Intro to Robotics
Exercise 3

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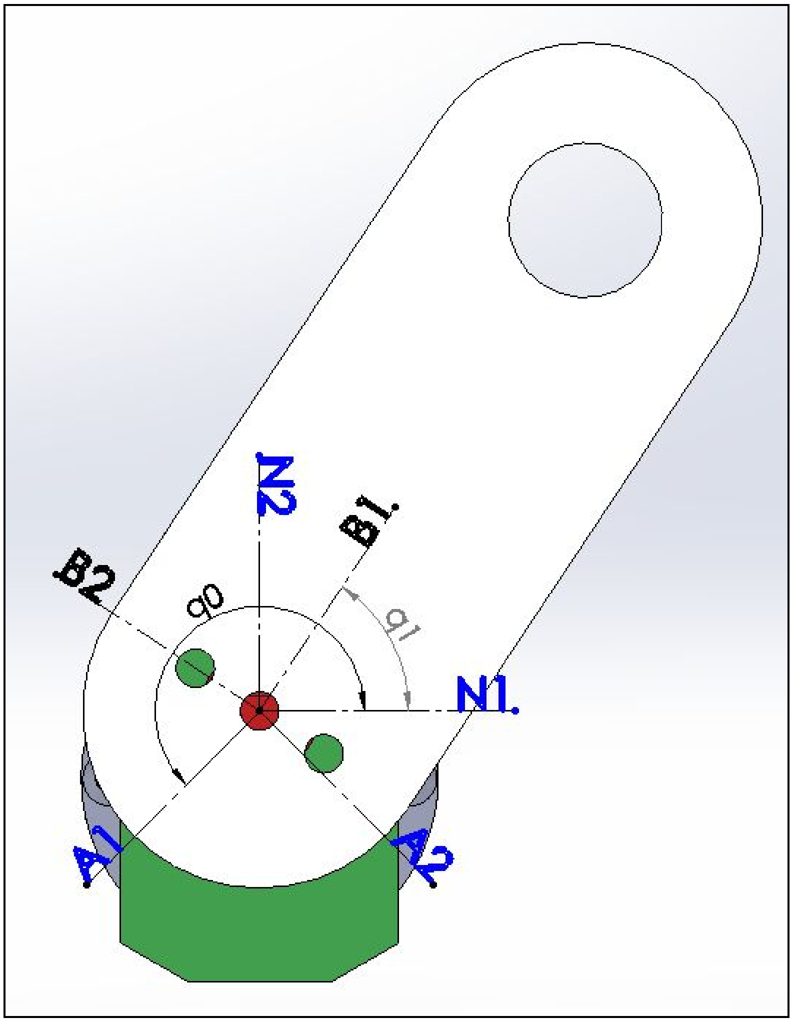
System Diagram



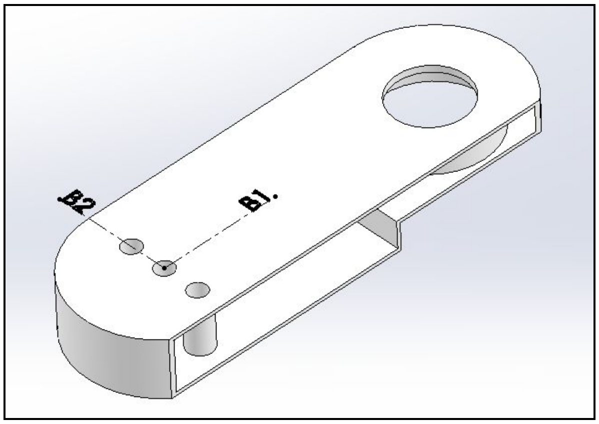
The system is shown below. The shaft (the top of which is a red circle) is considered body A, and the link is body B. The shaft's A.3 axis is in the same direction as the N.3 and B.3 axis, however, the A.1 axis is at an angle q0 from the N.1 axis, not an angle q1. The COM of body A is assumed to be on its polar axis. The COM of body B is assumed to be at the dot next to the text B1. which is a distance LB away from point N, which is located at the origin.

