SPTF SCHEDULING PROJECT

Traffic Theory Project 2022-2023

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Sommario

TRACK

1. Implement a M/G/1 system with a SPTF discipline where G is Uniform(0,L);
2. All input parameters must be NED parameters in omnetpp.ini;
3. Collect statistics on:
    1. Average queueing time;
    2. Average queueing time conditioned to the packet's length;
    3. Average response time;
    4. Utilization factor of the server;
    5. Queue length over time.
4. Compare experimental and theoretical values of:
    1. Average conditional queueing time;
    2. Average response time;
    3. Utilization factor of the server.

1 – Implement an M/G/1 system

We implemented an M/G/1 system in which service time G is distributed as a Uniform distribution between 0 and L.

The system has tree modules. Source, Queue and Sink.

Source generates messages with an exponential distribution with average λ, then it is assigned to each message a computation time distributed uniformly between 0 and L. This value is attached to a message by using the class Mail, which is an extension of the class cMessage.

Queue receive Mails and process it if server is empty. If it is not, mails are inserted into a cQueue. Right after the insertion of a new mail, the cQueue is sorted, to guarantee the SPTF scheduling. Surely this could be done in a more efficient way, but this was the easiest to implement and on a stable system it does not impact so much on the simulation.

Sink receive the packets and collects some statistics. Then packets are destroyed.

Full project is available in a private repository on GitHub[1], which will be set as public repository when projects will e evaluated.

2 – omnetpp.ini file

In the file omnetpp.ini is possible to make a custom simulation by editing the parameters. Is it possible to edit the end of uniform distribution L, and the average of packets arrival λ.

It is also possible to edit the start of the uniform (0 by default), but since it was not required it is just a parameter inside *queue.cc* file.

Default simulation processing time and simulation waiting time can be edited too in *omnetpp.ini*.

3 – Statistics collection

Statistics and data are collected in each module by Omnet.

Omnet editor was enough for compute most of the statistics, but the simulation generates also a log.txt file with a format that fit well with *Matlab importdata()* function. This is required to answer the 3.1 and 4.1 questions about conditional queue time.

In particular, the log is useful to associate a packet with its length (process time) and it's waiting time in the queue. *Omnet* generate collects this data individually and it was not possible to associate that to the same packet. This is necessary to compute packet queue time conditioned to their size.

All the following statistics refer to a simulation with the following values:

$$G \sim U(0, 0.8) \qquad \lambda = 2\ \frac{usr}{s}$$

so that $avgInterArrivalTime = 0.5\ s$, and $\rho = \lambda E[S] = 0.8$ which means system is stable.

3.1 – Average queueing time

Average queueing time is computed with the Omnet tool, and it is:

$$W_q = 1.13\ usr$$

3.2 – Average queueing time conditioned to packet length

As mentioned above, to compute the conditioned queue time, the simulation generates a log file, which is then computed with Matlab.

Matlab takes as input a table where each row contains a packet id, packet length, packet queue time.

Knowing this, it is possible to cluster the packets length in small interval and count the average waiting time of each packet of that cluster.

The result of the Matlab computation is then plotted on a histogram and the result is showed in *figure 1*.

It is clear, as we can expected, that short packet have a small queue time – since they "skip" the queue, and as the length increase, the queue time increase due to the increasing number of packet that are served before them.

