Theory of Electrical Circuits PROJECT

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Twin-T Notch Filter

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Contents

1 Summary

Noise reduction in electronic circuits is essential for signal integrity, particularly to mitigate 50 Hz or 60 Hz power line interference using notch filters. This report details the design, simulation, and analysis of a notch filter, initially targeting 60 Hz interference and subsequently adapted for 50 Hz. It also explores the impact of component tolerances through Monte Carlo simulations and includes an experimental setup to validate the theoretical results. This project was partially based on the Texas Instruments application note "High-speed Notch Filters" [1].

2 Methodology

2.1 Notch Filter Design

A Twin-T notch filter was designed to eliminate 50 Hz noise. The circuit components were selected to achieve the desired notch depth and quality factor (Q). The governing equation for the center frequency is:

$$f_0 = \frac{1}{2\pi R_0 C_0} \tag{I}$$

where R_0 and C_0 are the primary resistance and capacitance values.

2.2 Monte Carlo Analysis

Monte Carlo simulations were used to assess the impact of component tolerances on the filter's performance. Random variations were introduced into the resistor and capacitor values following a normal distribution. The frequency response was analyzed over multiple iterations to evaluate the statistical impact of these tolerances on key performance metrics such as notch depth and bandwidth.

2.3 Robust Circuit Approach

The robust circuit design method increases the circuit's tolerance to variations in component values, ensuring stable and reliable performance. One common technique involves replacing a single resistor with two resistors of half the original value connected in series. This approach reduces the overall deviation in resistance, as individual variations statistically average out, which improves robustness against manufacturing tolerances. Similar techniques can be applied to capacitors and inductors, increasing the circuit's reliability in the presence of component tolerances, temperature variations, and aging effects.

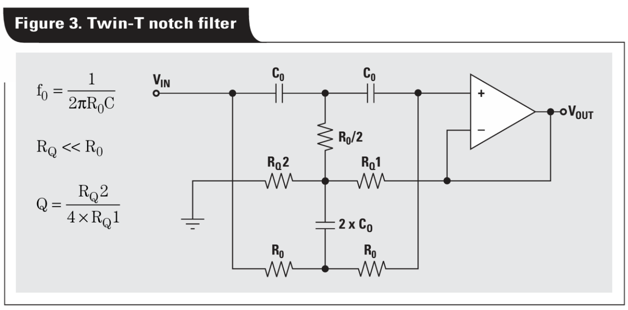


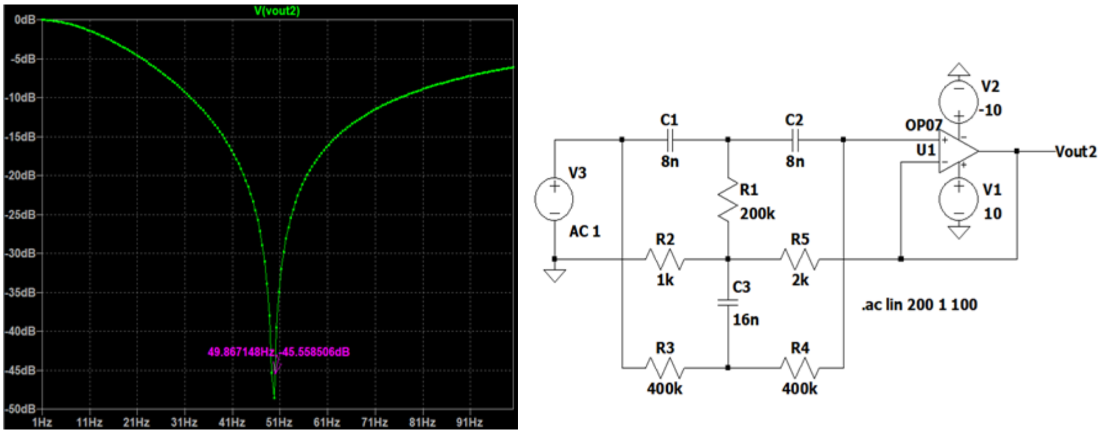
Figure 1: Twin-T Notch Filter with R_0 and C_0 as Parameters

3 Notch Filter Design and Simulation

3.1 Circuit Analysis

3.1.1 Notch Filter Design for 50 Hz Noise Elimination

In this section, the 60 Hz notch filter is converted to a 50 Hz filter by selecting appropriate values for the capacitors and resistors that influence the filter's center frequency and notch depth. Using eq. (I), a resistor value of 400 kΩ is chosen, which results in a corresponding capacitor value of 8 nanofarads.
It should be noted that these values may not be easy to find in practice, so adjustments might be needed. Components can also be combined in series or parallel to get closer to the desired values.

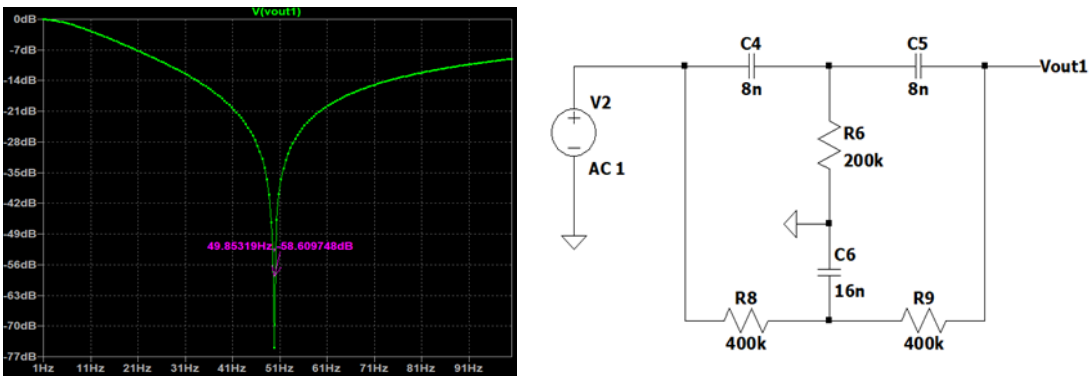


(a) Frequency Response (b) Circuit Design

Figure 2: 50 Hz Notch Filter

3.1.2 Q and Its Effect on the Filter

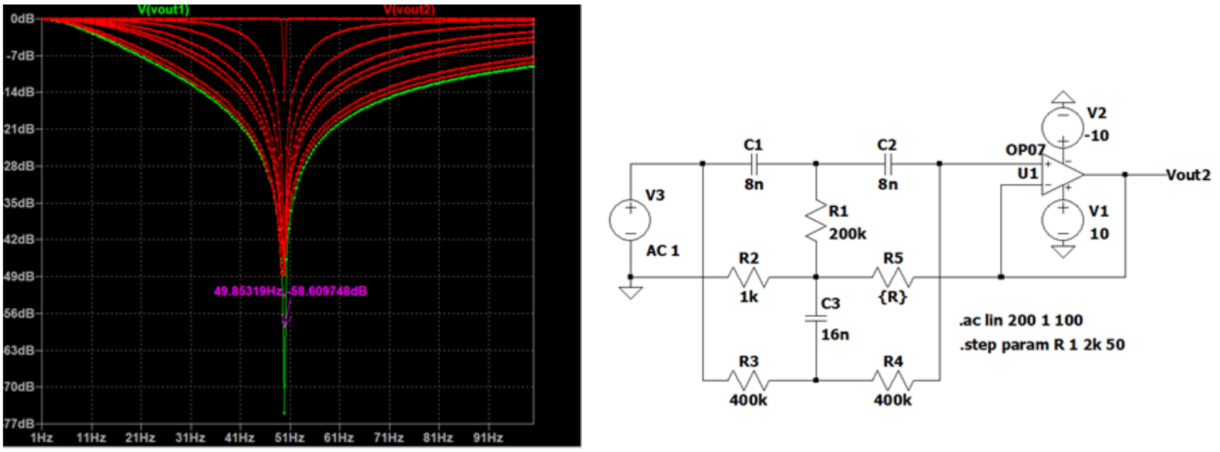
Using simulation and the formula $Q = \frac{R_5}{4R_2}$, we analyze how changes in resistance affect the circuit's frequency response. First, we simulate the initial circuit, where Q cannot be adjusted, and present its frequency response.



(a) Frequency Response (b) Circuit Design

Figure 3: Fixed Q Notch Filter

Next, we present the frequency response and the corresponding circuit with a modified Q on the notch filter.

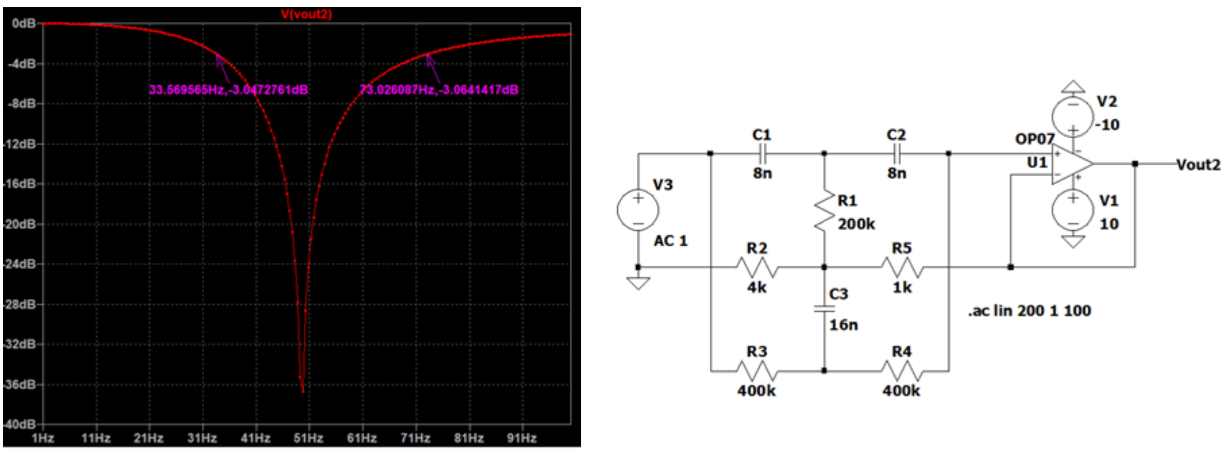


(a) Frequency Response (b) Circuit Design

Figure 4: Adjustable Q

It can be observed that increasing Q decreases the filter's bandwidth. By adjusting the resistors R_5 and R_2, the desired bandwidth can be achieved.

