cosine-Similarity*:

$$\rho_{ij} = \arccos \left(\frac{\mathbf{a}_i \mathbf{a}_j}{\|\mathbf{a}_i\|_2 \cdot \|\mathbf{a}_j\|_2} \right),$$

where vectors $\mathbf{a}_i, \mathbf{a}_j \in \mathbb{R}^n$ denote the point corresponding to vertex v_i, v_j respectively (Fortunato 2010). Of course this is not quite as connected to the other methods given that it already assumes a full embedding which itself is a very involved procedure, in many instances similar to clustering methods.

2.3.2 Girvan-Newman

The clustering method deduced in Girvan and Newman 2001 was a clear revolution at the time and brought graph clustering / community detection methods closer to practitioners. It consists of multiple iterations which will each increase the number of clusters by exactly one. Each step removes the “most valuable edge”, usually given by the edge (v_i, v_j) with the highest *betweenness centrality*

$$b_i = \frac{2}{(n-1)(n-2)} \sum_{j=1, \neq i}^n \sum_{l=1, \neq i}^{j-1} \frac{\sigma_{jl}^i}{\sigma_{jl}},$$

where σ_{jl} denotes the number of shortest paths connecting vertices v_j and v_l , and σ_{jl}^i the number of such paths passing through vertex v_i (Grindrod and Lambiotte 2022). Girvan-Newman is classified as a divisive hierarchical clustering method.

2.3.3 Map Equation

This rather unusual clustering approach exploits the information-theoretic duality between community structures in networks and the length of a random walker’s movements on a graph. The map equation describes the theoretical limit of how closely a random walk can be described by

$$L(C) = qH(\mathcal{Q}) + \sum_{i=1}^{n_C} p_i H(P_i),$$

where $H(\mathcal{Q})$ and $H(P_i)$ are entropies, p_i is the fraction of time the random walker spends in cluster C_i and q the probability that a random walker switches to another cluster at any given step (Rosvall and Bergstrom 2008). See also Figure 1, which depicts a clustering of the semantic graph G_{12} .

2.3.4 Spectral Clustering Methods

Spectral community detection methods generally analyse the eigenvectors corresponding to the n_C smallest eigenvalues and extract community information from them. A common approach is to take the rows of the eigenvector matrix as positions in space and perform k-means clustering on them, which is a spatial clustering method in \mathbb{R}^n (Fortunato 2010). For the purposes of this report, we will only consider the interesting behaviour of the eigenvalues of the graph Laplacian $L = D - A$, which can tell us how many clusters C_i we may expect given the adjacency matrix A and degree matrix D of the graph $G = (V, E)$. In our case of an undirected (and connected) network, we always expect a zero eigenvalue $\lambda_1 = 0$ of the Laplacian and a number of positive eigenvalues $\lambda_i > 0$ following that. λ_2 is often referred to as the spectral gap of the graph, and its corresponding eigenvector is called the *Fiedler vector* (Grindrod and Lambiotte 2022).

3 Network Generation and Results

The text corpus and vocabulary database used for the analysis in this report is based on the database behind the [Latin is Simple Library](#) which is in the public domain. The authors of these works are Augustus, Caesar, Cato, Cicero, Ovid, Hyginus, Titus, Vergil, Seneca the younger and Seneca the elder. In order to avoid incorporating too many grammatical particles / utility words, according to common practice (Biemann 2006), we have removed the 2000 most frequent words from the dataset. As Latin is a

highly inflected language, we took great care to consider inflections of a Latin word as the same vocabulary entry / vertex on the graph.

We will consider the semantic graph built by iterating the text corpus with neighbouring radii $r = 4$, $r = 8$ and $r = 12$ and refer to the resulting graphs as G_4 , G_8 and G_{12} respectively. Note that a higher neighbouring radius r naturally yields a higher number of edges.

Table 2: Clustering results for the corpus-generated semantic networks G_4 , G_8 and G_{12} with 10843 nodes and 1.1 million, 1.8 million and 2.4 million edges, respectively.

