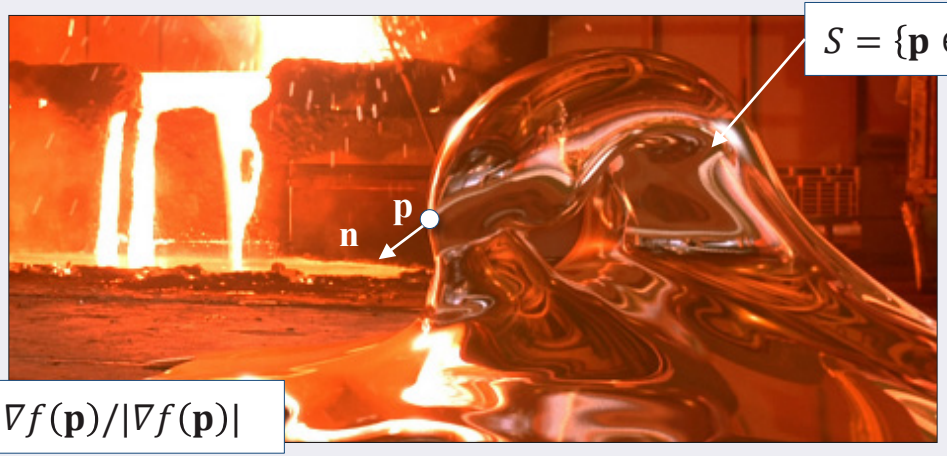
Computer Graphics

From mathematics …



… to the screen

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Computer Graphics

Core
Modeling
Ray Tracing
Meshing

Computer Graphics

Introduction

Visualization

$$S = \{\mathbf{p} \in \boldsymbol{R}^3 | f(\mathbf{p}) = 0\}$$

Polygonization [Araujo2015] converts model to **large** meshes

Direct **ray tracing** remains computationally **intensive**
Parallel implementation partially alleviate the problem

Ladybug
© Inigo Quilez

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Ray marching

Ray marching

Algorithm

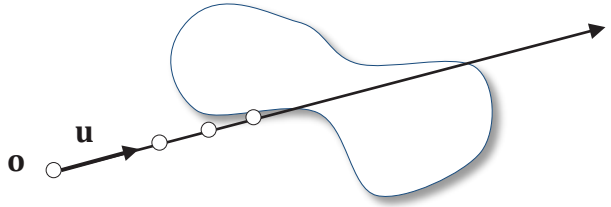
General but **slow** technique [Perlin1989]
No assumptions about the mathematical properties of f
Adapted for computing the intersection with fractal objects

Implicit surface S

$$S = \{\mathbf{p} \in \mathbf{R}^3 | f(\mathbf{p}) = 0\}$$

Ray Δ

$$\Delta: \boldsymbol{\delta}(t) = \mathbf{o} + \mathbf{u}t \mid t \in [0, \infty]$$



March along the ray at positions $\mathbf{p}_k$

Ray marching

General algorithm that steps and evaluates f at points $\mathbf{p}$ along the ray

Computationally **intensive** with **fixed step** [Perlin1989]
Robust Lipschitz techniques with **global** constants [Kalra1989]

K. Perlin, E. Hoffert. Hypertexture. *ACM SIGGRAPH Computer Graphics*, **23**(3), 1989.
D. Kalra, H. Barr. Guaranteed Ray Intersections with Implicit Surfaces. *ACM SIGGRAPH Computer Graphics*, **23**(3), 1989.

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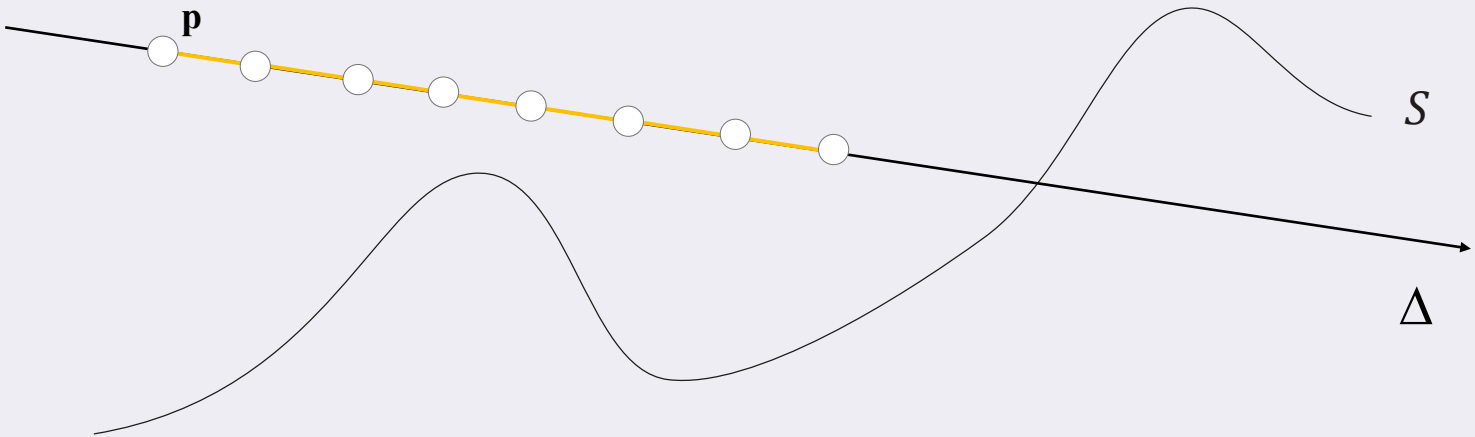
Ray marching: visual interpretation



