Unsupervised Semantic Field Analysis
using Community Detection Methods

Special Topic on NETWORKS
Candidate Number: 1072462

Abstract

This work will attempt to

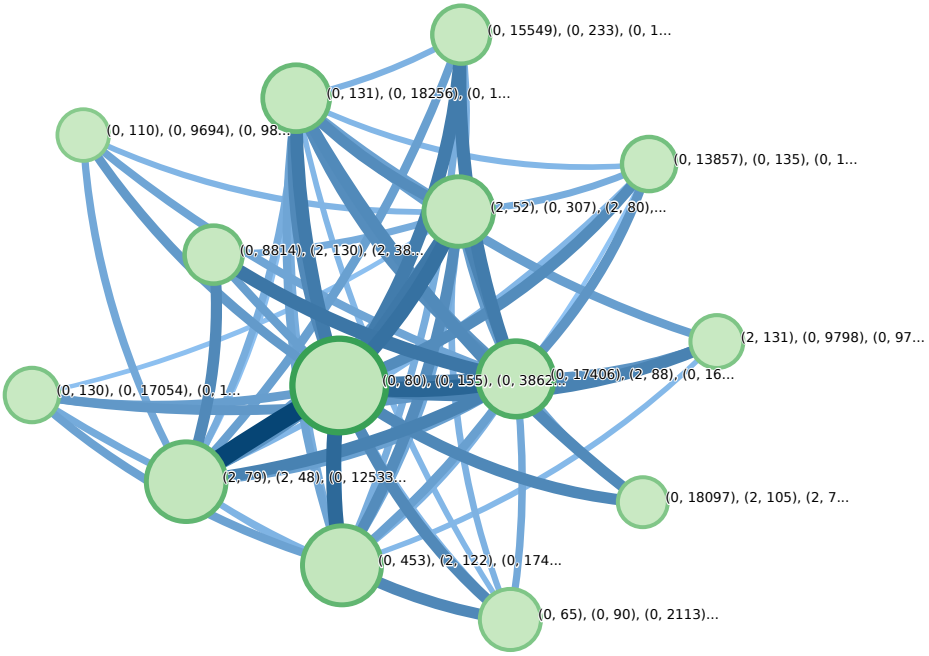


Figure 1: Visualisations of a clustering of the corpus generated semantic network G_8, obtained using the Map Equation approach (cf. Section 3.4.3). Visualised using the InfoMap visualisation package (Rosvall and Bergstrom 2008).

Contents

1 Motivation

In the context of Natural Language Processing, graph clustering methods can also be employed for various other tasks such as word sense induction or language separation. In this work however, we will only focus on semantic field tagging.

2 Introduction

Let $\mathbb{N} = \mathbb{Z}^+$ denote the positive integers and $N_0 := \{0\} \cup \mathbb{N}$ the nonnegative integers.

The methods we will discuss to identify semantic fields will be based on graph clustering algorithms applied to a text corpus word connectedness / neighbourhood network. As we will discuss later, different notions of connectedness can give us different insight into the structure of a natural language. We will focus our attention on methods for undirected graphs, "graphs without direction" (cf. Definition 2.1).

2.1 Definition: Undirected Graph

A graph $G = (V, E)$ with vertices V and edges $E \subseteq V \times V$ is undirected if and only if $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall\, v_i, v_j \in V$.

Vertices are also referred to as *nodes*. Every graph G is uniquely described by its adjacency matrix $A \in \{0,1\}^{n \times n}$ (Definition 2.2), which allows us to talk about "linear algebra" of graphs, which is especially relevant to spectral graph clustering methods (cf. Section 3.4.4).

2.2 Definition: Adjacency Matrix

Let $A \in \{0,1\}^{n \times n}$ denote the symmetric adjacency matrix of an undirected graph $G = (V, E)$. Its entries are given by $a_{ij} = \{A\}_{ij} = \mathbb{1}_{(v_i, v_j) \in E}$, so $a_{ij} = 1$ if vertex v_i is connected to v_j and 0 otherwise.

By construction, $A = A^T$ is symmetric and has all-0s in the diagonal, a definition that corresponds to the fact that you cannot be friends with yourself in a social network.

Further let $m := |E|$ and $n := |V|$ denote the number of edges and vertices, respectively. The degree d_i of a vertex $v_i \in V$ is defined by the number of edges connecting to it, so

$$d_i := \deg(v_i) = \Big| \{(v_j, v_k) \in E \mid v_j = v_i\} \Big|,$$

for an undirected graph $G = (V, E)$. The handshaking lemma (Lemma 2.1) tells us an important fact useful for normalisation.

