COST

Statistical Estimation Theory

Z Score of difference

- **Difference of Mean**

$$Z = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}}$$

- **Difference of Proportion**

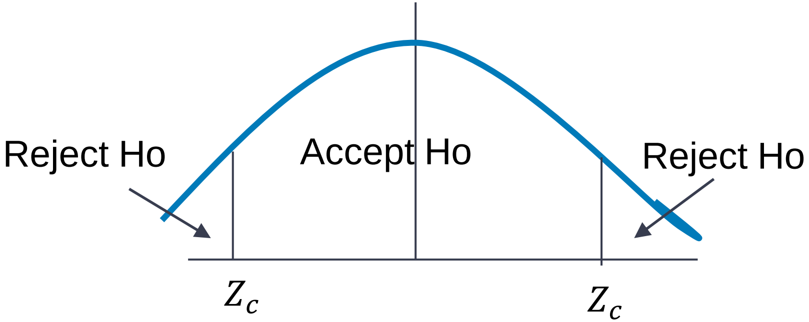
$$Z = \frac{(P_1 - P_2) - (p_1 - p_2)}{\sqrt{pq(\frac{1}{N_1} + \frac{1}{N_2})}}$$

$$Z = \frac{P - p}{\sqrt{\frac{pq}{N}}}$$

Where $p = \frac{N_1 P_1 + N_2 P_2}{N_1 + N_2}$

Steps for hypothesis testing

1. Write given values.
2. Propose Ho and H1.
3. Identify test-
   - one tailed (if < , >)
   - two tailed (if ≠)
4. Get table value Z_c according to LOS mentioned in the problem.
5. Find Z score using the formula.
6. Inference-
   - If Z < Z_c , accept Ho.
   - If Z > Z_c , reject Ho.



Question

Qn) Two groups A and B consist of 100 people each who have a disease. A serum is given to group A but not to group B, otherwise the two groups are treated identically. It is found that in group A and B, 75 and 65 people respectively, recover from the disease. Test the hypothesis that the serum helps to cure the disease at 1% level.

Step 1- Write given values

- $N_1 = 100$
- $P1 = \frac{75}{100} = 0.75$

- $N_2 = 100$
- $P2 = \frac{65}{100} = 0.65$

$$p = \frac{N_1 P_1 + N_2 P_2}{N_1 + N_2}$$

$q = 1\text{-}p = 0.3$

$LOS = \alpha = 0.01 = 1\ \%$

Cont..

- Step 2- Propose H0

$H_0: p_1 = p_2$ Serum does not help to cure the disease.

$H_1: p_1 > p_2$ Serum helps to cure the disease

- Step 3- Identify Test
  - As > sign is there, use One tailed Test

Cont….

	$\boldsymbol{\alpha = 0.05\ (5\ \%)}$	$\boldsymbol{\alpha = 0.01\ (1\ \%)}$
Two-tailed Test	Z_c=1.96	Z_c= 2.58
One-tailed Test	Z_c=1.645	Z_c= 2.33

- Step 4- Get table value of Z_c for LOS $\boldsymbol{\alpha = 0.01\ (1\ \%)}$

  Z_c= 2.33

- Step 5- Find Z score using formula-

$$Z = \frac{(P_1 - P_2) - (p_1 - p_2)}{\sqrt{pq(\frac{1}{N_1} + \frac{1}{N_2})}}$$

- $N_1 = 100$
- $N_2 = 100$
- $P1 = \frac{75}{100} = 0.75$
- $P2 = \frac{65}{100} = 0.65$
- p= 0.7
- q= 0.3

Z = 1.5430

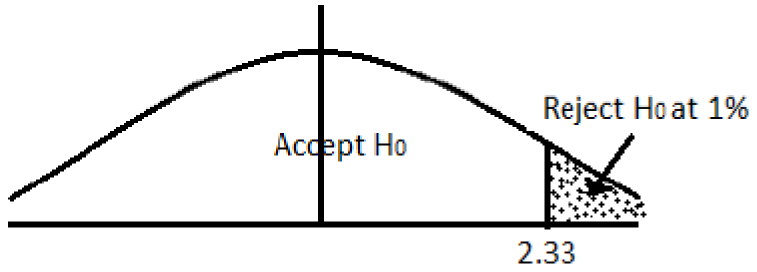
As per the claim, $p_1 = p_2$

Hence, $p_1 - p_2 = 0$

Cont. ..

