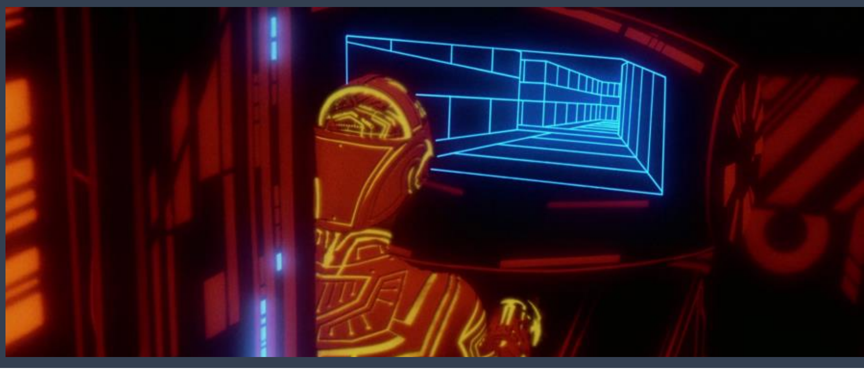
CGA

MODULE-: Two Dimensional & Three-dimensional Transformations



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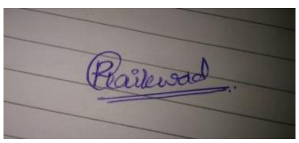
Certificate

This is to certify that the e-book titled <u>"Computer Graphics and Animation"</u> comprises all elementary learning tools for a better understating of the relevant concepts. This e-book is comprehensively compiled as per the predefined eight parameters and guidelines.



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Date: 28-01-2021

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Unit II: Two Dimensional & Three-Dimensional Transformations

❖ Contents

Two Dimensional Transformations : Transformations and Matrices, Transformation Conventions, 2D Transformations, Homogeneous Coordinates and Matrix Representation of 2D Transformations, Translations and Homogeneous Coordinates, Rotation, Reflection, Scaling, Combined Transformation, Transformation of Points, Transformation of The Unit Square, Solid Body Transformations, Rotation About an Arbitrary Point, Reflection through an Arbitrary Line, A Geometric Interpretation of Homogeneous Coordinates, The Window-to-Viewport Transformations.

Three dimensional Transformations : Three-Dimensional Scaling, Three-Dimensional Shearing, Three-Dimensional Rotation, Three-Dimensional Reflection, Three-Dimensional Translation, Multiple Transformation, Rotation about an Arbitrary Axis in Space, Reflection through an Arbitrary Plane, Matrix Representation of 3D Transformations, Composition of 3D Transformations, Affine and Perspective Geometry, Perspective Transformations, Techniques for Generating Perspective Views, Vanishing Points, the Perspective Geometry and camera models, Orthographic Projections, Axonometric Projections, Oblique Projections, View volumes for projections.

❖ Objectives :

To familiarize students with the two dimensional and three-dimensional transformations.

❖ Prerequisites and Linking

Basic knowledge of C++ Programming

❖ Bridge :

The contents of this chapter are related to Programming languages like core Java in Sem IV and ASP.Net with C#, Advanced Java in Sem V and final year Project in semester VI.

Introduction

This chapter introduces 2D transformation operations used in computer graphics. These transformations are often referred to as transformation in planes since they have 2D definition of objects. The major geometric operations we discuss in this chapter include translation, scaling, rotation, reflection, and shearing.

The word we live in is three-dimensional (3D). Therefore, if we want to construct a realistic computer model of it, the model should be 3D as well. Contrary to images that are described in two dimensions (2D) or planes, an object is described in 3D or space. Though we have been using 2D display systems, correct and realistic visual effects of objects of nature require us to define them in 3D.

Two-Dimensional Transformation :

Transformation Matrix

Suppose the matrix definition of original image and transformed images be A and B respectively. The transformed image B is obtained by operating a matrix T represented by B = A.T, where the matrix T is called the geometrical operator matrix or transformation matrix.

We can also write a system of equation in a matrix form, where (x, y) and (x′, y′) are the coordinates of the original point P′, respectively. The general matrix of transformation is given by

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \ldots(1)$$

where a, b, c and d are individual elements of the matrix for transformation capable of producing effects like scaling, reflection, rotation and shearing.

Consider a point P(x, y) with matrix definition given by [x y]. A transformation on this point is nothing but multiplication of this point and 2 $\times$ 2 transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and $$[P]\,[T] = [x\ y] \begin{bmatrix} a & b \\ c & d \end{bmatrix} = [ax + cy \quad bx + dy] = P[x'\ y'] \quad \ldots(2)$$

This operation transforms the initial point P(x, y) to point P(x′, y′), where x′ and y′ are calculated.

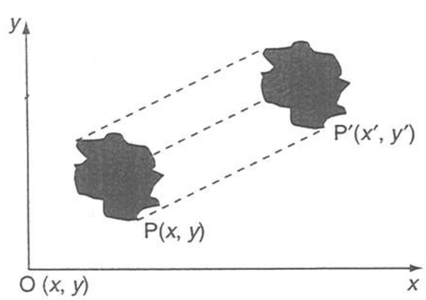


Fig. : An image in a 2D coordinate system.

This image can be considered as a set of points, where every point in the image has coordinates (x, y). If the image is transformed and moved to a new position then the point P(x, y) transforms into P′ (x′, y′). The coordinates of P′ can be obtained from the original point P of original image using geometrical definition of the transformation.

We need to represent the above transformation. This transformation is represented by a point-to-point transformation. The transformation is applied to x and y to get transform point x′ and y′. While representing transformation, we need to have relation between (x, y) and (x′, y′). This relation may be function, matrix, or system of equation.

The most general transformation is obtained using the relations.

$$x = f(x, y), \quad y = g(x, y)$$

where f and g are function of x and y. We assume that the f and g are linear function of x and y for which we get a system of equation for the transformation as

$$x' = ax + by, \quad y' = cx \text{ and } dy$$

When written as function of transformation, it is called **transformation function** given by

$$T(x', y') = (ax + by, cx + dy)$$

This transformation function may be used for phenomenon like scaling, reflection required for performing operation in 2D and 3D coordinate systems.

TRANSFORMATION CONVENTIONS

Transformation means changing some graphics into something else by applying rules. We can have various types of transformations such as translation, scaling up or down, rotation, shearing, etc. When a transformation takes place on a 2D plane, it is called 2D transformation.

Transformations play an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

Different transformation conventions are used in 2D graphics.

TYPES OF TRANSFORMATIONS IN TWO–DIMENSIONAL GRAPHICS

- Almost all graphical systems allow the programmer to define the picture that include a variety of transformations.
- E.g. A programmer is able to magnify a picture so that detail appears more clearly. Or reduce it so that more of the picture is visible.
- The programmer is also able to rotate the picture so that he can see it in different angles.
- Operations that are applied to the geometric description of an object to change its position, orientation, or size are called geometric transformations.

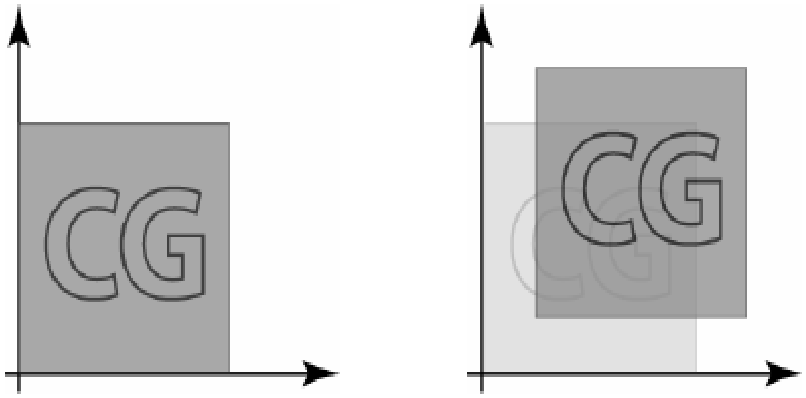
- So the geometric-transformation functions that are available in some system are following:

1. Translation
2. Rotation
3. Scaling
4. reflection
5. shear.

2D TRANSFORMATION (TRANSLATION,SCALING,ROTATION,SHEARING)

https://www.youtube.com/watch?v=Juo-Kua_eQc

<u>1 Two-Dimensional(2D) Translation</u>



- We perform a translation on a single coordinate point by adding offsets to its coordinates so as to generate a new coordinate position.
- Similarly, a translation is applied to an object that is defined with multiple coordinate positions by relocating all the coordinate positions by the same displacement along parallel paths.
- Suppose tx and ty is the translation distances, (x, y) is the original coordinates, is the new coordinate position.

$$\mathrm{x}' = \mathrm{x} + \mathrm{tx} \qquad\qquad \mathrm{y}' = \mathrm{y} + \mathrm{ty}$$

- Express the translation use matrix equation as following:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

- Translation is a rigid-body transformation that moves objects without deformation.

2. Two–dimensional scaling

In Euclidean geometry, scaling is an affine, linear transformation that can enlarge or diminish an object by certain factors. Scaling is changing the size of an image without changing its shape.

Let us consider another case where matrix elements a or d or both are not zero and c=b=0. The transformation operator T operates on the point P(x, y) to produce scaling operation in x–direction, y–direction or in both the directions, depending on the case as it may be. We discuss the individual cases as given in the Figure 1.

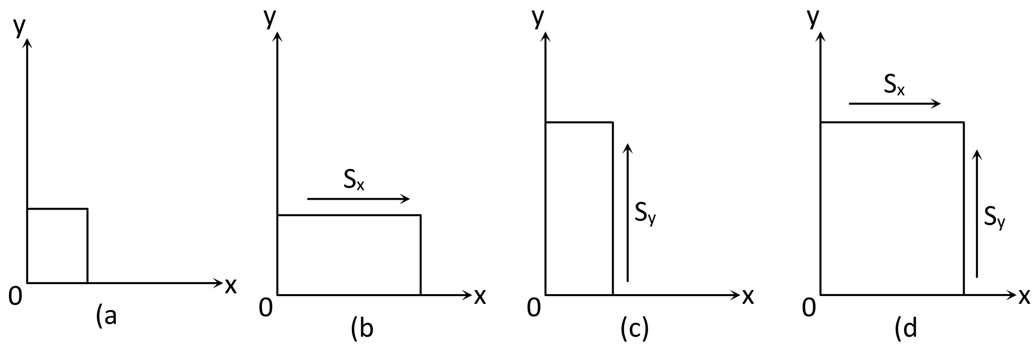


Fig.1 : Scaling transformations: (a) Original image; (b) scaling in x–direction; (c) scaling in y–direction; (d) scaling xy–direction.

Let the matrix element $a = s_x \neq 0$ and $d = 1$ with $b = c = 0$. The transformation matrix reduces to matrix of transformation representing scaling in x–direction by magnitudes s_x. Therefore,

$$P'(x', y') = [x' \;\; y'] = [P].[T_y] = [x \;\; y] \begin{bmatrix} 1 & 0 \\ 0 & s_y \end{bmatrix} = \begin{bmatrix} x \, s_y y \end{bmatrix}$$

That is, $\quad x' = s_x x$ and $y' = y$ and $T_x = \begin{bmatrix} s_x & 0 \\ 0 & 1 \end{bmatrix}$

is the scaling transformation matrix in x–direction. The operational significance of this operation implies that the points of the image are scaled in x–direction by magnitude s_x and y remains same (see Figure 1b).

Now let the matrix element $a = 1$ and $d = s_y \neq 0$ with $b = c = 0$. The transformation matrix reduces to matrix of transformation representing scaling in y–direction by magnitude s_y (see Figure 1c). Therefore,

$$P'(x', y') = [x' \;\; y'] = [P].[T_y] = [x \;\; y] \begin{bmatrix} 1 & 0 \\ 0 & s_y \end{bmatrix} = \begin{bmatrix} x \, s_y y \end{bmatrix}$$

That is, $\quad x' = x$ and $y' = s_y y$ and $T_y = \begin{bmatrix} 1 & 0 \\ 0 & s_y \end{bmatrix}$

is the scaling transformation matrix in y direction. The operational significance of this operation implies that the points of the image are scaled in y–direction by magnitude s_y and x remains same.