Now, we examine the frequency response at Q = 1 and calculate the -3 dB bandwidth.



(a) Frequency Response (b) Circuit Design

Figure 5: Notch Filter with Q = 1

According to the results, the bandwidth is 40 Hz, ranging from 33 Hz to 73 Hz. Since the noise frequency in Iran varies between 49.8 Hz and 50.2 Hz, increasing Q too much will significantly reduce the bandwidth, narrowing the oscillation range. This can cause the filter to malfunction and allow noise to pass through.

3.1.3 Suitable Operational Amplifier (Op-Amp)

For designing a Twin-T Notch Filter aimed at eliminating 50 Hz noise, the op-amp should have the following characteristics:

1. **Suitable Frequency Response:** The op-amp should have a wide enough bandwidth to operate effectively at low frequencies (50 Hz). This ensures the filter will perform well in noise elimination.

2. **High Signal-to-Noise Ratio (SNR):** The op-amp should generate minimal noise to maintain signal quality. This feature is crucial for filters where precision is important.

3. **Stability at High Frequencies and High Resistor Values:** The op-amp must remain stable in circuits with high resistances (several hundred kΩ) without causing oscillations or distortion.

4. **Ability to Operate Accurately with High Resistor Values:** In this project, high-value resistors (several hundred kΩ) will be used, so the op-amp should be well-suited for working with these resistances.

5. **High Gain and Linearity:** For precise filter design, the op-amp must provide suitable gain and maintain linearity at low frequencies.

Two Suggested Models:

- **NE5532**
  It has a high signal-to-noise ratio and a wide frequency response, offering much more precise performance in 50 Hz noise filters. It also exhibits high stability when using high resistances, making it particularly suitable for designing sensitive and precise filters.

- **TL072**
  It has a good frequency response and an acceptable signal-to-noise ratio, which is sufficient for most projects. It is suitable for general circuits and analog filters with medium precision, and compared to the NE5532, it is more cost-effective.

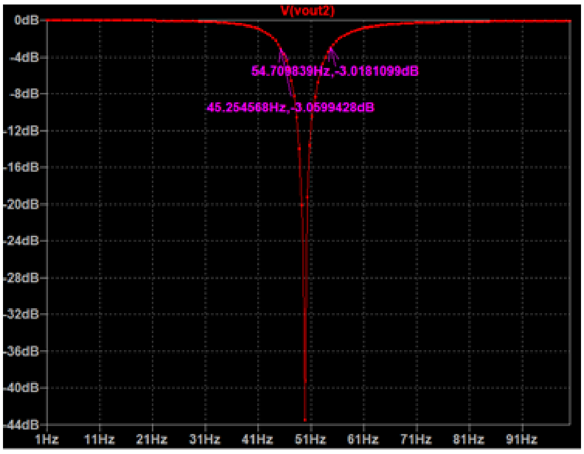
Final Conclusion: If high precision and stability in sensitive circuits are a priority, the **NE5532** is the best choice. However, if cost and adequate performance within the medium precision range are more important, the **TL072** would be a suitable option.

3.2 Circuit Redesign

3.2.1 Meeting the Desired Conditions

Based on the findings from the previous section, we have calculated the circuit components according to the following conditions:

$$\mathrm{BW}_{-3\,\mathrm{dB}} < 10\,(\mathrm{Hz}), \quad f_0 = 50\,(\mathrm{Hz}), \quad \mathrm{Gain}(f = f_0) < -40\,(\mathrm{dB})$$



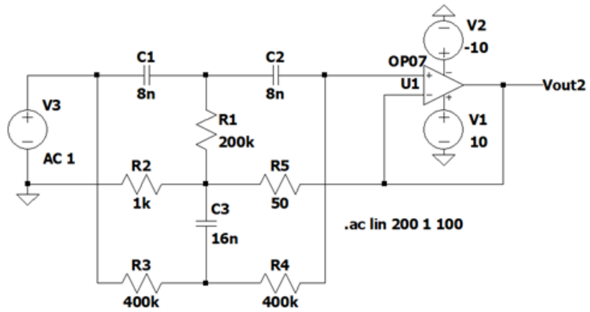
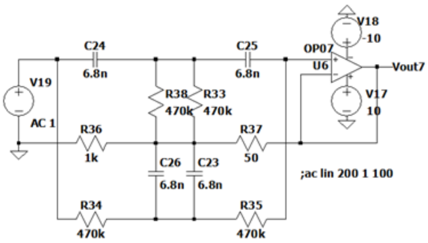


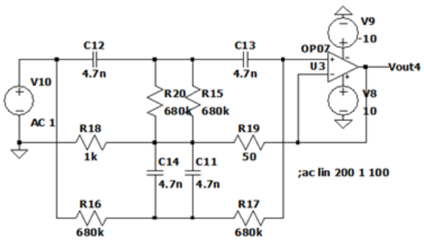
Figure 6: Frequency Response and Circuit Design for Notch Filter with Desired Conditions

3.2.2 Design Based on the Standard and Available Values

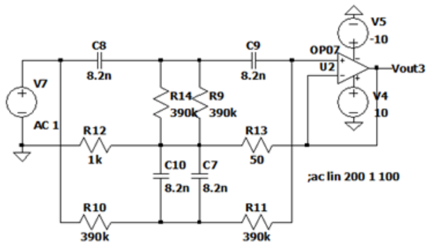
Now, we modify the main components of the circuit that form the notch filter at 50 Hz based on standard market-available values to achieve the desired result with the fewest number of resistors and capacitors. The circuit can be selected from the three options below. It should be noted that the values of the capacitors and resistors are based on the E24 series, and their values can be adjusted by multiplying or dividing them by factors of 10.



(a)



(b)



(c)

Figure 7: Circuit Designs for Notch Filter with Standard Components

4 Transfer Function

From the circuits designed in the previous section, we have selected fig. 7b. In the subsequent sections, the naming conventions will be based on the following circuit.

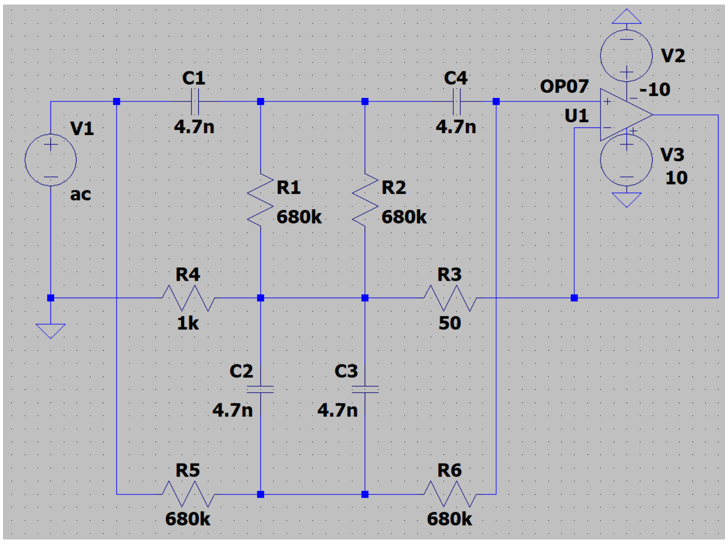


Figure 8: The Selected Circuit for Implementation

4.1 Transfer Function Calculation

To calculate the transfer function, we used the admittance matrix approach in MATLAB. Instead of directly solving the system of equations, we represented the circuit using its admittance matrix, which is a more efficient method for handling complex systems. This method allows us to perform matrix multiplication and inversion to derive the transfer function.
The component values were set as $R_3 = 50\,\Omega$ and $R_4 = 1\,\mathrm{k}\Omega$. Kirchhoff's Current Law (KCL) equations were formulated for the circuit and converted into admittance matrix form. The resulting admittance matrix is:

$$\begin{bmatrix} -Y_{C4} & Y_{C1}+Y_1+Y_2+Y_{C4} & -Y_1-Y_2 & 0 \\ Y_{C4}+Y_6 & -Y_{C4} & 0 & -Y_6 \\ -Y_3 & -Y_1-Y_2 & (Y_{C2}+Y_{C3})+Y_1+Y_2+Y_3+Y_4 & -(Y_{C2}+Y_{C3}) \\ -Y_6 & 0 & -(Y_{C2}+Y_{C3}) & Y_5+Y_6+(Y_{C2}+Y_{C3}) \end{bmatrix}$$

Finally, the transfer function $H(s)$ of the circuit, derived from the admittance matrix, is given by:

$$H(s) = \frac{21\,{C_0}^2\,{R_0}^2\,s^2 + 4000\,{C_0}^2\,R_0\,s^2 + 4000\,C_0\,s + 21}{21\,{C_0}^2\,{R_0}^2\,s^2 + 4000\,{C_0}^2\,R_0\,s^2 + 4\,C_0\,R_0\,s + 4000\,C_0\,s + 21}$$

Using the admittance matrix method simplifies the process of obtaining the transfer function, making it easier to manipulate and analyze the system's behavior.

4.2 Bode Diagram

Now that we have obtained the transfer function, we can substitute the values calculated in the previous section. We set $R_0 = 680\,\mathrm{k\Omega}$, $C_0 = 4.7\,\mathrm{nF}$, and $s = j\omega$, where j represents the imaginary unit and ω is the angular frequency. With these values, we can plot the Bode diagram.

