MA102: Introduction to Discrete Mathematics

Endsem (Remote) Marks: 15

Course Instructor: Dr. Swapnil Lokhande Date: Aug 21, 2021

Instructions: Clearly write your name, roll number and sign on the top of each page. Solutions must be written clearly. **Answer of each question should start on new page. Scan solutions in sequence**

Problem 1 **2 marks**

If $F(n)$ denotes the Fibonacci numbers then prove the followings.

(i) $F(n-1)\cdot F(n+1)-(F(n))^2=(-1)^n$, holds for $n\geq 1$. [1]

(ii) $F(0)\cdot F(1)+F(1)\cdot F(2)+\cdots+F(2n-1)\cdot F(2n)=(F(2n))^2$ when n is positive integer. [1]

Problem 2 **2 marks**

If $F_n=2^{2^n}+1$, $n\geq 0$, then prove the followings.

(i) $\prod_{r=0}^{n-1} F_r = F_n - 2$. [1]

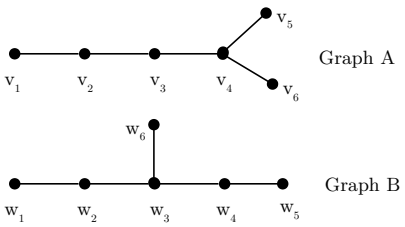
(ii) $\gcd(F_m, F_n)=1$ for all m, n with $m<n$. [1]

Problem 3 **1 mark**

Find the solution of the recurrence relation $a_n=2a_{n-1}+a_{n-2}-2a_{n-3}$ for $n=3,4,5,\ldots$ with the initial conditions $a_0=3$, $a_1=6$, and $a_2=0$.

Problem 4 **1 mark**

Consider the following two Graphs (Graph A and Graph B).



Prove that Graph A is not isomorphic to Graph B.

Problem 5 **1 mark**

Prove that in a collection of 16 distinct integers, there are distinct integers x and y such that 15 divides $x-y$.

Problem 6 **1 mark**

How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other?

Problem 7 **2 marks**

Construct 2 graphs with 5 vertices and 8 edges but are not isomorphic. Give reasons.

Problem 8 **1 mark**

Find the number of solutions of the inequality $x_1+x_2+x_3\leq 11$ with $x_1\geq 0$, $x_2\geq 0$, $x_3\geq 0$.

`Hint: Convert inequality to equality by introducing extra variable.`

Problem 9 **2 marks**

State and prove divisibility test of 22 and find remainder of your student id modulo 22.

Problem 10 **2 marks**

In a class with 9 students, each student sends cards to 3 others. Determine whether it is possible that each student receives cards from the same three students to whom he or she sent cards.