MAE 4345 Exercise 4

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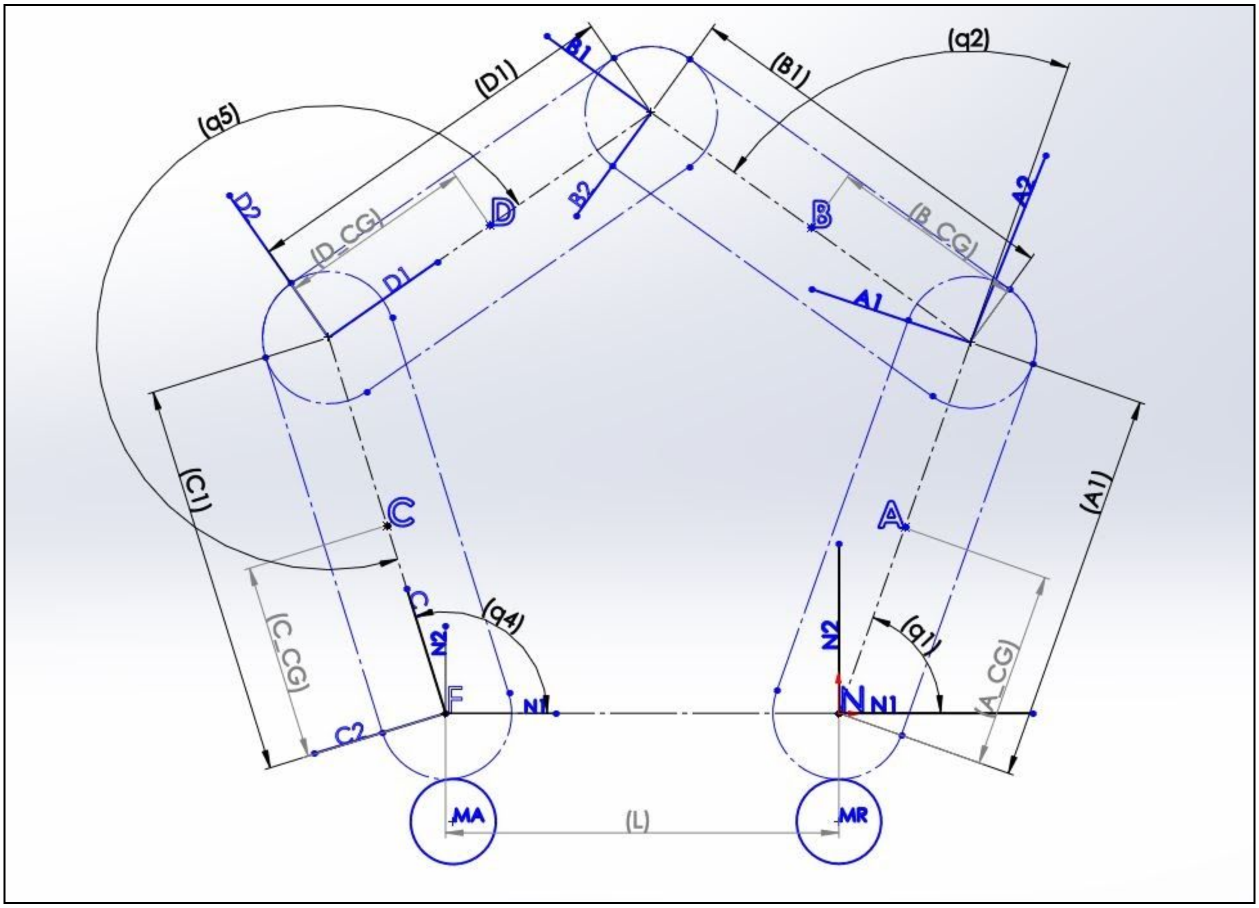
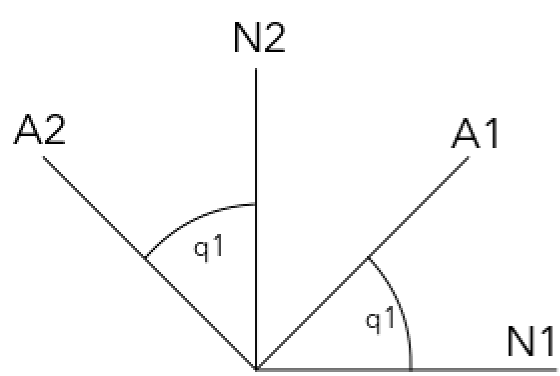