2.1 Lemma: Handshake

For every finite, undirected graph $G = (V, E)$ the individual vertex degrees sum up to exactly twice the number of edges, so

$$\sum_{i=1}^{n} d_i = \sum_{v \in V} \deg(v) = 2m\,.$$

The individual vertex degrees can be summarised in the so-called *degree matrix* $D := \text{diag}(d_1, ..., d_n)$, $D \in \mathbb{N}_0^{n \times n}$. The graph *Laplacian* is defined by $L := D - A$.

Given a graph, we are interested in performing **graph clustering**, also referred to as **community detection** or **graph partitioning**, the goal of which is to obtain a set of mutually exclusive clusters $C_i \subseteq V$ (cf. Definition 2.3). The term *graph partitioning* is more frequently used in the context of minimal cuts, where one aims to minimise each *cut size* referring to the number of edges in between clusters.

2.3 Definition: Graph Clustering

Let $C = \{C_i \subseteq V\}_{i=1...n_C}$ denote a clustering of $G = (V, E)$ into $n_C \in \mathbb{N}$ clusters where $C_i \cap C_j = \{\}\ \forall\, i, j \in \{1, ..., n_C\}$ and $\bigcup_{i=1}^{n_C} C_i = V$. Let $s_i \in \{1, ..., n_C\}$ denote the assigned cluster of vertex $v_i \in V$.

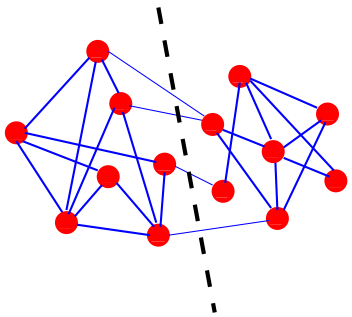


Figure 2: Partitioning of a graph using *cuts* (Fortunato and Castellano 2009).

These clusterings may be better or worse depending on the context, but a generally solid measure of "clustering goodness" is *modularity* (Definition 2.4).

2.4 Definition: Modularity

For a given undirected graph $G = (V, E)$ and clustering C, let

$$Q := \frac{1}{2m} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \delta(s_i, s_j) \,,$$

with $\delta(\cdot, \cdot)$ the Kronecker delta indicating whether two vertices v_i and v_j belong to the same cluster (Blondel et al. 2008).

Modularity is a measure of the quality of a clustering (also referred to as a partitioning) of G. It can also be written as the sum of individual cluster contributions

$$Q = \sum_{c=1}^{n_C} Q_c = \frac{1}{2m} \sum_{c=1}^{n_C} \left[\sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) \right] ,$$

which might make its purpose a bit clearer.

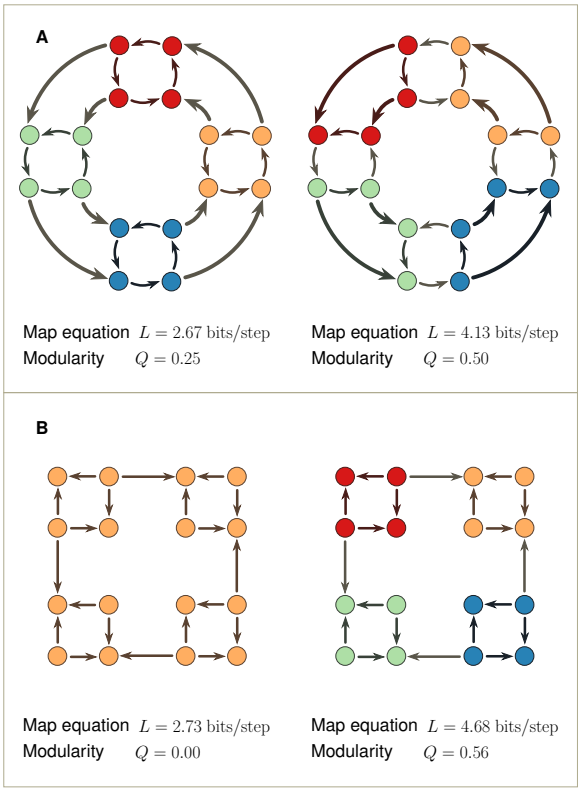


Figure 3: Four clustered graphs along with their modularity Q (and map equation value L, cf. Rosvall and Bergstrom 2008) to provide a visual illustration of modularity.