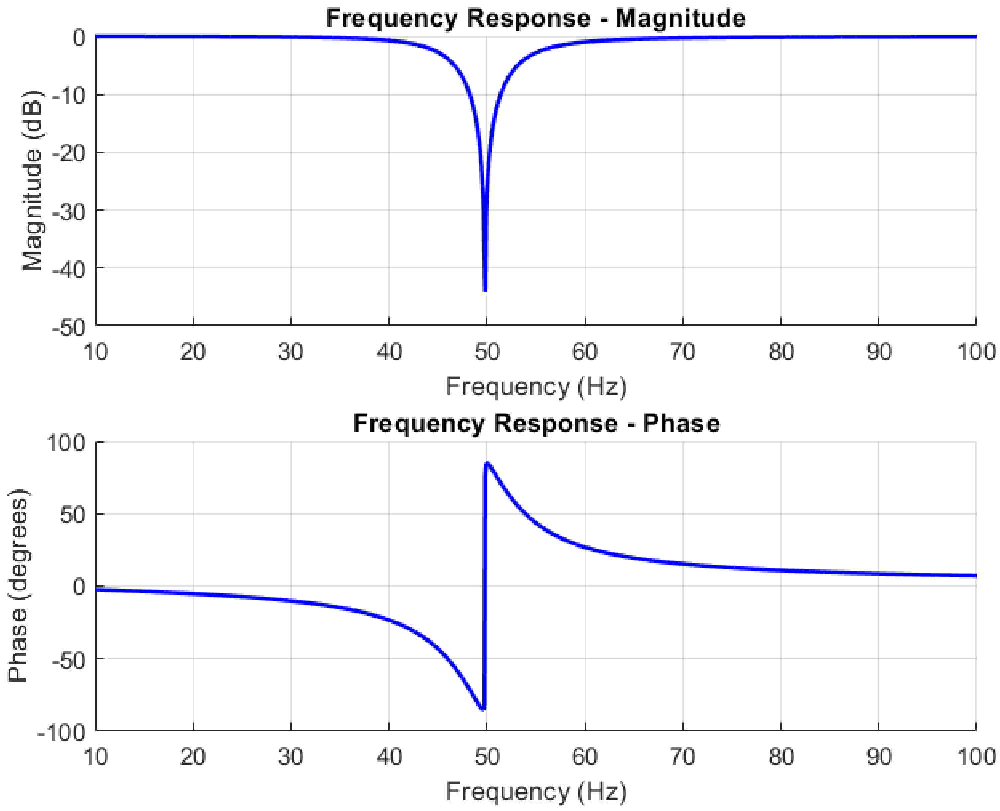


Figure 9: Bode Diagram of the Transfer Function

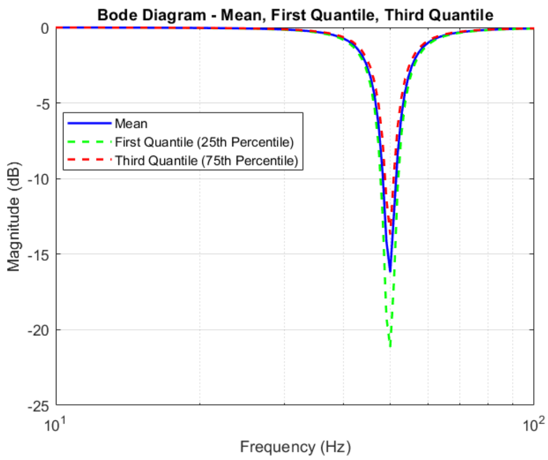
The magnitude plot of the frequency response reveals that the circuit attenuates the signal around 50 Hz, which aligns with the desired behavior for this filter.

5 Monte Carlo Analysis

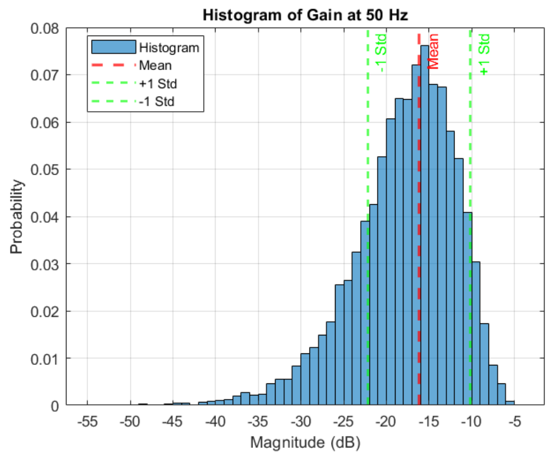
In our Monte Carlo analysis, we assume that the values of the circuit elements are random variables following a Gaussian distribution. The mean of each distribution is the ideal value of the respective component. The effect of different variances (tolerances) will be analyzed in the following sections.

5.1 Monte Carlo Simulation of $H(j\omega)$ with 2% Tolerance

By setting the tolerance for each element to 2% and running the simulation 10,000 times, we plot the mean, as well as the first and third quantiles of the aggregate transfer functions. R_3 and R_4 are considered ideal.



(a) Aggregate Transfer Function



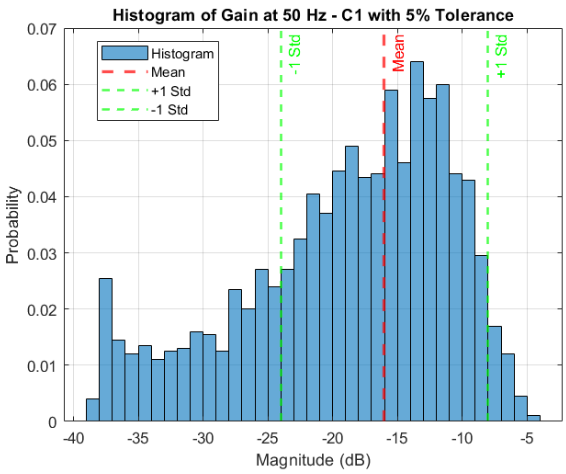
(b) Gain Histogram Around 50 Hz

Figure 10

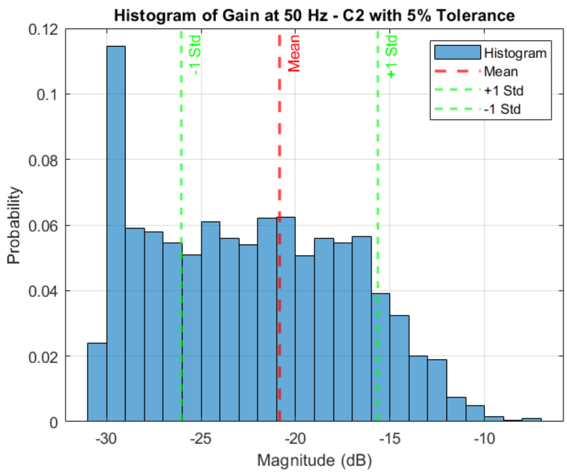
We can see that the circuit effectively attenuates the signal around 50 Hz, as intended, with the mean gain dropping to approximately -16 dB. The spread between the quantile curves reflects the variability introduced by component tolerances.

5.2 Effect of 5% Tolerance on Each Element

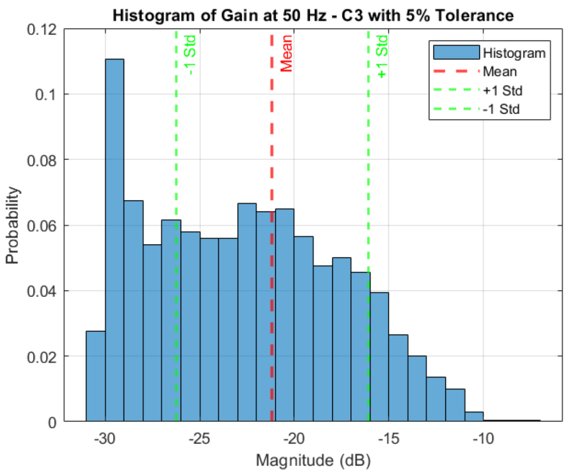
Here, we isolate individual circuit elements and plot the gain histogram by assuming a 5% tolerance for a single element while keeping all other elements at their ideal values. The simulation was performed with 2,000 samples.



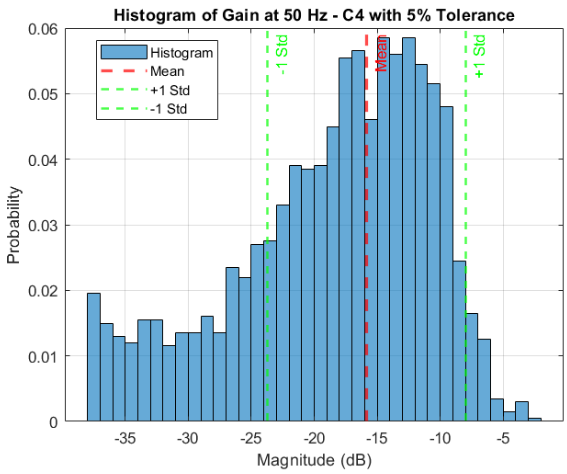
(a) C_1



(b) C_2

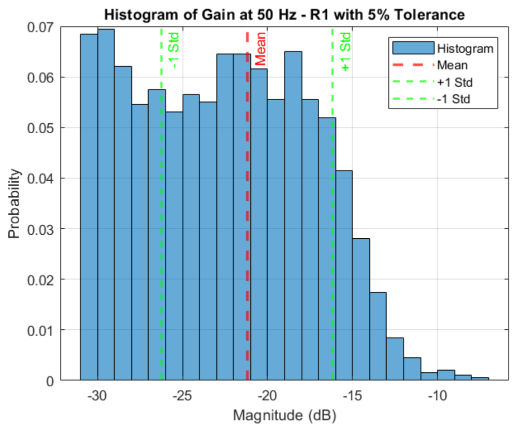


(c) C_3

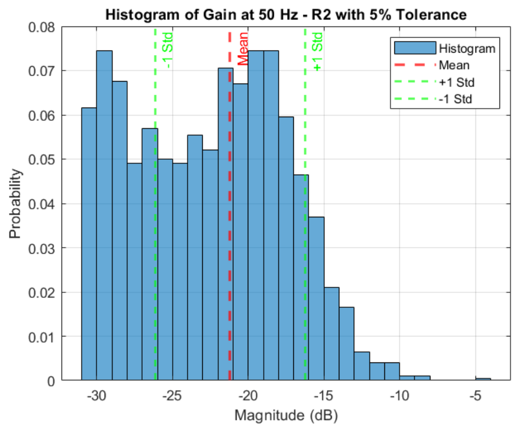


(d) C_4

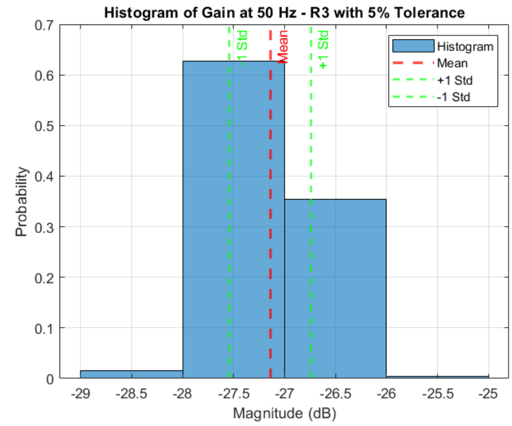
Figure 11: Gain Histograms for Capacitors with 5% Tolerance



