Engineering Mathematics Project

Instructor: Prof. Hamid Aghajan

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From Noise to Clarity: A Study of Filtering Techniques in Biomedical Imaging

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Preface

Notes on the project:

- **Due date:** 1403/11/12
- The project must be done individually. Each individual will present their results in an online session. Exact date of the online sessions will be announced.
- Please submit your project report as a `.pdf` file. Include all outputs and final results in the report. Make sure to list the practice test questions and provide a concise explanation of your problem-solving approach in each section.
- Ensure that all codes are provided in a separate `.m/.py/.ipynb` file. If a code cannot be tested accurately upon submission, the reported results will be considered invalid, and no points will be awarded in such cases.
- You have the flexibility to utilize either **MATLAB** or **Python** for your project.
- Ensure that you save all files, including your report, codes, helper functions, and any additional outputs, if required, in a compressed file format such as `.zip` or `.rar`. This compressed file should then be uploaded to the CourseWare (CW) submission platform.
- Your file names must be in the following format:

  `Project.#StudentID.zip/.rar/.pdf/.m/.py/.ipynb`

- The details of the grading system of this project will be provided in the coming days. Generally, the project is worth a total of 1 point, which could account for the grades lost in the final exam.
- In this project, it is essential to uphold the principles of academic integrity and refrain from any form of cheating or copying. Cheating undermines the learning process, diminishes personal growth, and compromises the trust placed in us as students/researchers/professionals. It is crucial to recognize that engaging in dishonest practices not only tarnishes our own reputation but also has serious consequences, both ethically and academically. **We emphasize that if anyone is found to have cheated, their results will not be accepted in this project, and they will receive a zero mark.**

Contents

1 Introduction

Biomedical image processing is a specialized field focused on the analysis and manipulation of medical and biological images to enhance their quality and extract valuable information. It plays a crucial role in diagnostics, treatment planning, and medical research by utilizing images from modalities such as MRI, CT scans, ultrasound, and microscopy. The primary steps involved include image acquisition, preprocessing (such as noise reduction and artifact correction), enhancement, segmentation, feature extraction, and classification.

Filters are integral to biomedical image processing, aiding in tasks like noise reduction, edge detection, and feature enhancement to improve image clarity and accuracy. Common types of filters used include smoothing filters (e.g., Gaussian and median filters), edge detection filters (e.g., Sobel and Canny), sharpening filters, and specialized filters like Gabor and wavelet transforms. These tools help in isolating relevant structures, highlighting important details, and ensuring that the images are suitable for accurate analysis and interpretation.

This project investigates various filtering techniques and their impact on biomedical images, focusing on noise reduction and feature preservation. The study also evaluates filter performance using quantitative measures, providing insights into optimal methods for specific image processing challenges.

1.1 Goal of the Project

The primary objectives of this project are:

- To explore the fundamental aspects of biomedical image processing, including noise modeling and filtering techniques.
- To implement and evaluate various filters such as mean, Gaussian, bilateral, and adaptive filters on noisy biomedical images.
- To analyze and compare the effectiveness of these filters using quantitative metrics like Signal-to-Noise Ratio (SNR) and Peak Signal-to-Noise Ratio (PSNR).

2 Aspects of an Image

In this part we want to investigate through different properties of an image. In image processing, analyzing different aspects helps us understand and manipulate visual information. Among these aspects we can mention Magnitude, Phase and Frequency domain. To gain a better understanding of this issue, experiments will be conducted.

2.1 Questions

Using pic1.jpg answer questions below:

1. Intuitively, What do low and high frequencies of an arbitrary image contain? Which one is more important? Do you think your answer depends on the image? Explain it.

2. Explain about low pass filter(LPF) and high pass filter(HPF) . In what applications LPF and HPF are used? Write two functions that apply LPF and HPF to an arbitrary image.

3. Apply LPF and HPF to pic1.jpg using functions written in the previous part. Explain the results.

Figure 1: Kobe Bryant (August 23, 1978 – January 26, 2020)

Using pic2.jpg and pic3.jpg answer questions below:

1) Plot the magnitude and phase of both images. Then, plot an image with magnitude of pic2.jpg and phase of pic3.jpg. Plot the other alternate.

2) Using the results above, Explain Why is the phase information more critical for reconstructing recognizable features of an image compared to the magnitude information?

(a) Wheat Field with Cypresses

(b) Irises

3 Noise

In this part we explore more about Noise. Noise in digital images and signals is an unwanted disturbance that affects quality and can originate from various sources. It can appear in different forms, such as salt-and-pepper noise, Gaussian noise and 50 HZ noise. Each type of noise requires specific filtering techniques for reduction, and understanding the nature of noise is crucial to effectively restoring signal or image quality. The following questions will help you better track the rest of the project.