Figure 1. Linkage with Frames, Lengths, and Angles

$$L_A = \{P_{NA}, {}^N_A R\} \qquad P_{NA} = A_{CG}\hat{\mathbf{a}}_{\mathbf{1}}$$

$${}^N_A R = \left\{ \begin{array}{c} \hat{a}_1 = \cos(q_1)\hat{n}_1 + \sin(q_1)\hat{n}_2 \\ \hat{a}_2 = -\sin(q_1)\hat{n}_1 + \cos(q_1)\hat{n}_2 \\ \hat{a}_3 = \hat{n}_3 \end{array} \right\}$$

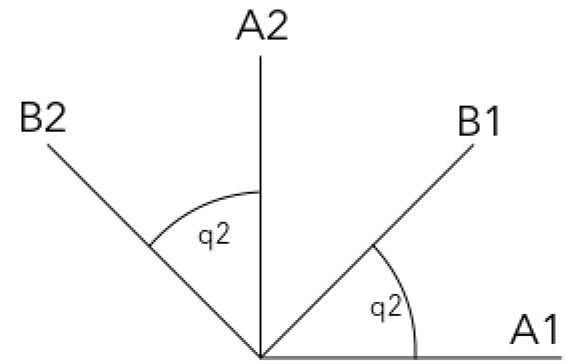
$${}^N_A R = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$L_B = \{P_{NB}, {}^N_B R\} \qquad P_{NB} = A_1 \mathbf{\hat{a}_1} + B_{CG} \mathbf{\hat{b}_1}$$

$${}^A_B R = \left\{ \begin{array}{c} \hat{b}_1 = \cos(q_2)\hat{a}_1 + \sin(q_2)\hat{a}_2 \\ \hat{b}_2 = -\sin(q_2)\hat{a}_1 + \cos(q_2)\hat{a}_2 \\ \hat{b}_3 = \hat{a}_3 \end{array} \right\}$$

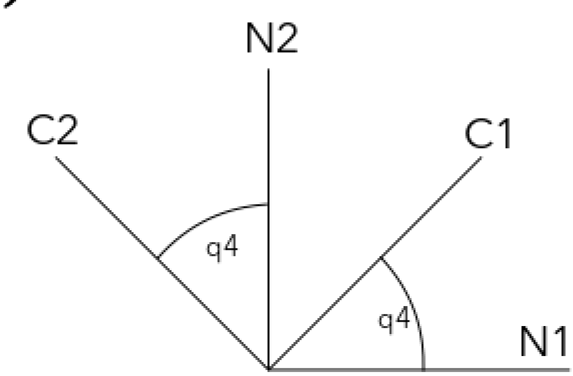
$${}^A_B R = \begin{bmatrix} \cos(q_2) & -\sin(q_2) & 0 \\ \sin(q_2) & \cos(q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$L_C = \{P_{NC}, {}^N_C R\} \qquad P_{NC} = C_{CG} \mathbf{\hat{c}_1} - L \mathbf{\hat{n}_1}$$

$${}^N_C R = \left\{ \begin{array}{c} \hat{c}_1 = \cos(q_4)\hat{n}_1 + \sin(q_4)\hat{n}_2 \\ \hat{c}_2 = -\sin(q_4)\hat{n}_1 + \cos(q_4)\hat{n}_2 \\ \hat{c}_3 = \hat{n}_3 \end{array} \right\}$$