3 Clustering Methods and Algorithms

In order to find ... Most clustering methods can be broadly categorised into spectral, partitional (such as k-means clustering), hierarchical, randomised, divisive and quality-optimisation algorithms. An interesting approach is Markov-Chain-Clustering, which may employ Simulated Annealing Fortunato 2010.

Community detection is, in principle, usually a "very hard" task given the vast number of possible system configurations as the graph grows in the number of edges or vertices, a statement which can be made more precise using complexity theory. In conventional complexity theory, problems are filed into different complexity classes when analysing their runtime and memory usage. There exist

1. NL (Nondeterministic Logarithmic space)
2. P (Polynomial time)
3. NP (Nondeterministic Polynomial time)
4. PSPACE (Polynomial space)
5. EXPTIME (Exponential time)
6. EXPSPACE (Exponential space)

computational complexity classes, sorted by the amount of problems contained in them (NL $\subseteq$ P $\subseteq$ NP $\subseteq$ PSPACE $\subseteq$ EXPTIME $\subseteq$ EXPSPACE). A particularly interesting open problem is whether P = NP, one of the millennium prize problems and the most important open problem in computer science.

3.1 Definition: NP-Hardness

A problem is referred to as *NP-hard* if and only if it is at least as hard as the hardest problems in the complexity class NP (nondeterministic polynomial time). Formally written,

$$\mathrm{NP} := \bigcup_{k \in \mathbb{N}} \mathrm{NTIME}(n^k)$$

the union of all decision problems with runtime bounded by $\mathcal{O}(n^k)$.

Many community detection algorithms or problems relating to it are NP-hard (Fortunato 2010). It is therefore often futile to employ exact algorithms as they quickly start to become infeasible for larger system sizes.

Note that here we only consider an unweighted, undirected graph, while most clustering algorithms also allow working with a weighted graph $\tilde{G} = (V, \tilde{E})$ where $\tilde{E} \subset V \times V \times \mathbb{R}^+$

is a set of three-tuples (v_i, v_j, w_{ij}) instead of vertex pairs. Or equivalently, one can define a function $w : \tilde{E} \mapsto \mathbb{R}^+$ which maps vertex pairs (edges) to their respective weight.

3.1 The Louvain Method

The popular algorithm introduced by Blondel et al. 2008 is based on modularity optimisation (cf. Definition 2.4) and has computational complexity of only $\mathcal{O}(m)$. It is referred to as the *Louvain method* named after the University of Louvain in Belgium, alma mater of the first author. The Louvain method is divided into two phases: Phase One (local optimisation) and Phase Two (merging of vertices). Grindrod and Lambiotte 2022.