Graph	Method	Q	Runtime [s]	n_C	Avg. size	Max size
G_4	louvain	0.1307	25.091	9	1204.78	4066
G_4	chinese-whispers	0.0168	126.342	1929	5.62	39
G_4	greedy-modularity	0.1150	1993.632	39	278.03	4836
G_8	louvain	0.1189	58.373	5	2168.60	3748
G_8	chinese-whispers	0.0142	179.585	1625	6.67	43
G_8	greedy-modularity	0.1074	2211.958	25	433.72	5375
G_{12}	louvain	0.1150	43.723	4	2710.75	3465
G_{12}	chinese-whispers	0.0011	242.934	365	29.71	9778
G_{12}	greedy-modularity	0.1033	2324.660	12	903.58	5520

Table 2 shows the individual runtimes of each algorithm on graphs G_4 , G_8 and G_{12} , along with the resulting modularity Q of a clustering into n_C clusters with average and maximum size given in the table as well. For graphs of this size, Girvan-Newman did not converge within a day, not even for the first iteration which would have only yielded a bipartition of the graph. From Table 2, one can clearly see that the Louvain method is not only the fastest but also yields the highest modularity Q , which is perhaps unsurprising given that it is based on optimisation of Q . However, it yields a very small number of very heterogeneously sized clusters which is not necessarily ideal for the application of semantic field analysis.

Followed closely after it, we have the Greedy Modularity method which also yields a fairly high modularity (slightly higher than 0.1), but has runtime of about half an hour, which is much longer than the runtime of Louvain which is on the order of seconds. This comparison may put the fantastic performance improvement of the

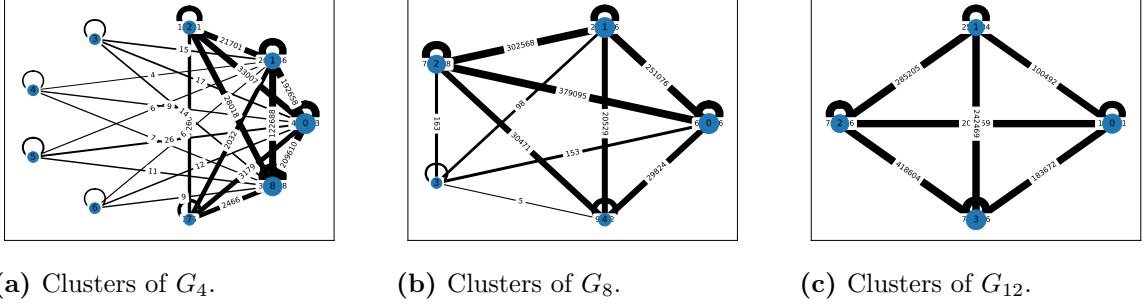


Figure 5: The weighted graphs of clusters and their number of inter-cluster edges connecting them for G_4 , G_8 and G_{12} , as obtained by the Louvain method. The graphs shown also contain edges from vertices (representing clusters) back to themselves (i.e. diagonal entries in the adjacency matrix) which represent the number of edges within the corresponding cluster.

Louvain method into perspective, given that it is based on the Greedy Modularity method.

The runner-up in terms of runtime is the Chinese Whispers algorithm which takes only a few minutes, but yields a very poor modularity Q . However, Chinese Whispers returns the most usable semantic fields which are of average size 6 for G_4 and G_8 and comprise around 30 words for G_{12} . Samples of the resulting semantic fields may be found in Table 4.

Table 3: Clustering results for Zachary’s Karate Club Graph (Zachary 1977).

Method	Q	Runtime [s]	n_C	Avg. size	Max size
greedy-modularity	0.4110	0.006	3	11.33	17
girvan-newman	0.3477	0.145	2	17.00	19
louvain	0.4439	0.002	4	8.50	14
chinese-whispers	0.3920	0.004	7	4.86	11

For reference, we also applied each algorithm to the standard Zachary’s Karate Club graph with runtimes, modularities and cluster sizes available in Table 3. The interested reader may find further runtime analysis tables and computational complexities in Ustalov et al. 2019 as well as Fortunato 2010.

Table 4: Some resulting clusters (semantic fields) in G_4 , the network of words whose edges correspond to distances of at most 4 words from one another in the Latin text corpus, extracted using the Chinese Whispers clustering method.

