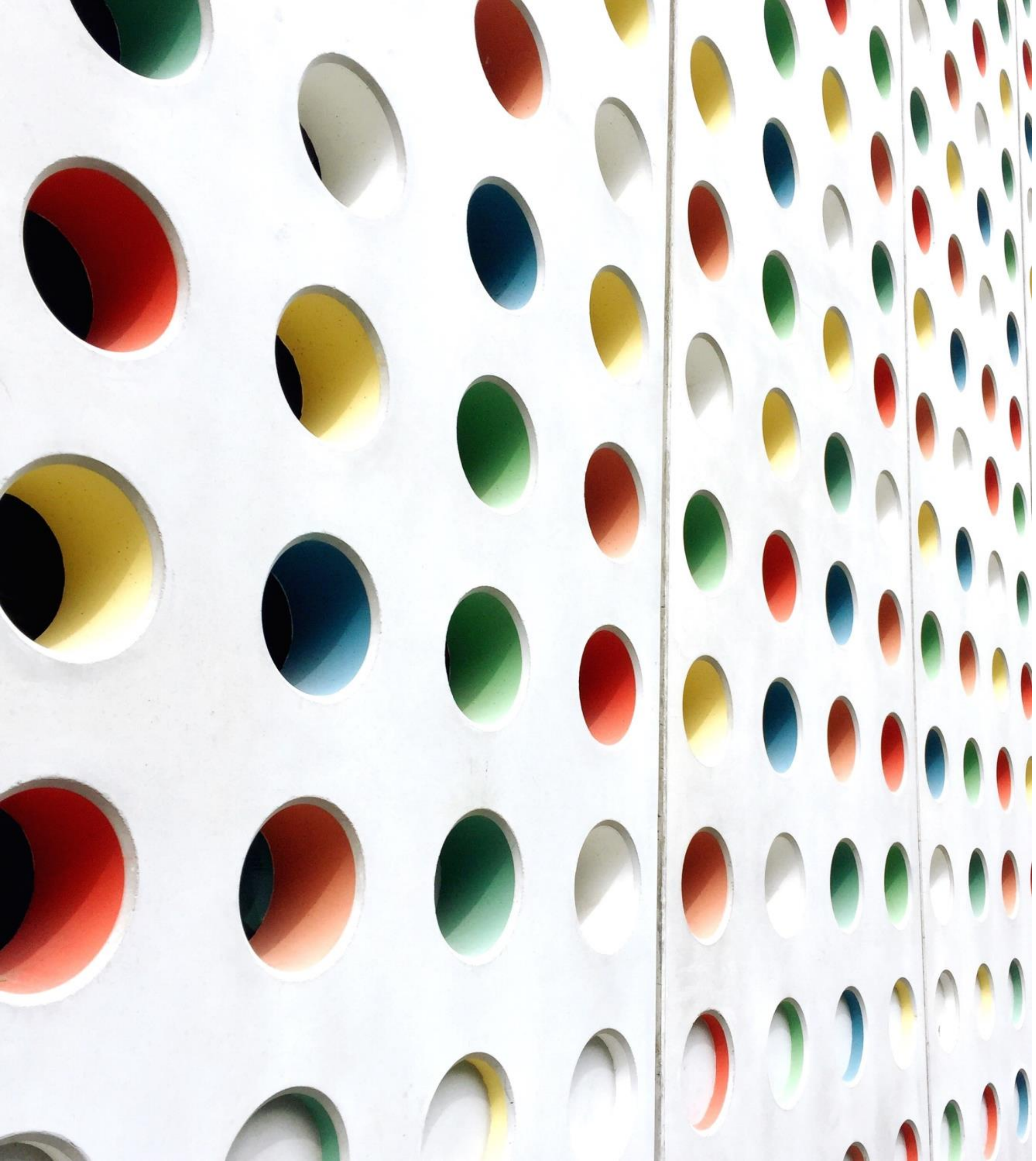


Homogeneous Co-ordinates

2D TRANSFORMATION

Old Scenario

TRANSLATION

ROTATION

SCALING

Homogeneous coordinates

Homogeneous Coordinates for Translation

$$\begin{bmatrix} X' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ y \\ 1 \end{bmatrix}$$

P`=T [tx, ty] . P

New Scenario

TRANSLATION

ROTATION

SCALING

Homogeneous coordinates for Rotation

$$\begin{bmatrix} X` \\ Y` \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$P` = R\ [\theta]\ .\ P$$

Homogeneous coordinates for Scaling

$$\begin{bmatrix} X` \\ Y` \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

P`=S [Sx, Sy] . P

Composite Transformation

Sequence of Transformations.