In general, if one wishes to produces scaling in both x and y simultaneously by magnitude s_x in x–direction and s_y in y–direction then the condition for such operation is $a \neq 1 = s_x$, $d \neq 1 = s_y$ and $b = c = 0$ (see Figure 1d). Therefore, the matrix of transformation for scaling in xy is

$$T_{xy} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

where $a = s_x$ and $d = s_y$, s_x, and s_y are the scaling factors chosen in x and y directions respectively.

Example 1 : Consider a line AB with coordinates A(0, 0) and B(4, 5) in a 2D plane. Obtain a matrix of transformation for scaling for the line AB in x–direction by the factor 3 and plot the same.

Solution :

From the above information, we have coordinates of the line AB as A = [0 0] and B = [4 5]. The matrix of transformation is scaling matrix in x–direction with $s_x = 3$, $s_y = 1$ and b = c = 0 written as

$$T_x = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

The scaling operation, when individually performed on the point A (0, 0), result to

$$A'(x', y') = [A][T_x] = [0\ 0] \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = [0\ 0]$$

Hence the point remains unchanged with A′(0, 0). Similarly for the point B(4, 5), we have

$$B'(x', y') = [B][T_x] = [4\ 5] \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = [12\ \ 5]$$

Therefore, the line AB transforms to A′B′ with coordinates A′ (0, 0) and B′(12, 5) as shown in the figure 2.

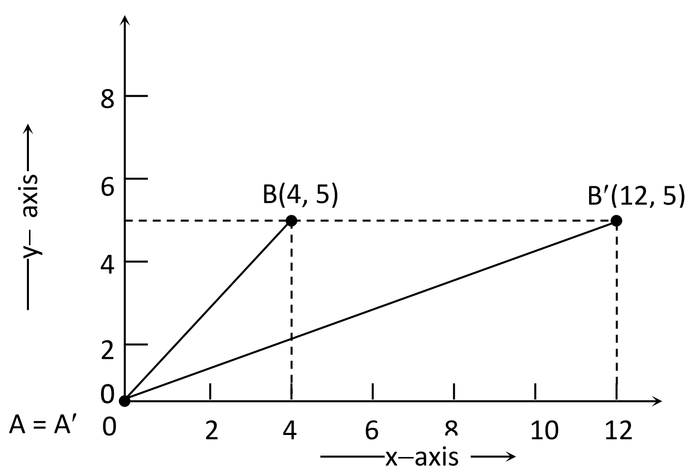


Fig.2: Figure for Example 1.

Note that the above operation could also be performed by an alternative approach where the line AB can be represented as a single matrix, in which each row represents the coordinates of the points of the image or image primitive. Only one operation would be sufficient to produce the result in such representation. Let the line AB be represented by

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & 5 \end{bmatrix}$$

Therefore, the transformation operation becomes,

$$A'B' = \begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}[T_x] = \begin{bmatrix} 0 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 12 & 5 \end{bmatrix}$$

Therefore, $A'(x', y') = (0, 0)$ and $B'(x', y') = (12, 5)$.

3.Two-Dimensional Reflection

Reflection is a phenomenon that has typically great significance in computer graphics because of its behaviour about an axis. Mathematically, a transformation in which the direction of one axis is reversed is a reflection. In order sense, a reflection refers to a representation of the components of a coordinate system at an equal distance but in opposite direction about the axis. For example, a point P(0, 4) when reflected about x–axis becomes P′(0, –4), i.e. x–coordinate is transformed to its negative value.

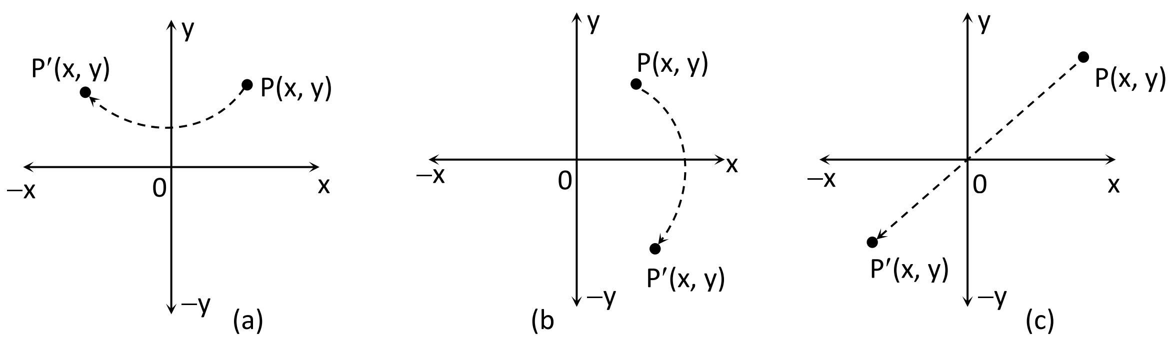


Fig. 3 : Scaling transformations: (a) Reflection about y–axis; (b) reflection about x–axis;

Reflection about y–axis

Let us consider the case when a = –1, d = 1, and b = c = 0. The matrix of transformation becomes,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

The transformed point P′ for the point P with coordinated (x, y) produces (x′, y′) given by

$$P'(x', y') = [x' \ y'] = [P][T] = [x \ \ y] \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [-x \ \ y]$$

This results in reflection about y–axis with x′ = –x and y′ = y, as shown in Figure 3(a).

Reflection about x–axis

Let us consider the case when a = 1, d = –1 and b = c = 0. The matrix of transformation becomes,

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The transformed point P′ for the point P with coordinates (x, y) produces (x′, y′) given by

$$P'(x', y') = [x' \ y'] = [P][T] = [x \quad y] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [x \quad -y]$$

This results in reflection about x–axis with x′ = x and y′ = –y, as shown in figure 3(b).

Example 2 : Consider a point P(2, 3) in a coordinate plane. Perform reflection of the point P through y–axis and draw the same.

Solution :

To perform reflection through y–axis, we have the matrix of reflection as $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Applying the reflection matrix to the point P(2, 3), we obtain

[–2 3] as given below,

$$P'(x', y') = [2 \ \ 3] \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= [-2, 3]$$

The point P with coordinates (2, 3) is reflected about y–axis, and new coordinates of the transformed (reflected) points becomes (–2, 3), as shown in figure 4.

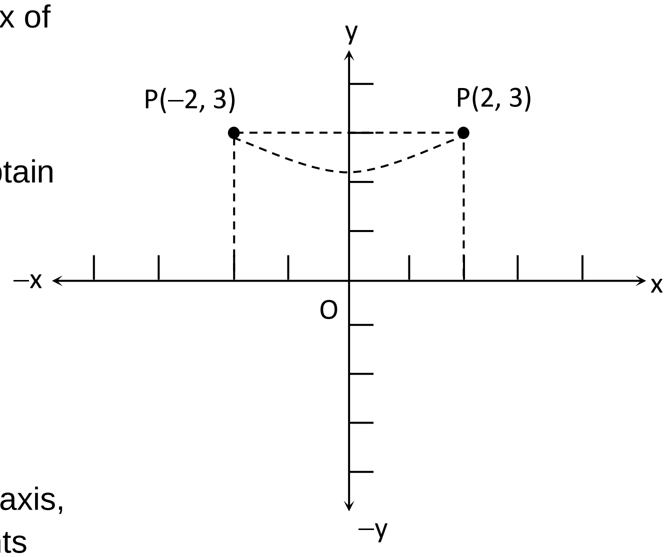


Fig. 4 : Figure for Example 2.

Example 3 : Consider a point P(2, 3) in a coordinate plane. Perform reflection of the point P through x–axis and draw the same.

Solution :

To perform reflection through x–axis, we have the matrix of reflection as

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Applying the reflection matrix to the point P(2, 3), we obtain [2 –3] as given below

$$P'(x', y') = [2 \ \ 3] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [2 \ \ -3]$$

The point P with coordinates (2, 3) is reflected about y–axis and new coordinates of the transformed (reflected) points becomes (2, –3), as shown in Figure 5,

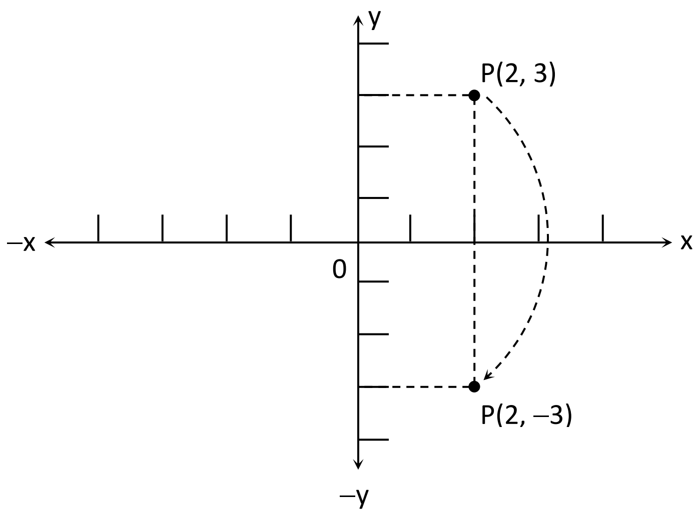


Fig. 5 : Figure for Example 3.

Example 4 : Consider the line AB with coordinates of the line A(2, 3) and B(4, 5) in the coordinate plane. Perform reflection of the point P about origin and draw the same.

Solution :

To perform reflection about the origin (xy plane), we have the matrix of reflection as

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

We apply the reflection matrix to the line AB to obtain A′ B′:

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}[T] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -4 & -5 \end{bmatrix}$$

Therefore, A′ = (–2, –3) and B′ = (–4, –5). The line is reflected about the origin, as shown in Figure 6.

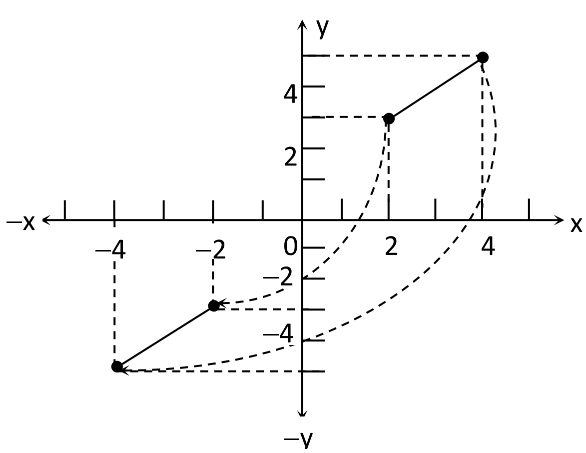


Fig. 6 : Figure for example 4

4 Two–dimensional rotation

Rotation refers to the movement of a body in such a way that the distance between a certain fixed point and any given point of that body remains constant. In mathematics, rotation is a transformation in which the coordinate axes are rotated by a fixed angle about the origin. Rotation is a very useful transformation technique used in rendering and animation nowadays. Let's us understand the mathematical implications of the rotation before applying it to any objection or the image (see Figure 9).

