

COST

Statistical Estimation Theory



Z Score of difference

► Difference of Mean

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}}$$

► Difference of Proportion

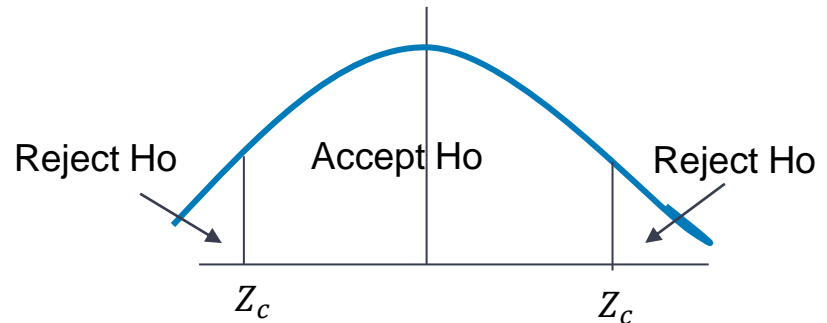
$$Z = \frac{(P_1 - P_2) - (p_1 - p_2)}{\sqrt{pq\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

$$Z = \frac{P - p}{\sqrt{\frac{pq}{N}}}$$

$$\text{Where } p = \frac{N_1 P_1 + N_2 P_2}{N_1 + N_2}$$

Steps for hypothesis testing

1. Write given values.
2. Propose H_0 and H_1 .
3. Identify test-
 - ▶ one tailed (if $<$, $>$)
 - ▶ two tailed (if \neq)
4. Get table value Z_c according to LOS mentioned in the problem.
5. Find Z score using the formula.
6. Inference-
 - ▶ If $Z < Z_c$, accept H_0 .
 - ▶ If $Z > Z_c$, reject H_0 .



Question

Qn) A sample of 100 electric light bulbs produced by manufacturer A showed a mean lifetime of 1190hrs with a standard deviation of 90hrs. A sample of 75 bulbs produced by manufacturer B showed a mean lifetime of 1230hrs with a standard deviation of 120hrs. Is there a difference between the mean lifetime of the two brands at significance level of 0.01?

Step 1- Write given values

- ▶ $N_1 = 100$
- ▶ $\bar{X}_1 = 1190 \text{ hrs}$
- ▶ $\sigma_1 = 90 \text{ hrs}$
- ▶ $N_2 = 75$
- ▶ $\bar{X}_2 = 1230 \text{ hrs}$
- ▶ $\sigma_2 = 120 \text{ hrs}$

$$LOS = \alpha = 0.01 = 1 \%$$

▶ Cont..

- ▶ Step 2- Propose H_0

$H_0: \mu_1 = \mu_2$ There is no difference between mean lifetime of two brands.

$H_1: \mu_1 \neq \mu_2$ There is a difference between mean lifetime of two brands.

- ▶ Step 3- Identify Test

- ▶ As \neq sign is there, use Two tailed Test

	$\alpha = 0.05$ (5 %)	$\alpha = 0.01$ (1 %)
Two-tailed Test	$Z_c=1.96$	$Z_c= 2.58$
One-tailed Test	$Z_c=1.645$	$Z_c= 2.33$

Cont....

- Step 4- Get table value of Z_c for LOS $\alpha = 0.01$ (1 %)

$$Z_c = 2.53$$

- Step 5- Find Z score using formula-

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

$$Z = -2.4209$$

- $N_1 = 100$
- $N_2 = 75$
- $\bar{X}_1 = 1190 \text{ hrs}$
- $\bar{X}_2 = 1230 \text{ hrs}$
- $\sigma_1 = 90 \text{ hrs}$
- $\sigma_2 = 120 \text{ hrs}$

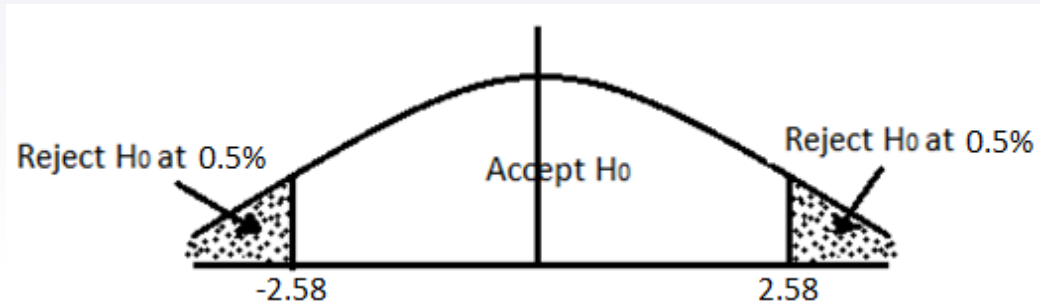
As per the claim, $\mu_1 = \mu_2$

Hence, $\mu_1 - \mu_2 = 0$

Cont. ..

- ▶ Step 6 – Inference

$$Z = -2.4209, Z_c = -2.58$$



- ▶ As Z lies between ± 2.58 , Accept H_0 .
- ▶ Therefore, we can support the claim at 0.01 LOS. i.e., there is no significant difference in mean lifetimes of two bulb manufacturers.

Question

- ▶ On an elementary school spelling examination, the mean grade of 32 boys was 72 with a standard deviation of 8, while the mean grade of 36 girls was 75 with a standard deviation of 6. It is claimed that there is no difference in boys and girls at spelling. Test the claim at 5% LOS.