Forming of products of transformation matrix is also called as **Concatenation** or **composition of matrices.**

Problem Based Learning

Example 1

Give a 3*3 homogeneous coordinate transformation matrix for each of the following translations:

A. Shift the image to the right 3 units

B. Shit the image up by 2 units

C. Move the image down by ½ unit and right by 1 unit

D.

Example 2

Find the transformation of triangle A(1,0), B(0,1) C(1,1) by

A. Rotating 45 degree about the origin and then translating one unit in x direction and y direction.

B. Translating one unit in x direction and y direction and then rotating 45 degree about the origin

Take Home Task

1. Write the matrix representation for Homogeneous coordinates for Reflection.

2. Write the matrix representation for Homogeneous coordinates for Shearing.

3. Using homogeneous coordinate transformation matrix apply following sequence of transformation to a unit square centred at origin with translation by factor (1,1) and rotation angle is 90 degree

Rotation about an Arbitrary Point

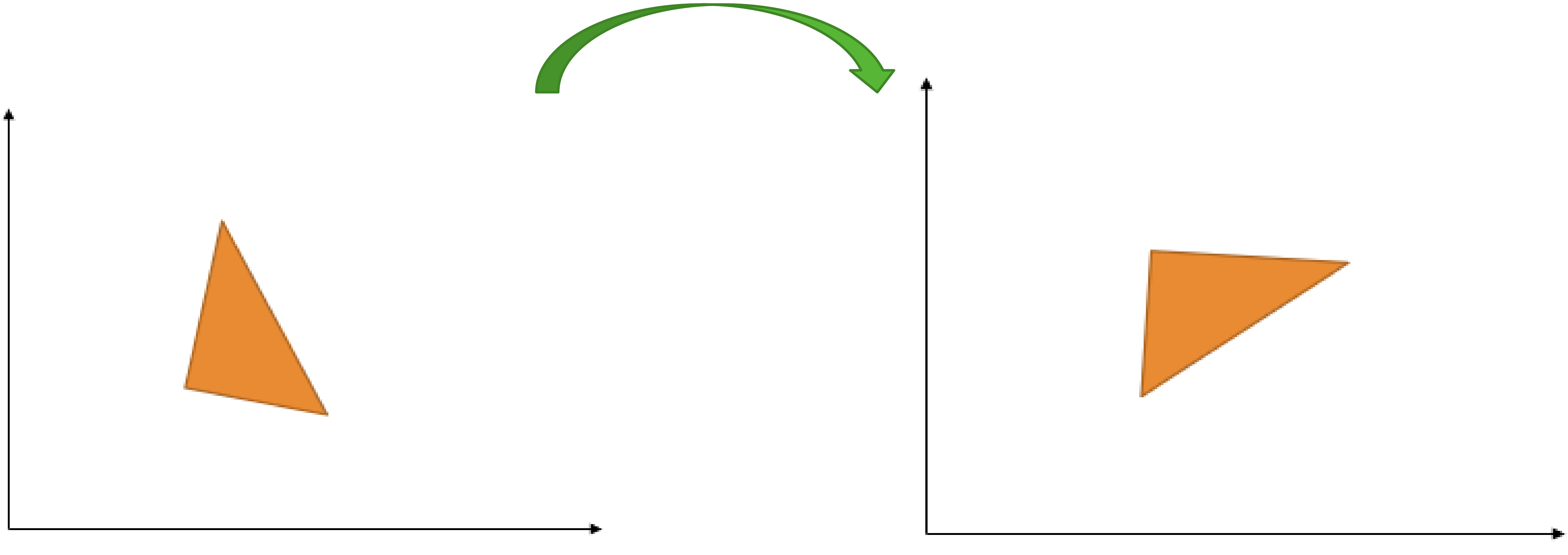
To rotate an object about an arbitrary point (xp, yp) we have to perform three steps:

