SML 201 Project 3 Predicting House Sold Prices for Seattle and Greater Area

Stuart Duffield and James Armstrong
2019-01-14

Project 3 is due by 11:59pm on Tuesday January 15. Please submit both a .Rmd and a .pdf file on Blackboard by the deadline and drop off a hard copy of the pdf file at 26 Prospect Avenue by 11am of the next day of the due date. To look for the drop-off cabinet, after you enter the building turn to the left to enter the lounge area and the file cabinet will then be on your right; you can just drop your report into the open slot of the cabinet labeled "SML 201 Homework"; note that the building might be locked after 6pm and on weekends. You are also welcome to drop off the pdf copy in advance of the deadline.

Late **projects** will be penalized at intervals rounded up to multiples of 24 hours. For example, if you are 3 hours late, 10% off or if you are 30 hours late, 20% off.

Make sure that you have all your digital signatures along with the honor pledge in each of these documents (there should be two signatures if you work in groups).

This project can be completed in groups of up to 2 students. It is okay to work by yourself, if this is preferable. You may not work with a given student on more than one project. In other words, if you work with Student_A on Project 1, then you cannot work with Student_A on any other projects. You must work with a new group mate for every project.

When working in a group it is your responsibility to make sure that you are satisfied with all parts of the report and the submission is on time (e.g., we will not entertain arguments that deficiencies are the responsibility of the other group member). We expect that the work on any given problem set or project contains approximately equal contributions from both members of the group; we expect that you each work independently first and then compare your answers with each other once you all finish or you work together. Failing to make contributions and then putting your name on a project will be considered a violation of the honor code. Also, please do not divide work between group mates. For all parts of this problem set, you MUST use R commands to print the output as part of your R Markdown file. You are not permitted to find the answer in the R console and then copy and paste the answer into this document.

In general you are not allowed to get help on projects from other people except from your group partner. There is an exception for this project: you are allowed to get help from the instructors if you need help to understand the definitions of the variables of the dataset or the procedure of the experiment.

You are always welcome to ask the instructors clarification questions. *Please treat projects as take-home exams*. Do not make suggestions on how to solve or the possible approach to a question to other students—please keep this in mind when asking questions on Piazza.

For all parts of this problem set, you **MUST** use R commands to print the output as part of your R Markdown file. You are not permitted to find the answer in the R console and then copy paste the answer into this document.

If you are completing this project in a group, please have only one person in your group turn in the .Rmd and .pdf files; the other person in your group should turn in the list of the names of the people in your group in the *Text Submission* field on the submission page. This means that **everyone should make** a **submission**—either a file-upload or a text submission—regardless of whether you are working in a group or not.

Please type your name(s) after "Digitally signed:" below the honor pledge to serve as digital signature(s). Put the pledge and your signature(s) at the beginning of each document that you turn in.

I pledge my honor that I have not violated the honor code when completing this assignment.

Digitally signed: Stuart Duffield and James Armstrong

In order to receive full credits, please

- avoid hard-coding and do not round intermediate calculations;
- round all final numerical answers to four digits after the decimal.
- have sensible titles and axis labels for all your graphs;
- adjust values for all the relevant graphical parameters so that your plots are informative.
- annotate your code and have answers in the write up section, not in the code chunks:
- have all answers written in complete sentences.
- drop off a **color** copy of the pdf by 11am the day after the due date.

Objective of this project

(Hypothetical) Congrats! You have been hired as an intern at Redfin (Redfin.com) in Seattle. As for your first project your manager would like you to build a new model for Redfin to predict the sold prices for houses in the King county area of the Washington state.

To see the prices predicted by Redfin's current model, you can see the number shown near the top of the web page of a listing; e.g., for this house (https://www.redfin.com/WA/Seattle/132-NE-95th-St-98115/unit-B108/home/2316), the Redfin estimate is \$401,144.)

Background info and the dataset

We will use a subset of the dataset kc_house_data.csv to build the model. The dataset contains sold prices for houses in King County (in Washington state), including Seattle, for transactions made between May 2014 and May 2015. We will use only a subset of the variables in kc_house_data.csv because we only want to include variables that have clear definitions and seem relevant for house prices. A description of the original dataset kc_house_data.csv and the complete list of the variable definitions can be found here (https://www.kaggle.com/harlfoxem/housesalesprediction/data).

We will use the dataset subset_kc_house_data.csv and the definitions for the variables in the dataset are listed below (see the website address provided above for the complete list of the variables).

- date The date the house was sold
- **price** Sold price of the house
- bedrooms Number of bedrooms in the house
- bathrooms Number of bathrooms in the house
- sqft_living Square footage of the house

- sqft_lot Square footage of the lot
- floors Total number of floors (levels) in house
- waterfront Does the house have a view to a waterfront (0-No; 1-Yes)
- condition How good the overall condition is
- grade Overall grade given to the housing unit, based on King County grading system (see table for BLDGGRADE on https://www5.kingcounty.gov/sdc/FGDCDocs/RESBLDG_EXTR_faq.htm)
- sqft_above Square footage of house apart from the basement
- sqft_basement Square footage of the basement
- yr_built Year when house was built
- yr_renovated Year when house was renovated
- zipcode Zip code of the house address

Question 1 (1 pt)

Read the dataset subset_kc_house_data.csv into R and name it house. house should have 21613 rows and 15 columns.

house has 21613 rows and 15 columns.

Exploring the relationship between price and other variables.

```
Question 2 (20 pts)
```

```
Part a (9 pts)
```

Use ggpairs() in the GGally package and make matrices of scatterplots to investigate the pairwise relationships between the variables;

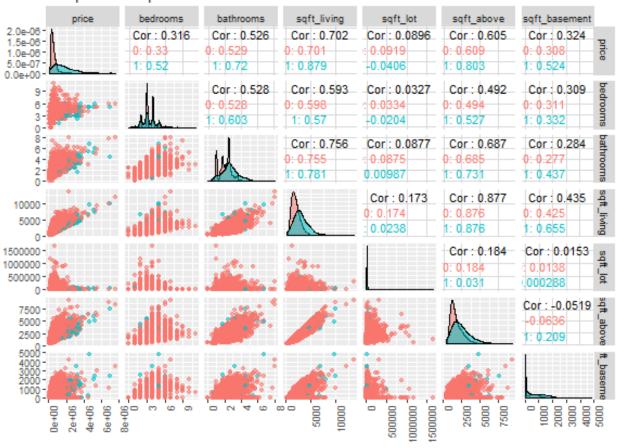
Group sqft_living, sqft_lot, sqft_above and sqft_basement in the same plot to see if any of these variables are correlated. In general, it is good to have x-variables that are highly correlated (i.e., with correlation close to -1 or 1) among themselves in your model? Explain why yes or why no.

