1 Solving an oscillator circuit equations

The problem to be considered involves the electronic circuit shown in Fig. 1. This circuit is a Colpitts oscillator (it was studied in Engi. 3013 and its frequency of oscillations were obtained there.) The project involves setting up the differential and algebraic equations for the circuit with an approximate model of the transistor and solving the equations to visualize the voltage waveforms and frequency appropriately.

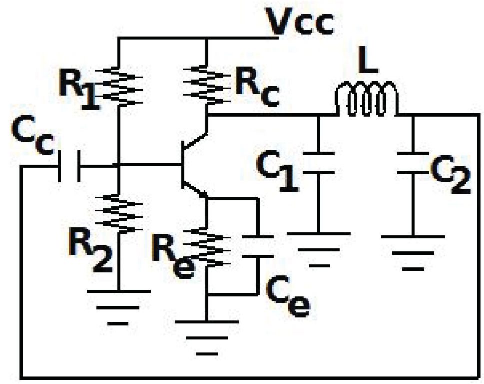

Figure 1: An Oscillator Circuit

The differential equations are set up using the following variables: current through the inductor, the voltage across the capacitor C_1 which is the voltage across the collector of the transistor, the voltage across the capacitor C_e which is the voltage at the emitter of the transistor, and the voltages across the capacitors C_c and C_2 which together make up the voltage across the base of the transistor.

The final five (corresponding to the 5 energy storage elements in the circuit) first order differential equations will have the collector, base and emitter voltages of the transistor, the capacitor voltage across C_c and the inductor current as the variables and in addition will have the transistor base current, emitter current and collector current in them as unknowns. The power supply voltage V_{cc} will be present in these equations along with the resistors, capacitors and inductor in symbolic form. Obtain these equations and document them in your report.

2 The Transistor Model

The transistor will be modeled by the simple Ebers-Moll model which neglects the internal capacitances of the transistor. This model (NPN Transistor) is shown in Fig. 2 and provides equations for the base current, collector current and emitter current of the transistor in terms of the voltages at the base, collector and emitter as given below and eliminates these variables from the differential equations set up in the previous Section in terms of the base, emitter and collector voltages and other constant parameters.

The Equations for Figure 2 (Ebers-Moll Equations) are:

$$I_C = \alpha_F I_F - I_R$$

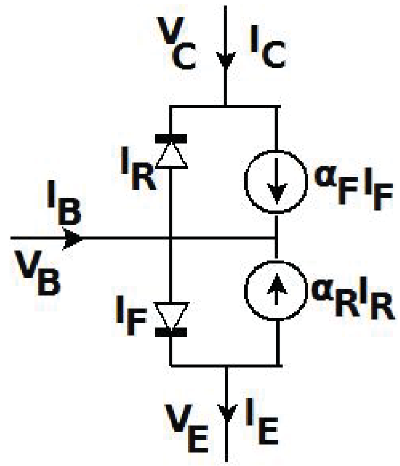

Figure 2: Ebers-Moll Model of NPN Transistor

$$
\begin{aligned}
I_E &= I_F - \alpha_R I_R \\
I_B &= (1-\alpha_F)I_F + (1-\alpha_R)I_R = I_E - I_C \\
I_F &= I_{ES}\left(e^{\frac{V_B - V_E}{V_T}} - 1\right) \\
I_R &= I_{CS}\left(e^{\frac{V_B - V_C}{V_T}} - 1\right) \\
\alpha_F I_{ES} &= \alpha_R I_{CS} = I_S
\end{aligned} \tag{1}
$$

3 The composite system

Once the differential equations are set up and the transistor current expressions obtained in terms of the unknown transistor terminal voltages, you may proceed to solve them using Matlab differential equations solver using appropriate prepackaged routines or using your own coded solvers.

Results to be obtained by solving these equations are:

1. Obtain the frequency of oscillation in steady-state of the circuit for the nominal parameters given below in the numerical data from your simulations.

2. Is the waveform a sine wave at the collector of the transistor (relative to ground), or at the capacitor C_2? Comment on the waveforms and attach figures in the report of the steady state waveforms at these two points.

3. If the inductor value is increased by 10% from its nominal value in the numerical data what is the frequency of oscillations (if any) from the new simulations.

4. If the temperature of the circuit increased by 50°C from room temperature then what is the change in oscillation frequency (if any) for the nominal value circuit?

4 Report:

1. Provide the differential equations with the notation $V_C, V_B, V_E, V_{C_c}, I_L$ with the collector current and emitter current represented as I_C, I_E. Summarize the algebraic equations for these two currents from the Ebers-Moll model.

2. Provide a printout of your commented code and details on how to run it so the instructor can take your soft copy of the code (by D2L) and run it to verify the code runs as per your sequence on how to run it and generates the results claimed by you.

3. Provide figures with appropriate labels and titles of the waveforms obtained from your simulations in the steady state and answers to the questions above.

4. Summarize your conclusions including difficulties encountered and how they were resolved.

5 Numerical Data:

The nominal circuit parameters are: $C_1 = C_2 = 2$ pF, $C_c = 400$ pF, $C_e = 100$ pF, $L = 1$ μH, $R_1 = 8$ kΩ , $R_2 = 2$ kΩ , $R_c = 2.4$ kΩ , $R_e = 1.3$ kΩ , $V_{cc} = 12$ V, nominal circuit temperature = 25°C .

Ebers-Moll model transistor parameters are: $\alpha_F = 0.9900$, and $\alpha_R = 0.5$. $I_S = 0.01$ pA , $V_T = kT/q$ with k = Boltzmann constant, q = electron charge and T temperature in °K .