Size $ C_i $	Sample Words
4	conterritus (frightened), minitabundus (threatening), infensus (hostile), properus (quick)
6	hibernus (wintry), tardus (slow), upilio (shepherd), formosus (beautiful), hibernum (winter camp (pl.)), subulcus (swineherd)
4	fraus (deceit), fraus (fraud), infitatio (denial), furtum (theft)
6	fraxinus (ash-tree), laurea (laurel/bay tree), fraxinus (of ash), corylus (hazel-tree), edurus (very hard), abies (fir tree/wood)
4	vicina (neighbor), vicinus (nearby), vicinus (neighbor), vicinum (neighborhood)
7	repens (sudden), susurro (whisperer), nutrix (nurse), susurrus (whispering), immanitas (brutality), susurrus (whisper), diritas (frightfulness)
8	Aegyptius (Egyptian), Aethiopia (Ethiopia), siccitas (dryness), Aegyptus (Egypt), desertum (desert), Aegyptius (Egyptian), asp (asp), Arabia (Arabia)
8	Marius (Marius), Maria (Mary), caelia (kind of beer), citrum (wood of citron tree), alternatus (alternate), citrus (African citrus tree), series (row), citrus (lemon tree)
6	inexpectatus (invincible), armum (arms (pl.)), tribunicius (of/- belonging to tribune), intutus (defenseless), Arma (Arms), arma (weapons)
2	globosus (round), rotundus (round)

The clusters obtained by the Louvain method are depicted in Figure 5, using a similar visualisation method as InfoMap Figure 1. The semantic connectedness of the Louvain clusters is not immediately apparent due to the large size of the clusters.

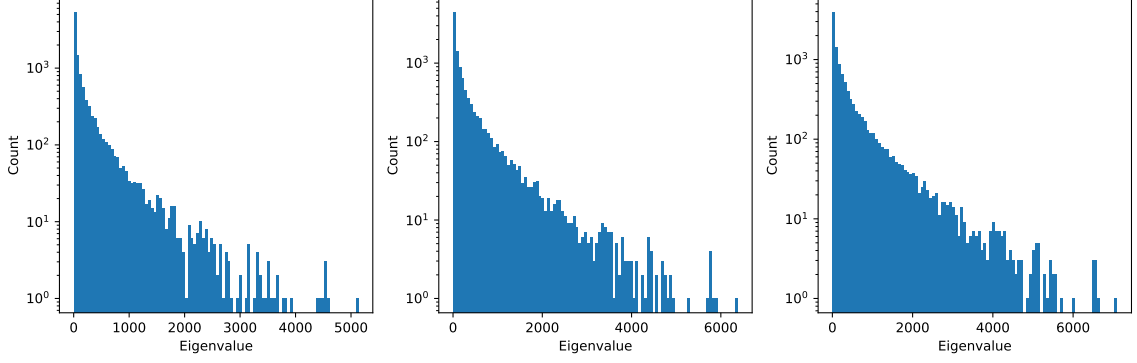


Figure 6: From left to right: Histograms of the eigenvalue spectrum of the graph Laplacians $L = D - A$ of G_4 , G_8 and G_{12} , respectively.

Figure 6 depicts a histogram of the spectra of the graph Laplacians L for the given three graphs. From here, one can make conclusions about the expected number of clusters in the network. G_4 , for instance, has a clear gap of eigenvalues around $\lambda = 2000$, which suggests a partitioning into about 2000 clusters of G_4 . Chinese Whispers, a probabilistic algorithm, yields almost exactly that number (within a 1 % error tolerance!). A similar argument can be made for both modularity-optimisation methods upon closer inspection of the spectrum, although by far not as clearly. The graphs G_8 and G_{12} are far more connected than G_4 and therefore yield a lower number of clusters. Which is expected, but not very easily visible in the eigenvalue spectrum.

4 Discussion and Outlook

In this report, we have introduced a few basic notions of network theory, discussed in detail two graph clustering approaches, the Louvain method (cf. Section 2.1) where we rederived the gain in modularity upon clustering of an isolated vertex and Chinese Whispers (cf. Section 2.2). Using a neighbourhood-based approach with parameter r , we constructed large networks G_4 , G_8 and G_{12} from an original classicist Latin literature corpus.