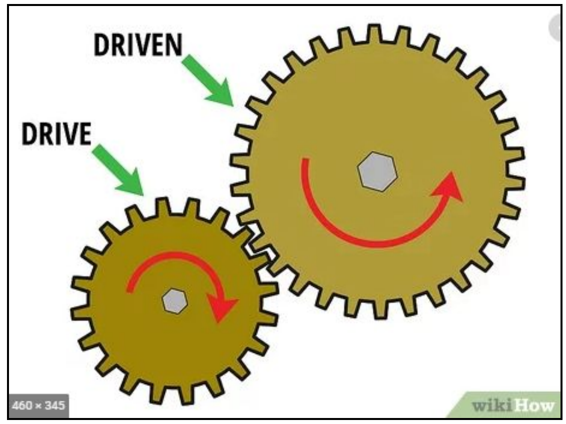
The constraints equation is $\Delta q_0 = -q_1 gr$ where the q's are the generalized coordinates and gr is the gear ratio between the motor shaft and the motor output spindle. The constraint equation carries to both the velocity and acceleration terms.

Thus the system has two generalized coordinates, q0 and q1, and a single gear constraint equation, thus the system has a single degree of freedom

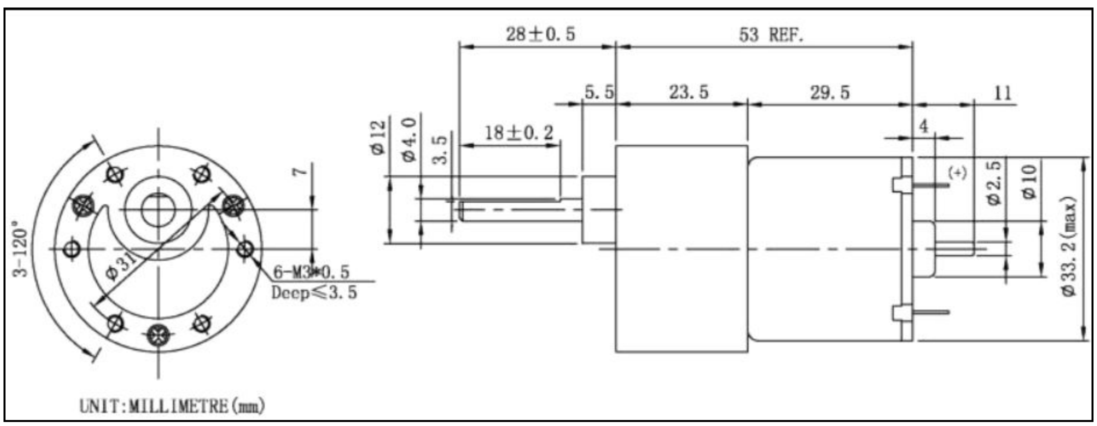
The gear ratio used in the gear constraint was printed on the side of the motor.

Mass Properties





The mass for the link, body B, was computed using ABS mass properties from SolidWorks applied to the CAD model. The model was hollowed out, and the shell thickness set to 0.90 mm, the same as was used to print the physical model. The internally printed structure was neglected as a sparse infill was used.



The mass of the axle that spins in body A was calculated using the drawing shown above. The axle was assumed to be 10.0 mm in diameter and 53 mm in length. A CAD model was generated, and the mass estimated with steel mass properties applied to the CAD model. The gears were assumed to interact in a manner similar to the ones shown below

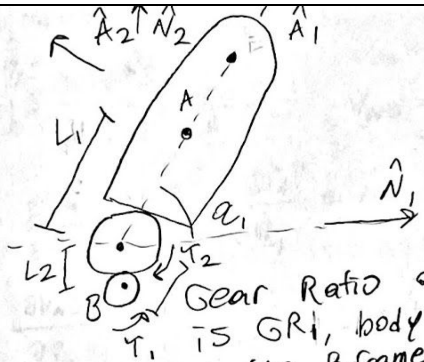
The moments of inertia for both bodies were evaluated in the same manner as the mass properties.

