

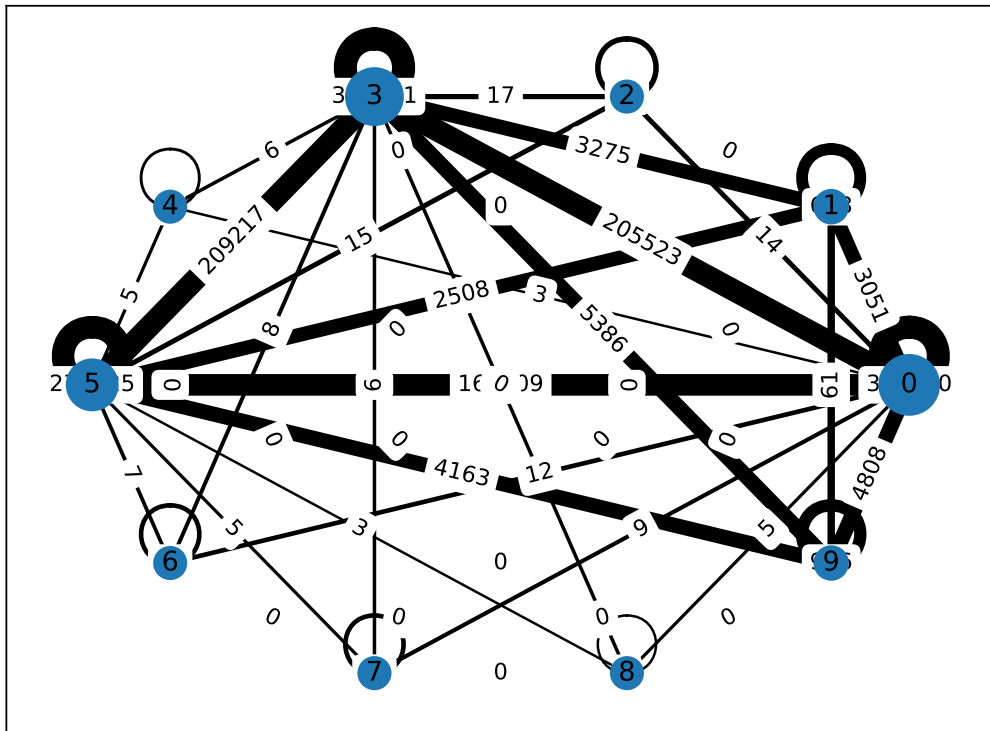
# Unsupervised Semantic Field Analysis by methods from community detection

Special Topic on [NETWORKS](#)

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## Abstract

This work will attempt to



**Figure 1:** Add some sort of graph plot here.

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# 1 Motivation

For non-periodic problem settings, Chebyshev series are a fantastic choice ([Biemann 2006](#)).

In the context of Natural Language Processing, graph clustering methods can also be employed for various other tasks such as word sense induction or language separation. In this work however, we will only focus on semantic field tagging.

## 2 Introduction

Let  $\mathbb{N} = \mathbb{Z}^+$  denote the positive integers and  $N_0 := \{0\} \cup \mathbb{N}$  the nonnegative integers.

The methods we will discuss to identify semantic fields will be based on graph clustering algorithms applied to a text corpus word connectedness / neighbourhood network. As we will discuss later, different notions of connectedness can give us different insight into the structure of a natural language. We will focus our attention on methods for undirected graphs, “graphs without direction” (cf. Definition 2.1).

### 2.1 Definition: Undirected Graph

A graph  $G = (V, E)$  with vertices  $V$  and edges  $E \subseteq V \times V$  is undirected if and only if  $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall v_i, v_j \in V$ .

Vertices are also often referred to as *nodes*. Every graph  $G$  is uniquely described by its adjacency matrix  $A \in \{0, 1\}^{n \times n}$  (Definition 2.2), which allows us to talk about “linear algebra” of graphs.

### 2.2 Definition: Adjacency Matrix

Let  $A \in \{0, 1\}^{n \times n}$  denote the symmetric adjacency matrix of an undirected graph  $G = (V, E)$ . Its entries are given by  $a_{ij} = \{A\}_{ij} = \mathbb{1}_{(v_i, v_j) \in E}$ , so  $a_{ij} = 1$  if vertex  $v_i$  is connected to  $v_j$  and 0 otherwise.