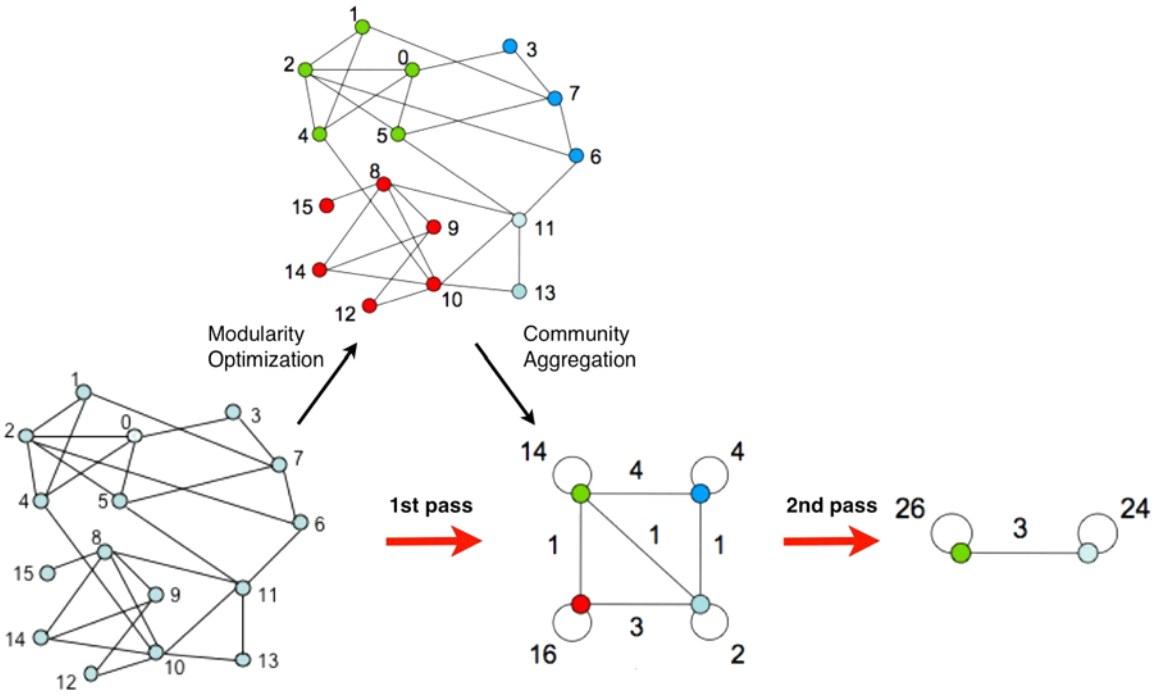


Figure 4: Louvain Graph Clustering Method Blondel et al. 2008.

Louvain is very effective, in part because a change (improvement) in modularity obtained by reassigning an **isolated** (not yet clustered) vertex v_i to a community C_j may be explicitly obtained by

$$\Delta Q = \left[\frac{\Sigma_{in}^{(j)} + k_{i,in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)} + k_i}{2m} \right)^2 \right] - \left[\frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 - \left(\frac{k_i}{2m} \right)^2 \right], \qquad (1)$$

(as given in the original paper) which is highly useful from a computational perspective (Blondel et al. 2008). Here, we have $d_{i,in}^{(j)} := \left|\left\{(v_x, v_y) \in (C_j \times C_j) \cap E \;\middle|\; v_x = v_i\right\}\right|$ the number of in-cluster edges connected to vertex v_i and the corresponding $\Sigma_{in}^{(j)} := \sum_{v_i \in C_j} d_{i,in}^{(j)} = |(C_j \times C_j) \cap E|$ represents the total number of edges between vertices within the same cluster C_j (so neglecting edges to vertices outside C_j). Likewise, $\Sigma_{tot}^{(j)} := \sum_{v_i \in C_j} d_i$ denotes the sum of vertex degrees within the cluster C_j (so including contributions to outside nodes).

In our case, the above weightings $k_j \in \mathbb{R}^+$ reduce to the degrees d_j alone given that we operate on an unweighted, undirected graph $G = (V, E)$. Note that the above Equation (1) simplifies to

$$\Delta Q = \frac{\cancel{\Sigma_{in}^{(j)}}}{\cancel{2m}} + \frac{d_{i,in}^{(j)}}{2m} - \frac{\cancel{\left(\Sigma_{tot}^{(j)}\right)^2} + 2\Sigma_{tot}^{(j)} d_i + \cancel{d_i^2}}{4m^2} - \frac{\cancel{\Sigma_{in}^{(j)}}}{\cancel{2m}} + \frac{\cancel{\left(\Sigma_{tot}^{(j)}\right)^2} + \cancel{d_i^2}}{4m^2} = \frac{d_{i,in}^{(j)}}{2m} - \frac{\Sigma_{tot}^{(j)} d_i}{2m^2} \,. \quad (2)$$

In more general terms, one may obtain the impact of moving a vertex $v_i \in V$ from cluster C_a to C_b by considering their individual contributions

$$Q_c = \sum_{v_i \in C_c} \sum_{v_j \in C_c} \left(A_{ij} - \frac{d_i d_j}{2m} \right) = \frac{\Sigma_{in}^{(j)}}{2m} - \left(\frac{\Sigma_{tot}^{(j)}}{2m} \right)^2 ,$$

to the total modularity $Q = \sum_{c=1}^{n_C} Q_c^{(c)}$, which are

$$\tilde{Q}_c^{(a)} = \frac{\Sigma_{in}^{(a)} - d_{i,in}^{(a)}}{2m} - \left(\frac{\Sigma_{tot}^{(a)} - d_i}{2m} \right)^2 ,$$
$$\tilde{Q}_c^{(b)} = \frac{\Sigma_{in}^{(b)} + d_{i,in}^{(b)}}{2m} - \left(\frac{\Sigma_{tot}^{(b)} + d_i}{2m} \right)^2 ,$$