1. Translate point (xp, yp) to the origin.

2. Rotate it about the origin.

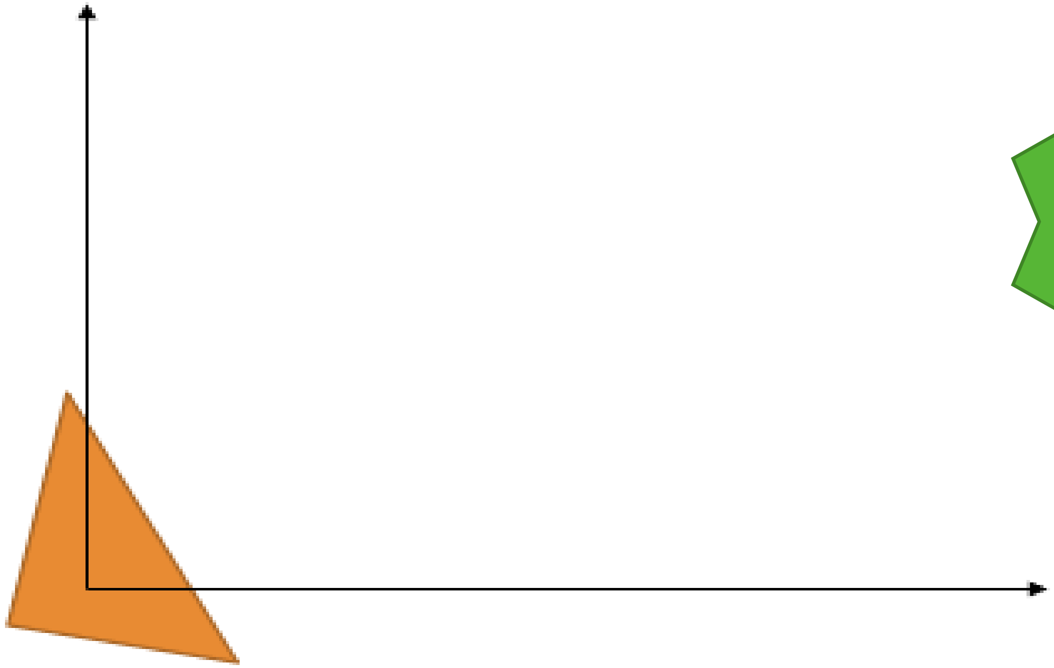
3. Translate the center of rotation back to original point.