3.1 Questions

1. What are the primary sources of salt-and-pepper noise in digital images, and what filtering techniques are most effective in mitigating this type of noise?

2. How does electrical interference introduce 50Hz noise into signal recordings, and what methods can be employed to eliminate or reduce this interference?

3. Identify the common sources of Gaussian noise in imaging systems and discuss the most suitable algorithms for its removal without significantly degrading image quality.

4. Explore SNR and PSNR. What roles do Signal-to-Noise Ratio (SNR) and Peak Signal-to-Noise Ratio (PSNR) play in evaluating the effectiveness of noise removal techniques in digital images? Compare and contrast the effectiveness of SNR and PSNR in different applications such as medical imaging, video compression, and wireless communications.

3.2 Noise Modeling

In practice, when working with real-world signals, it is often observed that the received signal differs from the theoretically expected one. To model this phenomenon, we can consider the received signal as the superposition of two signals:

- The original signal, which is theoretically expected.
- An unwanted signal, which is beyond our control and is referred to as noise.

If we denote the original signal as $x(t)$, the noise as $n(t)$, and the received signal as $x_n(t)$, we can model their relationship in two common ways, corresponding to additive and multiplicative noise:

$$x_n(t) = x(t) + n(t), \tag{1}$$
$$x_n(t) = x(t) \cdot n(t). \tag{2}$$

If the values of the noise $n(t)$ were known over time, we could easily recover the original signal using the relations $x(t) = x_n(t) - n(t)$ or $x(t) = \frac{x_n(t)}{n(t)}$. However, the main issue is that noise is inherently random, and its exact values at each moment cannot be precisely determined. Therefore, we model noise at each moment as a random variable with a specified distribution. For example, two proposed distributions for noise could be:

$$n(t) \sim U(-a, a), \tag{3}$$
$$n(t) \sim \mathcal{N}(0, \sigma^2). \tag{4}$$

In this section, we will first generate several types of noise and examine their properties as well as their superposition with noiseless signals.

3.2.1 Function Definition

Write a function named `noise` that takes three parameters a, L, and `type`, and works as follows:

- If the `type` input is the string `"uniform"`, the function returns a vector of length L where each component is a random variable from the distribution $U(-a, a)$.
- If the `type` input is the string `"normal"`, the function returns a vector of length L where each component is a random variable from the distribution $N(0, a^2)$.

3.2.2 Statistical Analysis of Noise Vectors

Before adding noise to the original signal, we observe some statistical properties of the noise and compare them with theoretical expectations.

(a) Generate and Plot Noise Vectors

Generate two noise vectors N_1 and N_2 with $a = 0.2$, using uniform and normal distributions respectively, with $L = 10000$. Plot the histogram of each vector (using MATLAB's `histogram` function). Do the resulting shapes confirm the expected probability distributions?

(b) Mean of Noise Vectors

For the noise vector N_1, define:

$$N_1 = \frac{1}{L} \sum_{i=1}^{L} N_1[i]. \tag{5}$$

Compute N_1 for $L \in \{1, 2, \ldots, 1000\}$ and plot it as a function of L. Repeat the same for N_2. What is the limit of N_1 and N_2 as $L \to \infty$? Which theorem or statement in probability theory precisely explains this result?

(c) Energy of Noise Vectors

Consider the following definition of the energy of a noise vector N:

$$E_N = \frac{1}{L} \sum_{i=1}^{L} (N[i])^2 . \tag{6}$$

For each of the two noise vectors N_1 and N_2, compute the energy as a function of L for $L \in \{1, 2, \ldots, 1000\}$, and plot it. What is the limit of E_{N_1} and E_{N_2} as $L \to \infty$? Verify your result through simulation.

(d) Cross-Correlation of Noise Vectors

For the two noise vectors N_1 and N_2, define:

$$c_{12} = \frac{1}{L} \sum_{i=1}^{L} N_1[i] N_2[i]. \tag{7}$$

Plot c_{12} as a function of L for $L \in \{1, 2, \ldots, 1000\}$. Also, compute the limit of c_{12} as $L \to \infty$ and verify it through simulation. Explain what information c_{12} provides about the two vectors. Justify how c_{12} serves as a measure of similarity.

3.2.3 Sinusoidal Signal and Noise Addition

In this question, the original signal used is a simple sinusoidal signal defined as:

$$x(t) = \sin(2\pi f_0 t), \quad f_0 = 1 \text{ Hz}.$$

The time range of the signal is considered as $[0, 2]$. Generate the signal $x(t)$ with an appropriate sampling frequency such that the plotted graph in MATLAB closely resembles a continuous sinusoidal curve. Plot $x(t)$ and specify the sampling frequency used in the report.