Body	Mass [kg]	Izz [kg m^2]
Body A	26.45×10^{-3}	3.30×10^{-7}
Body B	7.28×10^{-3}	6.39×10^{-6}

Equations of Motion

Per the assignment, the equations of motion were derived by hand. This is inconsistent with prior exercises, and points should not be taken off for not showing the equations of motion derived computationally. A direct quotation from the instructions is shown below.

> Write up this lab as a homework problem including the development of the equations of motion by hand. Make sure to include:

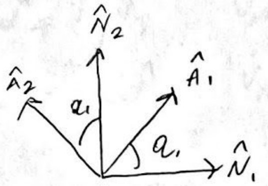


$L_A = \{\vec{P}_{NA},\ {}^{N}_{A}R\} + \text{geom}$

$\vec{P}_{NA} = L_1 \hat{A}_1$

$\vec{P}_{NB} = -L_2 \hat{N}_2$

Gear Ratio associated is GR_1, body B is attached to the B frame, which has a rotation of q_2

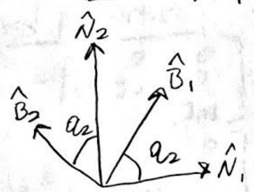


$\hat{A}_1 = c_1 \hat{N}_1 + s_1 \hat{N}_2$

$\hat{A}_2 = -s_1 \hat{N}_1 + c_1 \hat{N}_2$

$\hat{A}_3 = \hat{N}_3$

$${}^{N}_{A}R = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$\hat{B}_1 = c_2 \hat{N}_1 + s_2 \hat{N}_2$

$\hat{B}_2 = -s_2 \hat{N}_1 + c_2 \hat{N}_2$

$\hat{B}_3 = \hat{N}_3$

$${}^{N}_{B}R = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simplifications: None

Rigid Bodies: 2

Inertial Reference Frame and point: $N = (\hat{N}_1, \hat{N}_2)$

Other points and frames: $A = (\hat{A}_1, \hat{A}_2)$, $B = (\hat{B}_1, \hat{B}_2)$

Body attached points are A and B

Coordinates: 2, q_1 and q_2

Constraints: $GR_1 q_3 = q_1$

Degrees of Freedom: 2 coord − 1 const = 1 DOF

Velocity

$${}^{N}\omega^{A} = \dot{q}_1 \hat{N}_3 \qquad V_{NA} = L_1 \dot{q}_1 \hat{A}_2$$

$${}^{N}\omega^{B} = \dot{q}_3 \hat{N}_3 = \frac{\dot{q}_1}{GR_1} \hat{N}_3 \qquad V_{NB} = 0$$

$$\frac{\partial V_{NA}}{\partial \dot{q}_1} = L_1 \hat{A}_2 \qquad \frac{\partial\, {}^{N}\omega^{A}}{\partial \dot{q}_1} = \hat{N}_3$$

$$\frac{\partial V_{NB}}{\partial \dot{q}_1} = 0 \qquad \frac{\partial\, {}^{N}\omega^{B}}{\partial \dot{q}_1} = \frac{1}{GR_1} \hat{N}_3$$

Acceleration

$${}^{N}\dot{\omega}^{A} = \ddot{q}_1 \hat{N}_3 \qquad \dot{V}_{NA} = L_1 \ddot{q}_1 \hat{A}_2 - L_1 \dot{q}_1^{2} \hat{A}_1$$

$${}^{N}\dot{\omega}^{B} = \frac{\ddot{q}_1}{GR_1} \hat{N}_3 \qquad \dot{V}_{NB} = 0$$

Mass properties

$$I_{AA} = \begin{bmatrix} I_{A1} & 0 & 0 \\ 0 & I_{A2} & 0 \\ 0 & 0 & I_{A3} \end{bmatrix} \qquad \text{mass of body } A = m_A$$

$$I_{BB} = \begin{bmatrix} I_{B1} & 0 & 0 \\ 0 & I_{B2} & 0 \\ 0 & 0 & I_{B3} \end{bmatrix} \qquad \text{mass of body } B = m_B$$