By construction,  $A = A^T$  is symmetric and has all-0s in the diagonal, a definition that corresponds to the fact that you cannot be friends with yourself in a social network.

Further let  $m := |E|$  and  $n := |V|$  denote the number of edges and vertices, respectively. The degree  $d_i$  of a vertex  $v_i \in V$  is defined by the number of edges connecting to it, so

$$d_i := \deg(v_i) = \left| \{(v_j, v_k) \in E \mid v_j = v_i\} \right|,$$

for an undirected graph  $G = (V, E)$ . The handshaking lemma (Lemma 2.1) tells us an important fact useful for normalisation.

### 2.1 Lemma: Handshake

For every finite, undirected graph  $G = (V, E)$  the individual vertex degrees sum up to exactly twice the number of edges, so

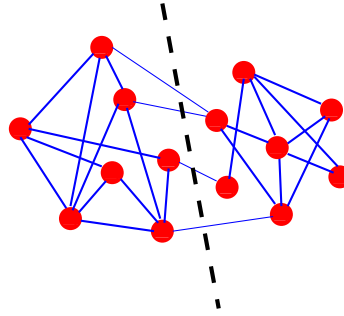
$$\sum_{i=1}^n d_i = \sum_{v \in V} \deg(v) = 2m.$$

The individual vertex degrees can be summarised in the so-called *degree matrix*  $D := \text{diag}(d_1, \dots, d_n)$ ,  $D \in \mathbb{N}_0^{n \times n}$ . The graph *Laplacian* is defined by  $L := D - A$ .

Given a graph, we are interested in performing **graph clustering**, also referred to as **community detection** or **graph partitioning**, the goal of which is to obtain a set of mutually exclusive clusters  $C_i \subseteq V$  (cf. Definition 2.3). The term *graph partitioning* is more frequently used in the context of minimal cuts, where one aims to minimise each *cut size* referring to the number of edges in between clusters.

### 2.3 Definition: Graph Clustering

Let  $C = \{C_i \subseteq V\}_{i=1 \dots n_C}$  denote a clustering of  $G = (V, E)$  into  $n_C \in \mathbb{N}$  clusters where  $C_i \cap C_j = \{\}$   $\forall i, j \in \{1, \dots, n_C\}$  and  $\bigcup_{i=1}^{n_C} C_i = V$ . Let  $s_i \in \{1, \dots, n_C\}$  denote the assigned cluster of vertex  $v_i \in V$ .



**Figure 2:** Partitioning of a graph using *cuts* (Fortunato and Castellano 2009).

These clusterings may be better or worse depending on the context, but a generally solid measure of “clustering goodness” is *modularity* (Definition 2.4).

### 2.4 Definition: Modularity

For a given undirected graph  $G = (V, E)$  and clustering  $C$ , let

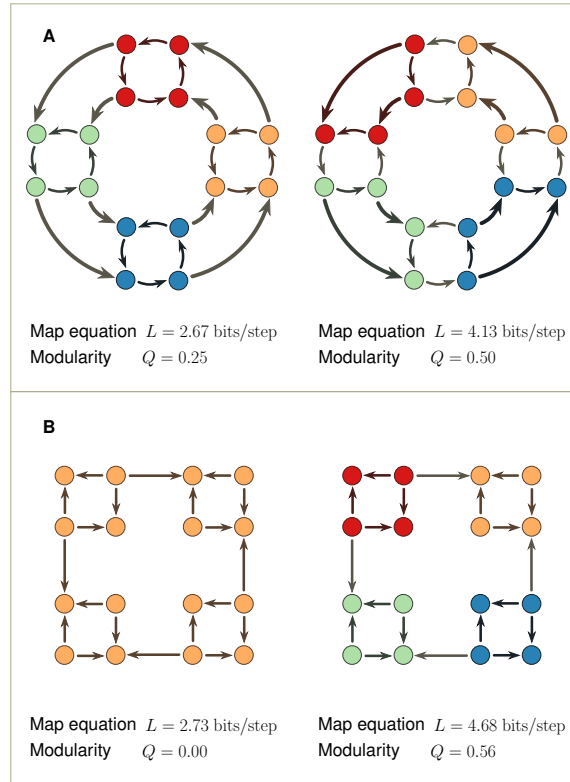
$$Q := \frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n \left( A_{ij} - \frac{d_i d_j}{2m} \right) \delta(s_i, s_j),$$

with  $\delta(\cdot, \cdot)$  the Kronecker delta indicating whether two vertices  $v_i$  and  $v_j$  belong to the same cluster (Blondel et al. 2008).

