Date of Exam - 04.01.2022

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY VADODARA

END-SEMESTER EXAM: AUTUMN 2021

B.TECH. III SEMESTER (GANDHINAGAR & DIU CAMPUS)

MA 201: PROBABILITY AND STATISTICS

MAX MARKS: 15
Duration: 60 MINUTES

INSTRUCTIONS -

1. ATTEMPT **ANY THREE** QUESTIONS. ALL QUESTIONS CARRY **ALMOST** EQUAL MARKS **(5+ 5 + 5 Marks).**
2. USE OF SCIENTIFIC-CALCULATOR/MATLAB/OCTAVE IS ALLOWED ONLY FOR CALCULATION PURPOSES. THEREFORE, WRITE THE STEPS PROPERLY.
3. ATTEMPT ONE QUESTION AT ONE PLACE. TRY TO MAINTAIN THE QUESTION ORDERING.
4. WRITE YOUR ROLL NO AND PAGE NO WITH SIGN.
5. SCAN THE PDF FILE, RENAME IT AS **MA201_ENDEXAM_YOURINSTITUTEID_NAME.PDF**
6. UPLOAD YOUR **MA201_ENDSEMEXAM_** FILE TO THE GOOGLE-FORM. 15 MINUTES IS GIVEN FOR UPLOADING.

Qus - 1: A manager evaluates effectiveness of major hardware upgrade by running a certain process 5 times before the upgrade (sample $X = [12, 8, 6, 29, 57]$) and 5 times after it (sample $Y = [5, 3, 15, 74, 2]$). Based on these data, the average running time is 22.4 minutes before the upgrade, 19.8 minutes after it. Historically, the standard deviation has been 2.0975 minutes, and presumably it has not changed. **(Assume x = 95% or x = 90% confidence interval if your roll number is odd or even).**

(a) Construct a $\mathbf{x}\%$ confidence interval for population mean of the sample X and as well as for Y .

(b) Construct a $\mathbf{x}\%$ confidence interval for difference of population mean of the samples X and Y. Why the margin for confidence interval in (c) should be more than that in (a)?

(c) Looking at (c), was the hardware up-gradation successful with $\mathbf{x}\%$ confidence (or $(\mathbf{1} - \mathbf{x})\%$ level of significance)?

(d) Find the Pvalue and claim whether hardware upgrade was successful or not?

(WRITE THE STEPS CLEARLY, BEFORE CALCULATIONS)

Qus - 2: A sample of n observations is collected from a continuous distribution with density $f(x) = \lambda^2 x e^{-\lambda x}$ for $x > 0$.

(a) Find the estimate of λ by maximum likelihood method.

(b) Name the distribution f(x) and estimate λ by moment method (Use the mean formula directly for the distribution).

(c) Are both the estimates in (a) and (b) the same? Find the error estimation $std(\widehat{\lambda})$.

(d) Calculate $\widehat{\lambda}$ and $std(\widehat{\lambda})$ for the sample $\mathbf{X}$ **of size 4** generated from your roll number. If your roll number is **202051001** then generate the sample of size **$X = [20, 20, 51, 101]$**.

(WRITE THE STEPS CLEARLY, BEFORE CALCULATIONS)

Qus - 3: Suppose a discrete random variable X is the outcome of a three-faced dice (sample space of X is $\Omega \equiv \{0, 1, 2\}$).

The PDF of X is given by $\Pr\{X = 0\} = 1/5, \Pr\{X = 1\} = 3/5, \Pr\{X = 2\} = 1/5.$

(a) Find probability of the outcome $X = 1$ occurs 2 times, and $X = 2$ occurs 3 times in 6 trials?

(b) Find probability that the outcome $X = 1$ occurs second time in $6th$ trial.

(c) If this dice is rolled $n = 90$ times, find $\Pr\{M_n \geq 424/375\}$ using the central limit theorem where $M_n = (X_1 + X_2 + ...X_n)/n$ and X_k is the kth trial of the random variable X.

(For (a), (b) , WRITE THE FORMULAS ONLY AND OMIT CALCULATIONS. For (c), WRITE THE STEPS CLEARLY)

Qus - 4: Are the continuous random variables X and Y, whose joint PDF is given as $f_{X,Y}(x, y) = C$ for $0 < x < y < 1$, independent? Find $E[X]$, $E[Y]$ and $E[XY]$

OR

Qus - 4: Find $E[X]$, $Var[X]$, $E[Y]$, $Var[Y]$, $Cov[X, Y]$ of the discrete random variables X and Y, whose joint PDF is given as follows:

$\mathbf{P_{X,Y}(x,y)}$		**x** = **0**	**x** = **1**
y	**0**	0.24	0.26
y	**1**	0.12	0.38

(WRITE SOME STEPS. YOU CAN USE MATLAB OR https://octave-online.net/ in checking the answers.)