Forces and moments

$$(\Sigma F)_A = -g m_A \hat{N}_3 \qquad (\Sigma M)_{AA} = \tau_2 \hat{N}_3$$

$$(\Sigma F)_B = -g m_B \hat{N}_3 \qquad (\Sigma M)_{BB} = -\tau_1 \hat{N}_3$$

$$\frac{\partial H_{AA}}{\partial t} = I_{AA}\,{}^{N}\dot{\omega}^{A} + {}^{N}\omega^{A} \times I_{AA}\,{}^{N}\omega^{A} = \begin{bmatrix} I_{A1} & 0 & 0 \\ 0 & I_{A2} & 0 \\ 0 & 0 & I_{A3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} \times \begin{bmatrix} I_{A1} & 0 & 0 \\ 0 & I_{A2} & 0 \\ 0 & 0 & I_{A3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}$$

$$\frac{\partial H_{AA}}{\partial t} = I_{A3}\ddot{q}_1 \hat{N}_3$$

$$\frac{\partial H_{BB}}{\partial t} = I_{BB}\,{}^{N}\dot{\omega}^{B} + {}^{N}\omega^{B} \times I_{BB}\,{}^{N}\omega^{B} = \begin{bmatrix} I_{B1} & 0 & 0 \\ 0 & I_{B2} & 0 \\ 0 & 0 & I_{B3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1/GR_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1/GR_1 \end{bmatrix}\begin{bmatrix} I_{B1} & 0 & 0 \\ 0 & I_{B2} & 0 \\ 0 & 0 & I_{B3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{q}/GR_1 \end{bmatrix}$$

$$\frac{\partial H_{BB}}{\partial t} = \frac{I_{B3}\ddot{q}_1}{GR_1}\hat{N}_3$$

$$F_1 = \left[\Sigma F_A \cdot \frac{\partial V_{NA}}{\partial \dot{q}_1} + \Sigma M_A \cdot \frac{\partial\,{}^{N}\omega^{A}}{\partial \dot{q}_1}\right] + \left[\Sigma F_B \cdot \frac{\partial V_{NB}}{\partial \dot{q}_1} + \Sigma M_B \cdot \frac{\partial\,{}^{N}\omega^{B}}{\dot{q}_1}\right]$$

$$\boxed{F_1 = -T_2 + \frac{T_1}{GR_1}}$$

$$F_1^* = \left[m_A \dot{V}_{NA} \cdot \frac{\partial V_{NA}}{\partial \dot{q}_1} + \frac{\partial H_{AA}}{\partial t} \cdot \frac{\partial\,{}^{N}\omega^{A}}{\partial \dot{q}_1}\right] + \left[m_B \dot{V}_{NB} \cdot \frac{\partial V_{NB}}{\partial \dot{q}_1} + \frac{\partial H_{BB}}{\partial t} \cdot \frac{\partial\,{}^{N}\omega^{B}}{\partial \dot{q}_1}\right]$$

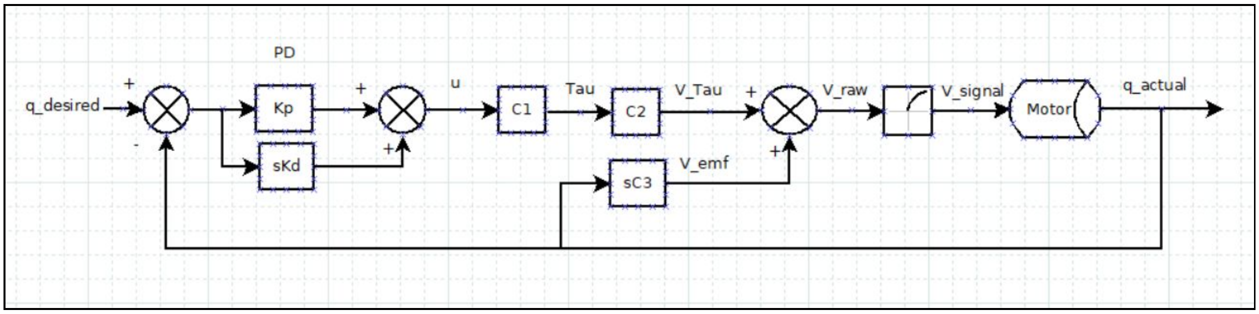
$$F_1^* = \left[m_A\left(L_1\ddot{q}_1\hat{A}_2 - L_1\dot{q}_1^2\hat{A}_1\right) \cdot L_1\hat{A}_2\right] + \left[I_{A3}\ddot{q}_1\hat{N}_3 \cdot \hat{N}_3\right] + \left[m_B(0) \cdot 0\right] +$$