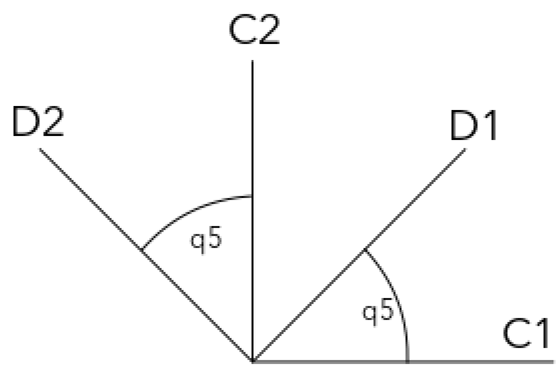
$${}^N_C R = \begin{bmatrix} \cos(q_4) & -\sin(q_4) & 0 \\ \sin(q_4) & \cos(q_4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



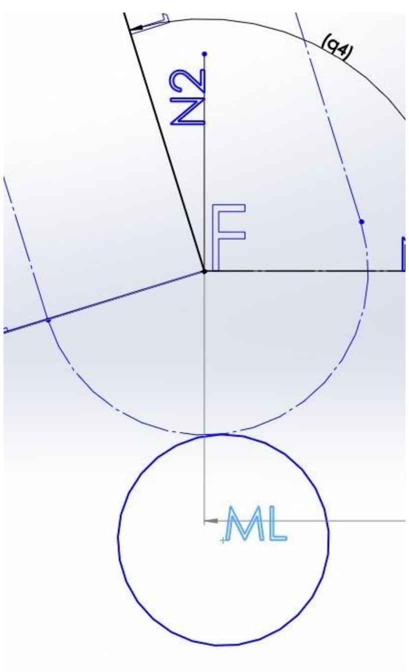
$$L_D = \{P_{ND}, {}^N_D R\} \qquad P_{ND} = C_1\hat{\mathbf{c}}_1 + D_{CG}\hat{\mathbf{d}}_1 - L\hat{\mathbf{n}}_1$$

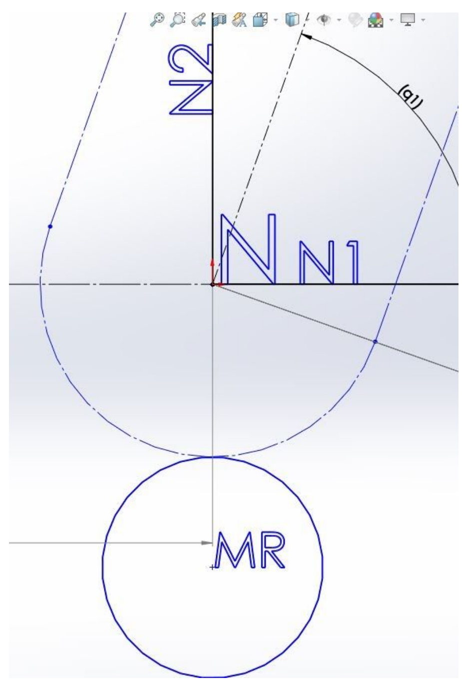
$${}^C_D R = \left\{ \begin{array}{c} \hat{d}_1 = \cos(q_5)\hat{c}_1 + \sin(q_5)\hat{c}_2 \\ \hat{d}_2 = -\sin(q_5)\hat{c}_1 + \cos(q_5)\hat{c}_2 \\ \hat{d}_3 = \hat{c}_3 \end{array} \right\}$$

$${}^C_D R = \begin{bmatrix} \cos(q_5) & -\sin(q_5) & 0 \\ \sin(q_5) & \cos(q_5) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Motor Properties





The diagrams above are for the left and right motor respectively. The left motor is body ML, with a body attached frame ML that has a rotation of q3. The right motor is body MR , with a body attached frame MR that has a rotation of q0. Both motors are located at a distance D from their respective link's rotational point. The position and orientation for both motors and the constraints for the left and right motor are:

$$L_{ML} = \{P_{NML}, {}^{N}_{ML}R\} \qquad P_{NML} = -L\hat{n}_1 - D\hat{n}_2$$

$$L_{MR} = \{P_{NMR}, {}^{N}_{MR}R\} \qquad P_{NMR} = -D\hat{n}_2$$

$${}^{N}_{ML}R = \left\{ \begin{array}{c} \hat{ML}_1 = \cos(q_3)\hat{n}_1 + \sin(q_3)\hat{n}_2 \\ \hat{ML}_2 = -\sin(q_3)\hat{n}_1 + \cos(q_3)\hat{n}_2 \\ \hat{ML}_3 = \hat{n}_3 \end{array} \right\} = \begin{bmatrix} \cos(q_3) & -\sin(q_3) & 0 \\ \sin(q_3) & \cos(q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{N}_{MR}R = \left\{ \begin{array}{c} \hat{MR}_1 = \cos(q_0)\hat{n}_1 + \sin(q_0)\hat{n}_2 \\ \hat{MR}_2 = -\sin(q_0)\hat{n}_1 + \cos(q_0)\hat{n}_2 \\ \hat{MR}_3 = \hat{n}_3 \end{array} \right\} = \begin{bmatrix} \cos(q_0) & -\sin(q_0) & 0 \\ \sin(q_0) & \cos(q_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\delta q_0 = -q_1 GR \qquad \delta q_3 = -q_4 GR$$

These constraints carry into both velocity and acceleration and with 6 generalized coordinates, and 4 constraints; two from motor constraints, and two from vector loops, we get the manipulator to have 2 degrees of freedom. The two vector loops are located below from Point N to Point N (P_{NN})

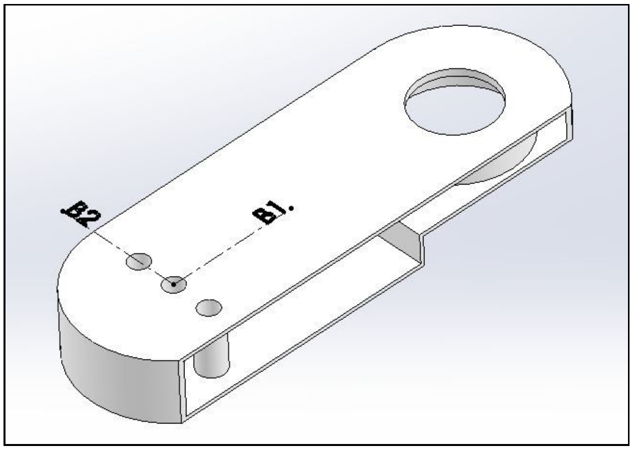
(1)

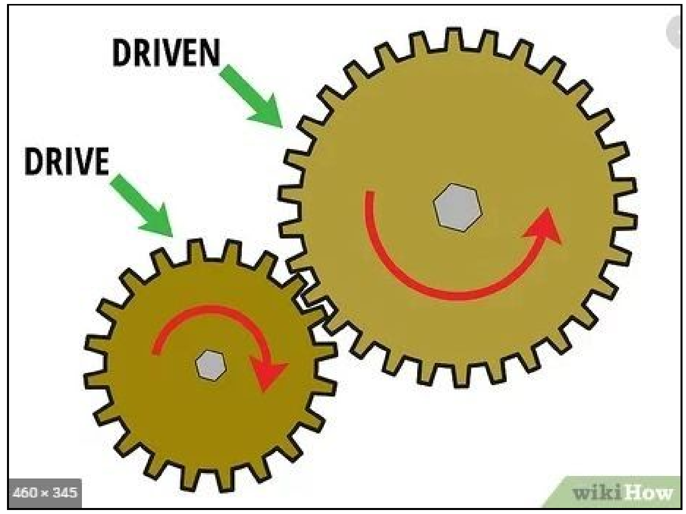
$$-L+LA_1\cos(q_1)+LB_1(-\sin(q_1)\sin(q_2)+\cos(q_1)\cos(q_2))-LC_1\cos(q_4)-LD_1(-\sin(q_4)\sin(q_5)+\cos(q_4)\cos(q_5))=0$$

(2)