Pivot Point Rotation or Rotation about an Arbitrary Point



Pivot Point Rotation or Rotation about an Arbitrary Point

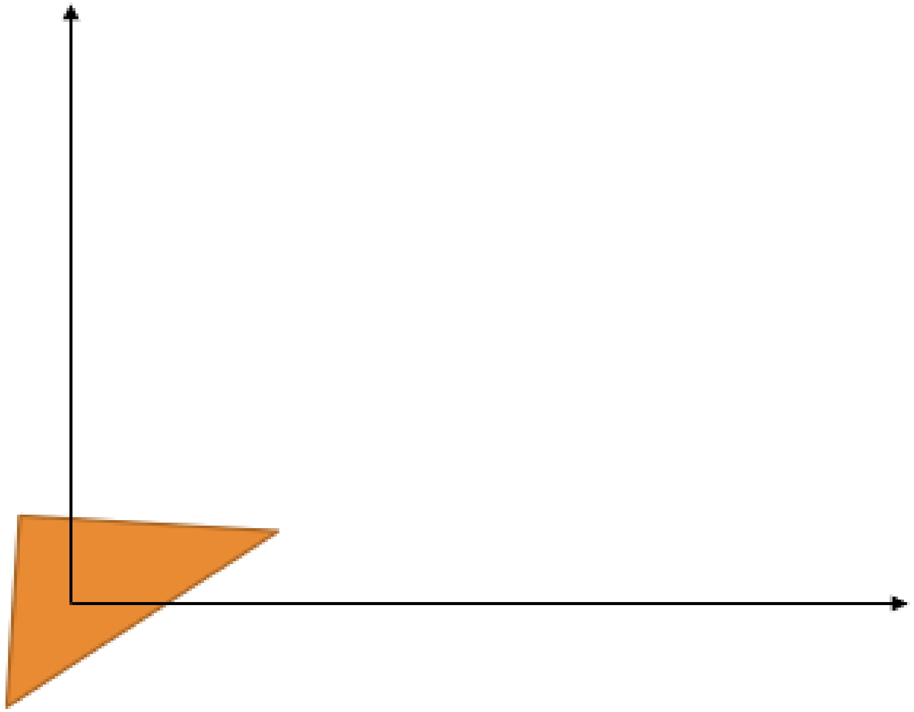
Bring it to Origin



Step 1

Pivot Point Rotation or Rotation about an Arbitrary Point

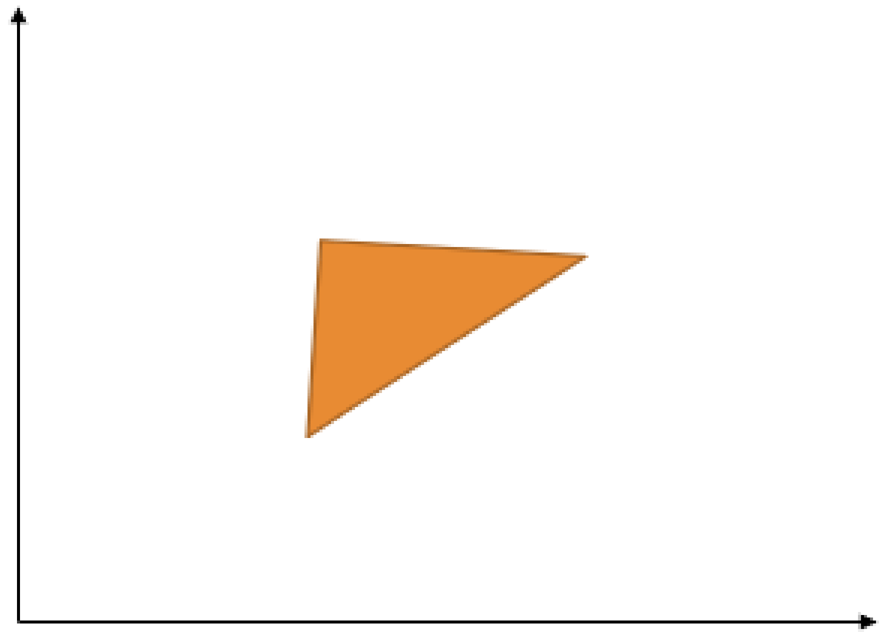
Rotate with given angle



Step 2

Pivot Point Rotation or Rotation about an Arbitrary Point

Translate back to its original location



Step 3

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

which can be expressed in the form

$$\mathbf{T}(x_r, y_r) \cdot \mathbf{R}(\theta) \cdot \mathbf{T}(-x_r, -y_r) = \mathbf{R}(x_r, y_r, \theta)$$

Example 3

Perform anticlockwise rotation of 45 degree to a triangle A(2,3), B(5,5), C(4,3) about a pivot point (1,1).

Fixed-Point Scaling

1. Translate object so that the fixed point coincides with the coordinate origin.
2. Scale the object with respect to the coordinate origin.
3. Use the inverse translation of step 1 to return the object to its original position.

Concatenating the matrices for these three operations produces the required scaling matrix

$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1 - s_x) \\ 0 & s_y & y_f(1 - s_y) \\ 0 & 0 & 1 \end{bmatrix} \tag{5-33}$$

or

$$\mathbf{T}(x_f, y_f) \cdot \mathbf{S}(s_x, s_y) \cdot \mathbf{T}(-x_f, -y_f) = \mathbf{S}(x_f, y_f, s_x, s_y) \tag{5-34}$$

Example 4

Find the transformation matrix that transforms the given square ABCD to half of its size with its centre still remaining at the same position. The coordinates of the square are: A(1,1), B(3,1), C(3,3), D(1,3) and centre at (2,2) also find the resultant coordinates of square.

Solution:

1. Translate the square so that its center coincides with the origin.

2. Scale the square with respect to origin.

3. Translate the square back to the original position.

Example 5

Magnify the triangle with vertices A(0,0) B(1,1) and C(5,2) to twice of its size while keeping C(5,2) as fixed.