after moving vertex v_i from cluster C_a to C_b. So the change in individual cluster modularity is given by the terms

$$\Delta Q_c^{(a)} = \tilde{Q}_c^{(a)} - Q_c^{(a)} = \frac{-d_{i,in}^{(a)}}{2m} - \frac{d_i^2 - 2\Sigma_{tot}^{(a)} d_i}{4m^2} ,$$
$$\Delta Q_c^{(b)} = \tilde{Q}_c^{(b)} - Q_c^{(b)} = \frac{d_{i,in}^{(b)}}{2m} - \frac{d_i^2 + 2\Sigma_{tot}^{(b)} d_i}{4m^2} ,$$

which yield the total modularity change $\Delta Q = \Delta Q_c^{(a)} + \Delta Q_c^{(b)}$ in the general case. Considering the special case when vertex $v_i \in V$ is still isolated, so when $C_a = \{v_i\}$ contains exactly that one vertex, we can further simplify to

$$\Delta Q = \frac{\cancel{d_i^2}}{\cancel{4m^2}} + \frac{d_{i,in}^{(b)}}{2m} - \frac{\cancel{d_i^2} + 2d_i \Sigma_{tot}^{(b)}}{4m^2} = \frac{d_{i,in}^{(b)}}{2m} - \frac{\Sigma_{tot}^{(b)} d_i}{2m^2} ,$$

because $\Sigma_{in}^{(a)} = 0$, $\Sigma_{tot}^{(a)} = d_i$ and therefore $\Delta Q_c^{(a)} = \frac{d_i^2}{4m^2}$, and we arrive at the same expression given by Blondel et al. 2008 (Equation (2)), for the special case when $C_a = \{v_i\}$. Note that the modularity contribution $Q_c = 0$ for an empty cluster.

This modularity gain of course is not restricted to the Louvain method in and of itself, it applies to all clustering methods and is relevant to those that operate on modularity.

Louvain is among the most widely used techniques for graph clustering and is available in numerous software packages.

3.2 Chinese Whispers

This method, like the Louvain method, also initialises the algorithm by assigning each vertex its own category (cluster).

The *Chinese Whispers* algorithm due to Biemann 2006

```
Input: an undirected graph G = (V, E).
Output: a graph clustering C = {C_i}_{i=1,...,n_C} into n_C classes.

Initialise with n_C = n classes, one per vertex.
while there are changes, do
  for v_i in shuffled(V), do
    Set s_i = arg max_j | {(v_k, v_l) ∈ E | v_k = v_i ∧ v_l ∈ C_j} |.
  end
end
```

It works

This algorithm can be thought of as an agent-based simulation of a social network Biemann 2006. Similar to Markov-Chain-Clustering Dongen 2000, Fortunato 2010.

The algorithm may equivalently be defined by considering the *class matrix* $\mathcal{D} \in \{0,1\}^{n \times n_C}$ of a graph $G = (V, E)$ in which rows represent nodes and columns represent clusters $C_j \in C$. So $G_{ij} = 1$ if v_i belongs to C_j and 0 otherwise. The algorithm operates iteratively and updates the class matrix $\mathcal{D}^{(k)}$ using

$$M : \{0,1\}^{n \times n_C} \mapsto \{0,1\}^{n \times n_C}, \quad \{M(\mathcal{D})\}_{ij} = \delta_{j, \arg\max_{k \in \{0 \dots n_C\}} \mathcal{D}_{ik}},$$

$$\mathcal{D}^{(k+1)} = M(\mathcal{D}^{(k)})A, \quad k = 0, 1, 2, ..., N$$

per iteration k, where $\mathcal{D}^{(0)} = I$ is the identity, corresponding to the initialisation of individual classes for each vertex. M is a row-wise operation on its argument, which

sets each entry in the row to zero except the (first appearing) largest one, which it sets to one.

The paper also discusses acquisition of word classes (semantic fields), where they ran Chinese Whispers against the British National Corpus (BNC) to identify clusters of similar words. In order to improve performance, they cut off the 2000 most frequent words in the corpus which we will also employ. This set of highly frequent words is likely to contain the *core vocabulary* of the language including, but not limited to, pronouns, conjunctions, prepositions and similar parts of speech. Biemann 2006 identified 282 clusters, 26 of which contained more than 100 words. Word clustering methods such as this one may even be used to improve part-of-speech tagging (Ushioda 1996).

