# On Job-Insertion for the Blocking-Job-Shop and its Application to the SBB Challenge

Master thesis of

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# References / Complete Definition / etc

https://github.com/MrPascalCase/SbbChallenge

# Job-Shop Problem

The ‘input’ data:

- A set of *jobs* $\mathcal{J} = \{J_1, \ldots, J_n\}$
- Each job is a sequence of *operations*, i.e. $J = (o_{J1}, \ldots, o_{Jn_J})$
- Each operation $o$ has a processing time $p_o$
- A set of *machines* $\mathcal{M} = \{M_1, \ldots, M_m\}$
- Each operation $o$ has a machine associated $M(o)$

The ‘problem’, define a time $t_o$ for every operation $o$:

$$\min_{t} \max_{J \in \mathcal{J}} \{t_{Jn_J} + p_{Jn_J}\}$$

$$0 \leq t_o \qquad \forall o \in \mathcal{O}$$

$$t_{Jk} + p_{Jk} \leq t_{Jl} \qquad \forall J \in \mathcal{J}\ \forall k\ \forall l \mid 1 \leq k < l \leq n_J$$

$$t_{o_1} + p_{o_1} \leq t_{o_2} \vee t_{o_2} + p_{o_2} \leq t_{o_1}$$

$$\forall \{(o_1, o_2) \mid o_1 \in \mathcal{O}, o_2 \in \mathcal{O}, o_1 \neq o_2, M(o_1) = M(o_2)\}$$

# Blocking-Job-Shop Problem

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$$t_{succ(o_1)} \le t_{o_2} \vee t_{succ(o_2)} \le t_{o_1}$$

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# Precedence constraint graph

# Disjunctive graph

**Definition.** A tuple $(V, A, E, \mathcal{E}, l)$ defines a disjunctive graph, if

(i) $(V, A \cup E)$ is a directed graph.

(ii) $(V, A)$ is an acyclic directed graph.

(iii) $\mathcal{E}$ is a set of unordered pairs of arcs; $E$ exactly contains all arcs of $\mathcal{E}$, i.e.:

$$\forall (e, \bar{e}) \in \mathcal{E} \ \big(e \in E \wedge \bar{e} \in E\big).$$

Furthermore, adding both arcs of a pair to $(V, A)$ yields a cyclic graph. i.e.:

$$\forall (e, \bar{e}) \in \mathcal{E} \ \big(V, A \cup \{e, \bar{e}\}\big) \text{ is cyclic.}$$

For $(e, \bar{e}) \in \mathcal{E}$, we call $\bar{e}$ the mate of $e$ and vice versa.

(iv) $l : (A \cup E) \to \mathbb{R}_{>0}$, defines the length of an arc.

# Selection

**Definition.** $S$ is a selection in a disjunctive graph $G = (V, A, E, \mathcal{E}, l)$ if

- $S \subset E$
- $S$ contains at most one element of every pair of $\mathcal{E}$.

We call $S$ complete (else partial) if $S$ contains one element of every pair of $\mathcal{E}$.

$S$ is called feasible if $(V, A \cup S)$ is acyclic.

# Selection (ii)

**Goal:**
Given a complete feasible selection, create ‘neighbour’ selections, such that we can apply a meta search heuristic (such as a taboo-search).

- In the (classical-)job-shop exchanging any $e$ with $\bar{e}$ yields such a neighbour.
- Not in the blocking-job-shop. How can we ‘repair’ a selection after such a change?
- Context:
  $\max_x\{f(x) \mid x \in \{0,1\}^n\}$,
  $N_1(x) = \{y \mid \|x - y\|_1 = 1\}$.

# Critical arcs

# Selection $S \to$ entry times $t$

- A selection corresponds to a set of feasible schedules. One of which, the *earliest-starting-date-schedule*, is among the best.
- For all $v \in V$, in topological order, set:

$$t_v = \max\{\, t_u + l(u,v) \mid (u,v) \in (A \cup S) \,\}$$

- Disregarding other schedules, we have a one-to-one relation.

# Job-Insertion Graph

**Definition.** Given a blocking-job-shop problem with jobs $\mathcal{J}$, a specific job $J \in \mathcal{J}$ and a complete feasible selection $S$ to the problem $\mathcal{J} \setminus J$. For $S$ i.e.:

- $\forall (e, \bar{e}) \in \mathcal{E}$ not adjacent to $J$, $e \in S \vee \bar{e} \in S$.
- $\forall (e, \bar{e}) \in \mathcal{E}$ adjacent to $J$, $e \notin S \wedge \bar{e} \notin S$.