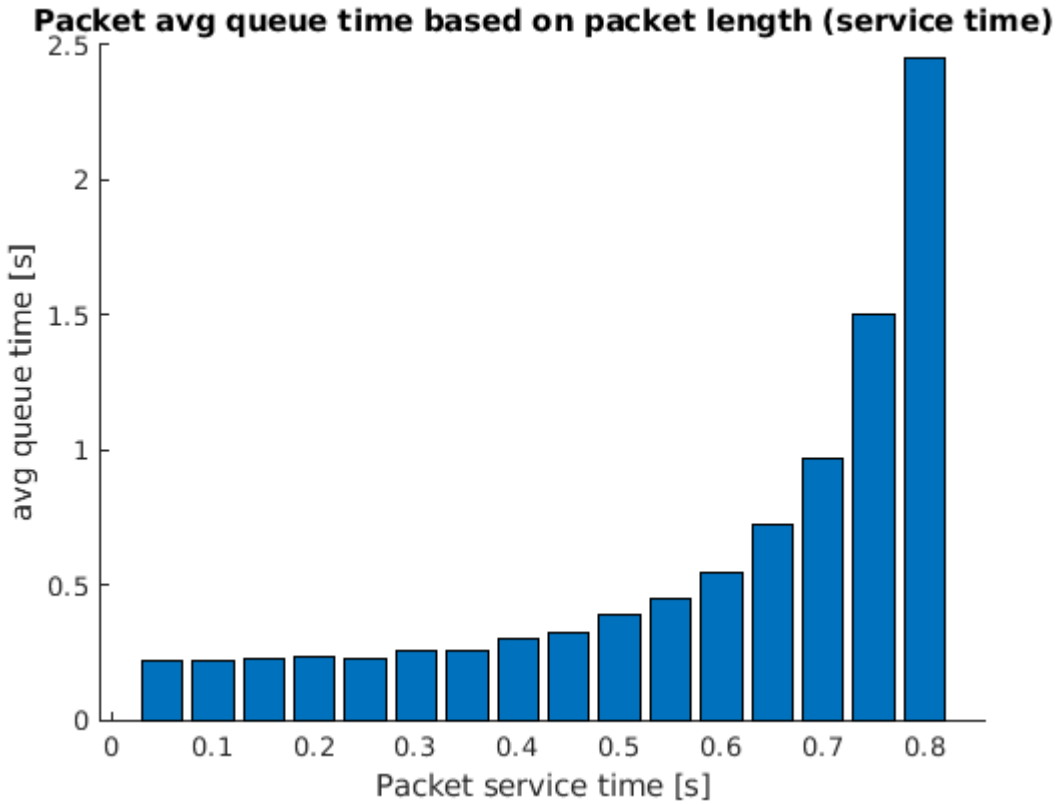

Figure 1: avg queue time conditioned to packet length.

This is the graph of matrix R generated with the script *analyzer.m* in *analysis* folder.

3.3 – Average response time

Average response time is computed from when the packet is created, to when the packet is destroyed. In the simulation it is computed by Omnet, and it is:

$$W = 0.97\ s$$

3.4 – Utilization factor of the server

Utilization factor of the server is the percentage of time the server is working over all the simulation time.

This also is computed by Omnet, and it is:

$$\rho = 0.78$$

3.5 – Queue length over time

Queue length over time data is collected by Omnet and are plotted in *figure 2*.