Table 1: Table ordered by size Biemann 2006.

Size $\|C_i\|$	Sample Words
18432	secret, officials, transport, unemployment, farm, county, wood, procedure, grounds, ...
4916	busy, grey, tiny, thin, sufficient, attractive, vital, ...
4192	filled, revealed, experienced, learned, pushed, occurred, ...
3515	White, Green, Jones, Hill, Brown, Lee, Lewis, Young, ...
2211	Ian, Alan, Martin, Tony, Prince, Chris, Brian, Harry, Andrew, Christ, Steve, ...
1855	Central, Leeds, Manchester, Australia, Yorkshire, Belfast, Glasgow, Middlesbrough, ...

A more recent work Ustalov et al. 2019 describes a meta-algorithm for graph clustering in the context of Natural Language Processing, which can be based on Chinese Whispers and also other methods (hence the term 'meta'). Their algorithm (`WATSET`) is designed for *fuzzy* graph clustering which exceeds the scope of this special topic, but represents the state of the art in graph-based applications in Natural Language Processing. Fuzzy clustering algorithms are capable of producing *soft* clusterings, which relax the requirement that all clusters are strictly disjoint (so $C_i \cap C_j = \{\}\ \forall\, i, j$, as introduced in Definition 2.3).

3.3 Greedy Modularity

This modularity-optimisation based clustering method takes a very similar route to the Louvain method Clauset, Newman and Moore 2004.

3.4 Further Approaches

3.4.1 Embeddings

Given a graph embedding, in which vertices are mapped to a point in $\mathbb{R}^n$, a frequently used measure of similarity is *Cosine-Similarity*:

$$\rho_{ij} = \arccos\left(\frac{\boldsymbol{a_i a_j}}{\|\boldsymbol{a_i}\|_2 \cdot \|\boldsymbol{a_j}\|_2}\right),$$

where vectors $\boldsymbol{a_i}, \boldsymbol{a_j} \in \mathbb{R}^n$ denote the point corresponding to vertex v_i, v_j respectively (Fortunato 2010). Of course this is not quite as connected to the other methods given that it already assumes a full embedding which itself is a very involved procedure, in many instances similar to clustering methods.

3.4.2 Girvan-Newman

The clustering method deduced in Girvan and Newman 2001 was a clear revolution at the time and brought graph clustering / community detection methods closer to practicians. It consists of multiple iterations which will each increase the number of clusters, and in between iterations optimises centrality and betweenness.

3.4.3 Map Equation

The map equation. See also Figure 1.

3.4.4 Spectral Clustering Methods

Fortunato 2010.

4 Network Generation and Results

The text corpus and vocabulary database used for the analysis in this report is based on the database behind the Latin is Simple Library which is in the public domain. In order to avoid incorporating too many grammatical particles / utility words, according

to common practice (Biemann 2006), we have removed the 2000 most frequent words from the dataset.

- G_4
- G_8
- G_{12}

Girvan Newman did not converge within a day, and if it had it would have only been a bipartition.

Table 2: Clustering results for the corpus-generated semantic networks G_4, G_8 and G_{12}.

Graph	Method	Q	Runtime [s]	n_C	Avg. size	Max size
G_4	louvain	0.1307	25.091	9	1204.78	4066
G_4	chinese-whispers	0.0168	126.342	1929	5.62	39
G_4	greedy-modularity	0.1150	1993.632	39	278.03	4836
G_8	louvain	0.1189	58.373	5	2168.60	3748
G_8	chinese-whispers	0.0142	179.585	1625	6.67	43
G_8	greedy-modularity	0.1074	2211.958	25	433.72	5375
G_{12}	louvain	0.1150	43.723	4	2710.75	3465
G_{12}	chinese-whispers	0.0011	242.934	365	29.71	9778

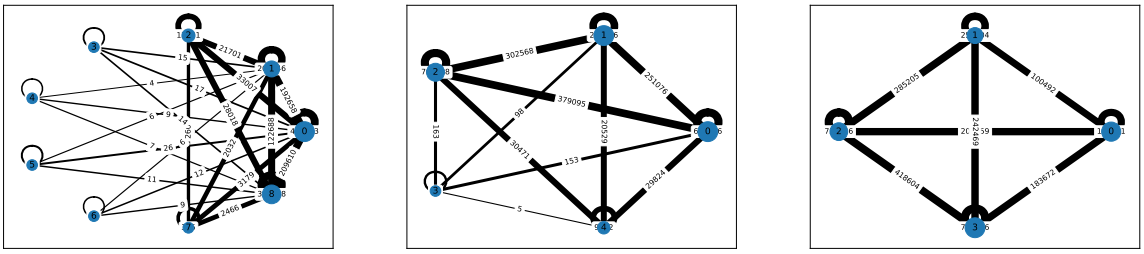
Table 3: Clustering results for Zachary's Karate Club Graph (Zachary 1977).