Since the rotation is performed about the origin, distances from the origin to P and to Q labelled r are equal. Considering figure 3.8b, by rules of trigonometry, we have

$$x = r\cos\phi \quad \text{and} \quad y = r\sin\phi \tag{4}$$

where ϕ is the angle between position vector for P and x–axis. The position vector for P becomes,

$$P = [x \quad y] = [r\cos\phi \quad r\sin\phi] \tag{5}$$

As the line OQ makes an angle $(\theta + \phi)$, we get,

$$y' = r\cos(\theta + \phi),\ x' = r\sin(\theta + \phi) \tag{6}$$

and

$$P'(x', y') = [r\cos(\theta + \phi)\ r\sin(\theta + \phi)] \tag{7}$$

where r is the length of OQ. Now, from Eq. (4)

$$x' = r\cos(\theta + \phi) = r(\cos\phi\cos\theta - \sin\phi\sin\theta)$$

$$y'\ r\sin(\theta + \phi) = r(\sin\theta\cos\phi + \cos\theta\sin\phi) \tag{8}$$

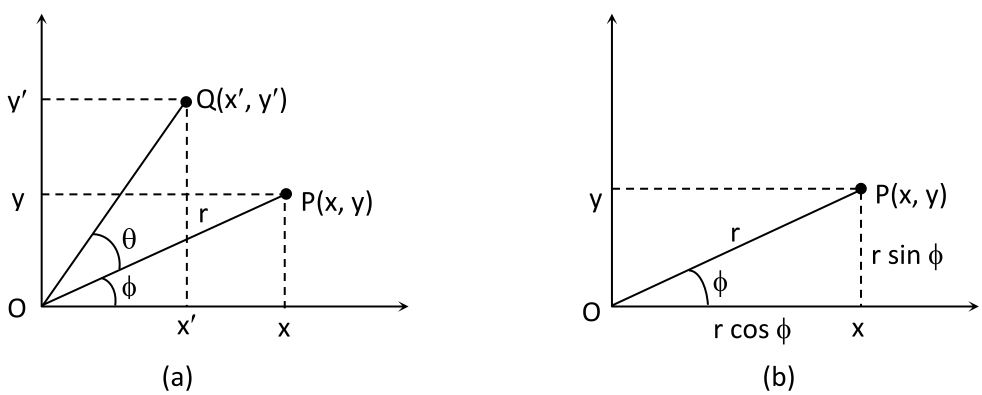


Fig.9 : (a) A 2D rotation of a point; (b) x and y coordinates of the point P(x, y).

Now replacing $r\cos\phi$ by x and $r\sin\phi$ by y from Eq. (7), we get

$$x' = x\cos\theta - y\sin\theta,\ y'\ x\sin\theta + y\cos\theta$$

$$Q = [P][M_\theta] = [x' \quad y'] = [x \quad y]\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

Example 5 : Rotate the triangle ABC by an angle 30°, where the triangle has the coordinates A(0, 0), B(10, 2), and C(7, 4).

Solution :

The triangle ABC can be represented by a matrix of vertices given by

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 10 & 2 \\ 7 & 4 \end{bmatrix}$$

The matrix of rotational transformation with the angle 30° is given by

$$M_{90^\circ} = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

The new coordinates of the rotated triangle A′B′C′ is obtained as

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} . M_{30^\circ} = \begin{bmatrix} 0 & 0 \\ 10 & 2 \\ 7 & 4 \end{bmatrix} . \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 5\sqrt{3}-1 & 5+\sqrt{3} \\ (7\sqrt{3}/2)/2 & (7/2)+2\sqrt{3} \end{bmatrix}$$

Hence, we get,

$$A' = (0, 0),\ B' = (5\sqrt{3} - 1, 5 + \sqrt{3}),\quad C' = \left(\frac{7\sqrt{3}}{2} - 2, \frac{7}{2} + 2\sqrt{3}\right)$$

5) Two–dimensional shear transformations :

Shearing is a deformation of an object in which parallel planes remain parallel but are shifted in a direction parallel to them. (The shear changes a quadrilateral to a parallelogram.) Shearing refers to change in the shape or simply deformation of an object or an image. It is often visualized as deformations produced on the object after applying force or pressure on the object in a particular direction.

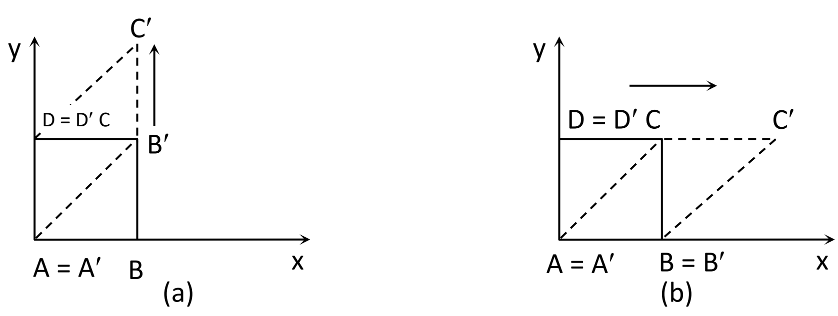


Fig.7 : Transformation for shearing operation on a unit square:

For the first case (Figure 7a), let a = d = 1 and c = 0. The matrix of transformation becomes,

$$\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

The point P(x, y) transforms to [x (bx + y)] as obtained below,

$$P'(x', y') = [x' \; y'] = [P][T] = [x \quad y] \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = x(bx + y)] \qquad \ldots(3)$$

It is important to note that the x coordinate of the point P remains unchanged while y′ coordinate changes and is linearly dependent on the original coordinates. This is called shearing proportional to x–coordinates. Similarly, when a = d = 1 and b = 0 (Figure 8a), transformation results into shearing proportional to y coordinates, having the shear transformation matrix.

$$\begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix}$$

The point P(x, y) in the figure transforms to [(x + cy)   y] as obtained below:

$$P'(x', y') = [x' \;\; y'] = [P][T] = [x \quad y] \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} = [(x + cy) \quad y]$$

i) Keeping all points on x-axis fix

(a) Shearing in the positive direction of x-axis result (see figure 8(a))

$$T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(b) Shearing in the negative direction of x-axis result (see figure 8(b))

$$T = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

ii) Keeping all point son y-axis fixed

Shearing in the positive direction of y-axis results (see figure 8(c)) $T=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

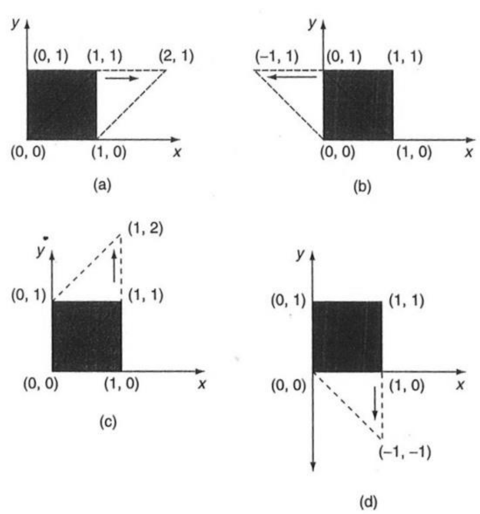


Fig. 8 : Shearing in the (a) Positive direction of x-axis; (b) negative direction of x-axis; (c) positive direction of y-axis; (d) negative direction of y-axis.

University Question

Shear a unit cube situated at origin with a shear transformation matrix

$$T_{shear} = \begin{bmatrix} 1 & 1.5 & 3 & 0 \\ 0.8 & 0 & 1 & 0 \\ 0.5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution :-

Matrix for a unit cube.

$$V = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

We have to multiply V with Shear matrix

$$V' = V \cdot T_{shear}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1.5 & 3 & 0 \\ 0.8 & 0 & 1 & 0 \\ 0.5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 1.5 & 5 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Homogeneous coordinates

Construction and design of sequence transformations involve transformations like scaling, rotation, reflection, and translation. The basic transformation discussed in previous sections can be expressed in general form,

$$[P'] = [P].\ [T_1] + [T_2]$$

For translation, we have $$[P'] = [P].\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + [t_x \quad t_y]$$

where t_x and t_y are translation factors in x and y directions respectively. T_1 is taken to be the identity matrix.

For rotation, we have $$[P'] = [P].\ \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + [0 \quad 0] \qquad \ldots(3)$$

This is, T_1 is the scaling matrix and $T_2 = 0$.

In order to produce a sequence of transformation, including above equations, one must compute transformed coordinates one at a time, say translation following rotation followed by scaling. The transformed coordinates is scaled thereafter. From the above discussion, we see the impossibility of representing a translation transformation as a 2×2 matrixes. This sequential transformation process is proved to be inefficient. A more general approach to overcome these intermediate computations is required that gives the final coordinates directly from initial coordinates.

The objective is achieved by introducing a homogeneous coordinate representation, which is a technique based on projective geometry. The coordinate representation of a point [x y] in the homogeneous representation is given by a triplet $[x_h\ y_h\ h]$, were,

$$x = \frac{x_h}{h} \text{ and } \frac{y_h}{h} \qquad \ldots(4)$$

For a 2D transformation system, the homogeneous parameter h can be any nonzero value, For simplicity and convenience, h is taken to be 1 which represents a point as $[x_h \quad y_h \quad 1]$. The transformation matrix is now represented by order 3×3 instead of 2×2.

1) Translation in homogeneous coordinates

The homogeneous transformation matrix for translation is written as

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

Therefore, the translated coordinates are,

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} = \begin{bmatrix} x+t_x & y+t_y & 1 \end{bmatrix}$$

2) Rotation in homogeneous coordinates

The homogeneous transformed matrix for rotation is written as

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots(7)$$

Therefore, the rotated coordinates are

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [x \cos\theta - y \sin\theta \quad x \sin\theta - y \cos\theta \ 1]$$

3) Scaling in homogeneous coordinates

The homogeneous transformation matrix for scaling is written as

$$T = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the scaled coordinates are

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} xS_x & yS_y & 1 \end{bmatrix}$$

University Question

3) Obtain the general combined matrix for scaling about an fixed point $P(xf, yf)$.

- Scaling with respect to a fixed point (xf, yf).

1) Translate the object so that the fixed point coincides with the coordinate origin

ii) ~~Scale~~ Scale the object with respect to the coordinate origin.

iii) Perform inverse of the translation in step 1 to return the object to its original position.

$$T_1 = \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}$$

The matrices for these three operations produces the required scaling matrix.

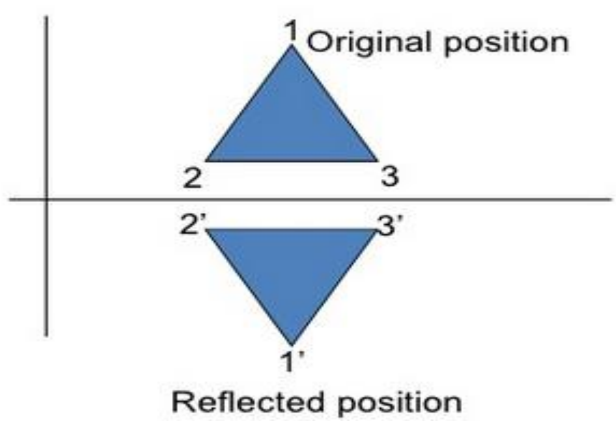
$$= T_1 \cdot S \cdot T_2$$

$$= \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$$

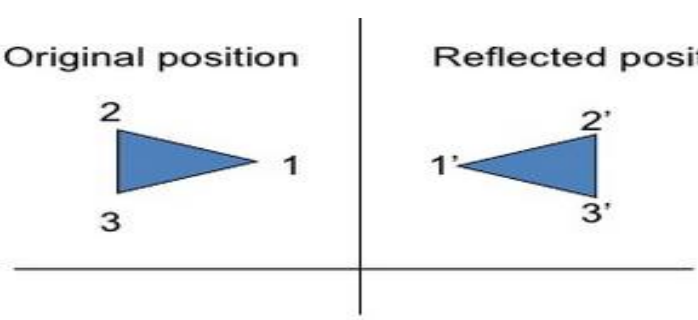
4) REFLECTION

<u>Reflection</u> is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180 deg about the reflection axis



Reflection about the line y=0, the X- axis , is accomplished with the transformation matrix

