Q. P. Code: 20944

(Time: 2½ hours)

Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt *any three* of the following: 15

a. Show that $\begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$ is an orthogonal matrix.

b. For different values of k, discuss the following equations:
$x + 2y - z = 0;\ 3x + (k + 7)y - 3z = 0;\ 2x + 4y + (k - 3)z = 0$

c. Find the eigen values of the matrix
$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

d. Express $\frac{-1}{2} + \frac{\sqrt{3}}{2} i$ in polar form.

e. Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -2^8$

f. Show that $\sec h^{-1}(\sin \theta) = \log \cot\left(\frac{\theta}{2}\right)$

2. Attempt *any three* of the following: 15

a. Solve: $(D^2 - 4D + 1)y = \cos 2x + x$

b. Solve $\sin 2x \frac{dy}{dx} = y + \tan x$

c. Solve: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

d. Solve $p^2 - py + x = 0$

e. Solve: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = \sin(\log x^2)$

f. Solve: $\frac{du}{dx} + v = \sin x;\ \frac{dv}{dx} + u = \cos x.$ given at $x = 0,\ u = 1 \quad and \quad v = 0$

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3. Attempt *any three* of the following: **15**

a. Find the Laplace Transformation of $f(t) = t^3 e^{2t}$

b. $L[f(t)] = \dfrac{8 + 12s - 2s^2}{(s^2+4)^2}$ then find $L[f(2t)]$

c. Find L[y(t)] of the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t} \;;\; y(0) = 1 \; and \quad y'(0) = 2$$

d. Find the inverse Laplace transform of : $\dfrac{5s+3}{(s+1)(s^2+2s+5)}$

e. Find the Laplace transform of : $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t) = f(t+2a)$

f. Solve the following differential equation by using Laplace transform method:

$\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 5y = e^{-t}\sin t$. Given $y(0) = 0, y'(0) = 1$

4. Attempt *any three* of the following: **15**

a. Evaluate $\displaystyle\int_0^1\int_0^1 \frac{dx\ dy}{\sqrt{(1-x^2)(1-y^2)}}$

b. Evaluate $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x\ dx\ dy}{\sqrt{x^2+y^2}}$ by changing polar co-ordinates.

c. Evaluate $\displaystyle\iint_R r^4\cos^3\theta\ dr\ d\theta$ where R is the region of curve $r = 2a\cos\theta$

d. Evaluate $\displaystyle\iiint \frac{dx\ dy\ dz}{\sqrt{1-x^2-y^2-z^2}}$ taken throughout the volume of the sphere $x^2+y^2+z^2 = 1$ in the positive octant.

e. Evaluate $\iint y\ dx\ dy$ over the area bounded by $y = x^2,\ x + y = 2$

f. Find the volume bounded by the cylinder $y^2 = x$ and $x^2 = y^2$ and the planes $z = 0$ and $x + y + z = 1$

5. Attempt *any three* of the following: **15**

a. Evaluate $\displaystyle\int_0^{1/2} x^3\sqrt{1-4x^2}\ dx$

b. Evaluate $\displaystyle\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\,d\theta$

c. Show that: $\displaystyle\int_0^1 \frac{x^a - x^b}{\log x}\ \log\left(\frac{a+1}{b+1}\right)$ using DUIS.

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d. If $y = \int_0^x f(t)\sin[a(x-t)]\,.\,dt$ then show that, $\frac{d^2y}{dx^2} + a^2y = af(x)$

e. Find $\frac{d}{dx}[erf(x) + erf_c(ax)]$

f. Define error function and prove that error function is an odd function.
