EC427 - Optimization Techniques
IIIT Vadodara

Autumn 2022 | Mid-Semester Examination
September 8, 2022 | 35 Marks

Note:

1. You can bring notes written in two A4 size pages (4 sides) to the exam. The notes should be handwritten. Photocopies and printouts are not allowed. Your names and ID numbers should be clearly written on each page.
2. Answers directly written without proper derivation/justification/computation will not be awarded any marks, irrespective of whether they are correct or not.
3. Calculators are not allowed.

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1. Let there be n coal mines located in n different locations, and let there be m power plants in m different locations, each requiring $b_j, j = 1\ldots,m$ tonnes of coal. In order to ship one tonne of coal from mine i to power plant j, it costs the transport company $c_{ij}, i = 1,\ldots,n, j = 1,\ldots,m$ rupees. Mathematically represent the problem of minimizing the cost for the transport company while ensuring that the demand of the power plants are met. Classify the obtained optimization problem in categories of optimization problems as discussed in the first class. **[5]**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function $f(x) = x^T Q x + b^T x + c, \forall x \in \mathbb{R}^2$, where $Q = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, c = 427$. At the point $x = (2,1)$, **[6]**
   (a) Find the (unit) direction along which the rate of decrease of the function f is 1.
   (b) Find the (unit) directions along which the rate of change of the function f is zero.

3. From the information given in each of the questions below, classify these points into local minima, not local minima, may be local minima for the optimization problem $\arg\min_{x\in\Omega} f(x)$. This should be done only by first computing points that satisfy the first order necessary conditions, followed by second order necessary (or sufficient, if applicable) condition, and only then using other tools if required. **[12]**
   (a) $f(x,y) = x^3 - x^2y + 2y^2, \Omega = \{(x,y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0\}$.
   (b) $f(x,y) = x^2 - 10y^2$. $\Omega = \{(x,y) \in \mathbb{R}^2 \mid y = 0\}$.
   (c) $f(x) = x^3$. $\Omega = \{x \in \mathbb{R} \mid x > -2\}$.
   (d) $f(x,y) = x^2 + y^3$. $\Omega = \mathbb{R}^2$.

4. In the Newton's one dimensional search method, given a function $f : \mathbb{R} \to \mathbb{R}$, we try to find a point where $f'(x) = 0$, i.e., we look for points that satisfy the first order necessary conditions. What if we use the Newton's method to find the minimizer of the function $g(x) = (f'(x))^2$? Derive the expression for iterations of the Newton's method for minimizing g. Discuss whether this will also result in finding a minimizer of f? Will we have to make additional assumptions for the method to work? **[6]**

$f(x) = \frac{1}{2}x^T Q x + c$, with $Q \in SPD(n), x \in \mathbb{R}^n, c \in \mathbb{R}$ be the function to be min-
d. Find a point $x_0 \neq x^*$ such that the steepest descent algorithm starting from
converge to the minima x^* in one iteration. Do not provide only geometric **[6]**
6. Let n, but work out the relation mathematically.
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