Lec 1

Lec 1. Sect. 1.1 (1.1.1)

1.1.1 Definition A linear equation in n variables is an equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$ and b are real numbers.

Example Linear equations

$$x_1 + 2x_2 = 5$$

$$x_1 - x_2 = 3$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1 - 2x_2 + x_3 = 0$$

$$x_1 + x_2 + \cdots + x_n = 1$$

$$\sin\theta x_1 + \cos\theta x_2 = 0$$

Example Nonlinear equations

$$x_1 x_2 = 1$$

$$x_1^2 + x_2^2 = 1$$

$$\sin x_1 + x_2 = 0$$

$$\sqrt{x_1} + x_2 = 2$$

1.1.2 Definition A system of linear equations (or a linear system) is a finite collection of linear equations in the same variables.

Example Linear system

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned}$$

Example Nonlinear system

$$\begin{aligned} x_1 x_2 &= 1 \\ x_1^2 + x_2^2 &= 1 \end{aligned}$$

1.1.3 Definition The set of all ordered n-tuples $(s_1, s_2, \ldots, s_n)$ satisfying each equation in the system is the solution set.

Example Solution set

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned}$$

$$\{(2, -1)\}$$

1.1.4 Definition Two linear systems are equivalent if they have the same solution set.

Example Equivalent systems

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= -1 \end{aligned}$$

1.1.5 Definition A system of linear equations is consistent if it has at least one solution. Otherwise, the system is inconsistent.

Example Consistent system

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned}$$

Example Inconsistent system

$$\begin{aligned} x_1 &= 2 \\ x_1 &= 3 \end{aligned}$$

1.1.6 Definition In a linear system, the coefficients and constants are ordering. Thus, the system

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\ &\vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

is completely specified by the array

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{bmatrix}$$

1.1.7 Definition The coefficient matrix of the system is the array of the coefficients

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

Example Coefficient matrix

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

1.1.8 Definition The augmented matrix of the system is the array of the coefficients and constants

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{bmatrix}$$

Example Augmented matrix

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

1.1.9 Theorem Two linear systems are equivalent if and only if their augmented matrices are equivalent.

Example Equivalent systems

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= -1 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

1.1.10 Theorem Two linear systems are equivalent if and only if they have the same solution set.

Example Equivalent systems

$$\begin{aligned} x_1 - x_2 &= 3 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= -1 \end{aligned}$$

$$\{(2, -1)\} \qquad \{(2, -1)\}$$

1.1.11 Definition A linear equation in homogeneous if the constant of zero, so that it is of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$$

1.1.12 Theorem The set of solutions to a homogeneous system of m linear equations in n variables is a subspace of $\mathbb{R}^n$.

Proof Let S be the set of solutions to a homogeneous system of m linear equations in n variables. Let $(s_1, s_2, \ldots, s_n)$ and $(t_1, t_2, \ldots, t_n)$ be solutions. Then

$$a_1(s_1 + t_1) + a_2(s_2 + t_2) + \cdots + a_n(s_n + t_n) = (a_1s_1 + \cdots + a_ns_n) + (a_1t_1 + \cdots + a_nt_n) = 0 + 0 = 0$$

$$a_1(ks_1) + a_2(ks_2) + \cdots + a_n(ks_n) = k(a_1s_1 + \cdots + a_ns_n) = k0 = 0$$

Lec 2

Lec 2. Sect. 1.2 (1.2.1)

1.2.1 Definition The following three operations are called elementary row operations:

- Interchange two rows.
- Multiply a row by a nonzero number.
- Replace a row by the sum of that row and a multiple of another row.

1.2.2 Theorem Elementary row operations transform a linear system into an equivalent one.

1.2.3 Definition Two matrices are equivalent if one can be transformed into the other by a sequence of elementary row operations.

Example Equivalent matrices

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

1.2.4 Definition A matrix is in row echelon form if:

- The first nonzero number in each row is 1.
- If row k is not all zeros, then the first nonzero number in row $k+1$ is to the right of the first nonzero number in row k.
- All zero rows are below the rows having nonzero numbers.

Example Row echelon form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

1.2.5 Theorem Every matrix is equivalent to a matrix in row echelon form.

1.2.6 Definition A matrix is in reduced row echelon form if:

- It is in row echelon form.
- The first nonzero number in each row is the only nonzero number in its column.

Example Reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

1.2.7 Theorem Every matrix is equivalent to a matrix in reduced row echelon form.

1.2.8 Theorem Every matrix is equivalent to a unique matrix in reduced row echelon form.

1.2.9 Theorem Every linear system is equivalent to a system whose augmented matrix is in reduced row echelon form.

Example Algorithm

1.2.10 Theorem The reduced row echelon form of A is the matrix of the form

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}$$

1.2.11 Theorem The number r in the reduced row echelon form of A is the number of the matrix A.

Proof Let A be a matrix. Let r be the number of pivots in the reduced row echelon form of A. Let s be the number of pivots in the reduced row echelon form of A^T. Then

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}^T = \begin{bmatrix} I_s & G \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}^T = \begin{bmatrix} I_r & 0 \\ F^T & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_s & G \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I_r & 0 \\ F^T & 0 \end{bmatrix}$$

$$r = s$$

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}$$

1.2.12 Theorem Every matrix is equivalent to a unique matrix of the form

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}$$

1.2.13 Theorem Every linear system is equivalent to a system whose augmented matrix is of the form

$$\begin{bmatrix} I_r & F \\ 0 & 0 \end{bmatrix}$$

1.2.14 Theorem Every linear system is equivalent to a system whose coefficient matrix is of the form

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