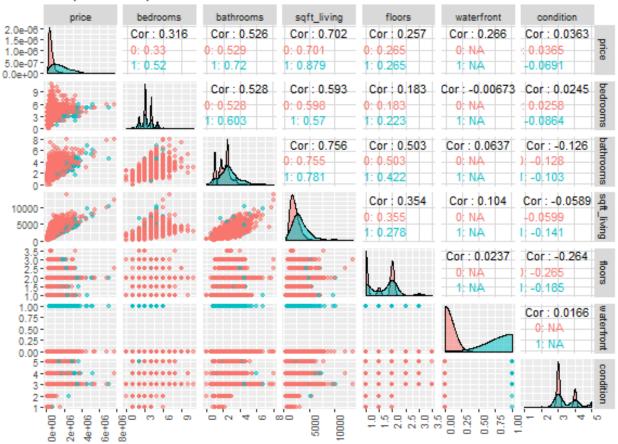
```
# Check squre footage of living area of 30+ bedroom house
house[house$bedrooms > 30, ]$sqft_living
[1] 1620
```

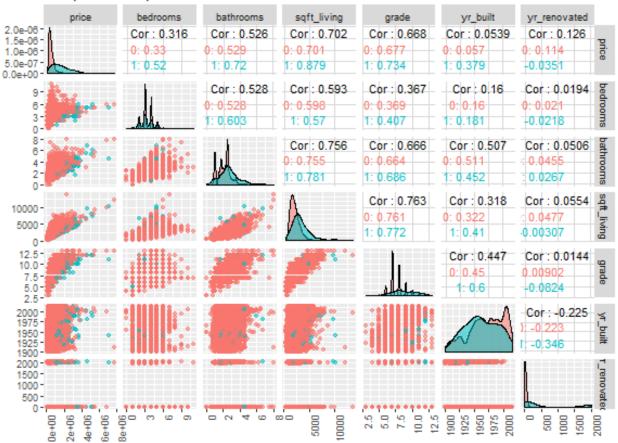
```
# Remove erroneous data
house = house[-which(house$bedrooms == 0 & house$bathrooms == 0 |
    house$bedrooms > 30), ]

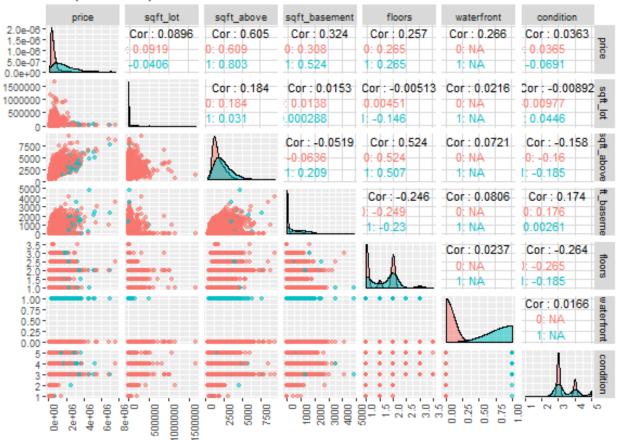
# Make Plots
library(ggplot2)
library(GGally)

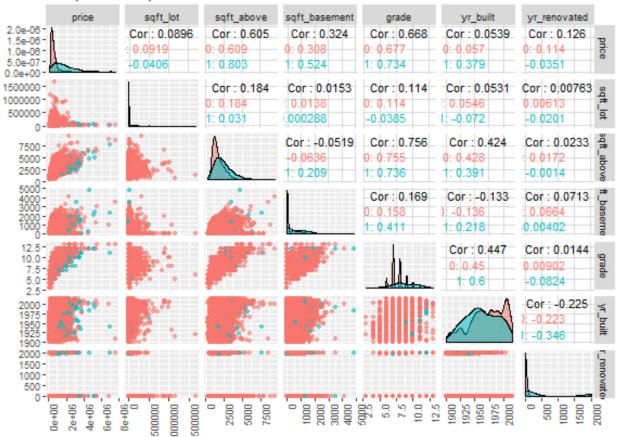
ggpairs(data = house[, c("price", "bedrooms", "bathrooms", "sqft_living",
    "sqft_lot", "sqft_above", "sqft_basement")], title = "Comparison Graph 1",
    aes(colour = as.factor(house$waterfront), alpha = 0.4)) + theme(axis.text.x = element_text(angle = hjust = 1))
```

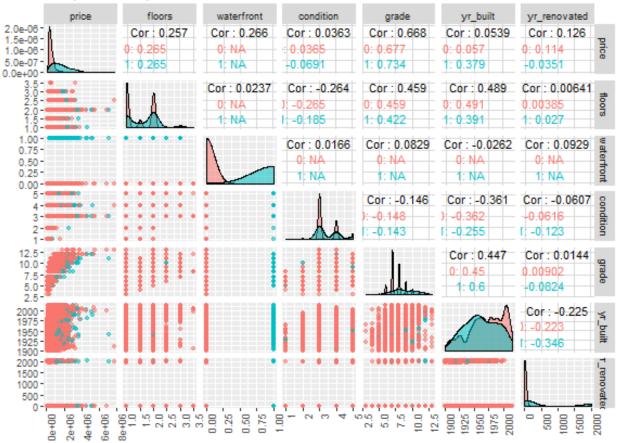


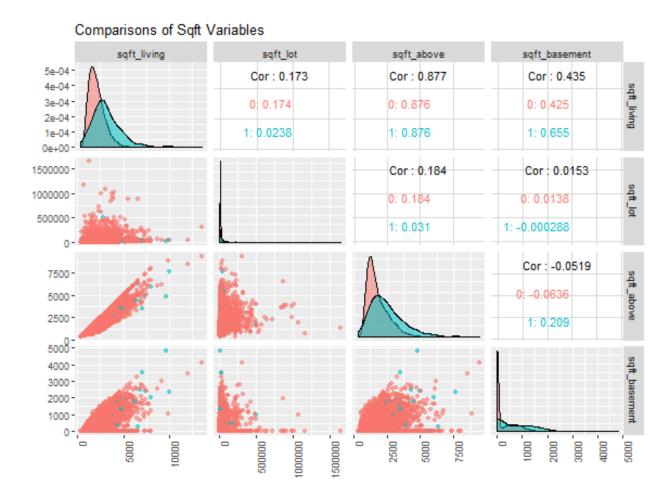












In the plot of Sqft Variables, sqft_living and sqft_above are highly correlated with an R-squared value of 0.877. In general, it is not good to have two x-variables that are highly correlated in the model for that might lead to explanatory redundancy.

Part b (2 pts)

If you could have only one predictor for your linear model, based on your scatterplot matrices in part a which variable is the most desirable one to be included in your model? Explain.

The most desirable single predictor for price is sqft_living, for it has the highest correlation with price, 0.702, amongst all the possible x-variables in house dataset. This means that the x-variable for sqft_living explains the most variation of price.

Part c (2 pts)

From the scatterplots in Part a we see that there is a house with more than 30 bedrooms; what is the living square footage of this house? Also, there are 7 houses recorded with 0 bedroom and 0 bathroom but with positive values for the square footage. All these 8 records are likely to be errors. Add a chunk of code in Part a before the code for making the matrix scatterplots to remove these 8 observations in house and assign the resulting data frame to the same name house; this way, your scatterplot matrices will not include these 8 observations. Check the dimension of the new data frame house to make sure that the replacement was done correctly.

```
# Check dimensions again
dim(house)
[1] 21605 15
```

Looking at Part a, the square footage of the 30+ bedroom house is 1620. The deletion of the erroneous data seems to have been done correctly, for the new dimensions of house has eight fewer observations than before (21613 to 21605).