Method	Q	Runtime [s]	n_C	Avg. size	Max size
greedy-modularity	0.4110	0.006	3	11.33	17
girvan-newman	0.3477	0.145	2	17.00	19
louvain	0.4439	0.002	4	8.50	14
chinese-whispers	0.3920	0.004	7	4.86	11

Apply to Karate Club

Table 4: Some resulting clusters in G_4, extracted using the Chinese Whispers clustering method.

Size $\lvert C_i \rvert$	Sample Words
4	conterritus (frightened), minitabundus (threatening), infensus (hostile), properus (quick)
6	hibernus (wintry), tardus (slow), upilio (shepherd), formosus (beautiful), hibernum (winter camp (pl.)), subulcus (swineherd)
4	fraus (deceit), fraus (fraud), infitiatio (denial), furtum (theft)
6	fraxinus (ash-tree), laurea (laurel/bay tree), fraxinus (of ash), corylus (hazel-tree), edurus (very hard), abies (fir tree/wood)
4	vicina (neighbor), vicinus (nearby), vicinus (neighbor), vicinum (neighborhood)
7	repens (sudden), susurro (whisperer), nutrix (nurse), susurrus (whispering), immanitas (brutality), susurrus (whisper), diritas (frightfulness)
8	Aegyptius (Egyptian), Aethiopia (Ethiopia), siccitas (dryness), Aegyptus (Egypt), desertum (desert), Aegyptius (Egyptian), aspis (asp), Arabia (Arabia)
8	Marius (Marius), Maria (Mary), caelia (kind of beer), citrum (wood of citron tree), alternatus (alternate), citrus (African citrus tree), series (row), citrus (lemon tree)
6	inexspectatus (invincible), armum (arms (pl.)), tribunicius (of/-belonging to tribune), intutus (defenseless), Arma (Arms), arma (weapons)
2	globosus (round), rotundus (round)



(a) Clusters of G_4. **(b)** Clusters of G_8. **(c)** Clusters of G_{12}.

Figure 5: Using the Louvain method. Clusters of G_4 are depicted in Figure 5a.

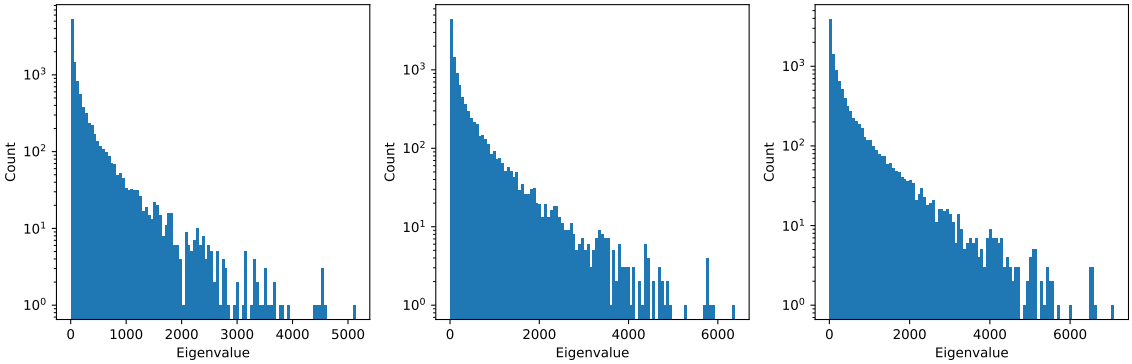


Figure 6: Histogram of the eigenvalue spectrum of the Graph Laplacian $L = D - A$ of G_4.

Computational complexity table from Ustalov et al. 2019, also referencing complexity section above. Fortunato also has complexities.

5 Discussion and Outlook

As the author describes in Biemann 2006, Chinese Whispers generally produces a fairly high number of clusters as compared to other methods, which one may circumvent by adjusting either the convergence condition or introduce a minimum cluster size through minor adjustments of the algorithm.

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