Modularity is a measure of the quality of a clustering (also referred to as a partitioning) of  $G$ . It can also be written as the sum of individual cluster contributions

$$Q = \sum_{c=1}^{n_C} Q_c = \frac{1}{2m} \sum_{c=1}^{n_C} \left[ \sum_{v_i \in C_c} \sum_{v_j \in C_c} \left( A_{ij} - \frac{d_i d_j}{2m} \right) \right],$$

which might make its purpose a bit clearer.



**Figure 3:** Bla bla todo (Rosvall and Bergstrom 2008).

### 3 Clustering Methods and Algorithms

In order to find ... Most clustering methods can be broadly categorised into spectral, partitional (such as k-means clustering), hierarchical, randomised, divisive and quality-optimisation algorithms. An interesting approach is Markov-Chain-Clustering, which may employ Simulated Annealing [Fortunato 2010](#).

Community detection is, in principle, usually a “very hard” task given the vast number of possible system configurations as the graph grows in the number of edges or vertices, a statement which can be made more precise using complexity theory. In conventional complexity theory, problems are filed into different complexity classes when analysing their runtime and memory usage. There exist

1. NL (Nondeterministic Logarithmic space)
2. P (Polynomial time)
3. NP (Nondeterministic Polynomial time)
4. PSPACE (Polynomial space)
5. EXPTIME (Exponential time)
6. EXPSPACE (Exponential space)

computational complexity classes, sorted by the amount of problems contained in them ( $NL \subseteq P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq EXPSPACE$ ). A particularly interesting open problem is whether  $P = NP$ , one of the millennium prize problems and the most important open problem in computer science.

#### 3.1 Definition: NP-Hardness

A problem is referred to as *NP-hard* if and only if it is at least as hard as the hardest problems in the complexity class NP (nondeterministic polynomial time).

Formally written,

$$NP := \bigcup_{k \in \mathbb{N}} NTIME(n^k)$$

the union of all decision problems with runtime bounded by  $\mathcal{O}(n^k)$ .

Many community detection algorithms or problems relating to it are NP-hard ([Fortunato 2010](#)). It is therefore often futile to employ exact algorithms as they quickly start to become infeasible for larger system sizes.

Note that here we only consider an unweighted, undirected graph, while most clustering algorithms also allow working with a weighted graph  $\tilde{G} = (V, \tilde{E})$  where  $\tilde{E} \subset V \times V \times \mathbb{R}^+$

is a set of three-tuples  $(v_i, v_j, w_{ij})$  instead of vertex pairs. Or equivalently, one can define a function  $w : \tilde{E} \mapsto \mathbb{R}^+$  which maps vertex pairs (edges) to their respective weight.

### 3.1 Embeddings

Given a graph embedding, in which vertices are mapped to a point in  $\mathbb{R}^n$ , a frequently used measure of similarity is *Cosine-Similarity*:

$$\rho_{ij} = \arccos \left( \frac{\mathbf{a}_i \mathbf{a}_j}{\|\mathbf{a}_i\|_2 \cdot \|\mathbf{a}_j\|_2} \right),$$

where vectors  $\mathbf{a}_i, \mathbf{a}_j \in \mathbb{R}^n$  denote the point corresponding to vertex  $v_i, v_j$  respectively.