Part d (7 pts)

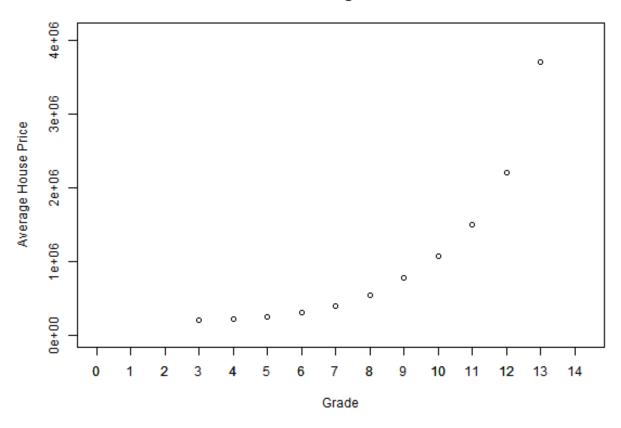
For each of the variables grade and bedrooms, calculate the average house price for each unique value of the variable and make a scatterplot for the average house prices v.s. the unique values of the variable; e.g., grade takes on the integer values 3 through 13 so your scatterplot for grade should have 11 points, the first point should have x-coordinate 3 and y-coordinate the average price for all the houses with grade = 3, the second point should have x-coordinate 4 and the y-coordinate the mean price for all the houses with grade = 4, and so on. Please include the origin for the graph for grade. Do the patterns on the two scatterplots look linear?

```
# Create average house price vectors
prices.grade <- tapply(X = house$price, INDEX = house$grade, FUN = mean)
prices.bedrooms <- tapply(X = house$price, INDEX = house$bedrooms,
    FUN = mean)

# Create Plot of Grade
plot(x = as.numeric(names(prices.grade)), y = prices.grade, main = "Grade vs. Average House Price",
    xlab = "Grade", ylab = "Average House Price", xlim = c(0, 1.1 *
        max(as.numeric(names(prices.grade)))), ylim = c(0, 1.1 *
        max(prices.grade)))

# Set Axis Ticks
axis(side = 1, at = c(0:max(as.numeric(names(prices.grade))))))</pre>
```

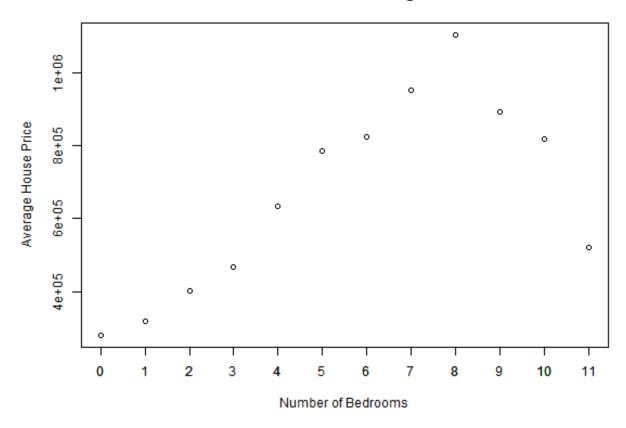
Grade vs. Average House Price



```
# Create Plot of Bedrooms
plot(x = as.numeric(names(prices.bedrooms)), y = prices.bedrooms,
    main = "Number of Bedrooms vs. Average House Price", xlab = "Number of Bedrooms",
    ylab = "Average House Price")

# Set Axis Ticks
axis(side = 1, at = c(0:max(as.numeric(names(prices.bedrooms)))))
```

Number of Bedrooms vs. Average House Price



The pattern for the grade scatterplot looks like there is an exponential relationship between grade and the average price.

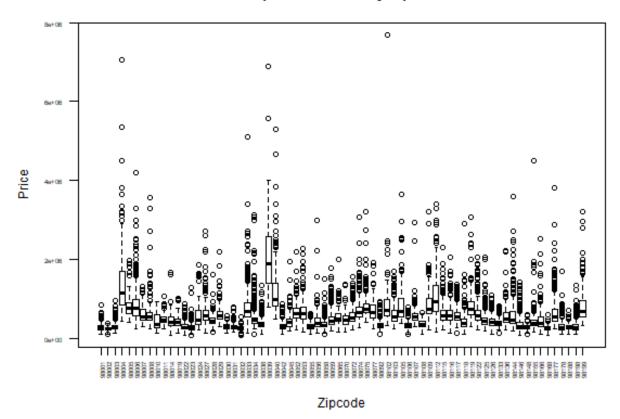
For the bedroom scatterplot, it seems there are two different linear relationships between bedroom and the average price, one positive linear relationship for the bedroom range of [0,8], and a second linear relationship for the bedroom range of (8,11].

Question 3 (24 pts) Zipcode variable

Part a (5 pts; 1 for answer; 2 for explanation)

Make side by side boxplots to compare the values in price by zip code. Based on your boxplots, is it good to include the dummy variables for some of the zip codes in your model? Explain.

Boxplots of Price by Zipcode



Yes, it is good to include the dummy variables for some zip codes because those zip codes seperate the values of price well.

Part b (8 pts; 1 for each answer, 3 for each interpretation)