Associated with the blocking-job-shop problem is the graph $G = (V, A, E, \mathcal{E}, l)$.
The job-insertion graph $G^J = (V, A^J, E^J, \mathcal{E}^J, l)$ is constructed as follows:

- $A^J := A \cup S$
- $\mathcal{E}^J := \mathcal{E}$ restricted to arcs adjacent to $J$.
- $E^J := E$ restricted to arcs adjacent to $J$.

# Short-Cycle Property (i)

**Definition.** A disjunctive graph $G = (V, A, E, \mathcal{E}, l)$ has the short-cycle property if for any cycle $Z$ in $(V, A \cup E)$, there exists a cycle $Z'$ in $(V, A \cup E)$ with $Z' \cap E \subseteq Z \cap E$ and $|Z' \cap E| = 2$.

# Short-Cycle Property (ii)

**Proposition.** The job-insertion graph $G^J = (V, A^J, E^J, \mathcal{E}^J, l)$ has the short cycle property.

- $(\mathbf{n = 1})$ Enter and leave $J$, $|Z \cap E| = 2$. ✓
- $(\mathbf{n} \rightsquigarrow \mathbf{n+1})$ Prove that: a cycle entering $J$ $n+1$ times has a short cycle.

  Let $Z$ be a cycle which enters $J$ $n+1$ times. We choose arbitrarily a vertex $a$ where the cycle $Z$ leaves $J$.
  Let $b$ be the first vertex after $a$ where $Z$ reenters $J$. We differentiate two cases:

  (i) $\mathbf{a \preceq b}$:
  Define a cycle $Z'$, equal to $Z$, with the path $a \to b$ replaced by the path $a \to b$ within $J$. Then $Z'$ enters $J$ $n$ times.
  From the induction hypothesis it follows that a short cycle exists. ✓

  (ii) $\neg(\mathbf{a \preceq b})$:
  $b \to a$ (within $Z$) followed by $a \to b$ (within $J$) is a valid short cycle. ✓

# Conflict graph (i)

- We define a conflict graph for a job-insertion graph $G^J$: $H_{G^J} = (E^J, U)$.
- Vertices of $H_{G^J}$ are elements to be selected: arcs.
- Vertices are connected, if they conflict, i.e. selecting all vertices connected by an arc $u$ leads to a cyclic graph $(V, A \cup u)$.
- $u \in U$ if $u$ is a partial infeasible selection.
- As we require a feasible solution, it suffices to consider minimally infeasible edges (if we avoid all of them, we are good).
- The short cycle property guarantees that all edges connect *two* vertices.
- Hence, $U = \{(e, f) \mid (V, A^J \cup \{e, f\}) \text{ cyclic}\}$.
- ($H_{G^J}$ is bipartite.)

# Conflict graph (ii)

- stable sets of size $|E^J|/2$ in the conflict graph correspond to complete feasible solutions.
- (bipartite $\implies$ at least 2 exist)

# Closure

used to generate a complete feasible selection $S'$ from $S$ which does not include $e \in S$.

**Idea:**

- Pick $J \in \mathcal{J}$ with $h(e) \in J$.
- Construct $G^J$.
- Construct $H_{G^J}$.
- Select $\bar{e}$ in $H_{G^J}$.
- For all $f$ such that $(e, f)$ in $U$, select $\bar{f}$. (*)
- Take the closure of (*).
- Complement this selection with $S$ for all pairs in $E^J$ where we did not make a choice due to the closure.
- done.

# Why do we need the conflict graph?

- We use the conflict graph to query information like 'enumerate the neighbours of $e$'.
- In the disjunctive graph, this is equivalent to 'find all disjunctive arcs $f \in E^J$ such that $(V, A^J \cup \{e, f\})$ is cyclic'.
- This corresponds to paths searches:
  for all $\{f \in E^J \mid h(f) \in J \wedge h(f) \preceq t(e)\}$ does a path $h(e) \to t(f)$ exist?

# ‘Left’ Closure (i)

- Constructing $G^J$ we picked $J$ such that $h(e) \in J$.
- (We could do the same thing with $J$ such that $t(e) \in J$ $\Longrightarrow$ ‘right’ closure.)
- In case of the left-closure: Arcs such as $e$ which *enter* $J$ are replaced by arcs $\bar{e}$ that *leave* $J$. Whenever we insert $\bar{e}$, we have to check if a path exists between $h(\bar{e}) \to t(\bar{e})$ $(t(\bar{e}) \in J,\ h(\bar{e}) \notin J)$. Of a potential cycle, only the disjunctive arc reentering $J$ is relevant. Hence we search paths from $h(\bar{e})$ into the vertex interval $[o_{J1}, \ldots, t(\bar{e})]$. This is done efficiently with a forward path search.