- Step 6 – Inference

  $Z = 1.543$, $Z_c = 2.33$



- As $Z < 2.33$, Accept Ho.

- Therefore, serum does not help to cure the disease.

Question

Qn) Random samples of 200 bolts manufactured by machine A and of 100 bolts manufactured by machine B showed 19 and 5 defective bolts respectively. Test the hypothesis that

a. The two machines are showing different qualities of performance

b. Machine B is performing better than A

Use 5% significance level.

Solution

Consider proportion of defective bolts

$$N_1 = 200, P_1 = \frac{19}{200} = 0.095, N_2 = 100, P_2 = \frac{5}{100} = 0.05$$

$$p = \frac{N_1P_1 + N_2P_2}{N_1 + N_2} = \frac{(200 \times 0.095) + (100 \times 0.05)}{200 + 100} = 0.08$$

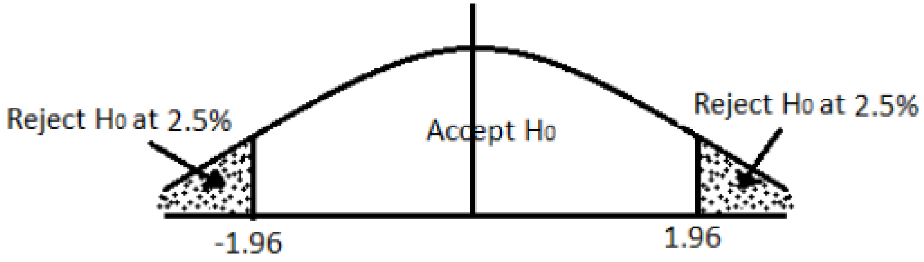
$$\Rightarrow q = 1 - p = 0.92$$

$$Z = \frac{(P_1 - P_2) - (p_1 - p_2)}{\sqrt{pq\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} = \frac{(0.095 - 0.05) - 0}{\sqrt{0.08 \times 0.92 \times \left(\frac{1}{200} + \frac{1}{100}\right)}} = 1.3543$$

Cont..

a. $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$



$\Rightarrow$ Accept H_0

There is no significant difference in the performance.

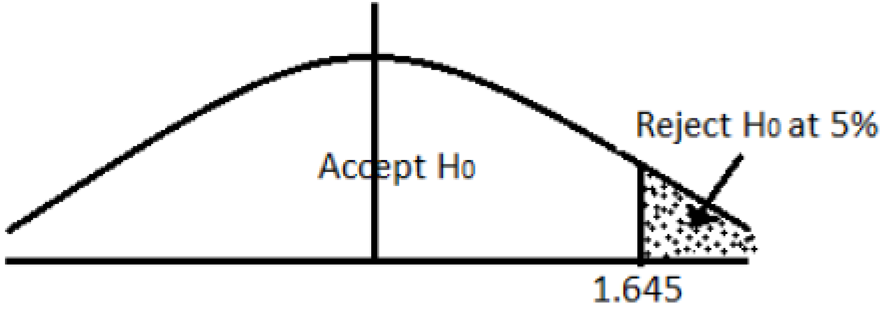
Cont..

b. $H_0 : p_1 = p_2$

$H_1 : p_1 > p_2$

One tailed test($l.o.s.\ \alpha = 5\%, table\ value = 1.645$)

$1.3543 < 1.645$



$\Rightarrow$ Accept H_0

There is no significant difference in the performance.