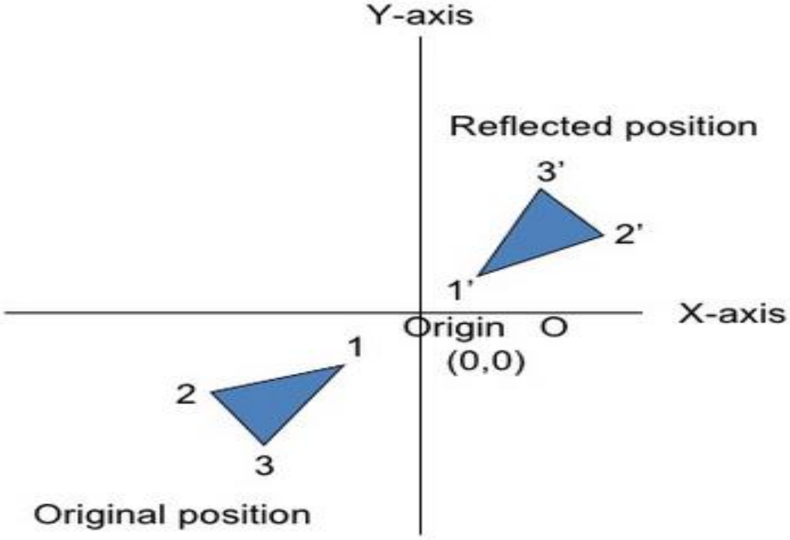
$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$



Reflection about the line x=0, the Y- axis , is accomplished with the transformation matrix

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Reflection of an object relative to an axis perpendicular to the xy plane and passing through the coordinate origin

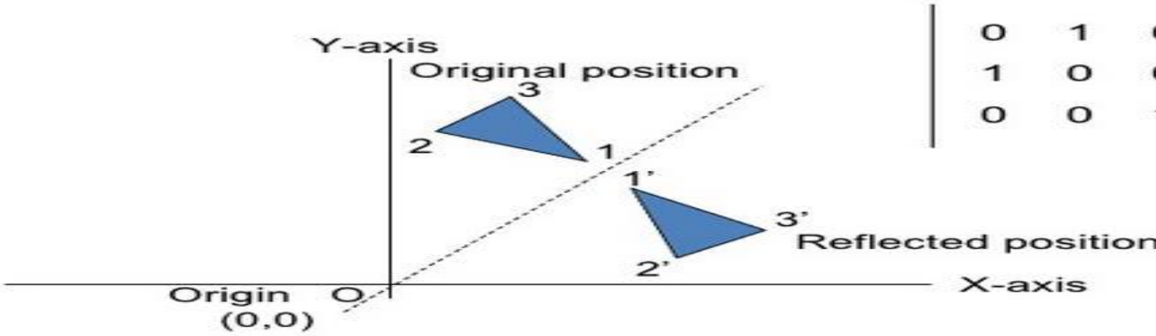


$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The above reflection matrix is the rotation matrix with angle=180 degree.

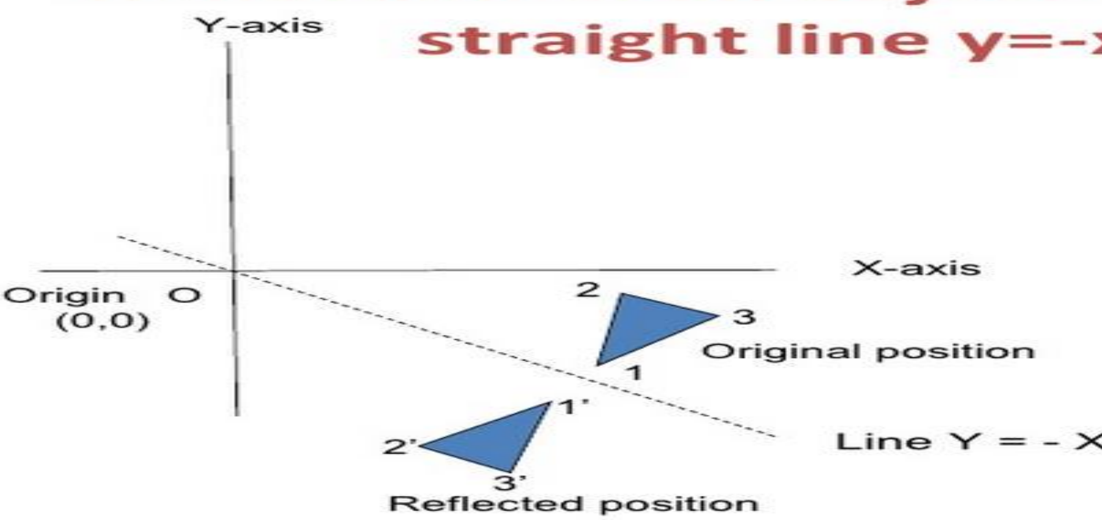
This can be generalized to any reflection point in the xy plane. This reflection is the same as a 180 degree rotation in the xy plane using the reflection point as the pivot point.

Reflection of an object w.r.t the straight line y=x



$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Reflection of an object w.r.t the straight line y=-x



$$\begin{vmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Combined transformation

In this method, the individual transformations are first combined and applied to bring overall transformation to the original object or the vector. For example, if T_1 and T_2 are two transformations to be applied in sequence then it is equivalent to perform $T_1.T_2$ to result T_3 that can be directly applied to the original definition of the object, or the vector, to produce an overall effect. If [x y] are the coordinates of any point in the image, and we apply transformations T_1, followed by $[T_2]$, to this point then it is equivalent to perform $T_1 . T_2 = T_3$ and

$$[x \quad y][T_3] = [x' \quad y']$$

$[x' \quad y']$ is the required coordinates after transformation similar to when perform sequentially.

Example 7 : Using homogeneous coordinate transformation matrix, apply the following sequence of transformations to a unit square centered at origin:

(a) Translation by factor (1/2, 1) and (b) Rotate by angle θ = 90°.

Solution :

The matrixes of transformations for translation and rotation are,

Translation matrix: $T_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$

Rotation matrix: $T_R = \begin{bmatrix} \cos 90^\circ & \sin 90^\circ & 0 \\ -\sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Therefore, combined homogeneous matrix of transformation is

$$T = T_T . T_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 1/2 & 1 \end{bmatrix}$$

The matrix representing the unit square ABCD is $\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} -1/2 & -1/2 & 1 \\ 1/2 & -1/2 & 1 \\ 1/2 & 1/2 & 1 \\ -1/2 & 1/2 & 1 \end{bmatrix}$

Now, the transformed unit square is,

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} -1/2 & -1/2 & 1 \\ 1/2 & -1/2 & 1 \\ 1/2 & 1/2 & 1 \\ -1/2 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 1/2 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 0 & 1 \\ 1/2 & 1 & 1 \\ -3/2 & 1 & 1 \\ -3/2 & 0 & 1 \end{bmatrix}$$

That is A′ =(–1/2, 0), B′= (1/2, 1), C′ =(–3/2, 1), and D′ = (–3/2, 0).

University Question

Q2 b) Using homogeneous coordinate transformation matrix, apply following sequence of transformation to a unit square centered at origin

i) Translation by factor (1, 1)

ii) Rotation by angle $\theta = 90°$.

→ i) Translation by factor (1, 1)

i.e $t_x = 1$ and $t_y = 1$

Homogeneous coordinates for translation are

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$T_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

ii) Rotation by angle $\theta = 90°$

$$T_R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T = T_T \cdot T_R$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

Unit square ABCD =

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} -1/2 & -1/2 & 1 \\ 1/2 & -1/2 & 1 \\ 1/2 & 1/2 & 1 \\ -1/2 & 1/2 & 1 \end{bmatrix}$$

Multiply T with Unit Square ABCD

A'B'C'D = T · Unit Square ABCD

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1/2 & -1/2 & 1 \\ 1/2 & -1/2 & 1 \\ 1/2 & 1/2 & 1 \\ -1/2 & 1/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 1/2 & 1 \\ -3/2 & 3/2 & 1 \\ -3/2 & 3/2 & 1 \\ -3/2 & 1/2 & 1 \end{bmatrix}$$

ie $A' = (-1/2, 1/2)$, $B' = (-3/2, 3/2)$, $C' = (-3/2, 3/2)$

$D' = (-3/2, 1/2)$.

Rotation about an arbitrary point

We considered rotation of any point P by arbitrary angle θ about the origin. However, many applications require a figure to be rotated around any arbitrary point rather than the origin O. In general, rotation about any arbitrary point can be performed by translating the point to origin followed by the required rotation and then translating back the obtained results to the original center of origin.

Consider the origin O(0, 0) of a coordinate system and $G(x_G, y_G)$ be the point about which the point $P(x_p, y_p)$ to be rotated by an angle θ. After rotation, the point P transforms to the point Q.

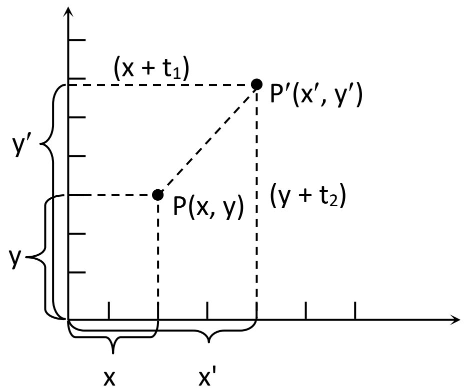


Fig. 10 : Translation of the origin

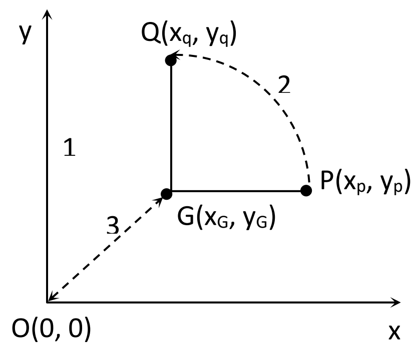


Fig. 11: Rotation of a point about an arbitrary

We follow the following naïve procedure to rotate any point P about any arbitrary point G by an angle θ as shown in Figure.

Step 1: Translate the origin O to G to obtain the point P′. The current coordinates of the point P after translation of the origin is given by P′ is

$$P' = [P] - [G] = [x_p - x_G \quad y_p - y_G] \tag{9}$$

Step 2:Apply the rotation matrix M_θ to P′ to obtain the rotated point P′′ given by

$$P'' = [P][M_\theta] = [x'' \quad y''] \tag{10}$$

The point P′′ (x′′, y′′) is the rotated coordinates with reference to the origin. Therefore, we need to translate back the coordinate system to the original system to obtain the actual rotation about the arbitrary point G.

Step 3: Translate the origin back to O by adding the coordinates of G to P′′ in order to obtain the translated point Q which is actually the rotation of point P about G by angle θ.

$$Q = [P''] + [G] = [x'' + x_G \quad y'' + y_G] \tag{11}$$

The method discussed above is equivalent to composite transformation with translation of the origin to the point G, rotational transformation with translation of the origin to the point G, rotational transformation followed by translation back to the origin.

Example 6 : Rotate a triangle ABC by and angle 90° about a point (–1, 1) where the triangle has the coordinates A(5, 0), B(10, 2) and C(7, 4).

Solution :

The matrix representation of the triangle ABC is,

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 10 & 2 \\ 7 & 4 \end{bmatrix}$$

Arbitrary points about which the triangle ABC are rotated be G = (–1, 1) and matrix of rotational transformation by angle 90° is

$$M_{90^\circ} = \begin{bmatrix} \cos 90^\circ & \sin 90^\circ \\ -\sin 90^\circ & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The problem requires sequence of transformations including translation to origin, rotation by 90° and finally translation back to origin. Translate the origin O to G to obtain the ΔA′B′C′.