$$\left[\frac{I_{B3}\ddot{q}_1}{GR_1}\hat{N}_3 \cdot \frac{1}{GR_1}\hat{N}_3\right]$$

$$F_1^* = \left(I_{A3} + m_A L_1^2\right)\ddot{q}_1 + \frac{I_{B3}}{GR_1^2}\ddot{q}_1$$

$$F_1 - F_1^* = 0$$

Block Diagram & Discussion



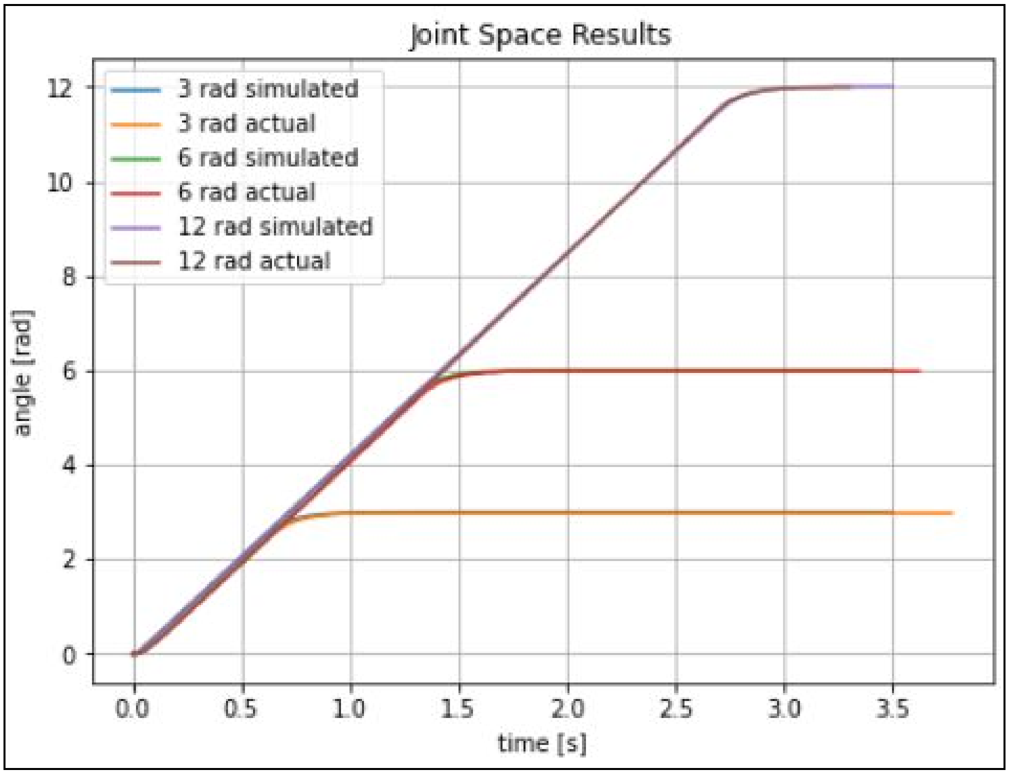
All simulations were performed using an RK45 integrator. The Kp value used was calculated based on the CPR of the encoder, which was set to 1000. The equation used is shown below, where Vmin is the minimum voltage required for the motor to begin moving. Vmin was determined via experimentation. For the joint space model, Kv was simply set so as to be critically damped. The saturation block between V_raw and V_signal indicates that the voltage signal was constrained between 0 and 12 volts.