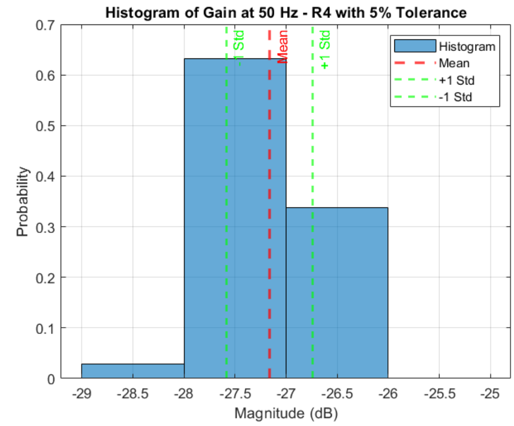
(a) R_1



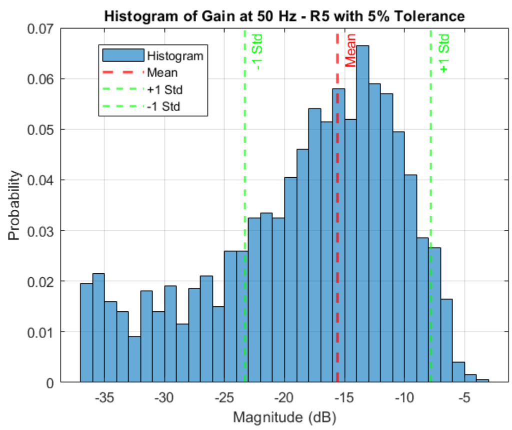
(b) R_2



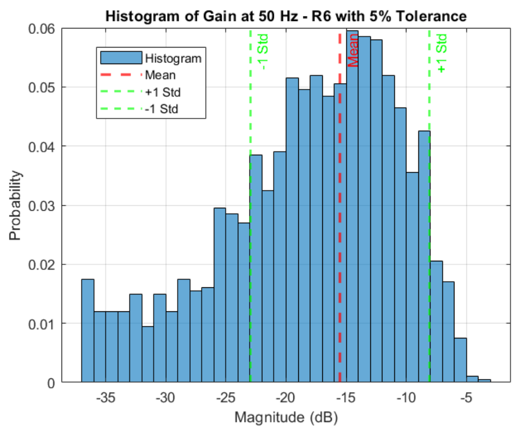
(c) R_3



(d) R_4



(e) R_5



(f) R_6

Figure 12: Gain Histograms for Resistors with 5% Tolerance

Based on the results, we observe that the circuit is more sensitive to tolerances in certain elements, specifically $\{C_1, C_4, R_5, R_6\}$, while variations in other elements do not have as much of an impact on the circuit's performance.

5.3 Adjusting Q to Achieve a More Reliable Circuit

Question: Now, based on the obtained results, try to determine an optimal tolerance for each element so that the following conditions are met. Guidance: You may need to adjust Q to satisfy some of the conditions. Use the new values of R_3 and R_4 in the continuation of the project.

$$BW_{-3\text{dB}} < 25\,\text{Hz}, \quad f_o = 50\,\text{Hz}, \quad P(\text{Gain at } 50\,\text{Hz} < -20\,\text{dB}) > 0.9 \tag{II}$$

Given the results in the previous section, we observe that the target frequency response of $P(\text{Gain at } 50\,\text{Hz} < -20\,\text{dB}) > 0.9$ is rarely achieved. There are multiple ways to bridge this gap. We can:

- Use elements with lower tolerances
- Lower the Q
- Use a second filter

The first option, using elements with lower tolerances, is not feasible due to availability or high cost. Since we are designing a single filter, the third option, adding a second filter, is also unsuitable. Therefore, our last resort is to reduce the Q, sacrificing some accuracy for improved tolerance insensitivity. A trade-off exists between accuracy and the -3 dB bandwidth. To achieve the best results within these constraints, we set $BW_{-3\text{dB}} = 25\,\text{Hz}$ and simulate different R_3 values. Using E24 resistor values, we test R_3 values of 130, 150, and 180 to find the closest match to a 25 Hz bandwidth.