Consider the following model (you will need to remove eval=F to run the code below):

```
summary(lm(price ~ factor(zipcode), data = house))
lm(formula = price ~ factor(zipcode), data = house)
Residuals:
     Min
                    Median
                                          Max
               1Q
                                  3Q
-1373107 -126733
                    -36653
                              64205
                                      6800605
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)
                       281195
                                    14898
                                           18.875 < 2e-16 ***
factor(zipcode)98002
                       -46911
                                    24992
                                           -1.877 0.060526 .
factor(zipcode)98003
                        12916
                                    22541
                                            0.573 0.566646
factor(zipcode)98004 1074732
                                          49.327
                                    21788
                                                  < 2e-16 ***
```

```
528970
                                            20.009
factor(zipcode)98005
                                    26437
                                                    < 2e-16 ***
factor(zipcode)98006
                        578490
                                    19567
                                            29.565
                                                    < 2e-16 ***
factor(zipcode)98007
                                            11.949
                                                    < 2e-16 ***
                        335910
                                    28111
factor(zipcode)98008
                        364313
                                    22474
                                            16.210 < 2e-16 ***
factor(zipcode)98010
                        142471
                                    31988
                                             4.454 8.47e-06 ***
factor(zipcode)98011
                        209157
                                    25157
                                             8.314 < 2e-16 ***
                                             5.920 3.27e-09 ***
factor(zipcode)98014
                        174422
                                    29464
factor(zipcode)98019
                        143594
                                    25371
                                             5.660 1.53e-08 ***
factor(zipcode)98022
                         34514
                                    23757
                                             1.453 0.146282
factor(zipcode)98023
                          5538
                                    19558
                                             0.283 0.777065
factor(zipcode)98024
                        304814
                                    34979
                                             8.714
                                                   < 2e-16 ***
factor(zipcode)98027
                        335796
                                    20407
                                            16.455 < 2e-16 ***
factor(zipcode)98028
                        181285
                                    22474
                                             8.066 7.61e-16 ***
factor(zipcode)98029
                        331459
                                    21716
                                           15.264
                                                  < 2e-16 ***
factor(zipcode)98030
                         14993
                                    23129
                                             0.648 0.516835
factor(zipcode)98031
                         19146
                                    22704
                                             0.843 0.399080
factor(zipcode)98032
                        -29899
                                    29376
                                            -1.018 0.308791
factor(zipcode)98033
                        522525
                                    20185
                                            25.887
                                                   < 2e-16 ***
factor(zipcode)98034
                        240458
                                    19209
                                            12.518 < 2e-16 ***
factor(zipcode)98038
                         85673
                                    18915
                                             4.529 5.95e-06 ***
factor(zipcode)98039
                       1879412
                                    42714
                                           44.000
                                                   < 2e-16 ***
factor(zipcode)98040
                        913035
                                    22496
                                            40.586
                                                  < 2e-16 ***
                         30437
                                    19188
                                             1.586 0.112689
factor(zipcode)98042
factor(zipcode)98045
                        158276
                                    24177
                                             6.547 6.02e-11 ***
factor(zipcode)98052
                        364037
                                    19014
                                            19.145
                                                   < 2e-16 ***
factor(zipcode)98053
                        395440
                                    20501
                                            19.289
                                                   < 2e-16 ***
factor(zipcode)98055
                         23067
                                    22824
                                             1.011 0.312189
factor(zipcode)98056
                                    20477
                                             6.822 9.21e-12 ***
                        139696
factor(zipcode)98058
                         72414
                                    19951
                                             3.630 0.000285 ***
factor(zipcode)98059
                        212358
                                    19828
                                            10.710
                                                   < 2e-16 ***
factor(zipcode)98065
                        247714
                                    21938
                                            11.292 < 2e-16 ***
factor(zipcode)98070
                        206285
                                    30016
                                             6.872 6.49e-12 ***
                        288764
                                    22704
                                            12.719
                                                   < 2e-16 ***
factor(zipcode)98072
factor(zipcode)98074
                        404411
                                    20091
                                            20.129
                                                   < 2e-16 ***
                                            24.143
                                                    < 2e-16 ***
factor(zipcode)98075
                        509382
                                    21098
factor(zipcode)98077
                        401580
                                    25033
                                            16.042
                                                    < 2e-16 ***
factor(zipcode)98092
                         53726
                                    21219
                                             2.532 0.011348 *
                                            19.624
factor(zipcode)98102
                        618201
                                    31502
                                                    < 2e-16 ***
factor(zipcode)98103
                        303633
                                    18849
                                            16.109
                                                    < 2e-16 ***
factor(zipcode)98105
                        581630
                                    23913
                                            24.322
                                                    < 2e-16 ***
                                             1.788 0.073858 .
factor(zipcode)98106
                         38387
                                    21474
factor(zipcode)98107
                        297859
                                    22873
                                           13.022
                                                  < 2e-16 ***
factor(zipcode)98108
                                             2.915 0.003556 **
                         74484
                                    25549
factor(zipcode)98109
                        598429
                                    30936
                                            19.344
                                                    < 2e-16 ***
                        814305
                                            35.716
                                                    < 2e-16 ***
factor(zipcode)98112
                                    22800
factor(zipcode)98115
                        338706
                                    18958
                                            17.866
                                                    < 2e-16 ***
factor(zipcode)98116
                        337439
                                    21558
                                            15.652
                                                    < 2e-16 ***
factor(zipcode)98117
                        295600
                                    19153
                                            15.433
                                                   < 2e-16 ***
factor(zipcode)98118
                        136443
                                    19485
                                             7.002 2.59e-12 ***
                                            22.163
                                                   < 2e-16 ***
factor(zipcode)98119
                        568253
                                    25640
factor(zipcode)98122
                        353165
                                    22322
                                            15.822
                                                    < 2e-16 ***
factor(zipcode)98125
                        188261
                                    20430
                                             9.215 < 2e-16 ***
factor(zipcode)98126
                                    21173
                                             6.778 1.25e-11 ***
                        143512
```

```
factor(zipcode)98133
                       105817
                                    19608
                                            5.397 6.86e-08 ***
factor(zipcode)98136
                       270494
                                    22948
                                           11.787
                                                  < 2e-16 ***
factor(zipcode)98144
                                           14.681 < 2e-16 ***
                       313353
                                    21344
factor(zipcode)98146
                        78288
                                    22364
                                            3.501 0.000465 ***
factor(zipcode)98148
                                    40344
                                            0.092 0.926659
                         3714
factor(zipcode)98155
                       142531
                                    20040
                                            7.112 1.18e-12 ***
factor(zipcode)98166
                       183037
                                    23182
                                            7.896 3.03e-15 ***
factor(zipcode)98168
                       -40867
                                    22800
                                           -1.792 0.073078 .
factor(zipcode)98177
                       394991
                                    23155
                                           17.058 < 2e-16 ***
factor(zipcode)98178
                        29418
                                    22973
                                            1.281 0.200376
factor(zipcode)98188
                         7884
                                    28480
                                            0.277 0.781931
factor(zipcode)98198
                        21684
                                    22541
                                            0.962 0.336079
factor(zipcode)98199
                       510626
                                    21788
                                           23.436 < 2e-16 ***
Signif. codes:
                0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 283100 on 21535 degrees of freedom
Multiple R-squared: 0.4074,
                                 Adjusted R-squared: 0.4055
F-statistic: 214.5 on 69 and 21535 DF, p-value: < 2.2e-16
```

What is the estimate for the y-intercept of the model? Interpret the meaning of the y-intercept. What is the coefficient estimate for the dummy variable for zip code 98004? interpret this number too.

The estimate of the y-intercept of the model is 281195. The y-intercept merely represents the average price of homes that are in the zipcode that does not have a dummy variable associated with it (This happens to be the zipcode with the lowest number in the dataset, 98001). The coefficient estimate for the dummy variable for zipcode 98004 is 1074732. This number is how much the average price of homes in the zipcode 98004 is greater than the estimate of the y-intercept.

Part c (4 pts)

The p-value on the third row of the table in the Coefficients table is 0.566646. State the Null hypothesis of the test that this p-value is for. Does the p-value suggest an inclusion or exclusion of the dummy variable $1_{zipcode=98003}$ to the model?

Null Hypoethsis: This dummy variable estimate is equal to 0.

The p-value suggests exclusion of the dummy variable from the model.

Part d (2 pts)

Answer TRUE or FALSE only for the following statement (no explanation is required):

The p-value (56.6665%) in part c above is the estimated chance that the coefficient for the dummy variable for zipcode 98003 is 0; i.e., the p-value is the chance that the Null hypothesis is true.

False.

Part e (5 pts)

For the model shown in Part b how many of the coefficient estimates are significant (i.e., with p-values less than $\alpha = .05$)? Use Bonferroni Correction factor and the method for controlling for False Discovery Rate (FDR) to achieve new cutoffs for the p-values. For each of the two methods, report the adjusted cutoff for the p-values and the number of coefficient estimates that are with p-values less than the new cutoff.

```
# Make vector of p-values
p.v.zip <- summary(lm(price ~ factor(zipcode), data = house))$coefficients[,</pre>
# How many p-values are significant?
sum(p.v.zip < 0.05)
[1] 55
# Bonferroni Correction Factor
sum(p.v.zip < (0.05/length(p.v.zip)))
[1] 53
# Cutoff P-value of Benferroni Correction
max(p.v.zip[p.v.zip < (0.05/length(p.v.zip))])</pre>
[1] 0.000465191
# FDR
sort.p.zip <- sort(p.v.zip)</pre>
length(sort.p.zip[sort.p.zip <= ((1:length(sort.p.zip)/length(sort.p.zip))) *</pre>
[1] 55
# P-value cutoff of FDR
max(sort.p.zip[sort.p.zip <= ((1:length(sort.p.zip)/length(sort.p.zip))) *</pre>
[1] 0.01134761
```

For the Bonferroni Correction Factor, 53 p-values are now significant, with a maximum p-value cutoff of 0.000465191. For the False Discovery Rate, 55 p-values are significant with a new p-value cutoff of 0.01134761.

Transforming some of the variables and creating additional ones

Question 4 (9 pts)

As we saw in question 2 some of the variables do not have a linear relationship with price. We will transform these variables in this question.