# ‘Left’ Closure (ii)

- We remove an arc $e$ which *enters* $J$ while adding an arc $\bar{e}$ that *leaves* $J$.
- A short cycle through $\bar{e}$ must therefore contain an arc $f$ which *enters* $J$.
- $f$ must be replaced with $\bar{f}$, which *leaves* $J$.
- The same argument holds for $\bar{f}$, indeed for the complete process. More and more arcs leave $J$, while fewer and fewer arcs enter $J$. This has the effect that $J$ moves backward in time through the time table, or, in a Gantt-chart, to the left. We call this version of the closure the <u>left-closure</u>.

**Algorithm 1:** Naive left-closure

**Input :** A graph $G$ corresponding to the current selection $S$: $G = (V, A \cup S)$, a disjunctive arc $a$ to remove from the selection.

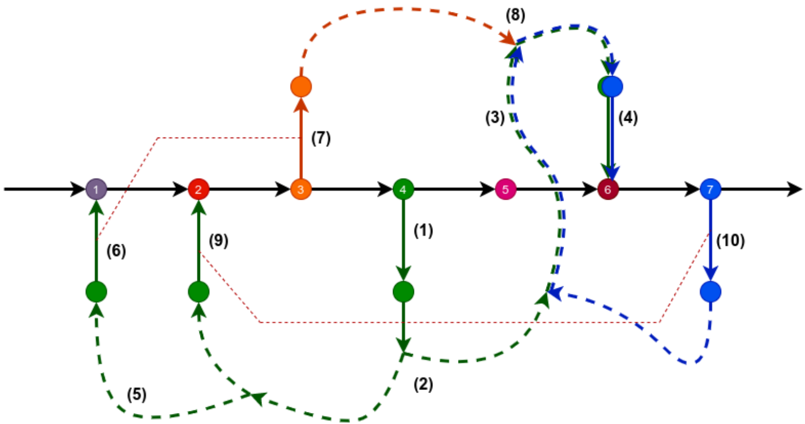
**Output:** A modified graph $G$, corresponding to a modified selection $S$, which is similar to the input $S$ but does not contain the arc $a$.

```
Set<Arc> arcsToRemove = {a}
while arcsToRemove.Count > 0 do
    Arc e = arcsToRemove.Pop()
    if G.ArcExists(e) then
        Arc ē = G.SwapInMate(e)
        foreach Arc f ∈
          G.ForwardPathSearchIntoRange(h(ē), [o_J1, t(ē)]) do
            arcToRemove.Add(f)
        end
    end
end
```

# My Contribution

Observe that, the later within $J$ the operation $t(\bar{e})$ occurs, the larger the target vertex interval of the path search. Indeed, $t(\bar{f}) \preceq t(\bar{e})$ implies $\{o_{J1}, \ldots, t(\bar{f})\} \subseteq \{o_{J1}, \ldots, t(\bar{e})\}$. Hence, for any vertex $v \in G$ such that a path exists form $h(\bar{f}) \to v$ and $h(\bar{e}) \to v$, the forward path search ($v$ onward) to be completed for $\bar{e}$ renders the path search for $\bar{f}$ irrelevant.

# My Contribution



**Algorithm 2:** All at once left-closure

**Input :** A disjunctive graph $G$ and a selection $S$: $G = (V, A \cup S)$, a disjunctive arc $a$ to remove from the selection.

**Output:** A modified graph $G$, corresponding to a modified selection $S$, which is similar to the input $S$ but does not contain the arc $a$.

$[\dots]$

**Algorithm 3:** All at once left-closure

[...]

Queue<Arc> Q = new BucketQueue()

let $\bar{a}$ be s.t. $\bar{a} \in$ G.SwapInMate(a) **and** $t(\bar{a}) \in J$

Q.Add($\bar{a}$, priority = any)

int[] P = new int[G.Count]

Initialize P: $P[o_{jk}] = \begin{cases} k, & \text{if } j = J \\ 0, & \text{otherwise} \end{cases}$

[...]