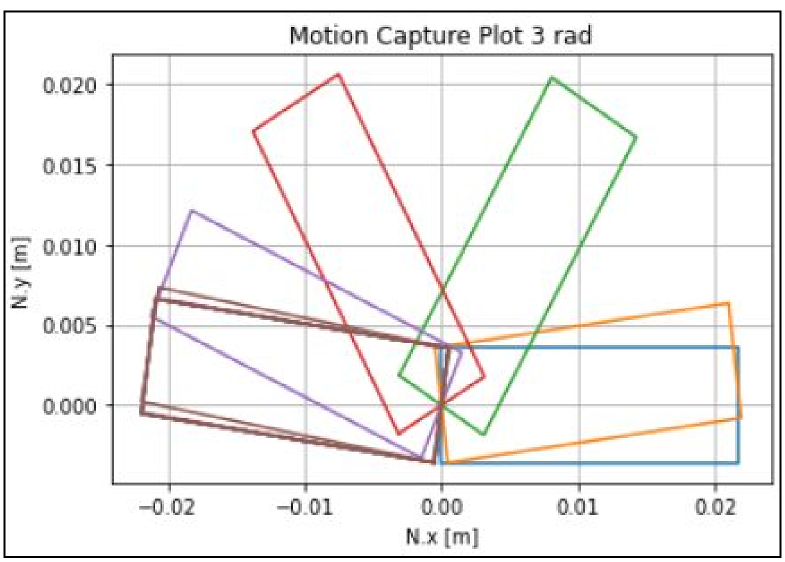
$$V_{min} = K_p(\frac{2\pi}{CPR})$$

The only change in the block diagram for the PD controller was that the C1 block was removed, thus eliminating any kind of compensation for mass properties. This meant that the controller had to be tuned in order to work correctly.

Joint Space Simulation & Experiment Results

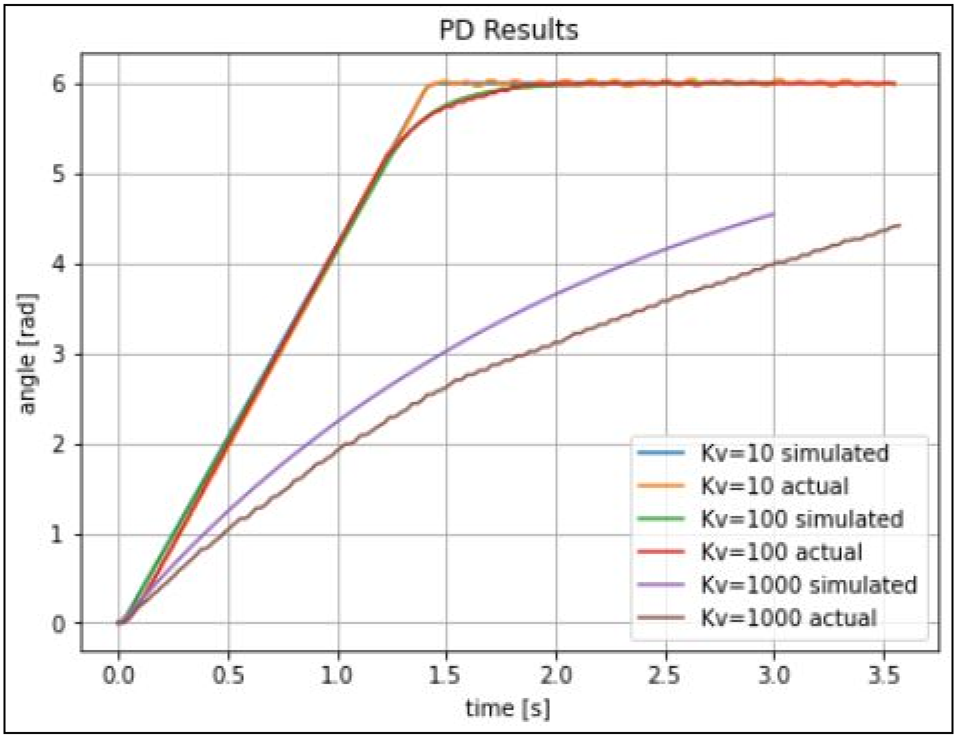
The joint space controller simulation closely matched the experimental results. That being said a slight DC gain of 4.5 had to be applied to the controller signal in order to overcome the static friction in the system and get the motor to not stop short.