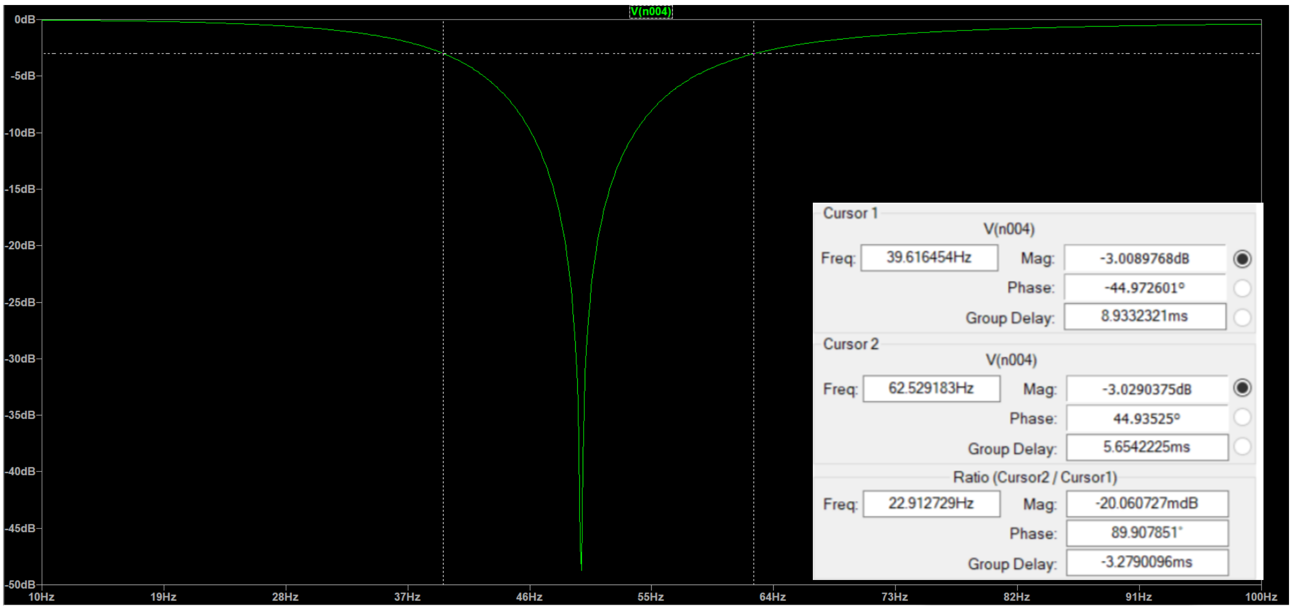


Figure 13: $R_3 = 130$

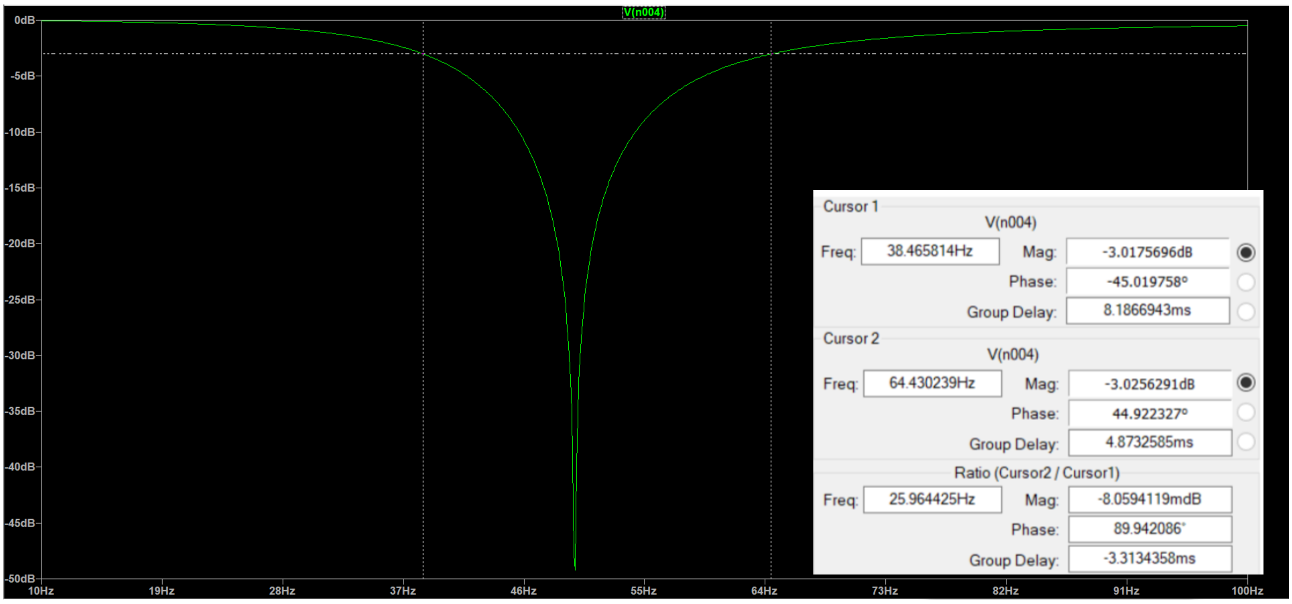


Figure 14: $R_3 = 150$

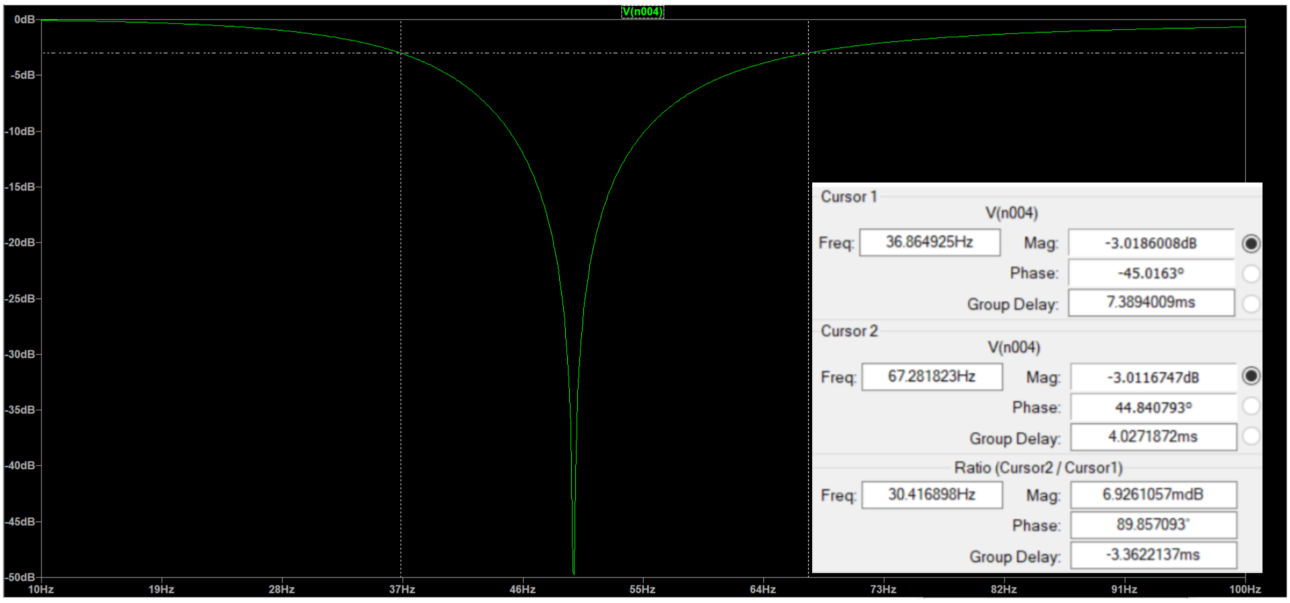


Figure 15: $R_3 = 180$

From the images above, we can summarize the results in the following table:

R_3 Value (Ω)	Bandwidth (Hz)
130	22.9
150	25.9
180	30.4

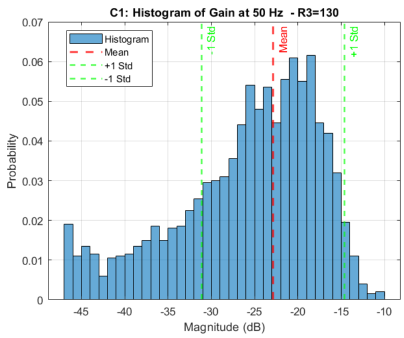
Table 1: $-3\,\mathrm{dB}$ bandwidth with different values of R_3

We see that $R_3 = 130\,\Omega$ and $R_3 = 150\,\Omega$ meet our requirements.

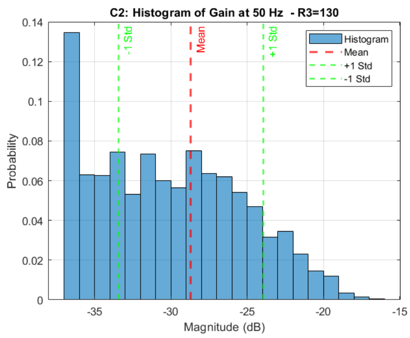
5.4 Simulation of R_3 Values

In the previous section, we observed that only $130\,\Omega$ and $150\,\Omega$ met our criteria for the $-3\,\mathrm{dB}$ bandwidth. In this section, we simulate the circuit using these values to determine which one provides a better response. The simulation is performed with 2,000 samples, assuming a 5% tolerance for a single element while considering all other elements as ideal.

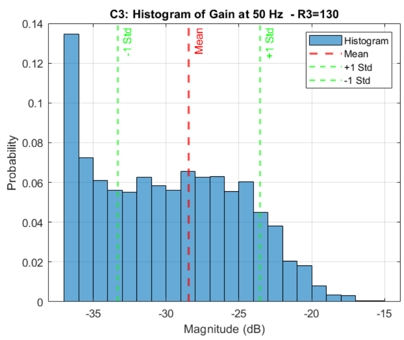
5.4.1 $R_3 = 130\Omega$



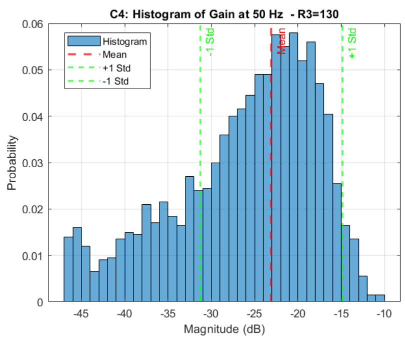
(a) C_1



(b) C_2

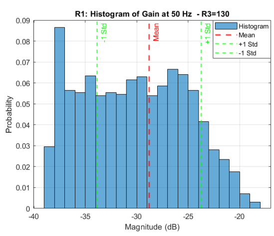


(c) C_3

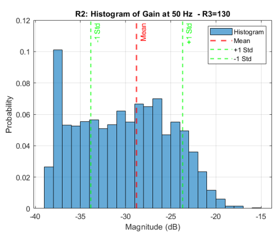


(d) C_4

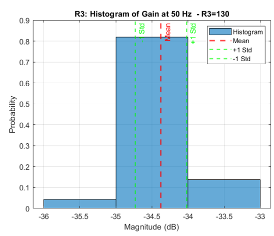
Figure 16: Gain Histograms for Capacitors with 5% Tolerance



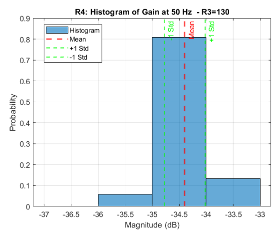
(a) R_1



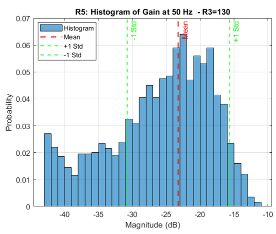
(b) R_2



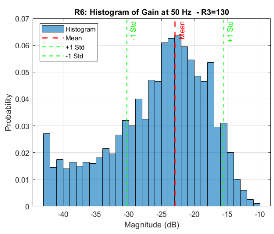
(c) R_3



(d) R_4



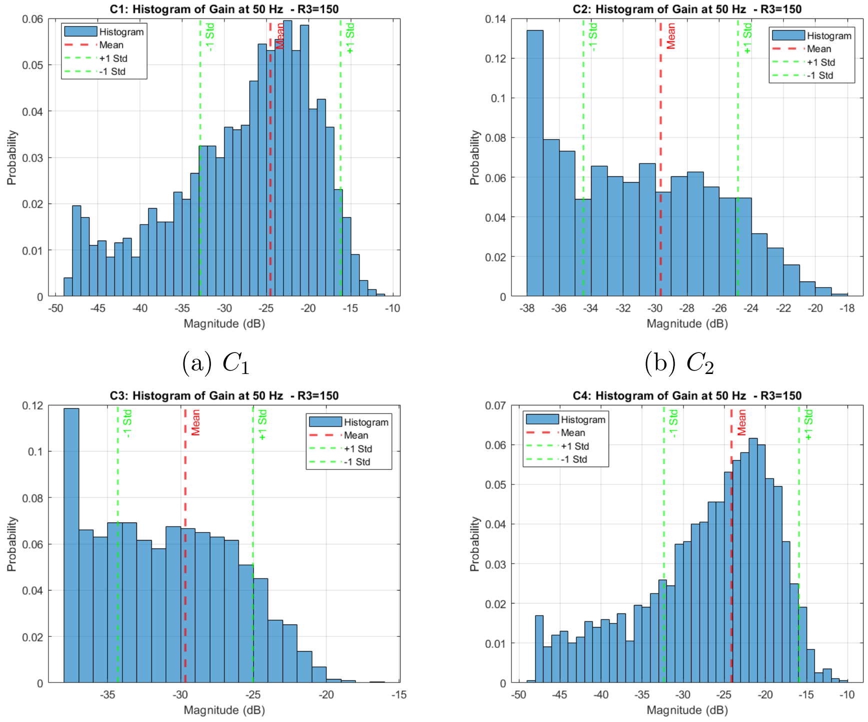
(e) R_5



(f) R_6

Figure 17: Gain Histograms for Resistors with 5% Tolerance

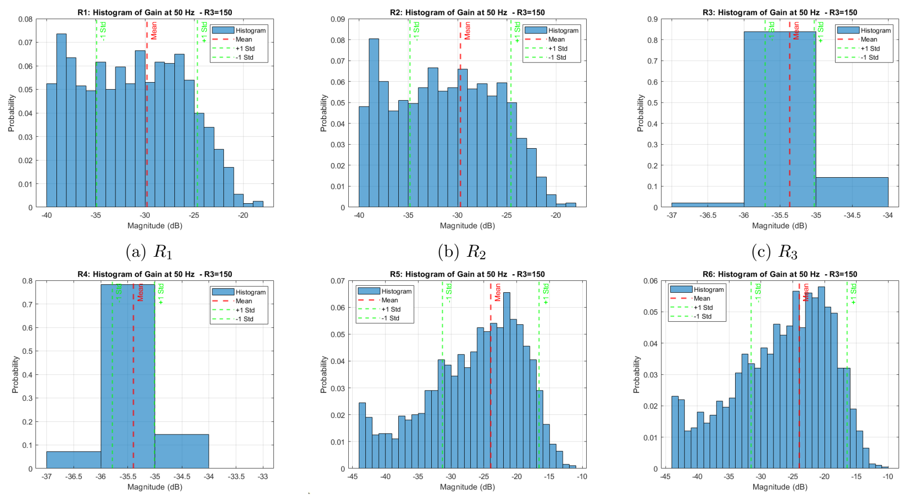
5.4.2 $R_3 = 150\Omega$



(a) C_1 (b) C_2

(c) C_3 (d) C_4

Figure 18: Gain Histograms for Capacitors with 5% Tolerance



(a) R_1 (b) R_2 (c) R_3

(d) R_4 (e) R_5 (f) R_6

Figure 19: Gain Histograms for Resistors with 5% Tolerance

5.4.3 Comparison

In this section, we determine $P(\text{gain at } 50\,\text{Hz} < -20\,\text{dB})$ by calculating the area under the histogram plot.