Next, add the noise vectors N_1 and N_2 to the generated signal to produce two noisy signals x_{n1} and x_{n2}:

$$x_{n1}(t) = x(t) + N_1(t), \quad x_{n2}(t) = x(t) + N_2(t). \tag{8}$$

Plot both noisy signals in the report.

3.2.4 Energy Function Definition

Write a function to compute the energy of a vector x, as defined by equation (6):

$$E = \texttt{energy}(x),$$

where the function takes a vector as input and outputs its energy.

3.2.5 Energy Computation of Signals

Using the `energy` function written in the previous section, compute the energy of the following signals:

- The original signal x.

- The noise vectors N_1 and N_2.
- The noisy signals x_{n1} and x_{n2}.

In total, compute the energies of five signals. Report the computed energy values.

Verify whether the relation $E_{x_n} = E_x + E_N$ approximately holds. For any two signals x and y, determine if the statement $E_{x+y} = E_x + E_y$ is true or false. Prove or disprove the statement and validate it for this specific example.

3.2.6 Signal-to-Noise Ratio (SNR)

The ratio of the energy (power) of the original signal to the energy (or power) of the noise is called the Signal-to-Noise Ratio (SNR):

$$\mathrm{SNR} = \frac{E_x}{E_N}.$$

Compute and report the SNR for the signals x_{n1} and x_{n2} generated in Section 3.2.3.

Subsequently, increase the value of a in the noise definitions until the smallest value of a is found such that there is no noticeable visual similarity between x and x_n. Report the corresponding value of a and the SNR for each type of noise separately.

4 Magnetic Resonance Imaging (MRI): A Comprehensive Overview

4.1 Introduction

Magnetic Resonance Imaging (MRI) is a non-invasive diagnostic technique that produces detailed images of the internal structures of the body. Utilizing strong magnetic fields and radio waves, MRI scans help in diagnosing a variety of conditions by revealing differences between healthy and diseased tissue.

4.2 How MRI works

MRI operates on the principles of nuclear magnetic resonance. Here's a step-by-step explanation:

- **Magnetic Alignment**: The human body is composed largely of water molecules, which contain hydrogen nuclei (protons). When a person enters the MRI scanner, a powerful magnetic field aligns these protons.
- **Radiofrequency Pulses**: Short bursts of radio waves are directed at the aligned protons, knocking them out of their equilibrium state.
- **Signal Emission**: When the radiofrequency pulse is turned off, the protons realign with the magnetic field, releasing energy in the process.
- **Image Formation**: This released energy is detected by coils in the scanner. Computers process these signals to construct detailed images of the body's interior.

Here is a brief explanation of the advantages of MRI for brain and spine, this information gives you a good insight into the rest of the project.

4.3 Brain MRI

MRI is particularly valuable for neurological applications due to its superior contrast resolution, which allows for:

- **Detection of Tumors and Lesions** : Identifying brain tumors, cysts, and other anomalies.
- **Stroke Evaluation**: Differentiating between ischemic and hemorrhagic strokes.
- **Multiple Sclerosis (MS)**: Visualizing demyelinating plaques characteristic of MS.
- **Aneurysm and Vascular Malformations**: Assessing blood vessel integrity and abnormalities.
- **Infection and Inflammation**: Detecting conditions like encephalitis or abscesses.

4.4 Spine MRI

Spine MRI provides critical information about spinal anatomy and pathology:

- **Disc Disorders**: Diagnosing herniated or degenerated discs.
- **Spinal Cord Compression**: Identifying causes of spinal cord pressure, such as tumors or bone fragments.
- **Inflammation and Infections**: Detecting osteomyelitis or epidural abscesses.
- **Congenital Anomalies**: Assessing birth defects affecting the spine.
- **Trauma**: Evaluating injuries to vertebrae, discs, ligaments, and the spinal cord itself.

4.5 Questions

Explore the main sources of noise in this method and suggest some ways to deal with this problem.