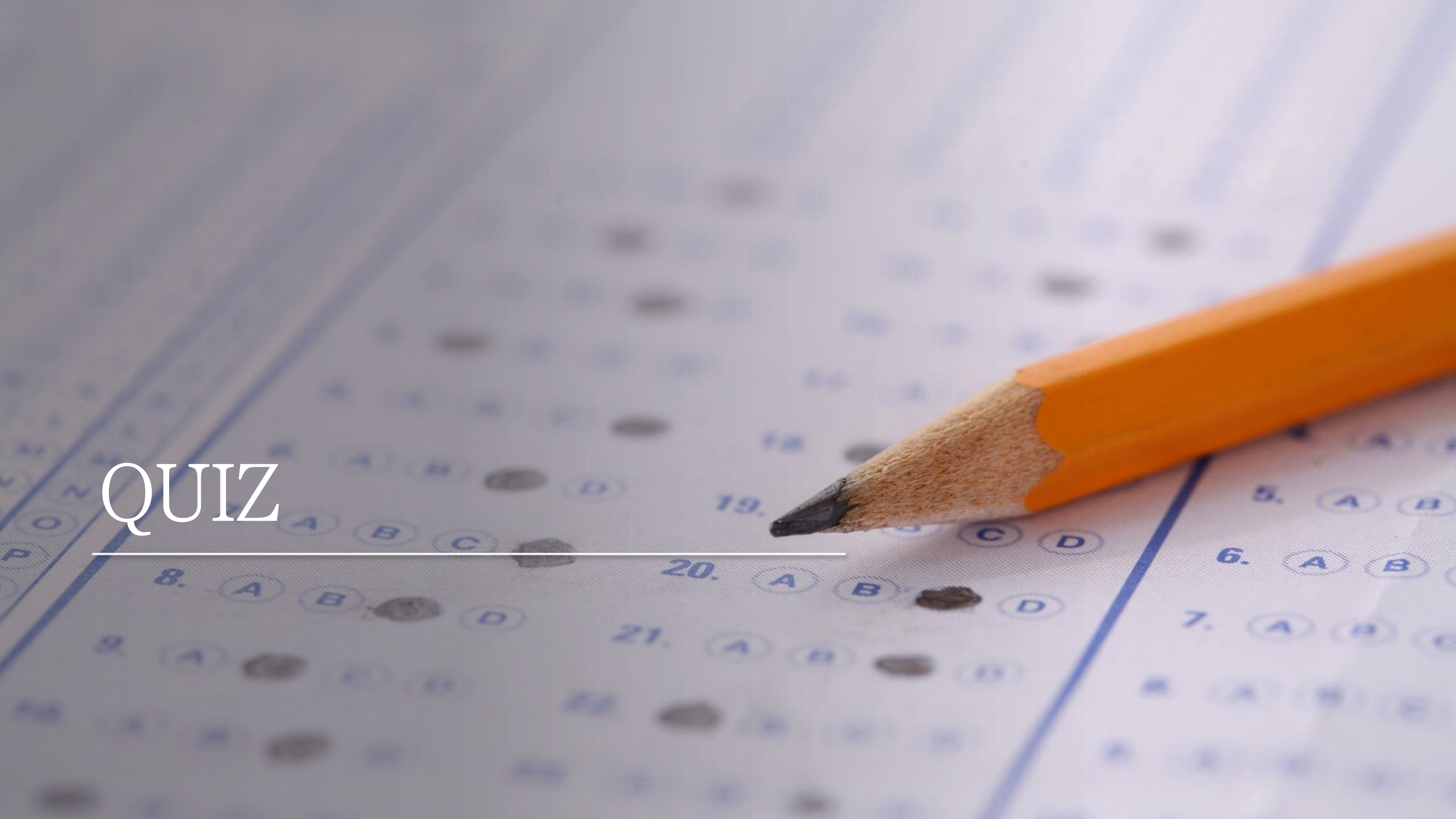
Steps:

1. Translate the triangle by Tx=5 and Ty=2.

2. Magnify the triangle by twice its size.

3. Again, Translate the triangle by Tx=-5 and Ty=-2

QUIZ



Thank You