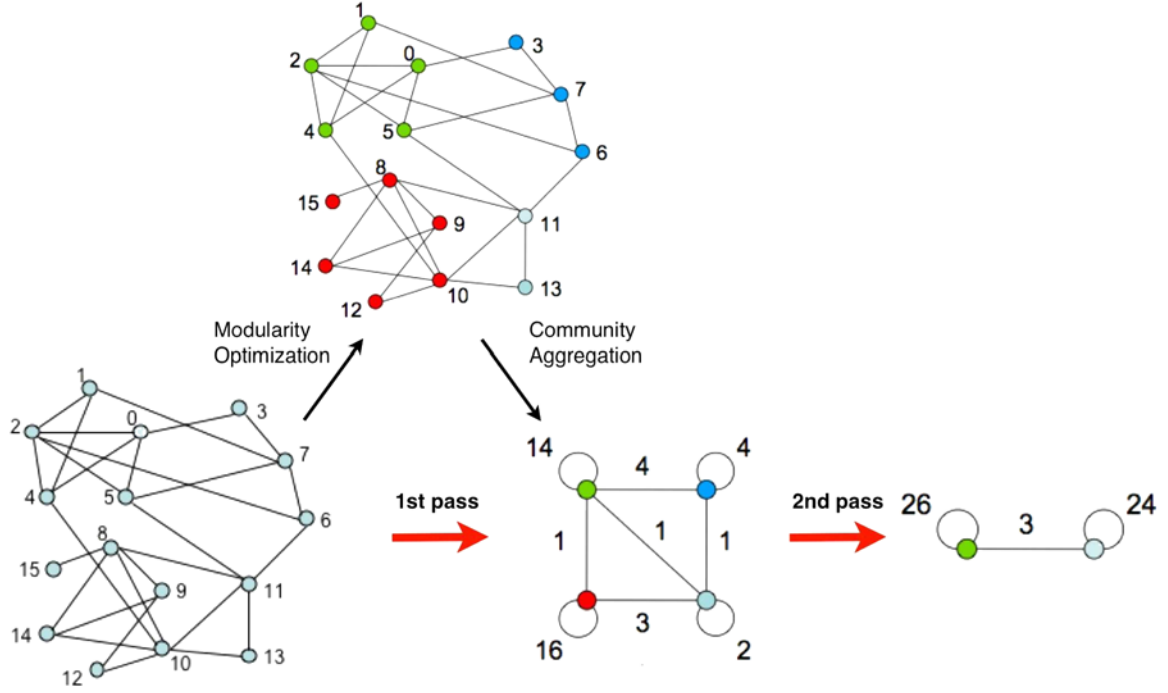
### 3.2 Girvan-Newman

The clustering method deduced in [Girvan and Newman 2001](#) was a clear revolution at the time and brought graph clustering / community detection methods closer to practitioners. Optimises centrality and betweenness.

### 3.3 Louvain

The popular algorithm introduced by [Blondel et al. 2008](#) is based on modularity optimisation (cf. Definition 2.4) and has computational complexity of only  $\mathcal{O}(m)$ . It is referred to as the *Louvain method* named after the University of Louvain in Belgium, alma mater of the first author. The Louvain method is divided into two phases: Phase One (local optimisation) and Phase Two (merging of vertices). [Grindrod and Lambiotte 2022](#).





**Figure 4:** Louvain Graph Clustering Method [Blondel et al. 2008](#).

Louvain is very effective, in part because a change (improvement) in modularity obtained by reassigning an **isolated** (not yet clustered) vertex  $v_i$  to a community  $C_j$  may be explicitly obtained by

$$\Delta Q = \left[ \frac{\Sigma_{in}^{(j)} + k_{i,in}^{(j)}}{2m} - \left( \frac{\Sigma_{tot}^{(j)} + k_i}{2m} \right)^2 \right] - \left[ \frac{\Sigma_{in}^{(j)}}{2m} - \left( \frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 - \left( \frac{k_i}{2m} \right)^2 \right], \quad (1)$$

(as given in the original paper) which is highly useful from a computational perspective ([Blondel et al. 2008](#)). Here, we have  $d_{i,in}^{(j)} := |\{(v_x, v_y) \in (C_j \times C_j) \cap E \mid v_x = v_i\}|$  the number of in-cluster edges connected to vertex  $v_i$  and the corresponding  $\Sigma_{in}^{(j)} := \sum_{v_i \in C_j} d_{i,in}^{(j)} = |(C_j \times C_j) \cap E|$  represents the total number of edges between vertices within the same cluster  $C_j$  (so neglecting edges to vertices outside  $C_j$ ). Likewise,  $\Sigma_{tot}^{(j)} := \sum_{v_i \in C_j} d_i$  denotes the sum of vertex degrees within the cluster  $C_j$  (so including contributions to outside nodes).

