# COST

Statistical Estimation Theory



## Z Score

Mean

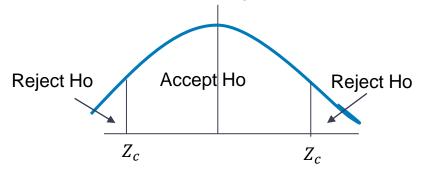
$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}}$$

Proportion

$$Z = \frac{P - p}{\sqrt{\frac{pq}{N}}}$$

## Steps for hypothesis testing

- 1. Write given values.
- 2. Propose Ho and H1.
- 3. Identify test
  - one tailed (if < , >)
  - two tailed (if ≠)
- 4. Get table value  $Z_c$  according to LOS mentioned in the problem.
- 5. Find Z score using the formula.
- 6. Inference-
  - ▶ If  $Z < Z_c$  , accept Ho.
  - If  $Z > Z_C$ , reject Ho.



**Qn)** The manufacturer of a patent medicine claims that it is 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people. Determine whether the manufacturer's claim is legitimate at 5% level of significance.

#### Step 1- Write given values

$$p = \frac{90}{100} = 0.9$$

$$q = 1 - 0.9 = 0.1$$

Population Parameter

- N = 200
- N 200

$$P = \frac{160}{200} = 0.8$$

$$LOS = \alpha = 0.05 = 5 \%$$

Sample data

Step 2- Propose HO

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H_0: p = 90\% = 0.9(Manufacturer's claim is valid)
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 $H_1: p < 0.9$  (Manufacturer's claim is not valid)

- Step 3- Identify Test
  - As < sign is there, use One tailed Test</p>

	$\alpha = 0.05 (5 \%)$	$\alpha = 0.01 (1 \%)$
Two-tailed Test	Z <sub>c</sub> =1.96	$Z_c$ = 2.58
One-tailed Test	Z <sub>c</sub> =1.645	$Z_c$ = 2.33

Step 4- Get table value of  $Z_c$  for LOS  $\alpha = 0.05$  (5 %)

$$Z_c = 1.645$$

Step 5- Find Z score using formula-

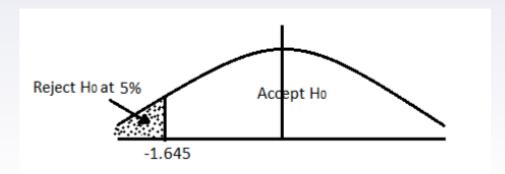
$$Z = \frac{P - p}{\sqrt{\frac{pq}{N}}}$$

$$Z = -4.714$$

p	0.9
q	0.1
P	0.8
N	200

Step 6 - Inference

$$Z = -4.714$$
,  $Z_c = -1.645$ 



- As Z falls in critical region, reject Ho.
- Therefore, we can not support the claim at 0.05 LOS. i.e., the medicine is not 90% effective.

The claim is made that 40% of tax filers use computer software to file their taxes. In a sample of 50, 24 used computer software to file their taxes. Test this claim at 5% LOS.

**Qn)** A pair of dice is tossed 100 times and it is observed that 23 times sum of numbers appearing on uppermost faces is 7. Test the hypothesis that the dice are fair by using a two-tailed test at 5% significance level.

#### Step 1- Write given values

- p = probability of getting sum 7
- $E = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$
- n(E) = 6
- -n(S) = 36
- $p = \frac{6}{36} = \frac{1}{6} = 0.167$

$$q = 1 - p = 0.833$$

$$N = 100$$

$$P = \frac{23}{100} = 0.23$$

$$LOS = \alpha = 0.05 = 5 \%$$

Step 2- Propose HO

$$H_0$$
:  $p = \frac{1}{6}$  (i.e. the dice are fair)

$$H_0: p \neq \frac{1}{6}$$
 (dice are not fair)

- Step 3- Identify Test
  - As  $\neq$  sign is there, use Two tailed Test

	$\alpha = 0.05 (5 \%)$	$\alpha = 0.01 (1 \%)$
Two-tailed Test	Z <sub>c</sub> =1.96	$Z_c$ = 2.58
One-tailed Test	Z <sub>c</sub> =1.645	$Z_c$ = 2.33

Step 4- Get table value of  $Z_c$  for LOS  $\alpha = 0.05$  (5 %)

$$Z_c = 1.96$$

Step 5- Find Z score using formula-

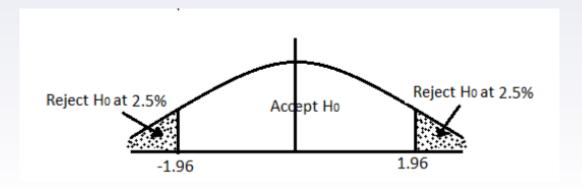
$$Z = \frac{P - p}{\sqrt{\frac{pq}{N}}}$$

$$Z = 1.689$$

p	0.167
q	0.833
P	0.23
Ν	100

Step 6 - Inference

$$Z = 1.689$$
,  $Z_c = 1.96$ 



As  $Z < Z_c$ , Accept Ho.

Therefore, we can support the claim at 0.05 LOS. i.e., the dice are fair.

A certain coin is tossed 500 times and showed up head in 270 occasions. Test the claim that the coin is unbiased at 5% LOS using two tailed test.