Part a (4 pts)

From the scatterplot for the average house prices v.s. the unique values in grade in question 2.d we see that the relationship between the two variables could be approximated by the equation

$$price = (grade)^b$$

for some constant b.

Taking log on both sides results in

$$log(price) = b \times log(grade)$$

Find out what the best value for b should be according to the data; this value should minimize $(log(price_i) - b \times log(grade_i))^2$ for the data points on average.

```
# Take logs of price vector and grade vector
log.price <- log(house$price)</pre>
log.grade <- log(house$grade)</pre>
# Create linear model without an intercept
summary(lm(log.price ~ log.grade - 1))
Call:
lm(formula = log.price ~ log.grade - 1)
Residuals:
    Min
            1Q Median
                             3Q
                                    Max
-2.4556 -0.4249 0.0314 0.4943 5.4853
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
log.grade 6.423823 0.002374
                                 2706 <2e-16 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 0.7083 on 21604 degrees of freedom
Multiple R-squared: 0.9971,
                              Adjusted R-squared: 0.9971
F-statistic: 7.321e+06 on 1 and 21604 DF, p-value: < 2.2e-16
```

The best value for b should be 6.4238

Part b (2 pts)

Use the value that you found for b in part a and create the variable trans.grade by transforming grade into $trans.grade = (grade)^b$.

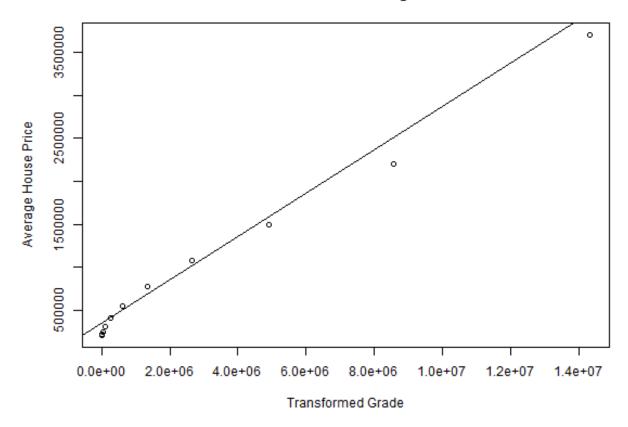
Make the scatterplot that you made for the average house prices v.s. the unique values in grade in question 2.d again, except that now replace the unique values of grade with that of trans.grade. Verify that the scatterplot now shows a more linear pattern.

```
# Create `trans.grade`
trans.grade = house$grade^summary(lm(log.price ~ log.grade - 1))$coefficients[1]

# Create Plot of Grade
plot(x = sort(unique(trans.grade)), y = prices.grade, main = "Transformed Grade vs. Average House Price xlab = "Transformed Grade", ylab = "Average House Price", xlim = c(0, max(trans.grade)))

# Add linear regression to verify linearity
abline(a = lm(house$price ~ trans.grade)$coefficients[1], b = lm(house$price ~ trans.grade)$coefficients[2])
```





The scatterplot now shows a more linear pattern with the transformed grade.

Part c (3 pts) Creating new variables for the model

Here, we will make a new data frame for all the variables that we will need for building the model.

We will create the new data frame mod.variables with all the transformed variables and some of the original variables. mod.variables should include price, sqft_living, sqft_basement, grade, bedrooms, bathrooms and date from the data frame house plus the following transformed variables:

- f.waterfront: the factor version of house\$waterfront;
- f.floor: the factor version of house\$floor:
- f.cond: the factor version of house\$condition;
- f.renov: a factor vector and each element in f.renov is 1 if the corresponding element in house\$yr_renovated does not equal to zero, and 0 otherwise;
- f.bdrm.less.eq.8: a factor vector and each element in f.bdrm.less.eq.8 is 1 if the corresponding element in house\$bedrooms is less or equal to 8, and 0 otherwise;
- f.zipcode: the factor version of house\$zipcode;

and also the transformed variable

• trans.grade: the vector trans.grade that you have already made.

```
# Create new dataframe `mod.variables`
mod.variables <- data.frame(price = house$price, sqft_living = house$sqft_living,
    sqft_basement = house$sqft_basement, grade = house$grade, bedroom = house$bedroom,
    bathroom = house$bathroom, date = house$date, f.waterfront = as.factor(house$waterfront),
    f.floor = as.factor(house$floor), f.cond = as.factor(house$condition),
    f.renov = as.factor(as.numeric(house$yr_renovated != 0)), f.bdrm.less.eq.8 = as.factor(as.numeric(house$zipcode), trans.grade = trans.grade)</pre>
```

Divide data into three subsets: one training set and two test sets.

For this part you just need to remove the eval=F argument for each of the code chunks below and run the code; no action is required from you other than that.

mod.variables covers the period from May 2014 to May 2015. We will prepare a vector date.format that we can use to extract out the observations that correspond to transactions closed in May 2015.

Then, the following lines will extract out all the observations with transactions closed in May 2015. We will use these observations for our out-of-time test set test2. test2 should be 646 by 13.

For the rest of the observations run the following code chunk; this will split the remaining observations into 2 sets: 85% training set and 15% in-time test set.

```
test1 = tmp[permu, ][(round(s * 0.85) + 1):s, ]
dim(test1)
[1] 3144 13
```

Model selection

We are now ready to build our model! We will use BIC and Adjusted R^2 as our criteria for our model selection. mod.variables has been divided into 3 sets:

- train.val: The 17815 by 13 training plus validation set
- test1: The 3144 by 13 in-time test set
- test2: The 646 by 13 out-of-time test set

For the rest of the project we will build our model with the training data, and test the performance of our chosen model on the 2 test sets.

Question 5 (27 pts)

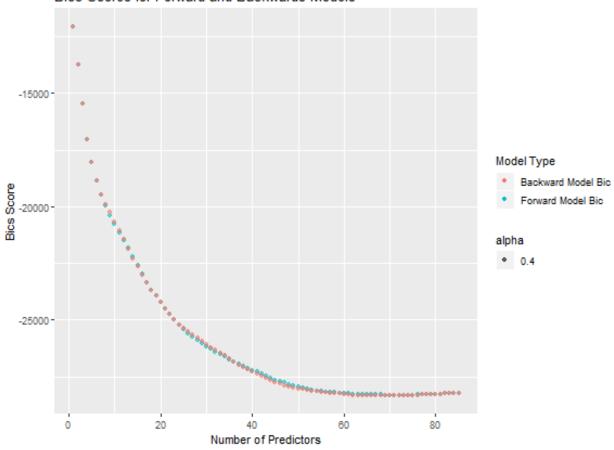
Part a (7 pts) Consider models with the original data only

In this part please consider only the predictors that already exist in the original dataset (i.e., all the variables in train except f.bdrm.less.eq.8 and trans.grade).

