

(2½ hours)

Total Marks: 75

- N. B.: (1) All questions are compulsory.
 (2) Make suitable assumptions wherever necessary and state the assumptions made.
 (3) Answers to the same question must be written together.
 (4) Numbers to the right indicate marks.
 (5) Draw neat labeled diagrams wherever necessary.
 (6) Use of Non-programmable calculators is allowed.

1. Attempt any three of the following:

15

- a. Find the adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

- b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$$

- c. Discuss the consistency of the following systems of equations and solve them whenever possible.

$$X_1 + 2X_2 + 2X_3 = 1$$

$$2X_1 + 2X_2 + 3X_3 = 3$$

$$X_1 - X_2 + 3X_3 = 5$$

- d. Express in $a + ib$ form $\cot(x + iy)$.

- e. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$.

- f. Prove that $(1 + \cos x + i \sin x)^n = 2^n \cos^n x/2 (\cos nx/2 + i \sin nx/2)$

2. Attempt any three of the following:

15

- a. Solve the Differential Equation $(1 - 2xy - x^3) dy - (1 + y^2 + 3x^2y) dx = 0$

- b. Solve the Differential Equation $x^2 dy/dx = 3x^2 - 2xy + 1$

- c. Solve the following Equation $\sec x dy/dx = y + \sin x$

- d. Solve the following Equation $p^2 x(x-2) + p(2y - 2xy - x + 2) + y^2 + y = 0$

- e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$.

- f. Find the General Solution of the equation $(D^3 + 3D)y = \cos x$

3. Attempt any three of the following:

15

- a. Evaluate $\int_0^\infty e^{-3t} t \cos 2t dt$

- b. Find the inverse Laplace transform for the function

$$F(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$$

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[TURN OVER]

- c. Find Laplace transformation of the function

$$f(t) = t(2\sin 3t + e^{2t})$$

- d. Obtain the Laplace transform of each of the given function

$$F(t) = e^{-2t}\cos 4t + e^{3t}\sin 6t$$

- e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s^2}{(s^2 + a^2)^2}$$

- f. Using Laplace transform method solve the following differential equations with the given condition.

$$(D^2 + 3D + 2)y = 4t + e^{3t} \text{ if } y = 1, Dy = -1 \text{ at } t = 0.$$

4. Attempt any three of the following:

15

- a.

$$\text{Evaluate } \int_0^2 \int_x^{4-x} \int_0^1 e^{2x+2y} dx dy.$$

- b.

$$\text{Evaluate } \int_0^4 \int_0^{\sqrt{4x-x^2}} \frac{y dx dy}{(x^2 + y^2)^{1/2}}$$

- c.

$$\text{Evaluate } \int_0^1 \int_{y^*y}^1 \int_0^{1-x} x dx dy dz.$$

- d.

$$\text{Evaluate } \int_0^a \int_0^{(a^2-x^2)^{1/2}} \int_0^{(a^2-x^2-y^2)^{1/2}} (xyz) dx dy dz .$$

- e.

$$\text{Change the order of integration and evaluate } \int_{-1}^2 \int_{x^2}^{x+2} dx dy$$

- f.

$$\text{Change to polar coordinates and evaluate } \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy .$$

5. Attempt any three of the following:

15

- a.

$$\text{Evaluate } \int_0^\pi \sin^2 x (1 + \cos x)^4 dx.$$

- b.

$$\text{Evaluate } \int_0^\infty \frac{x^2 dx}{(1+x^6)^{7/2}}.$$

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[TURN OVER]

c.

Evaluate $\int_0^{\infty} e^{-ax} \sin x / x \, dx$

d.

Evaluate $\int_0^{\pi/2} \frac{\log(1+a \sin^2 x) dx}{\sin^2 x}$.

e.

Evaluate $\int_0^1 x^m (\log x)^n \, dx$.

f.

Define error function. Evaluate $\operatorname{erf}(\sqrt{x})$