**Algorithm 4:** All at once left-closure

```
[...]
while Q.Count > 0 do
    Arc a = Q.Pop()
    if not G.ArcExists(a) then
        continue
    else if h(a) ∈ J and P[t(a)] ≥ P[h(a)] then
        foreach Arc b ∈ G.SwapInMate(a) do
            if P[t(b)] > 0 then
                Q.Add(b, priority = P[t(b)])
            end
        end
    else if P[t(a)] > P[h(a)] then
        P[h(a)] = P[t(a)]
        foreach Arc b ∈ G.OutgoingArc(h(a)) do
            Q.Add(b, priority = P[t(b)])
        end
    end
end
```

# Another improvement

- For a time $t$, we split $(V, A \cup S)$ into

$$A := \{v \mid t_v \leq t\} \text{ and } B := \{v \mid t_v > t\}.$$

- No vertex of $A$ is reachable from any vertex in $B$. Assuming that the left-closure computation affects no arcs adjacent to or in $B$, we can establish that $A$ remains unreachable from $B$. If we then–during the forward path search of the left closure–encounter a vertex of $B$ we can skip this branch of the path search.
- We choose $t = \max_{a \in \delta(J)}\{ t_{h(a)} \}$.

**Algorithm 5:** All at once left-closure with termination criterion

[...]

Initialize B: $B[i] = \begin{cases} true, & \text{if } t_i > \max_{b \in \delta(J)}\{\ t_{h(b)}\ \} \\ false, & \text{otherwise} \end{cases}$

**while** $Q.Count > 0$ **do**

    Arc $a$ = Q.Pop()

    [...]

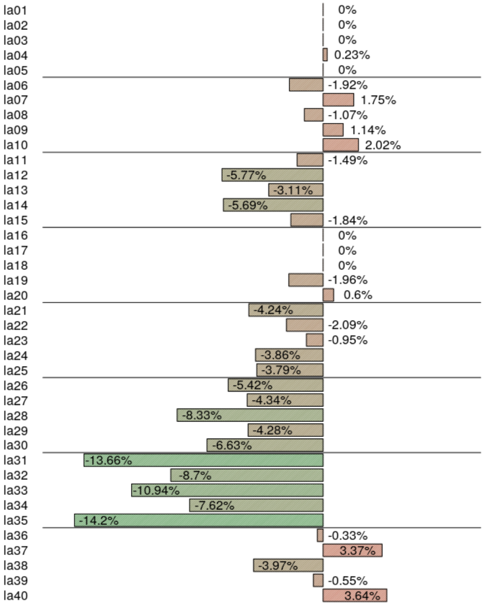
    **else if** $B[h(a)]$ **then**

        **continue**

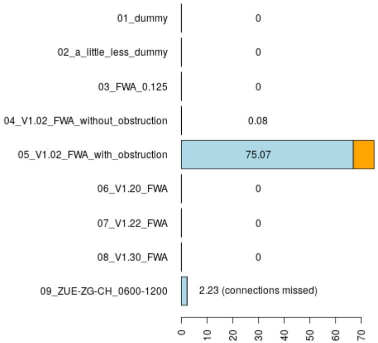
    [...]

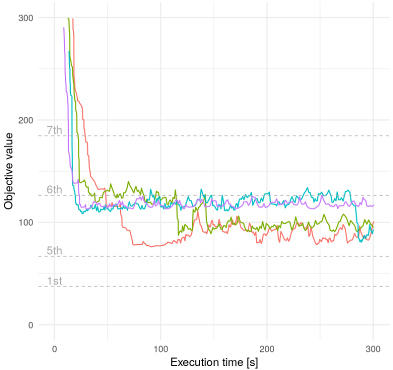
**end**

# Results (i)


la01 0%
la02 0%
la03 0%
la04 0.23%
la05 0%
la06 -1.92%
la07 1.75%
la08 -1.07%
la09 1.14%
la10 2.02%
la11 -1.49%
la12 -5.77%
la13 -3.11%
la14 -5.69%
la15 -1.84%
la16 0%
la17 0%
la18 0%
la19 -1.96%
la20 0.6%
la21 -4.24%
la22 -2.09%
la23 -0.95%
la24 -3.86%
la25 -3.79%
la26 -5.42%
la27 -4.34%
la28 -8.33%
la29 -4.28%
la30 -6.63%
la31 -13.66%
la32 -8.7%
la33 -10.94%
la34 -7.62%
la35 -14.2%
la36 -0.33%
la37 3.37%
la38 -3.97%
la39 -0.55%
la40 3.64%

# Results (ii)


01_dummy
0
02_a_little_less_dummy
0
03_FWA_0.125
0
04_V1.02_FWA_without_obstruction
0.08
05_V1.02_FWA_with_obstruction
75.07
06_V1.20_FWA
0
07_V1.22_FWA
0
08_V1.30_FWA
0
09_ZUE-ZG-CH_0600-1200
2.23 (connections missed)
0
10
20
30
40
50
60
70


Objective value
300
200
100
0
7th
6th
5th
1st
0
100
200
300
Execution time [s]