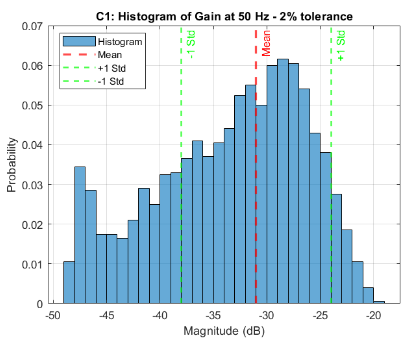
	R_1	R_2	R_3	R_4	R_5	R_6	C_1	C_2	C_3	C_4
$R_3 = 130$	99	99	100	100	75.2	75.2	72.6	98.3	98.5	74
$R_3 = 150$	99.5	99.5	100	100	80	79	83.5	99.5	99.7	80.4

Table 2: P(gain at 50 Hz < -20 dB)

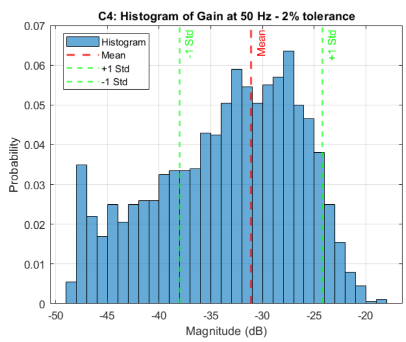
We can clearly see that $150\,\Omega$ has improved the system in terms of probability compared to $130\,\Omega$. Additionally, as we discussed earlier, variations in $\{R_5, R_6, C_1, C_4\}$ have a more significant impact on the circuit's performance.

5.5 Finding the Maximum Tolerance for Each Element ($R_3 = 150$)

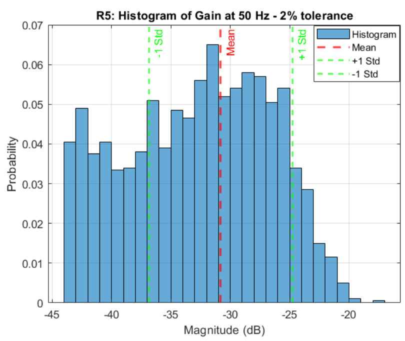
When looking at Table 2, we realize that not all of the elements met the requirement II. For the elements that did meet these criteria, we can conclude that a 5% tolerance is sufficient. However, for those that did not, we now perform a new simulation with a 2% tolerance to determine if it will suffice, as 2% is the next common tolerance after 5%.
It is worth mentioning that the results with a 5% tolerance were not significantly below the threshold, with accuracy values around 0.85.



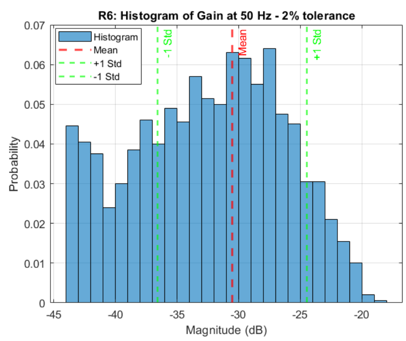
(a) C_1



(b) C_4



(c) R_5



(d) R_6

Figure 20: 2% Tolerance for Sensitive Elements

Looking at the graphs, it is clear that $P(\text{gain at } 50\,\text{Hz} < -20\,\text{dB})$ exceeds 99%. This indicates that a 2% tolerance is sufficient for the sensitive elements.

Now, we plot the histogram of the gain at 50 Hz using the newly determined tolerances.

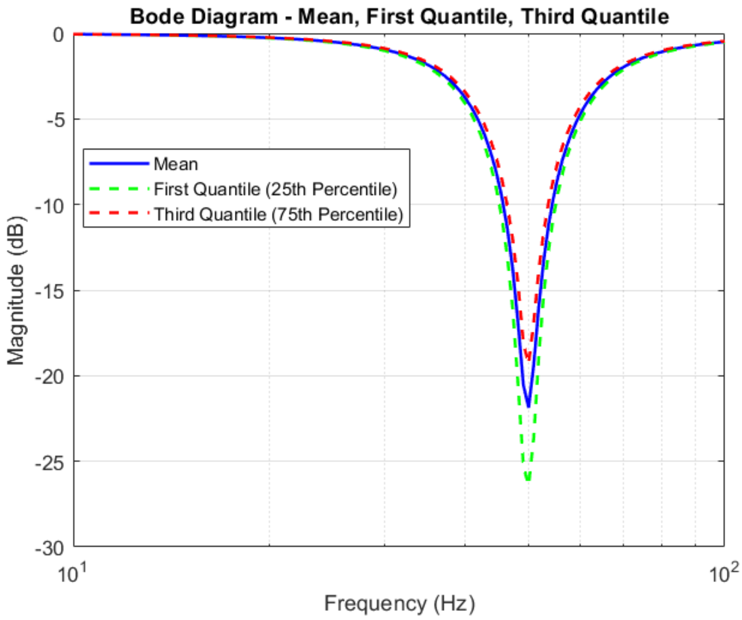


Figure 21: Magnitude of the Transfer Function with New Tolerances

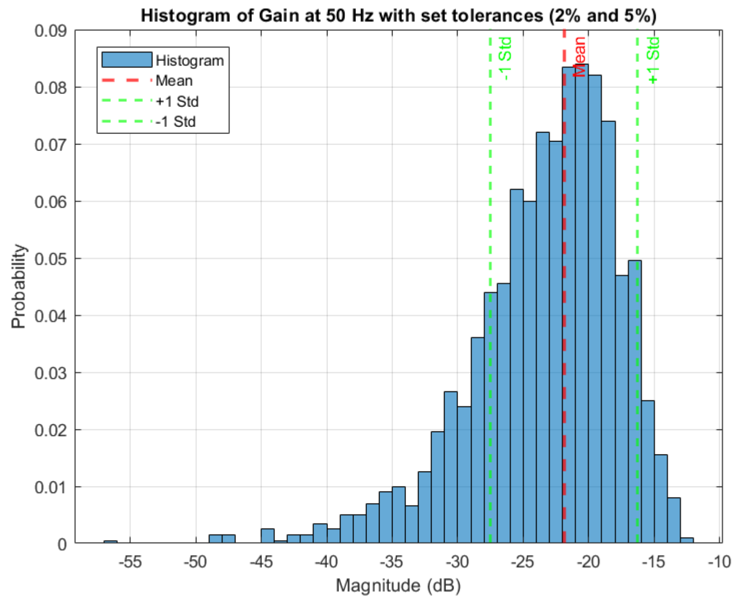
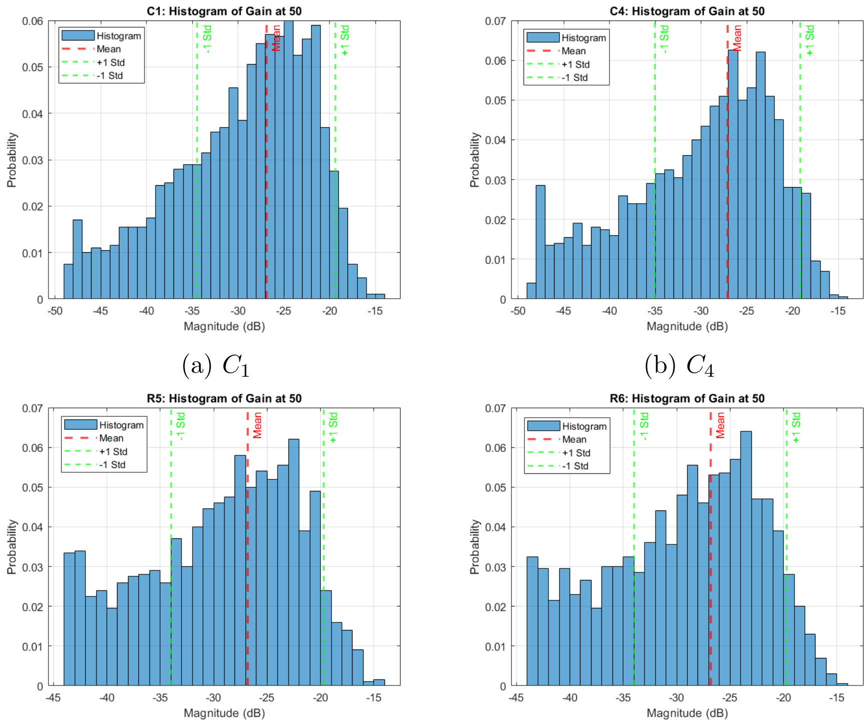


Figure 22: Gain Histogram Around 50 Hz with New Tolerances

When comparing with Figure 10 (all with 2% tolerance), we can see that our circuit has improved in terms of probability. The key observation is that by slightly increasing the bandwidth, we can increase the tolerance and achieve better results. In this case, 6 out of 10 elements have a 5% tolerance, while only 4 have a 2% tolerance.

5.6 Enhancing Circuit Robustness

We found that $\{R_5, R_6, C_1, C_4\}$ are more sensitive to changes. In the previous section, we addressed this by lowering their tolerance. In this section, we explore another approach. As explained in Section 2.3, we replace each resistor with two series-connected resistors of half the original value and each capacitor with two parallel-connected capacitors of half the original value with the same tolerance.