The coordinates of ΔA′B′C′ is obtained as shown below,

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \Delta ABC - [G] = \begin{bmatrix} 5 & 0 \\ 10 & 2 \\ 7 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 11 & 1 \\ 8 & 3 \end{bmatrix}$$

Applying rotation matrix M_{90° to ΔA′B′C′, we get ΔA′′B′′C′′

$$\begin{bmatrix} A'' \\ B'' \\ C'' \end{bmatrix} = \Delta A'B'C'.\left[M_{90^\circ}\right] = \begin{bmatrix} 6 & -1 \\ 11 & 1 \\ 8 & 3 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -1 & 11 \\ -3 & 8 \end{bmatrix}$$

Q2 c. Magnify the triangle with vertices A(0,0) B(1,1) and C(5,2) to twice its size while keeping C(5,2) fixed.

→ Solution:-

The required transformation is

$$S_{2,2,C} = T_{-V} \cdot S_{22} \cdot T_V$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix}$$

Representing a point P with coordinates (x, y) by the row vector (x y 1), we have.

$$A \cdot S_{22C} = (0\ 0\ 1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (-5\ -2\ 1)$$

$$B \cdot S_{22C} = (1\ 1\ 1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (-3\ 0\ 1)$$

$$C \cdot S_{22C} = (5\ 2\ 1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (5\ 2\ 1)$$

So $A' = (-5, -2)$, $B' = (-3, 0)$ and $C' = (5, 2)$.

Representing using 3×3 matrix as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

and applying S_{22c} this so

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot S_{22c} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 & 1 \\ -3 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}$$

Reflection through an arbitrary line

Reflection through an arbitrary line

Steps:

- Translate line to the origin
- Rotation about the origin
- Reflection matrix
- Reverse the rotation
- Translate line back

$$T_{GenRfl} = T_r \; R \; T_{rfl} \; R^T \; T_r^{-1}$$

The Window-to-Viewport Transformations.

COORDINATE SYSTEMS

Screen Coordinates: The coordinate system used to address the screen (device coordinates)

World Coordinates: A user-defined application specific coordinate system having its own units of measure, axis, origin, etc.

Window: The rectangular region of the world that is visible.

Viewport: The rectangular region of the screen space that is used to display the window.

WINDOW PORT TO VIEW PORT CONVERSION

https://www.youtube.com/watch?v=PHcgFLJGx3E

Window-to-Viewport Mapping

A window is specified by four world coordinates : wx_{min}, wx_{max}, wy_{min},and wy_{max} (see Fig. 5.1) Similarly, a viewport is described by four normalized device coordinates: vx_{min},vx_{max},vy_{min}, and vy_{max}. The objective of window – to – viewport mapping is to convert the world coordinates (wx, wy) of an arbitrary point to its corresponding normalized device coordinates (vx,vy). In order to maintain the same relative placement of the point in the viewport as in the window, we require:

$$\frac{wx - wx_{\min}}{wx_{\max} - wx_{\min}} = \frac{vx - vx_{\min}}{vx_{\max} - vx_{\min}} \quad \text{and} \quad \frac{wy - wy_{\min}}{wy_{\max} - wy_{\min}} = \frac{vy - vy_{\min}}{vy_{\max} - vy_{\min}}$$

Thus

$$\begin{cases} vx = \dfrac{vx_{\max} - vx_{\min}}{wx_{\max} - wx_{\min}}(wx - wx_{\min}) + vx_{\min} \\ vy = \dfrac{vy_{\max} - vy_{\min}}{wy_{\max} - wy_{\min}}(wy - wy_{\min}) + vy_{\min} \end{cases}$$

Since the eight coordinate values that define the window and the viewport are just constants, we can express these two,

formulas for computing (vx, vy) from (wx, wy) in terms of a translate-scale-translate transformation N Note that geometric distortions occur (e.g., squares in the window become rectangles in the viewport) whenever the two scaling constants differ.

$$\begin{pmatrix} vx \\ vy \\ 1 \end{pmatrix} = N.\begin{pmatrix} wx \\ wy \\ 1 \end{pmatrix}$$

where

$$N = \begin{pmatrix} 1 & 0 & vx_{min} \\ 0 & 1 & vy_{min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dfrac{vx_{max} - vx_{min}}{wx_{max} - wx_{min}} & 0 & 0 \\ 0 & \dfrac{vy_{max} - vy_{min}}{wy_{max} - wy_{min}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -wx_{min} \\ 0 & 1 & -wy_{min} \\ 0 & 0 & 1 \end{pmatrix}$$

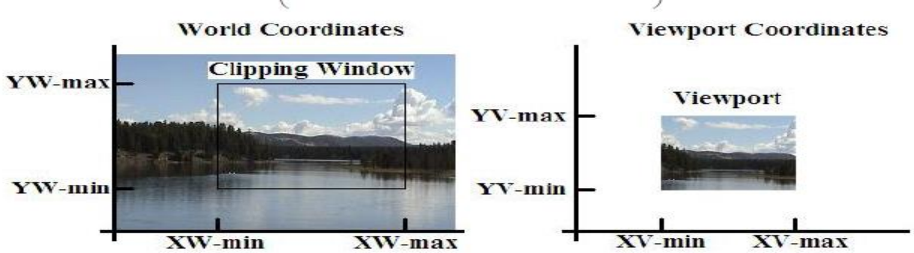


Fig. 5.1: Window-to-viewport mapping

THREE-DIMENSIONAL TRANSFORMATION

Objects in homogeneous coordinates

Points in a xyz space are expressed by vectors of homogeneous coordinates. These vectors are translated, rotated, scaled, and projected onto a 2D drawing surface by multiplying them by transformation matrices. We know that a translation matrix can be combined with a translation matrix, a scaling matrix with a scaling matrix and a rotation matrix with a rotation matrix. Since scaling and rotation matrices both are 3×3 matrices, they can be combined as well. That is, we can combine several scaling matrices with several rotation matrices. That is very useful, since we can pre-calculate very advanced transformations and just use a single matrix to perform them at runtime. But it would be even more useful if we could combine translation matrices with them too. But since translation matrices are added together it is a no-can-do thing. The solution to this is homogeneous coordinates. The trick is to make all the transformation into 4×4 matrices instead.

Basics of 3D - Co-ordinate system

https://www.youtube.com/watch?v=LwHfBAaSOns

Three-dimensional transformations

1. Scaling

In Euclidean geometry, a uniform scaling is a linear transformation that enlarges or diminishes objects. When the scale factor is same in all directions, it is called a homothety. The result of uniform scaling is similar (in the geometric sense) to the original. More general is scaling with a separate scale factor for each axis direction. A special case is a directional scaling (in one direction). Shapes may change, for example, a rectangle may change into a rectangle of a different shape.

A scaling can be represented by a scaling matrix. To scale an object by a vector $V = [s_x\ s_y\ s_z]$, each point $P = [x\quad y\quad z]$ would need to be multiplied with this scaling matrix:

Applying the scaling matrix on the point P. we have

$$P.S_v = [x\ y\ z] \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} = [s_x x \quad s_y y \quad s_z z] \qquad \ldots \qquad (1)$$

Such a scaling changes the diameter of an object by a factor between the scale factors, the area by a factor between the smallest and the largest product of two scale factors, and the volume by the product of all three.

It is useful to use homogeneous coordinates, since translation cannot be accomplished with a 3 × 3 matrix. To scale an object by a vector $V = [s_x\quad s_y\quad s_z]$, each homogeneous vector $P = [x\quad y\quad z\quad 1]$ would need to be multiplied with the scaling matrix :

$$S_v = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(2)$$

Now, we have the scaled point given by

$$P \cdot S_V = [x\quad y\quad z\quad 1] \cdot \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [s_x x \quad s_y y \quad s_z z \quad 1] \qquad \ldots(3)$$

It is useful to use homogeneous coordinates, since translation cannot be accomplished with a 3 × 3 matrix. To scale object by a vector $V = [s_x\quad s_y\quad s_z]$ each homogeneous vector. $P = [x\quad y\quad z\quad 1]$ would need to be multiplied with the scaling matrix:

$$S_V = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (4)$$

Now, we have the scaled point given by

$$P\ .\ S_V = [x\quad y\quad z\quad 1] \cdot \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [s_x x \quad s_y y \quad s_z z \quad 1]$$

Example : Write a scaling matrix with scaling factors 2, 3 and 1 in x, y and z directions, respectively. Apply the transformation matrix on a unit cube situated at the origin.

Solution :

Matrix of transformation for scaling with scaling factors 2, 3 and 1 or scaling vector V = (2, 3, 1) in x, y and z direction is given by

$$S_V = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A volume matrix, representing cube vertices in homogeneous coordinates, situated at origin is

$$V = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

2. Translation

A translation can also be interpreted as an addition of a constant vector to every point or shifting the origin of the coordinate system. If v is a fixed vector, then the translation T_V will work as

$$T_v(P) = P + v \qquad \ldots(5)$$

Translation is an affine transformation but not a linear transformation. Homogeneous co-ordinates are normally used to represent the translation operator by a matrix. Thus we write a 3D vector W = (w_x, w_y, w_z) using four homogeneous coordinates as w = (w_x, w_y, w_z, 1)

To translate an object by a vector V = [T_x T_y T_z], each homogeneous vector P = [x y z 1] would need to be multiplied by this translation matrix :

$$T_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix} \qquad \ldots(6)$$

The desired result is obtained, by applying the translation matrix, as

$$T_v \cdot P = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix} \qquad \ldots(7)$$

$$= \begin{bmatrix} x+T_x & y+T_y & z+T_z & 1 \end{bmatrix}$$

$$= P + v$$

To translate an object by a vector v =, $[T_x \quad T_y \quad T_z]$ each homogeneous vector $P = [x \quad y \quad z \quad 1]$ would need to be multiplied by this translation matrix:

$$T_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

The desired result is obtained, by applying the translation matrix, as

$$T_v \cdot P = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x+T_x & y+T_y & z+T_z & 1 \end{bmatrix}$$

$$= P + v$$

The above result shows that translation of the point $[x \quad y \quad z]$ by factors T_x, T_y, and T_z is merely the addition of the translation component in respective directions.

3. Rotation

In a 3D space, a coordinate rotation can be defined by three Euler angles, or by a single angle of rotation and the direction of a vector about which a body rotates. Rotations about the origin are most easily calculated using a 3 × 3 matrix transformation called a rotation matrix. Rotations about another point can be described by a 4 × 4 matrix acting on the homogeneous coordinates.

In general, rotation about the x-axis preserves the x coordinates and, in effect, occurs in the plane perpendicular to the x-axis.