In our case, the above weightings  $k_j \in \mathbb{R}^+$  reduce to the degrees  $d_j$  alone given that we operate on an unweighted, undirected graph  $G = (V, E)$ . Note that the above Equation (1) simplifies to

$$\Delta Q = \frac{\Sigma_{in}^{(j)}}{2m} + \frac{d_{i,in}^{(j)}}{2m} - \frac{(\Sigma_{tot}^{(j)})^2 + 2\Sigma_{tot}^{(j)}d_i + d_i^2}{4m^2} - \frac{\Sigma_{in}^{(j)}}{2m} + \frac{(\Sigma_{tot}^{(j)})^2 + d_i^2}{4m^2} = \frac{d_{i,in}^{(j)}}{2m} - \frac{\Sigma_{tot}^{(j)}d_i}{2m^2}. \quad (2)$$

In more general terms, one may obtain the impact of moving a vertex  $v_i \in V$  from cluster  $C_a$  to  $C_b$  by considering their individual contributions

$$Q_c = \sum_{v_i \in C_c} \sum_{v_j \in C_c} \left( A_{ij} - \frac{d_i d_j}{2m} \right) = \frac{\Sigma_{in}^{(j)}}{2m} - \left( \frac{\Sigma_{tot}^{(j)}}{2m} \right)^2,$$

to the total modularity  $Q = \sum_{c=1}^{n_C} Q_c^{(c)}$ , which are

$$\begin{aligned} \tilde{Q}_c^{(a)} &= \frac{\Sigma_{in}^{(a)} - d_{i,in}^{(a)}}{2m} - \left( \frac{\Sigma_{tot}^{(a)} - d_i}{2m} \right)^2, \\ \tilde{Q}_c^{(b)} &= \frac{\Sigma_{in}^{(b)} + d_{i,in}^{(b)}}{2m} - \left( \frac{\Sigma_{tot}^{(b)} + d_i}{2m} \right)^2, \end{aligned}$$

after moving vertex  $v_i$  from cluster  $C_a$  to  $C_b$ . So the change in individual cluster modularity is given by the terms

$$\begin{aligned} \Delta Q_c^{(a)} &= \tilde{Q}_c^{(a)} - Q_c^{(a)} = \frac{-d_{i,in}^{(a)}}{2m} - \frac{d_i^2 - 2\Sigma_{tot}^{(a)}d_i}{4m^2}, \\ \Delta Q_c^{(b)} &= \tilde{Q}_c^{(b)} - Q_c^{(b)} = \frac{d_{i,in}^{(b)}}{2m} - \frac{d_i^2 + 2\Sigma_{tot}^{(b)}d_i}{4m^2}, \end{aligned}$$

which yield the total modularity change  $\Delta Q = \Delta Q_c^{(a)} + \Delta Q_c^{(b)}$  in the general case. Considering the special case when vertex  $v_i \in V$  is still isolated, so when  $C_a = \{v_i\}$  contains exactly that one vertex, we can further simplify to

$$\Delta Q = \cancel{\frac{d_i^2}{4m^2}} + \frac{d_{i,in}^{(b)}}{2m} - \frac{\cancel{d_i^2} + 2d_i\Sigma_{tot}^{(b)}}{4m^2} = \frac{d_{i,in}^{(b)}}{2m} - \frac{\Sigma_{tot}^{(b)}d_i}{2m^2},$$

because  $\Sigma_{in}^{(a)} = 0$ ,  $\Sigma_{tot}^{(a)} = d_i$  and therefore  $\Delta Q_c^{(a)} = \frac{d_i^2}{4m^2}$ , and we arrive at the same expression given by Blondel et al. 2008 (Equation (2)), for the special case when  $C_a = \{v_i\}$ . Note that the modularity contribution  $Q_c = 0$  for an empty cluster.

This modularity gain of course is not restricted to the Louvain method in and of itself, it applies to all clustering methods and is relevant to those that operate on modularity.

Louvain is among the most widely used techniques for graph clustering and is available in numerous software packages.

### 3.4 Chinese Whispers

This method, like the Louvain method, also initialises the algorithm by assigning each vertex its own category (cluster).