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Sphere tracing

Sphere Tracing

Lipschitz functions

$\exists \lambda > 0 \ \ \forall (\mathbf{p}, \mathbf{q}) \in \mathbf{R}^3 \times \mathbf{R}^3 \ |f(\mathbf{p}) - f(\mathbf{q})| \leq \lambda |\mathbf{p} - \mathbf{q}|$

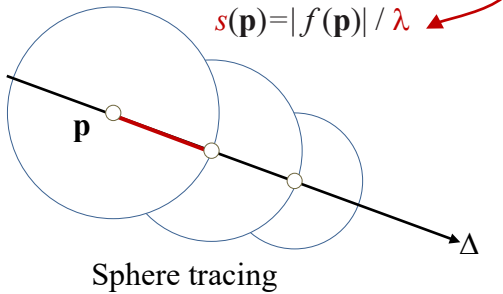
Exclusion criterion: $|f(\mathbf{p})| / \lambda$ is a signed distance bound to S

$\forall \mathbf{p} \in \mathbf{R}^3 \ B(\mathbf{p}, |f(\mathbf{p})/\lambda|) \cap S = \emptyset$

Global bound λ over $\mathbf{R}^3$

Sphere Tracing [Hart1996]

Start from ray origin $\mathbf{p} = \mathbf{o}$
At every step i
If $f(\mathbf{p}_i) < 0$ then
Intersection found
Otherwise step forward
$s(\mathbf{p}_i) = |f(\mathbf{p}_i)|/\lambda$



Sphere tracing

Sphere Tracing [Hart1989, Hart1996, Keinert2014] **adapts** step with **global** constants

J. Hart. Sphere Tracing: A Geometric Method for the Antialiased Ray Tracing of Implicit Surfaces. *The Visual Computer* **12**(10), 527-545,1996,

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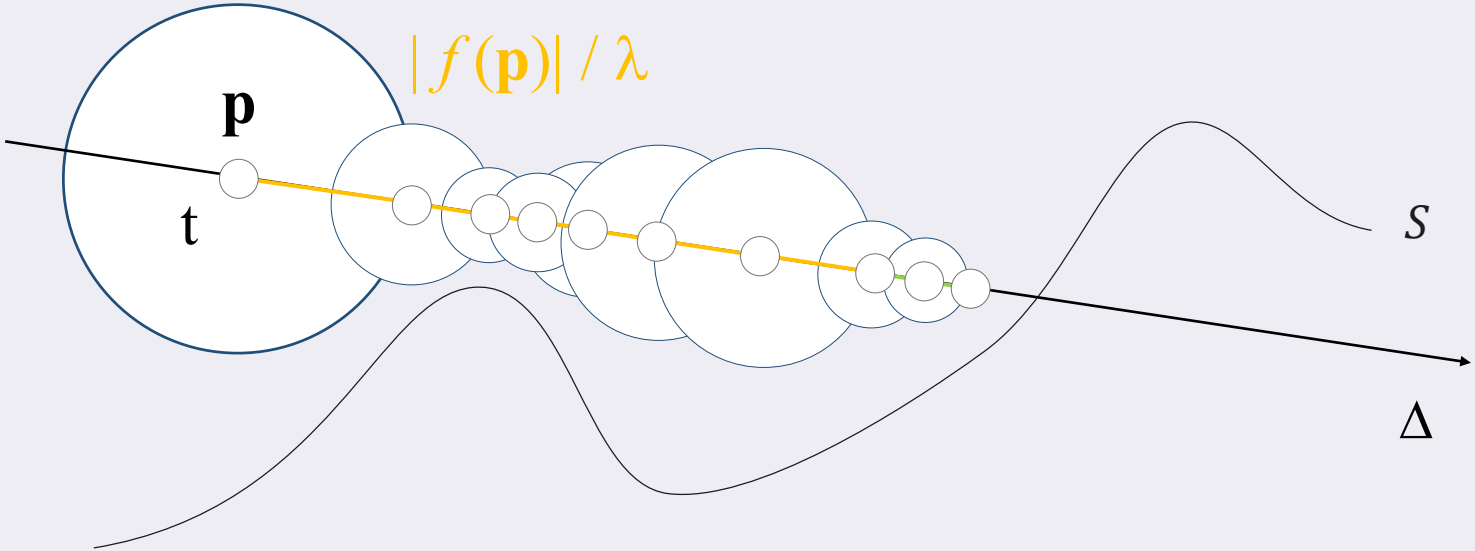
Sphere Tracing: visual interpretation



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