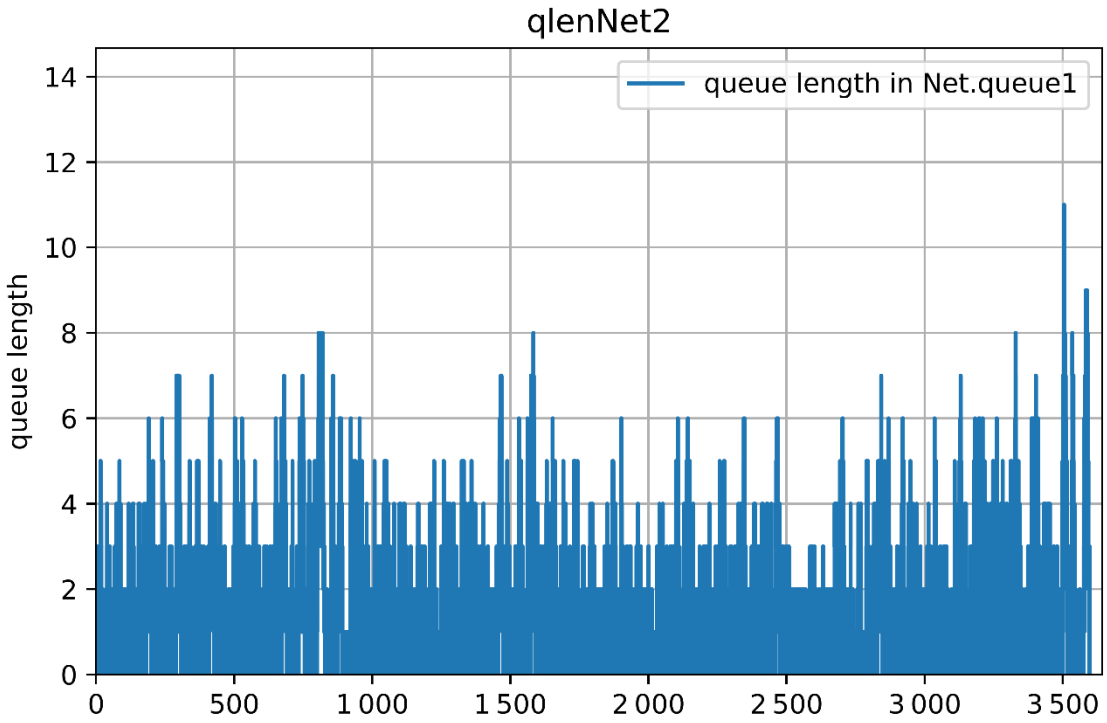

Figure 2: queue length over time.

4 – Compare experimental and theoretical values

Once experimental values are collected after running a simulation, they are compared to theoretical values calculated by solving equations of an M/G/1 system.

4.1 – Average conditional queue time

To compare the experimental and theoretical conditional queue time, we compared the result of the equation $P(A|B) = \frac{P(A \cap B)}{P(B)}$, where:

- $P(A = \alpha)$ is the probability of a packet to wait α seconds in the queue.
- $P(B = \beta)$ is the probability of a packet to have length β.

$P(A|B)$ is the conditioned probability computed in the request 3.1.

In file *conditional.m* are computed $P(A \cap B)$ and $P(B)$, and the result is showed in the heatmap in *figure 3*.

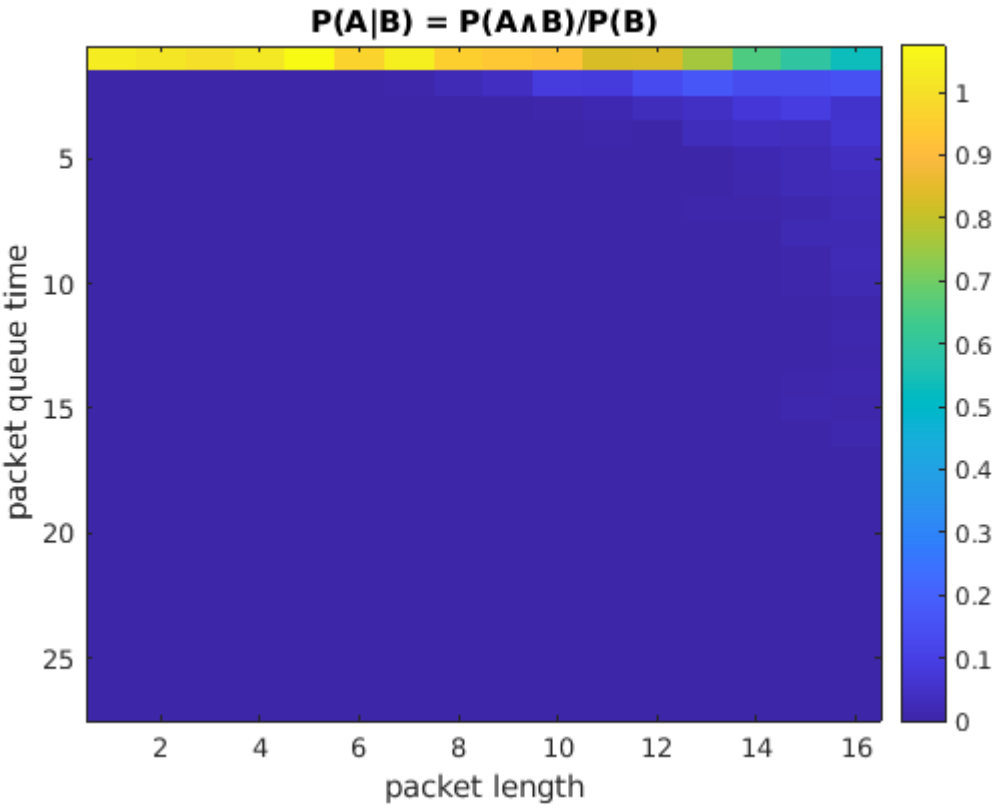

Figure 3: P(A and B)/P(B).