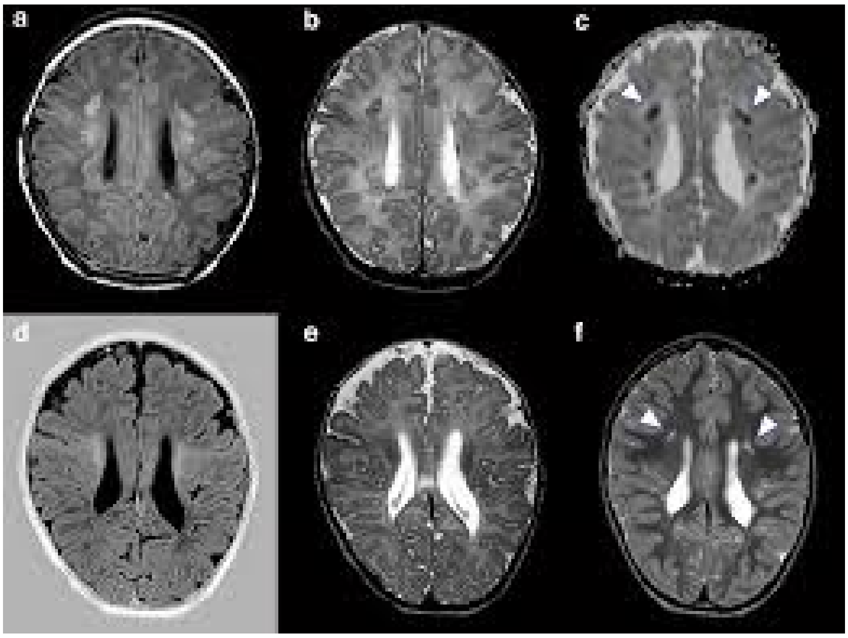

Figure 3: Brain MRI

5 Filters

In this part we learn about different filters, how they work and what's their impact on image. Then using these filters, you denoise two given noisy images and measure how effective each filter is.

Notice: you are not allowed to use built-in functions for this part.

5.1 Mean Filter

Investigate the Mean filter and implement a function to apply it to an input image. Explain about the effect of kernel size.

5.2 Gaussian Filter

Explore about Gaussian filter and write a function that applies a Gaussian filter to an input image. What is the effect of variance and kernel size on the output image?

5.3 Bilateral Filter

Explore about Bilateral filter and write a function that applies a Bilateral filter to an input image.

5.4 Evaluation

Evaluate the output of the three methods discussed in this section using SNR and PSNR criteria and present the results, compared with original_brain_MRI.jpg and original_spine_MRI.jpg in a table. Also, calculate these two criteria for the initial noisy image and declare which method performed better in noise reduction. Can you guess what kind of noise each picture has?

1/16

1	2	1
2	4	2
1	2	1

1/273

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

1/1003

0	0	1	2	1	0	0
0	3	13	22	13	3	0
1	13	59	97	59	13	1
2	22	97	159	97	22	2
1	13	59	97	59	13	1
0	3	13	22	13	3	0
0	0	1	2	1	0	0

Figure 4: Discrete approximation of the Gaussian kernels 3x3, 5x5, 7x7

6 Adaptive Filter

6.1 Introduction

Adaptive filters are dynamic signal processing tools that adjust their parameters in real-time to accommodate varying input signal characteristics. Unlike fixed filters, which have static coefficients, adaptive filters modify their behavior based on the changing conditions of the data they process. This flexibility makes them particularly effective in environments where signal properties fluctuate, such as in biomedical image processing. In this field, adaptive filters are used for tasks like noise reduction, artifact removal, and image enhancement, ensuring that important features are preserved while unwanted disturbances are minimized. They employ algorithms, such as the Least Mean Squares (LMS) or Recursive Least Squares (RLS), to continuously optimize filter parameters based on feedback from the output. This ability to learn and adapt makes adaptive filters invaluable for improving image quality and accuracy in medical diagnostics and research applications.

6.2 Questions

1. What are finite impulse response (FIR) Adaptive Filters, and What are their specific applications in Biomedical Image Processing? Explore about Wiener filters.

2. Explore the role of Stochastic Gradient Descent(SGD) in optimizing filter coefficients in real-time, its convergence properties, and its application in adaptive filter design.

3. Write a function that applies an Adaptive filter to an input image. Suggest a way to tune learning parameter. Apply this filter to original_brain_MRI.jpg and original_spine_MRI.jpg.

4. Evaluate the output of this method using SNR and PSNR criteria and present the results. Which of these four filters did better for each image? Can you explain why?

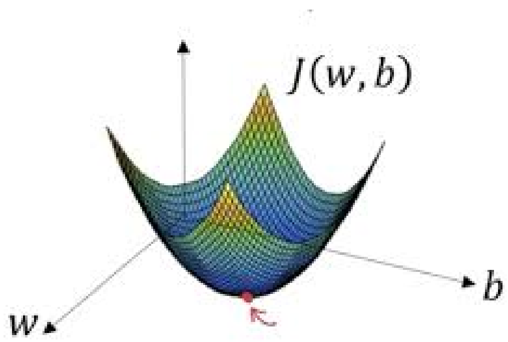

Figure 5: Gradient Descent Algorithm

7 References

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