We then analyzed three clustering methods from a computational perspective with results presented in Table 2, Table 4 and visualised in Figure 5. The Louvain method was by far the fastest algorithm for our application. The eigengap of the spectrum of the Laplacian then yielded a good estimate for the expected number of clusters.

As the author describes in [Biemann 2006](#), Chinese Whispers generally produces a fairly high number of clusters as compared to other methods, which one may circumvent by adjusting either the convergence condition or introduce a minimum cluster size through minor adjustments of the algorithm. Similar measures may be taken for the other algorithms. In our case however, the clusters produced make a reasonable impression.

References

- Biemann, Chris (June 2006). ‘Chinese whispers: an efficient graph clustering algorithm and its application to natural language processing problems’. In: *TextGraphs: Proceedings of the First Workshop on Graph Based Methods for Natural Language Processing*. Association for Computational Linguistics, pp. 73–80. DOI: [10.5555/1654758.1654774](https://doi.org/10.5555/1654758.1654774).
- Blondel, Vincent D, Jean-Loup Guillaume, Renaud Lambiotte and Etienne Lefebvre (Oct. 2008). ‘Fast unfolding of communities in large networks’. In: *Journal of Statistical Mechanics: Theory and Experiment* 2008.10, P10008. DOI: [10.1088/1742-5468/2008/10/P10008](https://doi.org/10.1088/1742-5468/2008/10/P10008).
- Clauset, Aaron, Mark E. J. Newman and Cristopher Moore (2004). ‘Finding community structure in very large networks’. In: *Physical review. E, Statistical, nonlinear, and soft matter physics* 70 6 Pt 2.
- Dongen, Stijn Marinus van (May 2000). ‘Graph Clustering by Flow Simulation’. In: *PhD thesis, Center for Math and Computer Science (CWI)*.
- Fortunato, Santo (2010). ‘Community detection in graphs’. In: *Physics Reports* 486.3, pp. 75–174. ISSN: 0370-1573. DOI: [10.1016/j.physrep.2009.11.002](https://doi.org/10.1016/j.physrep.2009.11.002).
- Fortunato, Santo and Claudio Castellano (2009). ‘Community Structure in Graphs’. In: *Encyclopedia of Complexity and Systems Science*. New York, NY, USA: Springer, New York, NY, pp. 1141–1163. DOI: [10.1007/978-0-387-30440-3_76](https://doi.org/10.1007/978-0-387-30440-3_76).
- Girvan, Michelle and Mark E. J. Newman (2001). ‘Community structure in social and biological networks’. In: *Proceedings of the National Academy of Sciences of the United States of America* 99, pp. 7821–7826.
- Grindrod, Peter and Renaud Lambiotte (5th Nov. 2022). *C5.4 Networks*. Lecture Notes for the course.
- Rosvall, Martin and Carl T. Bergstrom (Jan. 2008). ‘Maps of random walks on complex networks reveal community structure’. In: *Proc. Natl. Acad. Sci. U.S.A.* 105.4, pp. 1118–1123. DOI: [10.1073/pnas.0706851105](https://doi.org/10.1073/pnas.0706851105).
- Ushioda, Akira (June 1996). ‘Hierarchical Clustering of Words and Application to NLP Tasks’. In: *Fourth Workshop on Very Large Corpora*. Herstmonceux Castle, Sussex, UK: Association for Computational Linguistics. URL: <https://aclanthology.org/W96-0103>.

-
- Ustalov, Dmitry, Alexander Panchenko, Chris Biemann and Simone Paolo Ponzetto (Sept. 2019). ‘Watset: Local-Global Graph Clustering with Applications in Sense and Frame Induction’. In: *Computational Linguistics* 45.3, pp. 423–479. ISSN: 0891-2017. DOI: [10.1162/coli_a_00354](https://doi.org/10.1162/coli_a_00354).
- Zachary, Wayne (1977). ‘An Information Flow Model for Conflict and Fission in Small Groups’. In: *Journal of Anthropological Research* 33, pp. 452–473.