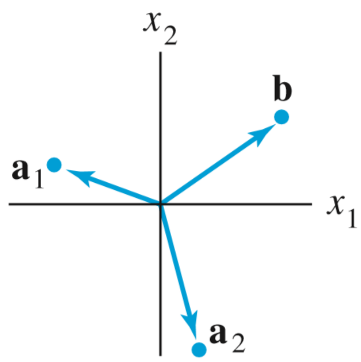
Indian Institute of Information Technology Vadodara
Mid-semester Examination
MA 101 (Mathematics I: Linear Algebra and Matrices)

1. Find the inverse of following matrix using block matrix inversion:

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 1 & 2 \\ 1 & 2 & -1 & 4 \\ 1 & 1 & 2 & 4 \end{bmatrix}$$

2. Is the next sentence true? If a vector w is a linear combination of $u, v \in \mathbb{R}^n$ then u is a linear combination of v, w. Give justification.

3. Let a_1, a_2, b be the vectors in $\mathbb{R}^2$ as shown in the figure and let $A = [a_1 \;\; a_2]$, here a_i are columns of A. Does the equation $AX = b$ have a solution? If so, is the solution unique? Explain.



4. Let V be a vector space of all polynomials of degree ≤ 4. Let $T : V \to V$ be defined by

$$T(p(x)) = \int_1^x p'(t)dt,$$

where $p(x) \in V$ and $p'(t)$ is derivative of $p(t)$.

   (a) Find a basis for V.

   (b) Check whether T is a linear transformation?

   (c) Find the matrix representing this linear transformation T with respect to the basis found in (a).

5. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and $T(e_1) = e_3$, $T(e_2) = e_1$ and $T(e_3) = e_2$, where $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.
   Then prove that T is invertible and $T^2 = T^{-1}$.

6. Let $\{v_1, v_2, ..., v_n\}$ be a basis for $\mathbb{R}^n$. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation which is one-one and onto.

   (a) Prove that $\{Tv_1, Tv_2, ..., Tv_n\}$ is also a basis for $\mathbb{R}^n$.

   (b) If T is only one-one but not onto then is $\{Tv_1, Tv_2, ..., Tv_n\}$ a basis for $\mathbb{R}^n$? Explain.