In the matrix form, the rotation transformations are rewritten as

$$R_{\theta x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos f & \sin f \\ 0 & -\sin f & \cos f \end{bmatrix}$$

$$R_{\theta y} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R_{\theta z} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_{\theta x}$, $R_{\theta y}$, and $R_{\theta z}$ are the matrices of transformation for rotation about x, y, and z directions respectively. The rotation matrix in a counterclockwise direction expressed in a homogeneous coordinate system is given as

$$R_{\theta x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For rotation about y-axis by an angle ϕ, the matrix of transformation in homogeneous coordinates is given by

$$R_{\theta y} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For rotation about z-axis by an angle ϕ, the matrix of transformation using homogeneous coordinates is given by,

$$= \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. REFLECTION

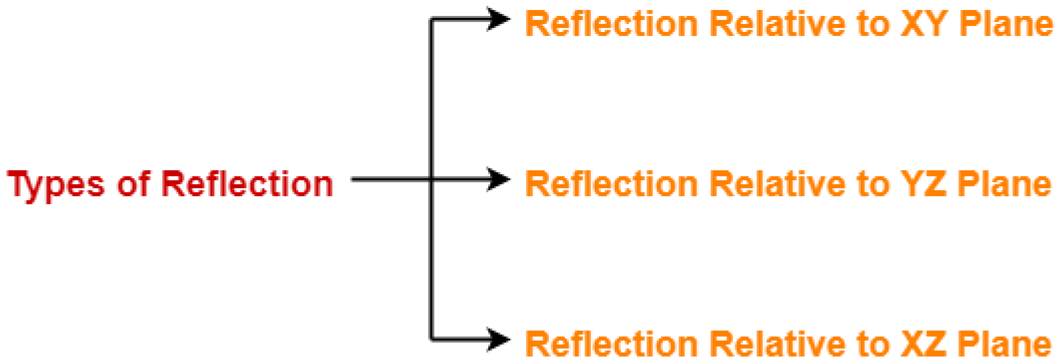
- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 3D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old}, Z_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new},Z_{new})

In 3 dimensions, there are 3 possible types of reflection-



1. <u>Reflection Relative to XY Plane:</u>

   This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old}$
- $Z_{new} = -Z_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XY plane)

2. <u>Reflection Relative to YZ Plane:</u>

   This reflection is achieved by using the following reflection equations-

- $X_{new} = -X_{old}$
- $Y_{new} = Y_{old}$
- $Z_{new} = Z_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to YZ plane)

3. **Reflection Relative to XZ Plane:**

This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$
- $Z_{new} = Z_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

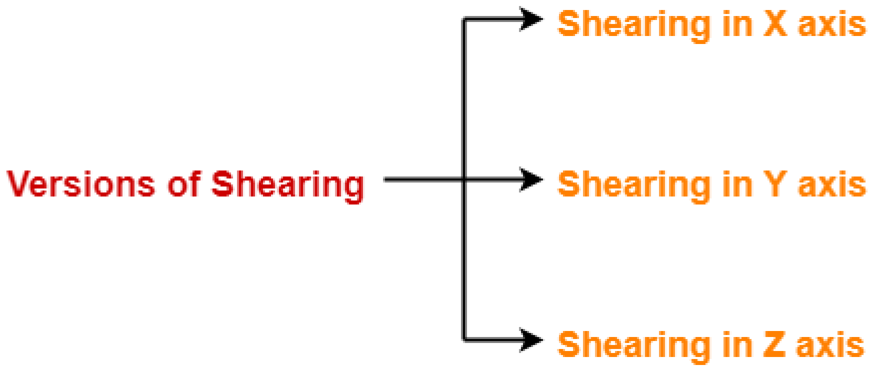
3D Reflection Matrix

(Reflection Relative to XZ plane)

5 Shearing

In a three-dimensional plane, the object size can be changed along X direction, Y direction as well as Z direction.

So, there are three versions of shearing-



1. Shearing in X direction
2. Shearing in Y direction
3. Shearing in Z direction

Consider a point object O has to be sheared in a 3D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old}, Z_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y
- Shearing parameter towards Z direction = Sh_z
- New coordinates of the object O after shearing = (X_{new}, Y_{new}, Z_{new})

1. <u>Shearing in X Axis-</u>

Shearing in X axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$
- $Z_{new} = Z_{old} + Sh_z \times X_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In X axis)

2. Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$
- $Z_{new} = Z_{old} + Sh_z \times Y_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Y axis)

3. Shearing in Z Axis-

Shearing in Z axis is achieved by using the following shearing equations-

- $X_{new} = X_{old} + Sh_x \times Z_{old}$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old}$
- $Z_{new} = Z_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

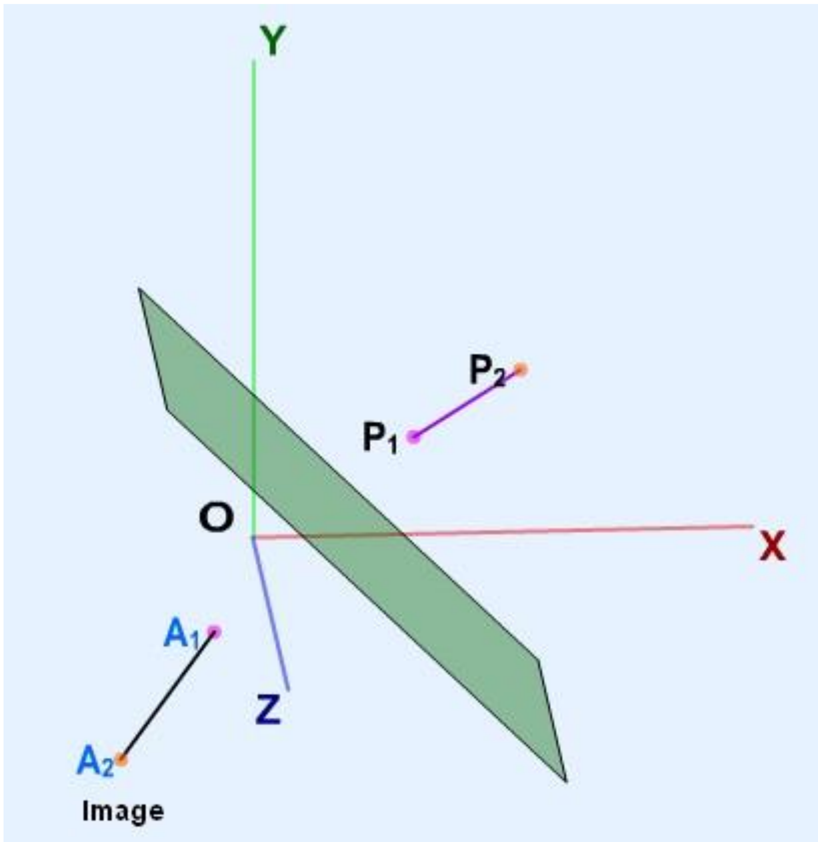
3D Shearing Matrix

(In Z axis)

Multiple Transformation

GENERAL 3-D REFLECTION

Some orientations of a three-dimensional object cannot be obtained using pure rotations; they require reflections. In three dimensions, reflection occur through a plane. By analogy with the previous discussion of two-dimensional reflection three-dimensional reflection through a plane is equivalent to rotation about an axis in three-dimensional space out in to four-dimensional space and back into three dimensional space. For pure reflection, the determinant of the reflection matrix is identically -1



<u>Steps to be performed</u>

1. Translate origin to A_1

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x & y & z & 1 \end{bmatrix}$$

2. Align vector with axis (say, *z*)

1. Rotate to bring vector in *yz plane*

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Rotate to bring vector along *z-axis*

$$R_y = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Reflect the line P_1P_2 through the standard *y-z plane.*
4. Reverse steps 2
5. Reverse step 1

Rotation About an Arbitrary Axis in 3 Dimensions

- A rotation matrix for any axis that does not coincide with a coordinate axis can be set up as a composite transformation involving combinations of translations and the co-ordinate – axes rotations.
- In a special case where an object is to be rotated about an axis that is parallel to one of the co-ordinate – axes we can obtain the resultant coordinates with the following transformations:
- Translate the object so that the rotation axis coincides with the parallel coordinate axis.
- Perform the specified rotation about that axis.
- Translate the object so that the rotation axis is moved back to the original position.

- When an object is to be rotated about an axis that is not parallel to one of the coordinate axes, we have to perform some additional transformations as follows:

1. Translate the object so that the rotation axis specified by unit vector u passed through the coordinate origin.

2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes. Usually, the z axis is preferred. To coincide the axis of rotation to z axis we have to first perform rotation of unit vector u about x axis to bring it into xz plane and then perform rotation about y axis to coincide it with z axis.

3. Perform the desired rotation Θ about the z axis.

4. Apply the inverse rotation about y axis and then about x axis to bring the rotation axis back to its original orientation.

5. Apply the inverse translation to move the rotation axis back to its original position.

Rotation about an arbitrary line

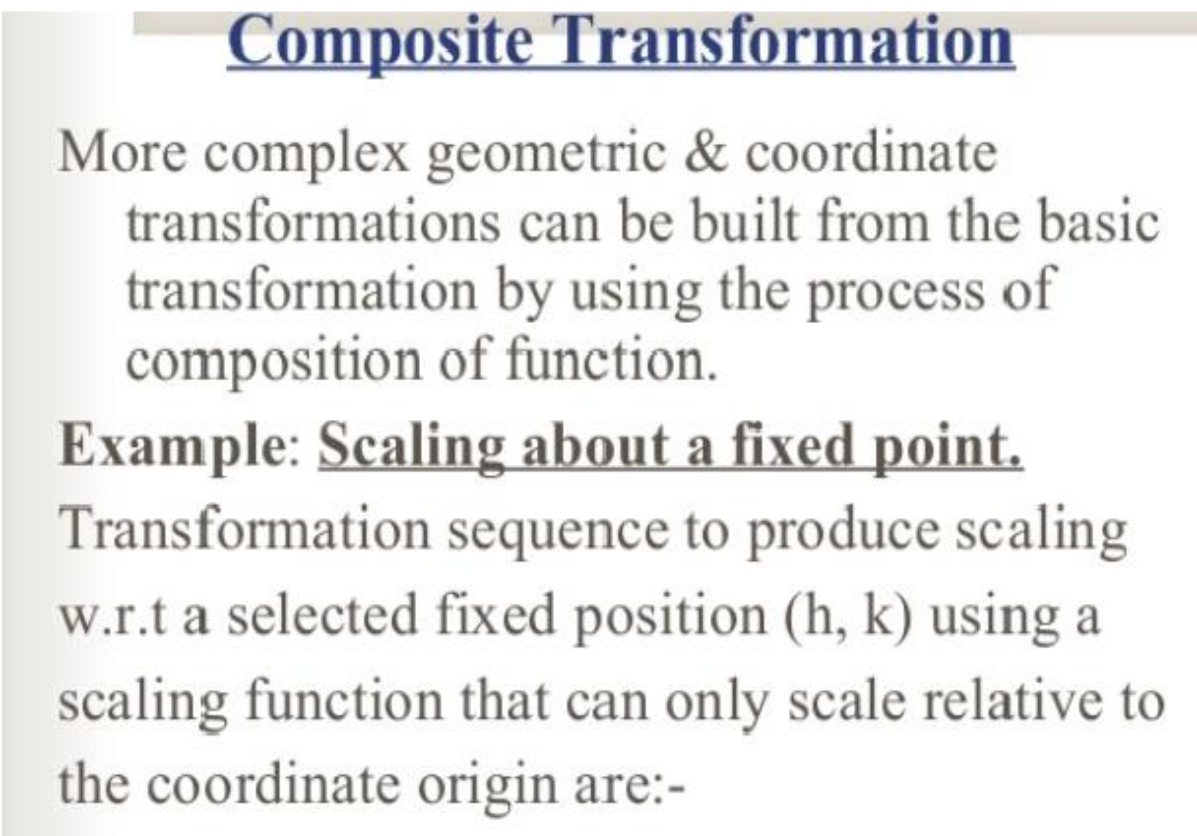
We will define an arbitrary line by a point the line goes through and a direction vector. If the axis of rotation is given by two points $P_1 = (a,b,c)$ and $P_2 = (d,e,f)$, then a direction vector can be obtained by $\langle u,v,w\rangle = \langle d-a, e-b, f-c\rangle$. We can now write a transformation for the rotation of a point about this line.