(a) C_1 (b) C_4

(c) R_5 (d) R_6

Figure 23: Robust Circuit Approach

Comparing our results with Figure 18 and Figure 19, we observe that although all elements are assumed to have a 5% tolerance in both cases, using the robust circuit method has significantly improved the results.

5.7 Question

Can you achieve a 20 dB attenuation with a probability of more than 90% for all components with a 2% tolerance and a bandwidth of 15 Hz? (If the answer is no, explain the reason for the limitation.)

First, we need to determine the value of R_3 for which the $-3\,\text{dB}$ bandwidth equals 15 Hz. Through experimentation, we find that $R_3 = 82\,\Omega$ is the closest value from the E24 series.

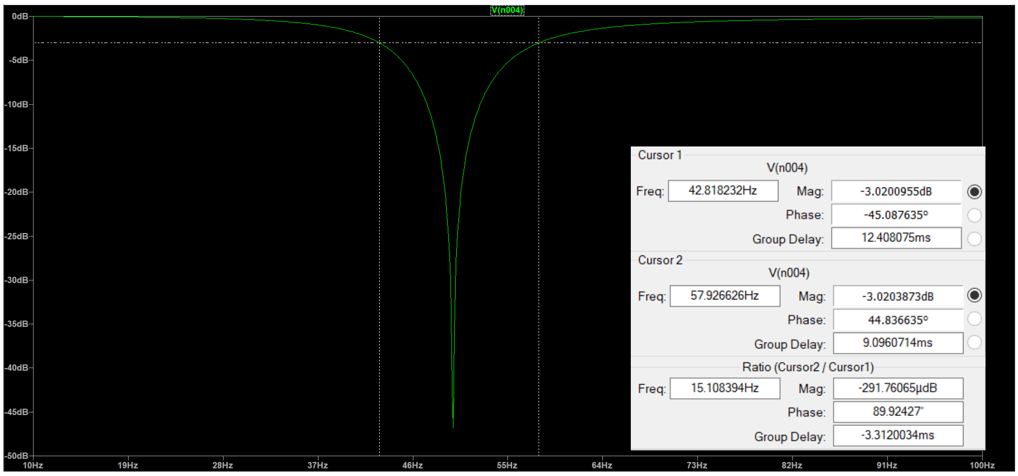


Figure 24: Magnitude of Transfer Function, $R_3 = 82\Omega$

To maximize our chances, we use the same method as in the previous section, replacing all elements with two series or parallel-connected components, each with half the original value, while maintaining a 2% tolerance.

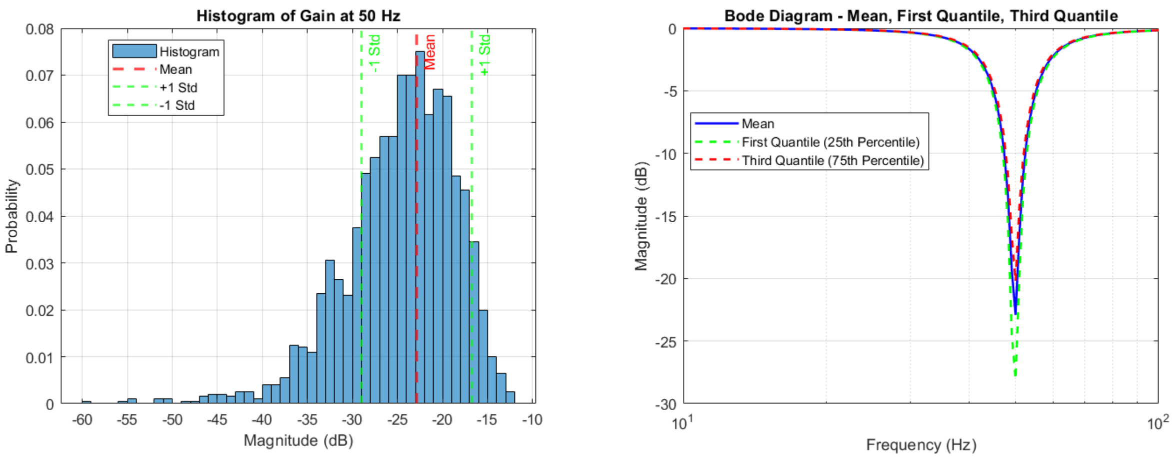


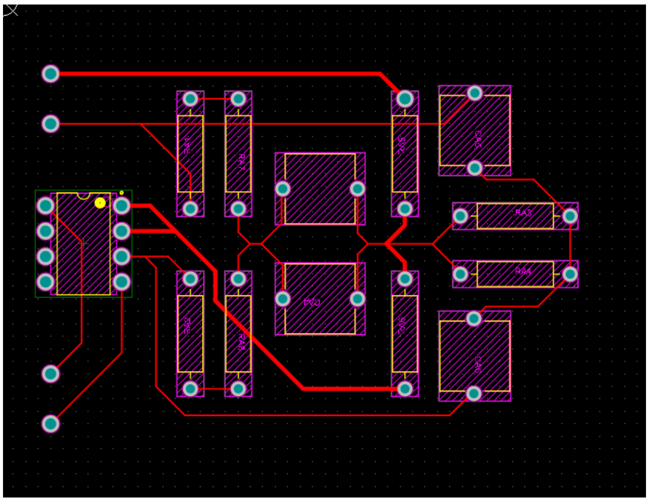
Figure 25: 2% Tolerance for All Elements

By examining the histogram plot, we observe that the probability of achieving more than 20 dB attenuation is approximately 58%. Therefore, the answer is no, as it does not meet the required probability of over 90%.

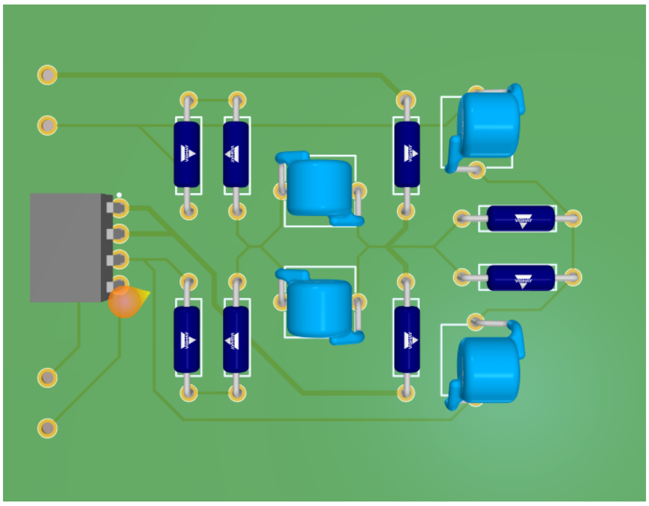
6 PCB Implementation

6.1 Design

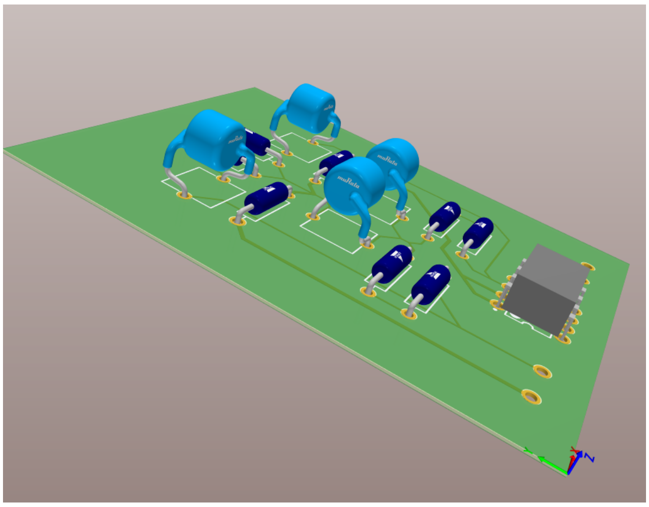
The images presented below show the PCB design of the notch filter circuit. The design was implemented using Altium Designer to create a compact and functional layout suitable for real-world implementation.



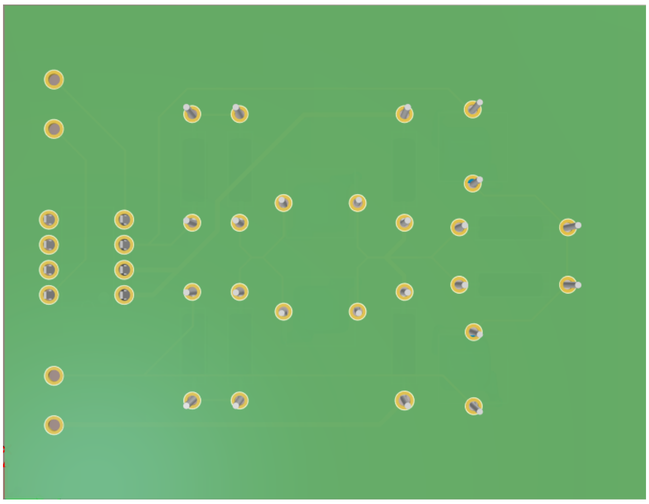
(a) Top-Side View, 2D Layout



(b) Top-Side View



(c) Isometric View



(d) Bottom Side

6.2 Implementation

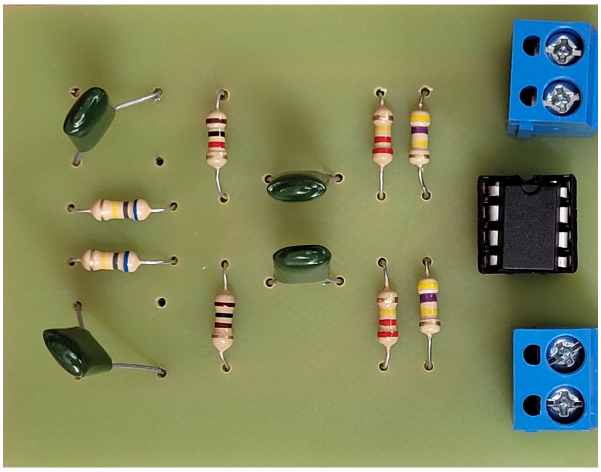
The resistors used in this project were purchased with a 5% tolerance, and the capacitors were polyester capacitors, with a 10% tolerance. Additionally, we used the NE5532 IC for the operational amplifier. Based on the available component values, the following resistor values were chosen for optimal performance:

Resistor	**Ideal Value (kΩ)**	**Value (kΩ)**	**Deviation from Ideal Value**
R_1	680	677	0.44%
R_2	680	683	0.44%
$R_1 \parallel R2$	340	339.993	no significant difference
R_3	0.150	0.1497	0.2%
R_4	1	0.989	1.1%
R_5	680	218.1 + 468 = 686.1	0.897%
R_6	680	217 + 464 = 681	0.147%

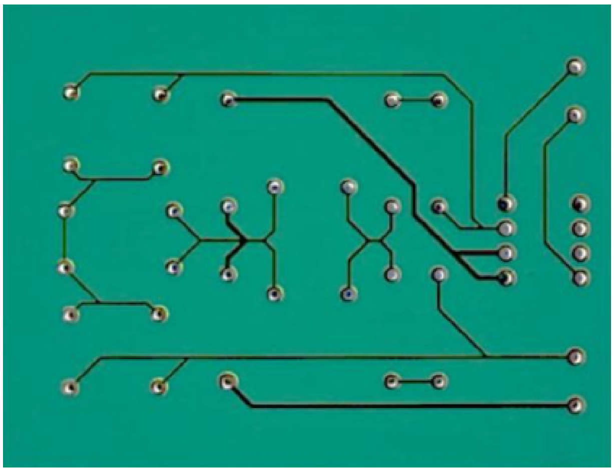
Table 3: Resistor Values and Deviations from Ideal Values

Capacitor	**Ideal Value (nF)**	**Value (nF)**	**Deviation from Ideal Value**
$C1$	4.7	4.68	0.43%
$C2$	4.7	4.74	0.85%
$C3$	4.7	4.68	0.43%
$C4$	4.7	4.71	0.21%
$C_2 \| \| C_3$	9.4	9.42	0.21%

Table 4: Capacitor Values and Deviations from Ideal Values



(a) Implemented Circuit



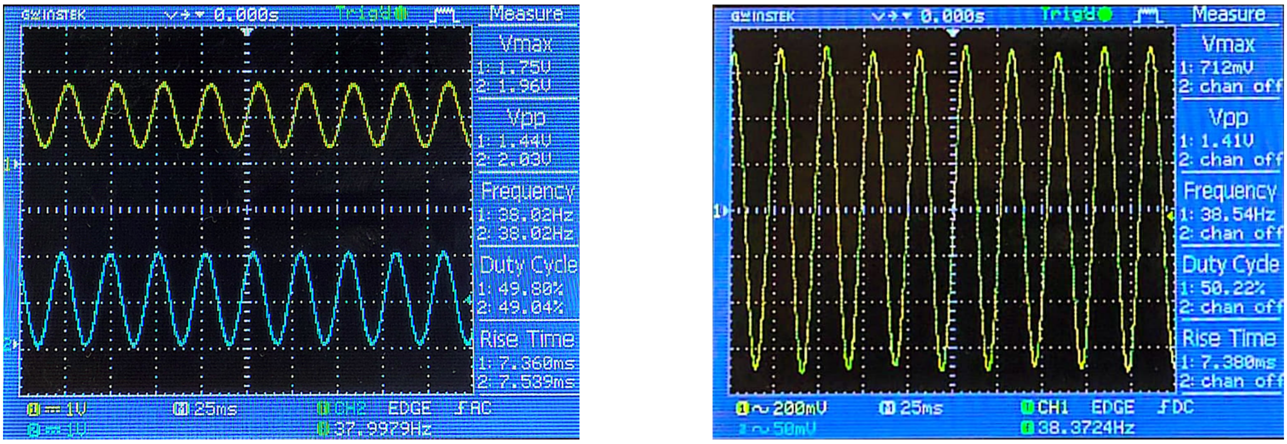
(b) Unassembled PCB Design

6.3 Experimental Results

In this section, we analyze the performance of the implemented circuit by measuring its -3 dB cutoff frequencies and the amplitude of the output across a range of input frequencies. The input signal used for testing was:

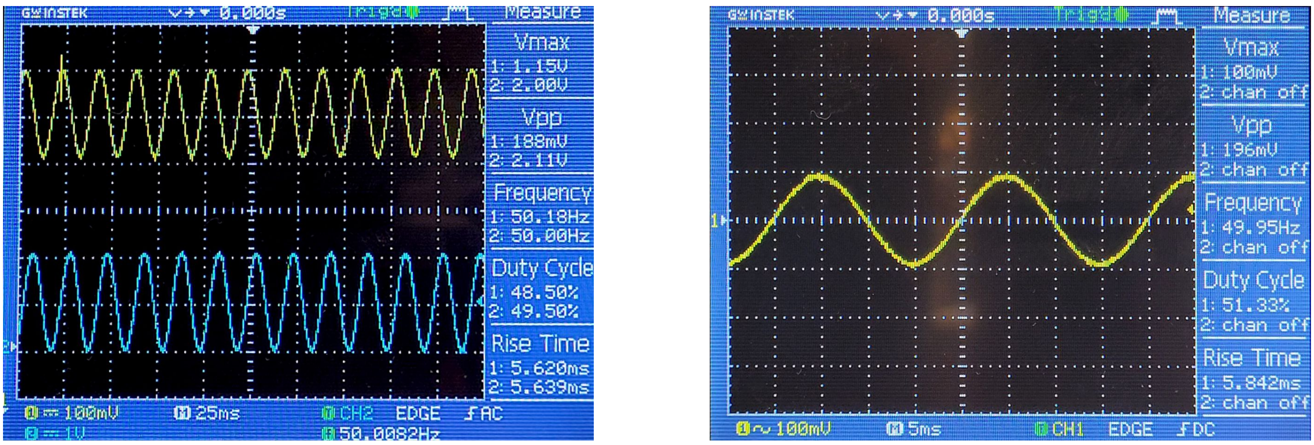
$$V_{\text{in}} = 1 + \sin(\omega t), \quad \omega = 2\pi f$$

where f represents the frequency of the input signal.
The **-3 dB cutoff frequency** is defined as the frequency at which the power of the output signal drops to half of its maximum value. This corresponds to a -3 dB reduction in amplitude, where the amplitude is measured in volts. Specifically, at the -3 dB point, the output voltage amplitude is approximately 0.707 times the maximum amplitude.
In the following figures, the blue signal represents the input, while the yellow signal shows the output from our circuit. These images illustrate the circuit's performance at various frequencies.



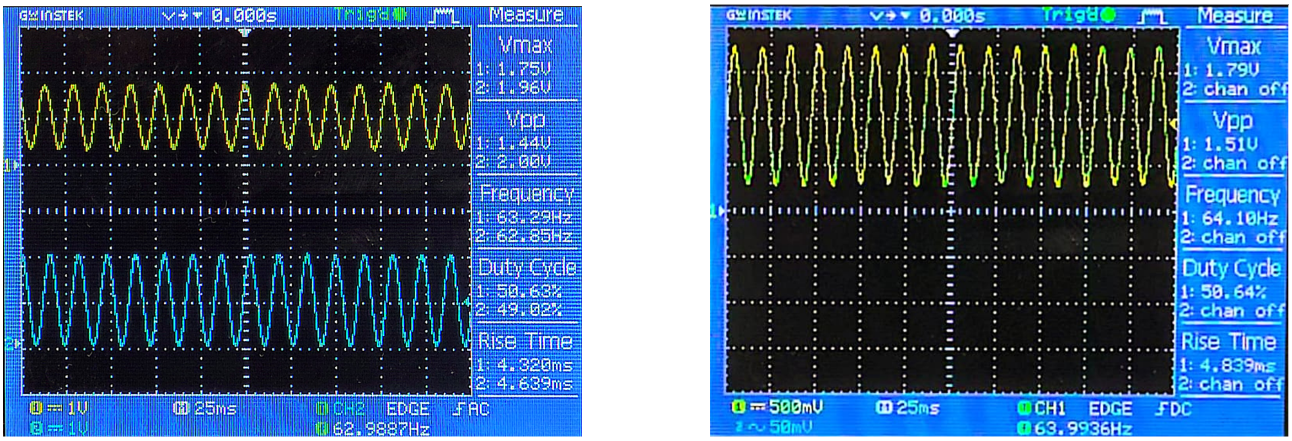
(a) DC-Coupled Output (b) AC-Coupled Output

Figure 28: Low-Frequency Cutoff, Approximately 38 Hz



(a) DC-Coupled Output (b) AC-Coupled Output

Figure 29: Notch Filter Frequency, Approximately 50 Hz



(a) DC-Coupled Output (b) DC-Coupled Output

Figure 30: High-Frequency Cutoff, Approximately 64 Hz

From the images presented above, we can have the following key observations:

- The low-frequency cutoff is observed at approximately 38 Hz.
- The notch filter is most effective around 50 Hz, where it attenuates the input signal by approximately -20.5 dB.
- The high-frequency cutoff is observed at approximately 63 Hz.
- -3 dB bandwidth is approximately 2 Hz.

Thus, we can conclude that the project requirements have been met.

$$\text{Requirements: } BW_{-3\,dB} \leq 25\,\text{Hz}, \quad f_o = 50\,\text{Hz}, \quad \text{Gain}(f = f_o) \leq -20\,\text{dB} \tag{III}$$

7 Appendix

7.1 Measurement Equipment

- **GDS-1062A Oscilloscope (GW Instek)**: To capture and record the input and output signals of the circuit.
- **GDM-396 Multimeter (GW Instek)**: To measure key circuit parameters such as voltage, current, and resistance.
- **SFG-1013 Function Generator (GW Instek)**: To generate the test input signal.
- **LCR-821 LCR Meter (GW Instek)**: To measure capacitor values accurately.

References

[1] T. I. Incorporated, "High-speed notch filters." https://www.ti.com/lit/pdf/slyt235, 2006. Accessed: 2025-02-05.