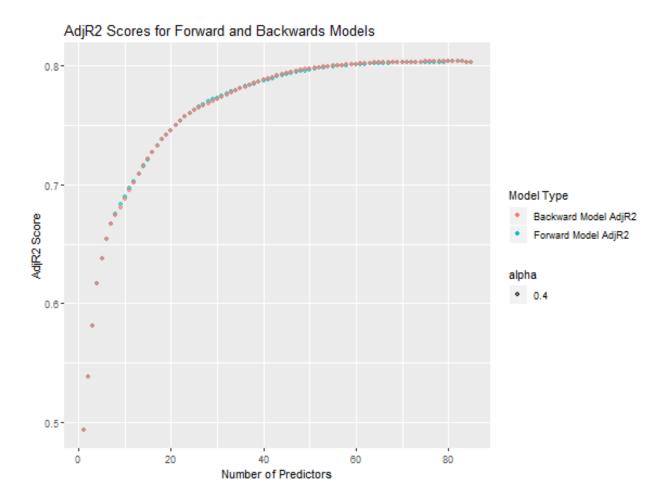
Use BIC and Adjusted R^2 as the criteria to evaluate the performance of your "best" models. Do backward and forward selection algorithms produce similar (in terms of the values for BIC and Adjusted R^2) results in this case?

```
# Calculate number of coefficients in the model besides the
# intercept
model.length <- length(lm(price ~ . + f.waterfront + f.floor + f.cond +
    f.renov + f.zipcode - trans.grade - f.bdrm.less.eq.8, data = train)$coef) -
# Load `leaps` package
library("leaps")
# Build Forward Model
model.forward <- regsubsets(price ~ . + f.waterfront + f.floor +</pre>
    f.cond + f.renov + f.zipcode - trans.grade - f.bdrm.less.eq.8,
    data = train, method = "forward", nvmax = model.length)
# Build Backward Model
model.backward <- regsubsets(price ~ . + f.waterfront + f.floor +</pre>
    f.cond + f.renov + f.zipcode - trans.grade - f.bdrm.less.eq.8,
    data = train, method = "backward", nvmax = model.length)
# Build a Dataframe for Easy GGplot Use
df.models <- data.frame(y = c(summary(model.forward)$bic, summary(model.backward)$bic,</pre>
```

Bics Scores for Forward and Backwards Models



```
# GGplot of AdjR2 Performance
ggplot(df.models[df.models$model.type == "Backward Model AdjR2" |
    df.models$model.type == "Forward Model AdjR2", ]) + geom_point(aes(x = x,
    y = y, color = model.type, alpha = 0.4)) + labs(title = "AdjR2 Scores for Forward and Backwards Mod
    xlab(label = "Number of Predictors") + ylab(label = "AdjR2 Score") +
    scale_color_discrete(name = "Model Type")
```



Looking at the plots of the Bic and Adjusted R-squared values of the forward and backward models, there seems to be little difference in terms of either metric between the 'best' models for any given number of predictors.

Part b (2 pts)

Print out the coefficient estimates for the best model with 7 predictors for each algorithm; is f.waterfront1 included in both models?

```
# Best Forward Model w/ 7 Predictors
coef(model.forward, id = 7)
   (Intercept)
                  sqft_living
                                       grade f.waterfront1
  -494755.0373
                                  87229.4192
                                                 893290.1133
                     162.1336
f.zipcode98004 f.zipcode98039 f.zipcode98040 f.zipcode98112
                 1115386.5017
   605666.9138
                                  355140.8267
                                                 427776.8863
# Best Backward Model w/ 7 Predictors
coef(model.backward, id = 7)
                                       grade
   (Intercept)
                  sqft_living
                                              f.waterfront1
  -494755.0373
                     162.1336
                                  87229.4192
                                                 893290.1133
f.zipcode98004 f.zipcode98039 f.zipcode98040 f.zipcode98112
   605666.9138
                 1115386.5017
                                 355140.8267
                                                 427776.8863
```

Yes, f.waterfront1 is included in both models.

Part c (3 pts)

Go back to modify your matrices of scatterplots in question 2.a to color the points by the case whether the house has a view to a waterfront. Note that now the plots indicates that it might be good to consider the following interaction terms for your model: f.waterfront:sqft_living, f.waterfront:bathrooms. Also, from question 2.d we see that it would be good to consider (f.bdrm.less.eq.8:bedrooms) too.

Part d (5 pts) Consider models with the additional variables

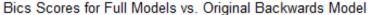
In this part we want to see if including additional variables that are not defined in the original dataset will improve the performance of the model. Repeat part a but now consider all the variables in the data frame train plus the interaction terms f.waterfront:sqft_living, f.waterfront:bathrooms and f.bdrm.less.eq.8:bedrooms.

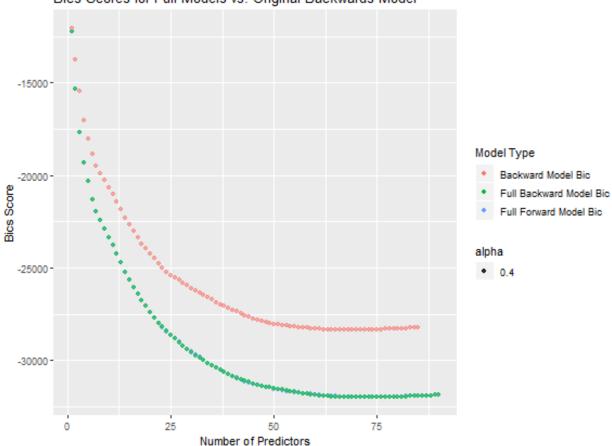
Pick the results from the better performing algorithm (if both algorithms perform similarly just pick either one) in part a and add the results to the plots for BIC and Adjusted R^2 for the current models too; this will allow you to see how much the performance of each of the best models improves by considering the additional variables.

As you can see from the graphs creating the right variables helps to improve your model significantly!

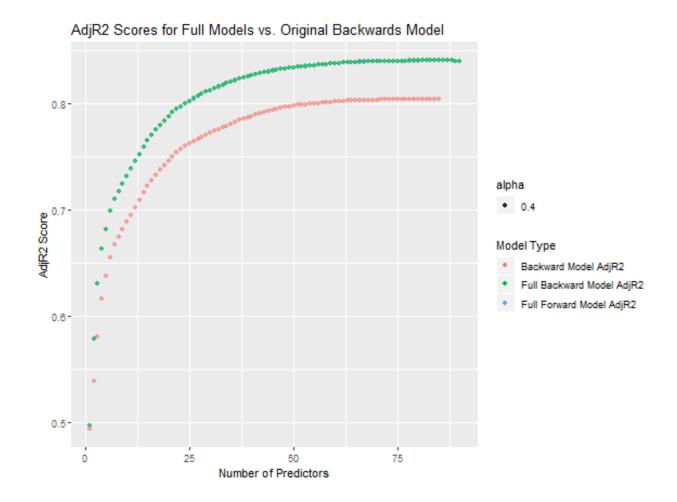
```
# Calculate Model Length of Full Model
num.var.full <- length(lm(price ~ . + f.waterfront + f.floor + f.cond +</pre>
   f.renov + f.zipcode + f.bdrm.less.eq.8 + f.waterfront:sqft_living +
   f.waterfront:bathroom + f.bdrm.less.eq.8:bedroom, data = train)$coef) -
# Create Forward Model w/ New Variables
model.forward.full <- regsubsets(price ~ . + f.waterfront + f.floor +</pre>
    f.cond + f.renov + f.zipcode + f.bdrm.less.eq.8 + f.waterfront:sqft_living +
   f.waterfront:bathroom + f.bdrm.less.eq.8:bedroom, data = train,
   method = "forward", nvmax = num.var.full)
# Create Backward Model w/ New Variables
model.backward.full <- regsubsets(price ~ . + f.waterfront + f.floor +</pre>
    f.cond + f.renov + f.zipcode + f.bdrm.less.eq.8 + f.waterfront:sqft_living +
   f.waterfront:bathroom + f.bdrm.less.eq.8:bedroom, data = train,
    method = "backward", nvmax = num.var.full)
# Create Dataframe for Easy GGplot Use
df.models.full = data.frame(y = c(summary(model.forward.full)$bic,
    summary(model.backward.full)$bic, summary(model.forward.full)$adjr2,
    summary(model.backward.full)$adjr2), x = c(rep(1:num.var.full,
    4)), model.type = c(rep("Full Forward Model Bic", num.var.full),
   rep("Full Backward Model Bic", num.var.full), rep("Full Forward Model AdjR2",
        num.var.full), rep("Full Backward Model AdjR2", num.var.full)))
# Plot Bic Scores of All Models
ggplot(df.models.full[df.models.full$model.type == "Full Backward Model Bic" |
   df.models.full$model.type == "Full Forward Model Bic", ]) +
    geom point(aes(x = x, y = y, color = model.type, alpha = 0.4)) +
   geom_point(data = df.models[df.models$model.type == "Backward Model Bic",
        ], aes(x = x, y = y, color = model.type, alpha = 0.4)) +
   labs(title = "Bics Scores for Full Models vs. Original Backwards Model") +
```

```
xlab(label = "Number of Predictors") + ylab(label = "Bics Score") +
scale_color_discrete(name = "Model Type")
```





```
# Plot AdjR2 Scores of All Models
ggplot(df.models.full[df.models.full$model.type == "Full Backward Model AdjR2" |
    df.models.full$model.type == "Full Forward Model AdjR2", ]) +
    geom_point(aes(x = x, y = y, color = model.type, alpha = 0.4)) +
    geom_point(data = df.models[df.models$model.type == "Backward Model AdjR2",
        ], aes(x = x, y = y, color = model.type, alpha = 0.4)) +
    labs(title = "AdjR2 Scores for Full Models vs. Original Backwards Model") +
    xlab(label = "Number of Predictors") + ylab(label = "AdjR2 Score") +
    scale_color_discrete(name = "Model Type")
```