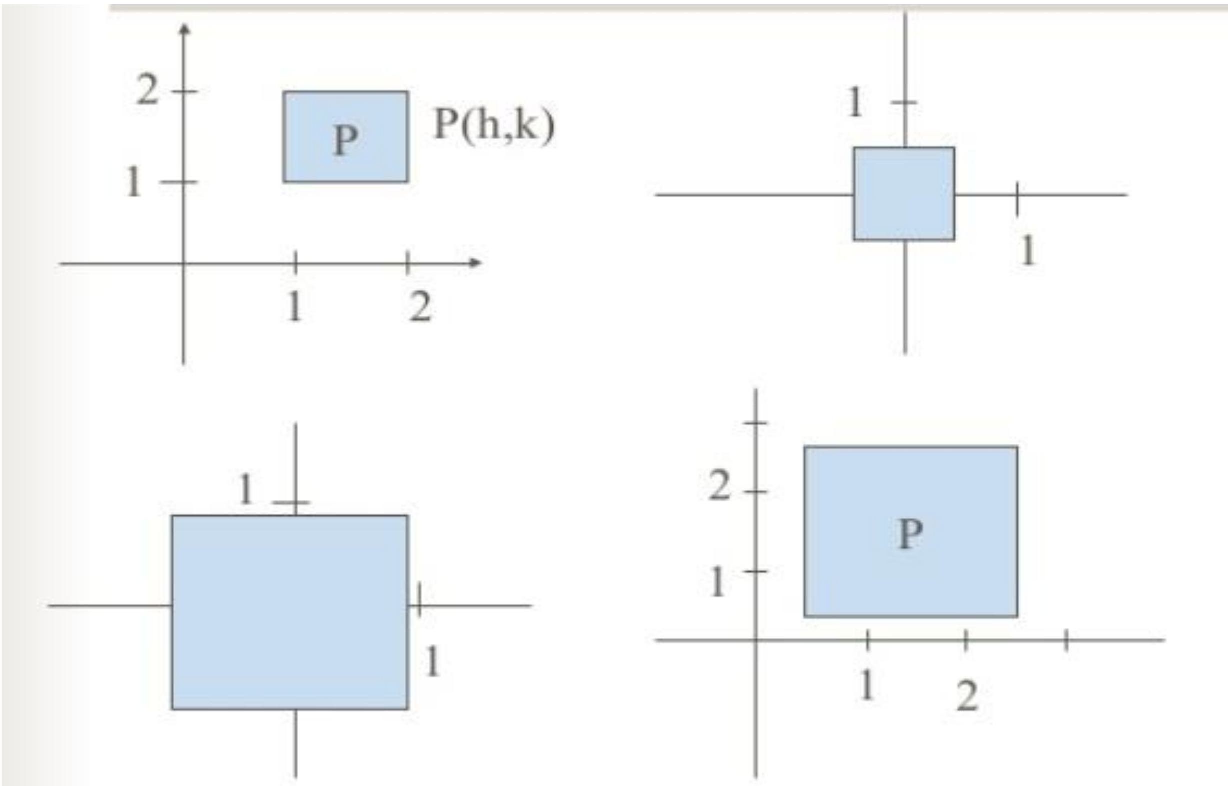
Reflection with respect to Any Plane

- It is necessary to reflect an object through a plane other than x=0(yz plane) , y=0(xz plane) or z=0(xy plane). To achieve such a reflection with respect to any plane steps to be followed are as follows:

1. Translate a known point Po, that lies in the reflection plane to the origin of the co-ordinate system.
2. Rotate the normal vector to the reflection plane at the origin until it is coincident with +ve z axis, this makes the reflection plane z=0 co-ordinate plane i.e xy plane
3. Reflect the object through z=0(xy plane) co-ordinate plane.
4. Perform the inverse transformation to those given to achieve the result.
5. Resultant Transformation matrix is: RT = T. Rxy . M. Rxy ^-1 .T^-1

Composition of 3D Transformations





Steps for doing composite transformation:-

1.) Translate the object so that its centre concides with the origin.

2.) Scale the object with respect to origin.

3.) Translate the scale object back to the original position.

Thus the scaling with respect to the point can be formed by transformation.

$$S_{sx,sy}, P = T_v \cdot S_{sx,sy} \cdot T_v^{-1}$$

Affine and Perspective Geometry

Affine geometry is a geometry studying objects whose shapes are preserved relative to affine transformations.

Perspective is the art and mathematics of realistically depicting three-dimensional objects in a two-dimensional plane, sometimes called centric or natural perspective to distinguish it from bicentric perspective. The study of the projection of objects in a plane is called projective geometry. Projective geometry is a geometry studying objects whose shapes are preserved relative to projective transformations.

Perspective Transformations,

When human eyes see near things they look bigger as compare to those who are far away. This is called perspective in a general way. Whereas transformation is the transfer of an object e.t.c from one state to another.

So overall, the perspective transformation deals with the conversion of 3d world into 2d image. The same principle on which human vision works and the same principle on which the camera works.

Objects which are near to you look bigger, while those who are far away, look smaller even though they look bigger when you reach them.

Projection

- The production of a 2D image of higher-dimensional object refers to graphical projection.
- Mathematically, a projection can be defined as a mapping of any point P $[x \quad y \quad z]$ to its image P′ $[x' \quad y' \quad z']$ onto the view plane, often called projection plane. The special relationship between the projectors and the view plane greatly affects the results of a projection.
- Parallel and perspective projections are two broad categories of projections used in computer graphics.

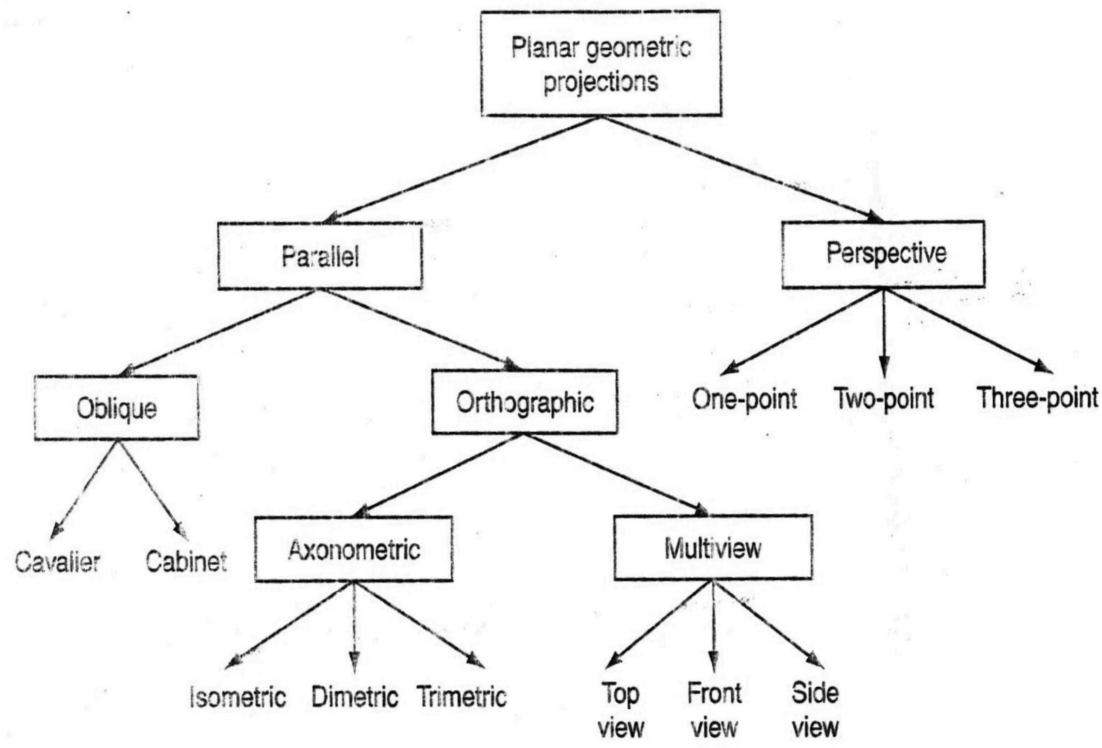


Fig.1 : Subclasses of planer geometric projections.

i) Parallel projection

Parallel projections are one of two major subclasses of planar geometric projections. Projections, within this subclass, have two characteristics in common. The first characteristic concerns the placement of the center of projection that represents a camera or viewing position. In a parallel projection, the camera is located at an infinite distance from the view plane.

By placing a camera at an infinite distance from the view plane, projectors to the view plane becomes parallel (the second characteristic of a parallel projection) in result of forming a parallelpiped view volume. Only objects within the view volume are projected to the view plane. Figure 2 shows the projection of a line AB to the view plane. In this case, the measurement of the line AB is maintained in the projected line A′B′. While the measurements of an object are not preserved in all parallel projections, the parallel nature of projectors maintains the proportion of an object along a major axis.

Therefore, parallel projections are useful in applications requiring the relative proportions of an object to be maintained.

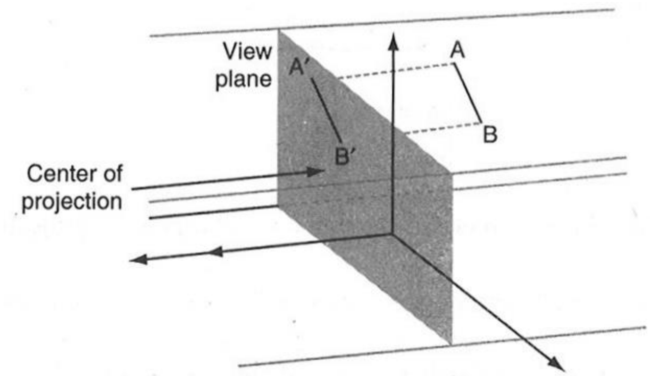
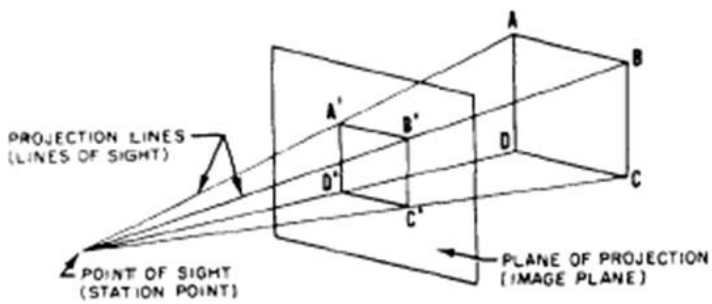


Fig. 2: Parallel projection defined by the center of projection placed at an infinite distance from the view plane.

A parallel projection transforms object positions to the view plane along parallel lines. A parallel projection preserves relative proportions of objects. Accurate views of the various sides of an object are obtained with a parallel projection, but not a realistic representation. A parallel projection can be further classified into orthographic and oblique parallel projections, respectively. If the direction of projection is perpendicular to the projection plane, it is an orthographic projection. If the direction of projection is not perpendicular to the projection plane, it is an oblique projection.

ii) Perspective projection



A. PERSPECTIVE PICTORIAL PROJECTION

Perspective projection is a type of projection where 3D objects are not projected along parallel lines, but along lines emerging form a single point. This has the effect that distant objects appear smaller than nearer objects. It also means that lines that are parallel in nature appear to intersect in the projected image. For example, if railways are pictured with perspective projection, they appear to converge towards a single point, called a vanishing point. A vanishing point is a point in a perspective drawing to which parallel lines appear to converge. The number and placement of the vanishing points determines which perspective technique is being used. Photographic lenses and the human eyes work in the same way. Therefore, a perspective projection looks most realistic.

Any perspective representation of a scene, including parallel lines, has one or more vanishing points in a perspective drawing. A one-point perspective drawing means that the drawing has a single vanishing point, usually (though not necessarily) directly opposite to the viewer's eye and usually (though not necessarily) on the horizon line. All lines parallel to the viewer's line of sight recede to the horizon towards the vanishing point. A two-point drawing would have lines parallel to two different angles. Any numbers of vanishing points are possible in a drawing: one for each set of parallel lines that are at an angle relative to the plane of the drawing.

View plane:

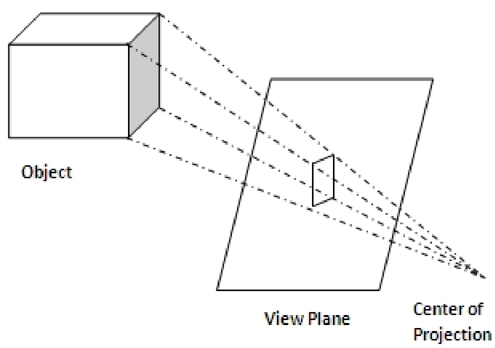
It is nothing but the film plane in a camera which is positioned & oriented for a particular shot of the scene.