On X axis there are length intervals, which must be multiplied by the delta used to divide the total length to retrieve the real packet length. In this case $delta\_length = 0.05$.

Since the value of time is between 0s and 27s, but almost all of the packet have queue time less than 5s, the heatmap is not well suited to make this comparison. It is still the most visually clear image we could generate to compare the results.

Despite this it is clear that the two computations bring to the same result.

To have a more specific proof of the equality of the two results it is reported in *figure 4* the matrix used to generate the heatmap, rounded to the 4^{th} decimal.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1.0385	1.0138	0.9980	1.0183	1.0767	0.9711	1.0407	0.9553	0.9351	0.9239	0.8317	0.8362	0.7598	0.6541	0.5934	0.5305
2	0	0	0	0	0	0.0022	0.0067	0.0225	0.0382	0.0877	0.0832	0.1326	0.1708	0.1371	0.1371	0.1551
3	0	0	0	0	0	0	0	0	0	0.0067	0.0135	0.0315	0.0450	0.0742	0.0899	0.0562
4	0	0	0	0	0	0	0	0	0	0.0022	0.0067	0.0022	0.0315	0.0382	0.0337	0.0629
5	0	0	0	0	0	0	0	0	0	0	0	0.0022	0	0.0157	0.0225	0.0405
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0112	0.0225	0.0292
7	0	0	0	0	0	0	0	0	0	0	0	0.0022	0.0045	0.0045	0.0090	0.0247
8	0	0	0	0	0	0	0	0	0	0	0	0	0.0022	0.0022	0.0180	0.0180
9	0	0	0	0	0	0	0	0	0	0	0	0.0022	0	0.0022	0.0067	0.0225
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0045	0.0180
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0045
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0022	0.0022	0.0090
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0067
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0045	0.0090
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0090	0.0045
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0112
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0022
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0022
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0022

Figure 4: P(A|B)/P(B) displayed as matrix.

This show well how few are packets with huge queue time. It is also much clear the increasing of queue time as packet length increase. The result is compatible with the graph computed in question 3.1.

We can conclude that $P(A|B) = \frac{P(A \cap B)}{P(B)}$ is true by comparing the graph in *figure 1* and the matrix in *figure 4*.

4.2 – Average response time

In an M/G/1 system, average response time is:

$$W = E[S] \cdot \left(\frac{1 + C_s^2}{2} \cdot \frac{\rho}{1 - \rho} \right)$$

where $E[S]$ is the expected value of a uniform distribution, and C_s^2 is the squared coefficient of variation.

Using numerical values in the formula we obtained:

$$W = 1.53\ s$$

which is a bit greater than the experimental result.

This is the starting point of a consideration.

Let's compute the average queue length of an M/G/1 system.

It is $L_q = \left(\frac{1+C_s^2}{2}\right)\frac{\rho^2}{1-\rho}$. The theoretical value is $L_q = 2.26\ usr$, and the experimental value computed with *Omnet* is $L_q = 1.13\ usr$.

This is fine, since the L_q formula for an M/G/1 does not take into account the scheduling policy. It is by default a FCFS[2]. Changing the scheduling policy to a SPTF make the shortest packet, which have a shortest processing time, go first. This cause longer packets to stay in the queue and let's shortest packet to quit sooner. As a result of this we obtain a shortest queue full of longer user.

This explain why experimental L_q is shorter that theoretical L_q.

As a consequence of this, since crossing time depends on L_q, it is normal that theoretical value do not match with experimental value. In fact, $W = \left(1 + \frac{L_q}{\rho}\right) E[S]$.

4.3 – Utilization factor of the server

As above, the formula for server utilization factor in an M/G/1 system is:

$$\rho = \lambda \cdot E[S] = 0.8$$

which is perfectly fine with the experimental result ($\rho = 0.78$). This because utilization factor does not depend on scheduling policy as soon as we assume there is no time between user ending service and next user starting service.

Notes

1. https://github.com/MrPratula/omnetpp_sptf_scheduling
2. First Come First Served