Part e (2 pt)

What is the purpose of using measures, such as BIC or Adjusted R-square, to evaluate the performance of a model? Why don't we just use RSS instead?

The purpose of using more advances measures such as BIC and Adjusted R-square is to reduce the chance of overfitting. Both of these measurements penalize the use of too many predictors, which RSS doesn't do.

Part f (4 pts: 3pts for the choice and 1 for the print out) The champion model

Propose your champion model and justify your choice. Print out the coefficient estimates for your model.

```
# Determine Minimum Bic Score of all the Backward Models
min(summary(model.backward.full)$bic)
[1] -31996.21

# Determine Minimum Bic Score of all the Forward Models
min(summary(model.forward.full)$bic)
[1] -31984.11

# Determine Which Model has the Lowest Bic Score
which.min(summary(model.backward.full)$bic)
```

```
[1] 73
# Print Coefficient Estimates
coef(model.backward.full, id = which.min(summary(model.backward.full)$bic))
              (Intercept)
                                         sqft_living
            -3.834670e+05
                                        1.701558e+02
            sqft_basement
                                            bathroom
            -3.952260e+01
                                        1.829877e+04
            f.waterfront1
                                            f.floor2
            -2.124835e+05
                                       -3.643444e+04
               f.floor2.5
                                            f.floor3
             6.236239e+04
                                       -8.321621e+04
                  f.cond4
                                             f.cond5
             3.244811e+04
                                        7.820127e+04
                 f.renov1
                                   f.bdrm.less.eq.81
             6.983655e+04
                                        3.198346e+05
           f.zipcode98004
                                      f.zipcode98005
                                        2.905656e+05
             7.384613e+05
           f.zipcode98006
                                      f.zipcode98007
             2.472365e+05
                                        2.334408e+05
           f.zipcode98008
                                      f.zipcode98010
             2.545519e+05
                                        5.057717e+04
           f.zipcode98011
                                      f.zipcode98014
             1.200722e+05
                                        1.030834e+05
           f.zipcode98019
                                      f.zipcode98023
             8.217335e+04
                                       -4.330957e+04
           f.zipcode98024
                                      f.zipcode98027
             1.664390e+05
                                        1.492438e+05
           f.zipcode98028
                                      f.zipcode98029
             1.266475e+05
                                        2.060299e+05
           f.zipcode98033
                                      f.zipcode98034
             3.464777e+05
                                        1.884824e+05
                                      f.zipcode98039
           f.zipcode98038
             2.748814e+04
                                        1.184619e+06
           f.zipcode98040
                                      f.zipcode98045
             4.844281e+05
                                        9.443042e+04
           f.zipcode98052
                                      f.zipcode98053
             2.217685e+05
                                        1.993487e+05
           f.zipcode98055
                                      f.zipcode98056
             3.518707e+04
                                        8.410252e+04
           f.zipcode98059
                                      f.zipcode98065
             6.969822e+04
                                        1.007930e+05
           f.zipcode98070
                                      f.zipcode98072
             9.883942e+04
                                        1.442673e+05
           f.zipcode98074
                                      f.zipcode98075
             1.685156e+05
                                        1.662235e+05
           f.zipcode98077
                                      f.zipcode98092
             1.095847e+05
                                       -3.277016e+04
           f.zipcode98102
                                      f.zipcode98103
             5.045550e+05
                                        3.341012e+05
           f.zipcode98105
                                      f.zipcode98106
             4.637457e+05
                                        1.086031e+05
           f.zipcode98107
                                      f.zipcode98108
```

```
3.443664e+05
                                     1.048291e+05
        f.zipcode98109
                                   f.zipcode98112
                                     5.913112e+05
          5.138048e+05
        f.zipcode98115
                                   f.zipcode98116
          3.339462e+05
                                     3.084508e+05
        f.zipcode98117
                                   f.zipcode98118
          3.163935e+05
                                     1.601251e+05
        f.zipcode98119
                                   f.zipcode98122
          4.974673e+05
                                     3.298756e+05
        f.zipcode98125
                                   f.zipcode98126
          2.031170e+05
                                     1.904924e+05
        f.zipcode98133
                                   f.zipcode98136
          1.474208e+05
                                     2.645969e+05
        f.zipcode98144
                                   f.zipcode98146
          2.890333e+05
                                     1.075490e+05
        f.zipcode98155
                                   f.zipcode98166
          1.296301e+05
                                     5.454169e+04
        f.zipcode98168
                                   f.zipcode98177
          4.488097e+04
                                     2.384070e+05
        f.zipcode98178
                                   f.zipcode98199
          4.819452e+04
                                     3.938892e+05
           trans.grade sqft_living:f.waterfront1
          1.020992e-01
                                     2.720194e+02
bathroom:f.waterfront1 bedroom:f.bdrm.less.eq.81
          6.801565e+04
                                    -1.241958e+04
```

I evaluate the best model here on the lowest BIC score alone, for BIC is motivated more by statistical theory than Adjusted R2 (StatLearn 213). Since the lowest BIC score of the backward models is lower than the lowest BIC score of the forward model (-31996.21 < -31984.11), I will elect to use this backward model.

Part g (4 pts)

Let k be the number of predictors that you chose for your champion model. Compare the best models with k predictors selected by the backward and forward algorithms. Do the algorithms pick the same set of predictors? Are there any predictors that are selected by the forward algorithm but not by the backward? List these variables if there are any. Similarly, are there any predictors that are selected by the backward algorithm but not by the forward? List the variables too if there are any.

```
[1] "bedroom" "f.cond3" "f.zipcode98003" [4] "f.zipcode98042"
```

The variables in the backward that are not in the forward model are "f.zipcode98010", "f.zipcode98038", "f.zipcode98055", and "bedroom:f.bdrm.less.eq.81". The variables in the forward model that are not in the backward model are "f.cond3", "f.zipcode98003", "f.zipcode98042", and "bedroom".