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The *Chinese Whispers* algorithm due to [Biemann 2006](#)

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```

1 Input: an undirected graph  $G = (V, E)$ .
2 Output: a graph clustering  $C = \{C_i\}_{i=1, \dots, n_C}$  into  $n_C$  classes.
3
4 Initialise with  $n_C = n$  classes, one per vertex.
5 while there are changes, do
6     for  $v_i$  in shuffle( $V$ ), do
7         Set  $s_i = 3$ 
8     end
9 end
```

---

Corresponds to an agent-based simulation of a social network [Biemann 2006](#). Similar to Markov-Chain-Clustering [Dongen 2000](#), [Fortunato 2010](#).

The algorithm may equivalently be defined by considering the *class matrix*  $\mathcal{D} \in \{0, 1\}^{n \times n_C}$  of a graph  $G = (V, E)$  in which rows represent nodes and columns represent clusters  $C_j \in C$ . So  $G_{ij} = 1$  if  $v_i$  belongs to  $C_j$  and 0 otherwise. The algorithm operates iteratively and updates the class matrix  $\mathcal{D}^{(k)}$  using

$$M : \{0, 1\}^{n \times n_C} \mapsto \{0, 1\}^{n \times n_C}, \quad \{M(\mathcal{D})\}_{ij} = \mathcal{D}_{ij}$$

$$\mathcal{D}^{(k+1)} = M(\mathcal{D}^{(k)})A, \quad k = 0, 1, 2, \dots, N$$

per iteration  $k$ , where  $\mathcal{D}^{(0)} = I$  is the identity, corresponding to the initialisation of individual classes for each vertex.

The paper also discusses acquisition of word classes (semantic fields), where they ran Chinese Whispers against the British National Corpus (BNC) to identify clusters of similar words. In order to improve performance, they cut off the 2000 most frequent words in the corpus which we will also employ. This set of highly frequent words is likely to contain the *core vocabulary* of the language including, but not limited to, pronouns, conjunctions, prepositions and similar parts of speech. [Biemann 2006](#) identified 282 clusters, 26 of which contained more than 100 words. Word clustering methods such as this one may even be used to improve part-of-speech tagging ([Ushioda 1996](#)).

**Table 1:** Table ordered by size [Biemann 2006](#).

| Size $ C_i $ | Sample Words  |
|--------------|---|
| 18432        | secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ... |
| 4916         | busy, grey, tiny, thin, sufficient, attractive, vital, ...                              |
| 4192         | filled, revealed, experienced, learned, pushed, occurred, ...                           |
| 3515         | White, Green, Jones, Hill, Brown, Lee, Lewis, Young, ...                                |
| 2211         | Ian, Alan, Martin, Tony, Prince, Chris, Brian, Harry, Andrew, Christ, Steve, ...        |
| 1855         | Central, Leeds, Manchester, Australia, Yorkshire, Belfast, Glasgow, Middlesbrough, ...  |