World coordinates positions in the scene are transformed to viewing coordinates, then viewing coordinates are projected onto the view plane.

View reference point: This point is the center of our viewing coordinate system.

The production of a 2D image of higher dimensional object refers to graphical projection.

A projection can be defined as a mapping of any point P[x,y,z] to its image P`[x`,y`,z`] onto the view plane, called as projection plane.



We can project 3D objects onto the two-dimensional view plane. There are two basic ways of projecting objects onto the view plane:

1. Parallel Projection

2. Perspective Projection

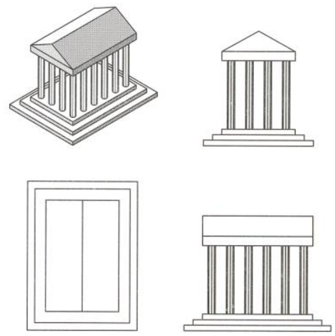
Types of Parallel projections.

- There are two main types of parallel projection.

1. Orthographic projection
   - If the direction of projection is perpendicular to the projection plane, then it is called as an orthographic projection.
   - It is categorized as

A) multi view projection -Front, side and top view

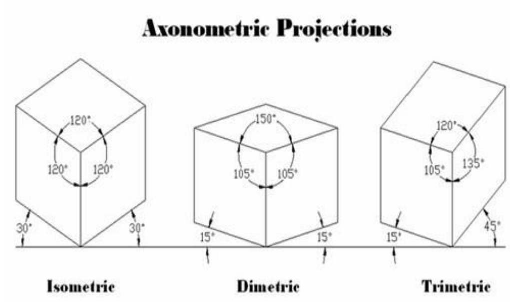
DOP perpendicular to view plane.



B) Axonometric projections

Axonometric projections allow the user to place the view-plane normal in any direction such that 3 adjacent faces of a cube like object are visible.

- These are divided as
- 1) isometric projections
- 2) Dimetric projections
- 3) Trimetric projections



2. Oblique Projection

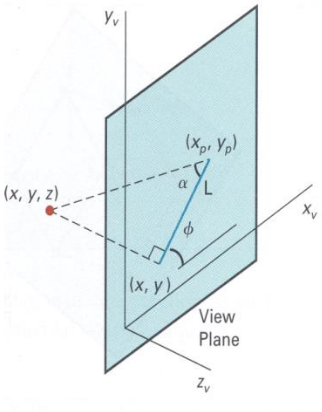
If the direction of projection is not perpendicular to the projection plane is called as oblique projection.

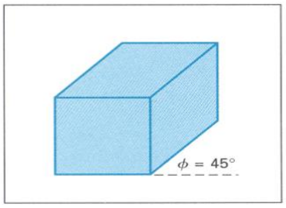
A multi-view projection displays a single face of a 3D object.

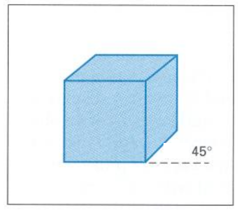
They are classified as

- a) cavalier projection.
- b) cabinet projection.

DOP not perpendicular to view plane.







Cavalier

(DOP α = 45o)

tan(a) = 1

Cabinet

(DOP α = 63.4o)

tan(a) = 2

T**ypes of Perspective projections.**

It is a type of projection where 3D objects are not projected along parallel lines, but along lines emerging from a single point.

A vanishing point is a point in a perspective drawing to which parallel lines appear to converge.

Produces realistic views but does not preserve relative proportions.

The lines of projection are not parallel.

The lines converge at a single point called the center of projection or projection reference point.

The center of projection lies below the view plane.

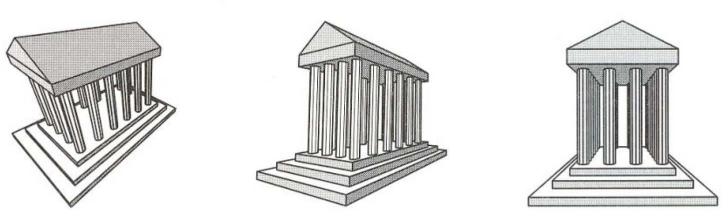
Objects closer to viewer look larger. Parallel lines appear to converge to single point

It is categorized as

a) One Point Projections

b) Two Point Projections

c) Three Point Projections



- Perspective projection
    - Size varies inversely with distance - looks realistic.
    - Distance and angles are not (in general) preserved.
    - Parallel lines do not (in general) remain parallel.
- Parallel projection
    - Good for exact measurements
    - Parallel lines remain parallel.
    - Angles are not (in general) preserved.
    - Less realistic looking

Questions

BASIC

1. What is meant by transformation matrix?
2. Describe Homogeneous coordinates.
3. Write translation equation in homogeneous coordinates.
4. Write rotation equation in homogeneous coordinates.
5. Define the following in 2-Dimensional- Translation, Rotation, Scaling, Shearing, and Reflection.
6. Define scaling transformation with the help of a diagram.
7. Consider a line AB with coordinates A(0, 0) and B(4, 5) in a 2D plane. Obtain a matrix of transformation for scaling for the line AB in x–direction by the factor 3 and plot the same.
8. Consider a point P(2, 3) in a coordinate plane. Perform reflection of the point P through y–axis and draw the same.
9. Consider a point P(2, 3) in a coordinate plane. Perform reflection of the point P through x–axis and draw the same.
10. Write scaling equation in homogeneous coordinates.
11. Consider the line AB with coordinates of the line A(2, 3) and B(4, 5) in the coordinate plane. Perform reflection of the point P about origin and draw the same.
12. What do you mean by Three-Dimensional Transformations?
13. Explain Homogeneous Coordinates.
14. Explain 3D scaling.
15. Write short note on 3D translation.
16. Explain 3D rotation.
17. Explain 3D Shear Transformations.
18. Explain 3D Reflection.

INTERMEDIATE

1. Explain Two–dimensional rotation with a diagram.
2. Explain Two-Dimensional Reflection.

3. Find the transformation of a triangle with coordinates A(1,0) B(0,1) and C(1,1) by rotating 450 about the origin and then translating two units in x and y direction.
4. Explain Two–dimensional shear transformations.
5. Rotate the triangle ABC by an angle 30°, where the triangle has the coordinates A(0, 0), B(10, 2), and C(7, 4).
6. Write short note on Rotation about an arbitrary point.
7. A point p(5,7,10) is moved 2 units in x-direction, 2 units in y-direction and 3 units in z-direction. Find P'.
8. A point P(4,6,8) is scaled 2 units in x-direction, 3 units in y-direction and 3 units in z-direction. Find P'.
9. Obtain transformation matrix for rotation about line joining the points (0,0,0) and (1,1,1) with the angle of rotation 45 degrees in counterclockwise sense.
10. Explain the different types of Parallel projections.
11. Difference between parallel and perspective projection.
12. Explain different types of projection.
13. Explain the different types of Perspective projections.
14. Explain the terms View plane, World coordinates, projection plane.

ADVANCE

15. Rotate a triangle ABC by and angle 90° about a point (–1, 1) where the triangle has the coordinates A(5, 0), B(10, 2) and C(7, 4).
16. Using homogeneous coordinate transformation matrix, apply the following sequence of transformations to a unit square centered at origin:
    (a) Translation by factor (1/2, 1)
    (b) Rotate by angle θ = 90°.
17. *A triangle is defined by 3 vertices A (0, 2, 1) B, (2, 3, 0), C (1, 2, 1). Find the final co-ordinates after it is rotated by 45° around a line joining the points (1, 1, 1) and (0, 0, 0).*
18. *A triangle is defined by 3 vertices A (0, 2, 1) B (2, 3, 0), C (1, 2, 1). Find the final co-ordinates after it is rotated by 45° around a line joining the points (2, 2, 2) and (1, 1, 1).*
19. *A cube defined by 8 vertices*

    A (0, 0, 0) B (2, 0, 0) C (2, 2, 0) D (0, 2, 0)

    E (0, 0, 2) F (2, 0, 2) G (2, 2, 2) H (0, 2, 2)

    Find the final co-ordinates after it is rotated by 45° around a line joining the points (2, 0, 0) and (0, 2, 2).

- **Multiple Choice Questions:**

1. A transformation that maps each point onto itself is called a/an ____________transformation.
   a) Uniform
   b) Unique
   c) Identity
   d) Reflection
2. Shear transformation can be expressed in terms of ____________.
   a) Rotation only
   b) Scaling only
   c) Reflection only
   d) **Product of scaling and Rotation**.
3. _______transformation do not change the size of the object.
   a) Shear
   b) Translation
   c) Scaling
   d) Reflection

4. Pixel can be arranged in a regular

   a) One dimensional grid
   b) **Two-dimensional grid**
   c) Three-dimensional grid
   d) None of these

5. Several graphics image file formats that are used by most of graphics system are ___.

.

   a) GIF
   b) JPEG
   c) TIFF
   d) **All of these**

6. RGB model are used for ___.

   a) **Computer display**
   b) Printing
   c) Painting
   d) None of these

7. Basic geometric transformation include ___.

   a) Translation
   b) Rotation
   c) Scaling
   d) **All of these**

8. Some additional transformation are ___.
a) Shear
b) Reflection
c) **Both a & b**
d) None of these

9. The transformation in which an object is moved in a minimum distance path from one position to another is called

a) **Translation**
b) Scaling
c) Rotation
d) Reflection

10. The transformation in which an object is moved in a minimum distance path from one position to another is called
a) **Translation**
b) Scaling
c) Rotation
d) Reflection

11. If the direction of projection is perpendicular to the projection plane, it is an _________projection.
a) O**rthographic**
b) Oblique
c) Cavalier
d) Cabinet

12. If the direction of projection is not perpendicular to the projection plane is called as _______projection.
a) **Oblique**
b) Orthographic
c) Cavalier
d) Cabinet

13. The Cavalier projection makes 45° angle with the projection plane.
a) 45 degree
b) 60 degree
c) 78 degree
d) 54 degree

14. The Cabinet projection makes 63.4° angle with the projection plane.
a) 45 degree
b) **63.4 degree**
c) 78 degree
d) 54 degree

15. It is a type of projection where 3D objects are not projected along parallel lines, but along lines emerging from a single point.
    a) **Perspective**
    b) Orthographic
    c) Parallel
    d) Cabinet
16. In parallel projection, we specify a direction of projection instead of center of projection.
    a) Perspective
    b) Orthographic
    c) **Parallel**
    d) Cabinet
17. In parallel projection, the distance from the center of projection to project plane is _____.
    a) Not Constant
    b) Constant
    c) **Infinite**
    d) finite
18. Which of the following is not a types of perspective projections?
    a) **seven**
    b) one
    c) two
    d) three
19. ______perspective projection gives better impression of depth.
    a) one point
    b) vanishing point
    c) three point
    d) two point
20. A ________point is a point in a perspective drawing to which parallel lines appear to converge.
    a) one point
    b) **vanishing point**
    c) three point
    d) two point

❖ E-Link to External E-books :

- ❖ http://www.slideshare.net/rhspcte/computer-graphics-ebookhearn-baker
- ❖ http://myitweb.weebly.com/computer-graphics1.html
- ❖ https://www.youtube.com/watch?v=vzCamh_rSdo
- ❖ https://www.youtube.com/watch?v=Xt-AkLPC3Iw
- ❖ https://www.youtube.com/watch?v=VpNJbvZhNCQ

❖ Chapter Outcomes :

learning how two and three-dimensional transformations take place.

❖ Recommended Books

- **TextBooks:**
  1) Computer Graphics R. K. Maurya, John Wiley.

- **References:**
  1) **Computer Graphics Ghodse**
  2) Computer Graphics Rajiv Chopra
  3) Computer Graphics By Apurvaa Desai`
  4) Principles of Interactive Computer Graphics, Willaim M. Newman, Robert F. Sproull,Tata McGraw-Hill.