Question 6 (10 pts)

Part a (6 pts) Checking assumptions

For your champion model plot the residual plot, the qq plot and the histogram for the distribution of the residuals. What are the assumptions on the distribution of the errors for a linear model? Do your plots suggest any possible violation of the assumptions? Explain.

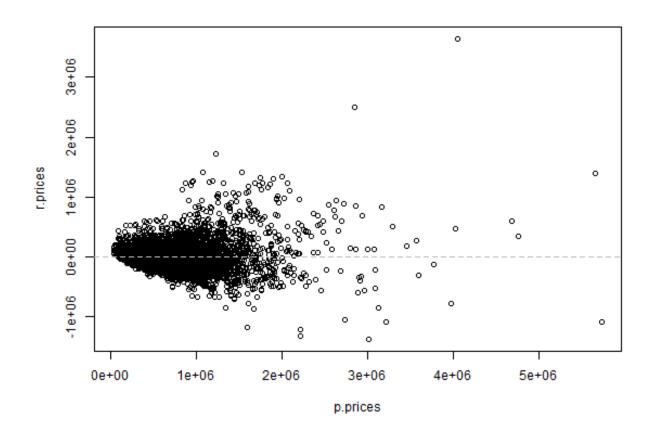
You can use the following function predict.reg to make prediction on a dataset with a model produced by regsubsets(). It works similarly to the predict() function except that it has an extra input argument id that specifies which best model you would like to use for making the predictions. E.g., say g is an output of regsubsets(), then predict.reg(object = g,newdata = d,id = 5) predicts for the data in dataset d by using the best model in g for 5 predictors.

You do not need to understand how the function is being coded

```
predict.reg = function(object, newdata, id) {
    form = as.formula(object$call[[2]]) # Extract the formula used when we called regsubsets()
    mat = model.matrix(form, newdata) # Build the model matrix
    coefi = coef(object, id = id) # Extract the coefficients of the id'th model
    xvars = names(coefi) # Pull out the names of the predictors used in the ith model
    mat[, xvars] %*% coefi # Make predictions using matrix multiplication
}
p.prices <- predict.reg(object = model.backward.full, newdata = train,
    id = as.numeric(which.min(summary(model.backward.full)$bic)))

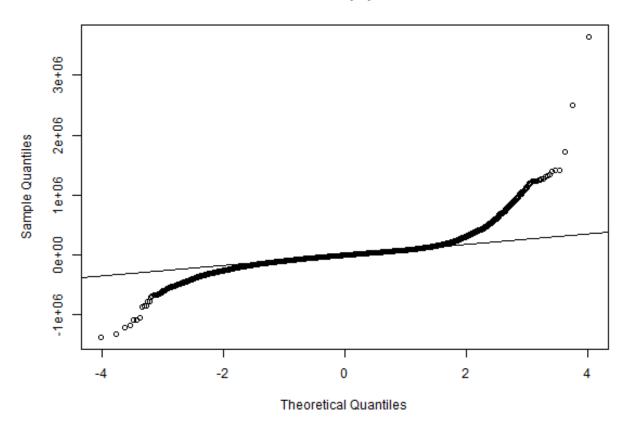
r.prices <- train$price - p.prices

plot(x = p.prices, y = r.prices)
abline(a = 0, b = 0, lty = 2, col = "grey")</pre>
```



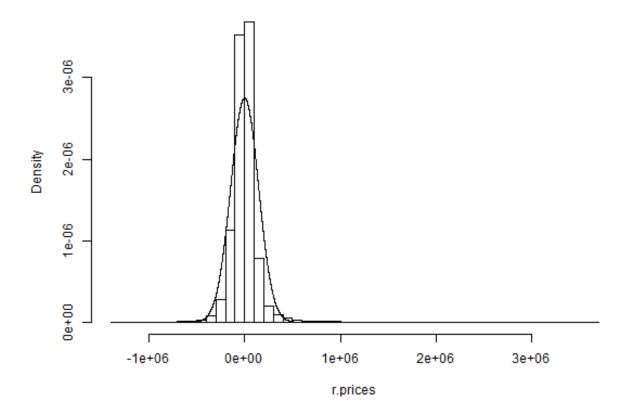
```
qqnorm(y = r.prices)
qqline(y = r.prices)
```

Normal Q-Q Plot



```
hist(r.prices, breaks = 60, freq = F)
r.prices.seq = seq(from = min(r.prices), to = max(r.prices), length = length(r.prices))
lines(y = dnorm(r.prices.seq, mean = mean(r.prices), sd = sd(r.prices)),
    x = r.prices.seq)
```

Histogram of r.prices



For a linear model, we assume that the errors are independent, and that the errors are normally distributed with a constand standard deviation. The plot of the residuals suggests that there is a non-constant standard deviation, which would indicate there is a violation of our assumptions. The qq-plot and histogram reaffirm this observation, as we do not observe a straight line with a slope of 1 in the qq-plot and we can see that the distribution of the residuals is not normal, with a greater concentration of residuals around the mean than in a normal distribution.

Part b (4 pts)

Estimate the mean squared errors for your champion model with the in-time and out-of-time test sets. Is the MSE estimated with the out-of-time test set bigger or smaller than the MSE estimated with the in-time test set? Is this result expected?

```
# Create Vector of Predicted Prices for In-Time Test
p.intime <- predict.reg(object = model.backward.full, newdata = test1,
    id = as.numeric(which.min(summary(model.backward.full)$bic)))

# Create Vector of Predicted Prices for Out-of-Time Test
p.outtime <- predict.reg(object = model.backward.full, newdata = test2,
    id = as.numeric(which.min(summary(model.backward.full)$bic)))

# Calculate MSE
mse.intime <- mean((test1$price - p.intime)^2)</pre>
```

```
mse.outtime <- mean((test2$price - p.outtime)^2)

# Take Ratio Between MSEs
mse.intime/mse.outtime
[1] 0.7190626</pre>
```

The MSE estimated with the out of time test is bigger than the MSE estimated with the in-time test. This result is expected, for data sampled from the same time frame as the training set should have similar prices as a function of their attributes compared to data sampled from another time frame.

Model interpretation

Question 7 (9 pts; 3 pts for each part)

For the following model answer TRUE or FALSE for the questions below; no explanation is required and you will only be graded based on the TRUE and FALSE part of your answer even if you provided explanations.

```
Call:
```

```
lm(formula = price ~ f.waterfront * sqft_living, data = mod.variables)
```

Coefficients:

```
(Intercept) f.waterfront1
-16206.9 -267588.0
sqft_living f.waterfront1:sqft_living
264.4 348.6
```

(i) **TRUE or FALSE** The model above gives different y-intercepts (for predicting the house price) for houses with and without a view to a waterfront.

True

- (ii) **TRUE or FALSE** The model above gives different slopes depending on if the house has a view to a waterfront or not.

 True
- (iii) **TRUE or FALSE** Using the model shown above to predict the **price** of a 1000 sqft house that has a view to a waterfront we calculate the estimated price this way:

```
Predicted price = -16206.9 - 267588 + 264.4 + 348.6 \times 1000.
```

False