A more recent work [Ustalov et al. 2019](#) describes a meta-algorithm for graph clustering in the context of Natural Language Processing, which can be based on Chinese Whispers and also other methods (hence the term 'meta'). Their algorithm (**WATSET**) is designed for *fuzzy* graph clustering which exceeds the scope of this special topic, but represents the state of the art in graph-based applications in Natural Language Processing. Fuzzy clustering algorithms are capable of producing *soft* clusterings, which relax the requirement that all clusters are strictly disjoint (so  $C_i \cap C_j = \{\}$   $\forall i, j$ , as introduced in Definition 2.3).

### 3.5 Map Equation

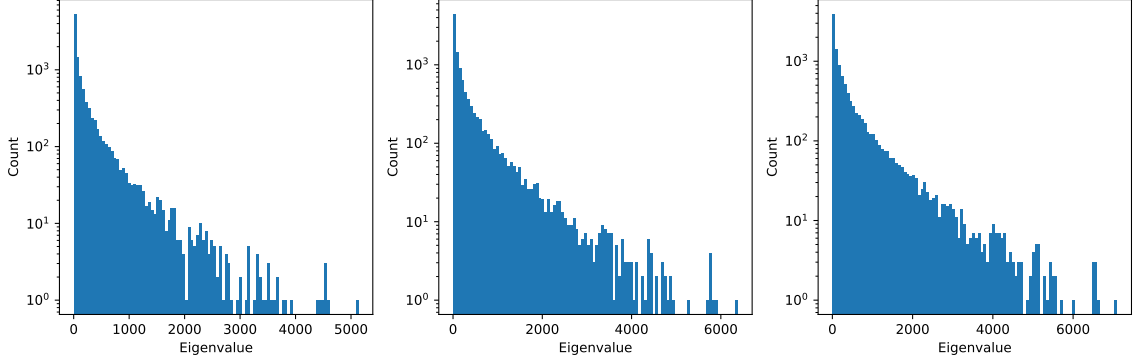
The map equation.

### 3.6 Spectral Clustering Methods

[Fortunato 2010](#).

### 3.7 Fiedler

[Fortunato 2010](#).



**Figure 5:** Histogram of the eigenvalue spectrum of the Laplacian  $L = D - A$  of the graph  $G_4$ .

**Table 2:** Clustering results for  $G_4$ .

| Method            | Modularity | Runtime [s] | $n_C$ | Average size | Max size |
|-------------------|------------|-------------|-------|--------------|----------|
| louvain           | 0.1307     | 25.091      | 9     | 1204.78      | 4066.    |
| chinese-whispers  | 0.0168     | 126.342     | 1929  | 5.62         | 39.      |
| greedy-modularity | 0.1150     | 1993.632    | 39    | 278.03       | 4836.    |

## 4 Method

We shall also remove the 2000 most frequent words

## 5 Results

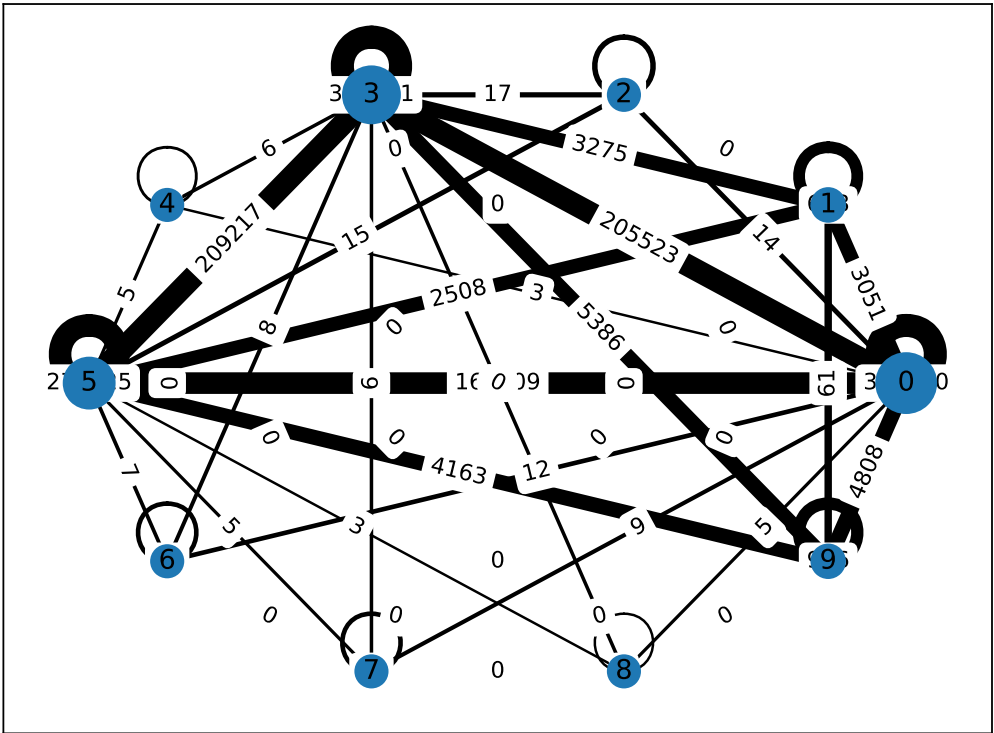
An itemize with some semantic field clusters we found. Girvan Newman did not converge within a day, and if it had it would have only been a bipartition.

**Table 3:** Clustering results for Zachary’s Karate Club Graph ([Zachary 1977](#)).

| Method            | Modularity | Runtime [s] | $n_C$ | Average size | Max size |
|-------------------|------------|-------------|-------|--------------|----------|
| greedy-modularity | 0.4110     | 0.006       | 3     | 11.33.       | 17.      |
| girvan-newman     | 0.3477     | 0.145       | 2     | 17.00.       | 19.      |
| louvain           | 0.4439     | 0.002       | 4     | 8.50.        | 14.      |
| chinese-whispers  | 0.3920     | 0.004       | 7     | 4.86.        | 11.      |

**Table 4:** Results ordered by size.

| Size $ C_i $ | Sample Words  |
|--------------|---|
| 18432        | secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ... |
| 3            | torture, executioner, butchering,   |



**Figure 6:** Plotty plot

Apply to Karate Club

Computational complexity table from [Ustalov et al. 2019](#), also referencing complexity section above. Fortunato also has complexities.

## 6 Author analysis?

Authors whose works circulate around these semantic fields: bla bla, maybe not that interesting

## 7 Discussion and Outlook

Bla