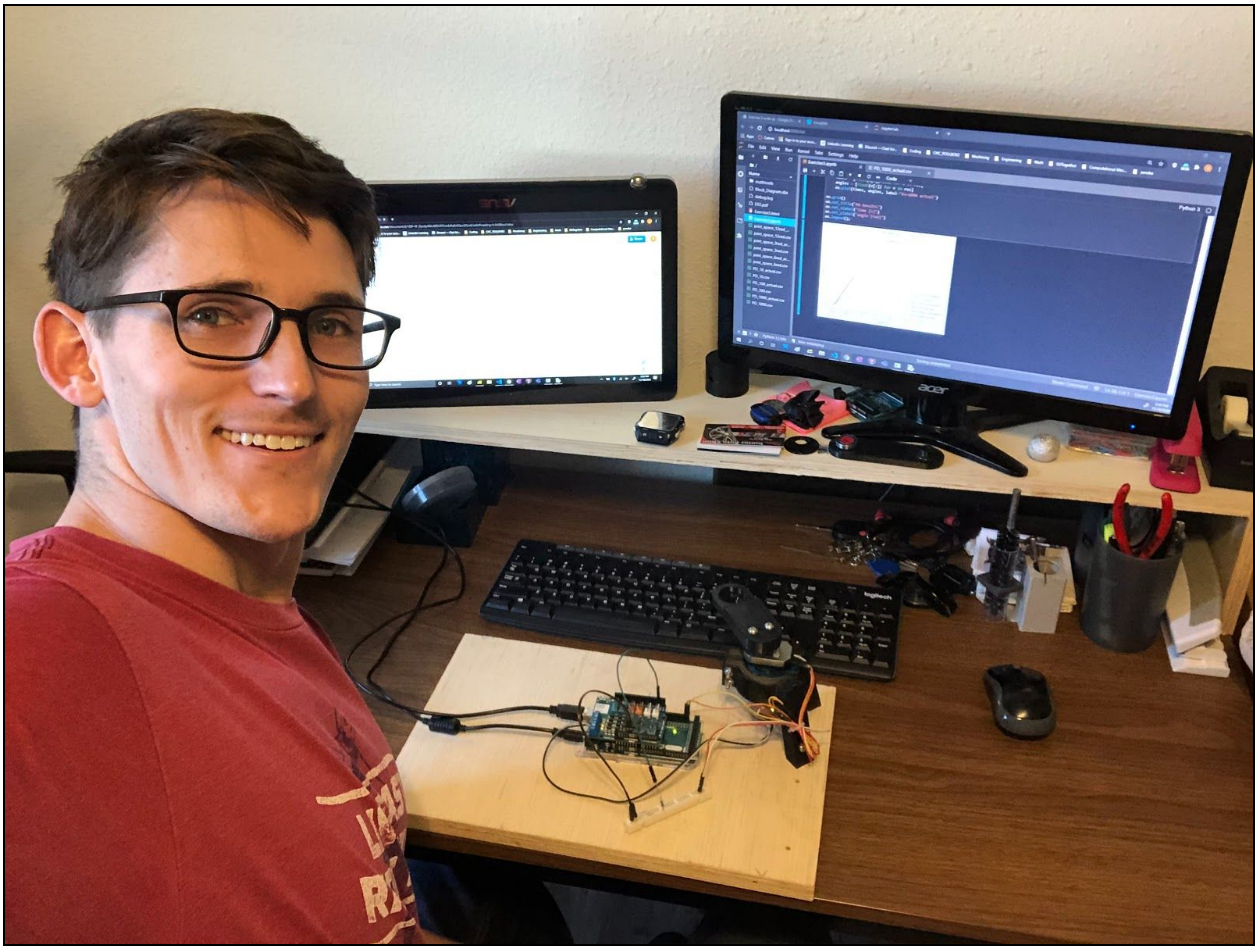


PD Simulation & Experiment Results

The PD controller did not behave as simulated. Due to the massive increase in the strength of the signal, as C1 was a very small number and now was absent from the controller, the controller required a DC gain of 0.5 to function in the realm of normal. There was also substantial vibration. A PID controller is recommended as it would allow for lower Kp and Kv values, while not undershooting the target.



Team Photo (note, only one person due to COVID)



References

[gears] https://www.wikihow.com/Determine-Gear-Ratio