As the author describes in [Biemann 2006](#), Chinese Whispers generally produces a fairly high number of clusters as compared to other methods, which one may circumvent by adjusting either the convergence condition or introduce a minimum cluster size through minor adjustments of the algorithm.



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## References

- Biemann, Chris (June 2006). ‘Chinese whispers: an efficient graph clustering algorithm and its application to natural language processing problems’. In: *TextGraphs: Proceedings of the First Workshop on Graph Based Methods for Natural Language Processing*. Association for Computational Linguistics, pp. 73–80. DOI: [10.5555/1654758.1654774](https://doi.org/10.5555/1654758.1654774).
- Blondel, Vincent D, Jean-Loup Guillaume, Renaud Lambiotte and Etienne Lefebvre (Oct. 2008). ‘Fast unfolding of communities in large networks’. In: *Journal of Statistical Mechanics: Theory and Experiment* 2008.10, P10008. DOI: [10.1088/1742-5468/2008/10/P10008](https://doi.org/10.1088/1742-5468/2008/10/P10008).
- Dongen, Stijn Marinus van (May 2000). ‘Graph Clustering by Flow Simulation’. In: *PhD thesis, Center for Math and Computer Science (CWI)*.
- Fortunato, Santo (2010). ‘Community detection in graphs’. In: *Physics Reports* 486.3, pp. 75–174. ISSN: 0370-1573. DOI: [10.1016/j.physrep.2009.11.002](https://doi.org/10.1016/j.physrep.2009.11.002).
- Fortunato, Santo and Claudio Castellano (2009). ‘Community Structure in Graphs’. In: *Encyclopedia of Complexity and Systems Science*. New York, NY, USA: Springer, New York, NY, pp. 1141–1163. DOI: [10.1007/978-0-387-30440-3\\_76](https://doi.org/10.1007/978-0-387-30440-3_76).
- Girvan, Michelle and Mark E. J. Newman (2001). ‘Community structure in social and biological networks’. In: *Proceedings of the National Academy of Sciences of the United States of America* 99, pp. 7821–7826.
- Grindrod, Peter and Renaud Lambiotte (5th Nov. 2022). *C5.4 Networks*. Lecture Notes for the course.
- Rosvall, Martin and Carl T. Bergstrom (Jan. 2008). ‘Maps of random walks on complex networks reveal community structure’. In: *Proc. Natl. Acad. Sci. U.S.A.* 105.4, pp. 1118–1123. DOI: [10.1073/pnas.0706851105](https://doi.org/10.1073/pnas.0706851105).
- Ushioda, Akira (1996). ‘Hierarchical Clustering of Words and Application to NLP Tasks’. In: *VLC@COLING*.
- Ustalov, Dmitry, Alexander Panchenko, Chris Biemann and Simone Paolo Ponzetto (Sept. 2019). ‘Watset: Local-Global Graph Clustering with Applications in Sense and Frame Induction’. In: *Computational Linguistics* 45.3, pp. 423–479. ISSN: 0891-2017. DOI: [10.1162/coli\\_a\\_00354](https://doi.org/10.1162/coli_a_00354).
- Zachary, Wayne (1977). ‘An Information Flow Model for Conflict and Fission in Small Groups’. In: *Journal of Anthropological Research* 33, pp. 452–473.

## A Appendix Things?