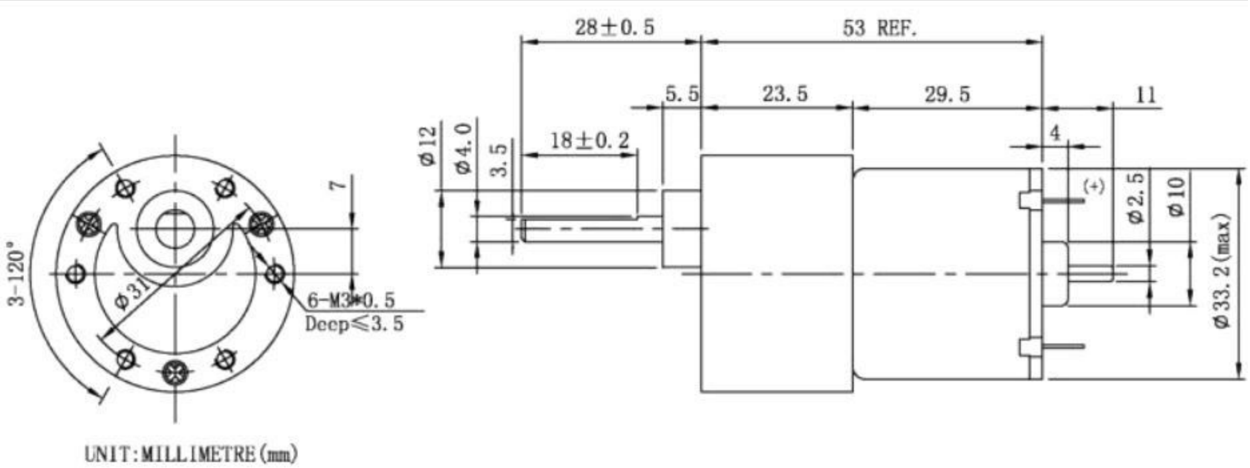
$$0=LA_1\sin(q_1)+LB_1(\sin(q_1)\cos(q_2)+\sin(q_2)\cos(q_1))-LC_1\sin(q_4)-LD_1(\sin(q_4)\cos(q_5)+\sin(q_5)\cos(q_4))$$

Mass Properties





The mass for the links were computed using ABS mass properties from SolidWorks applied to the CAD model. The model was hollowed out, and the shell thickness set to 0.90 mm,
the same as was used to print the physical model. The internally printed structure was neglected as a sparse infill was used.



The mass of the axle that spins in body A and body C were calculated using the drawing shown above. The axle was assumed to be 10.0 mm in diameter and 53 mm in length. A CAD model was generated, and the mass estimated with steel mass

properties applied to the CAD model. The gears were assumed to interact in a manner similar to the ones shown below.
The moments of inertia for both bodies were evaluated in the same manner as the mass Properties.

Body	Mass (kg)	Izz ($kg*m^2$)
Body A	26.45×10^{-3}	3.30×10^{-7}
Body B	7.28×10^{-3}	6.39×10^{-6}
Body C	26.45×10^{-3}	3.30×10^{-7}
Body D	7.28×10^{-3}	6.39×10^{-6}

Trajectory Planning

The hyperboloid path was planned initially in Exercise 1 by generating the shape using the mathematical definition, then discretizing the curve into points. An updated python code was used to generate a points matrix for x and y values, and the number of points could be increased or decrease to consequentially change the resolution. The points matrices satisfied the input for the operational space controller, as it performed run time inverse kinematics.

For the joint space simulation, a numerical solver was used to solve for the inverse kinematics at each desired point in the simulation, the simulation was then run using an RK45 integrator.

Once the separate controllers were completed, criteria had to be initialized that would tell the robot when to move to the next point. To accomplish this, an error value was chosen that would be a point of comparison to the joint angle error, or the path magnitude error, for joint space and operational space, respectively. This error value was chosen due to trial and error, but it was found that using a small error target would not be feasible, due to compliance in the robot structure, and inconsistencies in encoder reading.

The device did not perform as expected from the simulations. Error due to “slop” in the joint design, friction against the end effector and the paper, and encoder resolution and read speeds all drove the device to perform with less accuracy than expected.

Note, both team members not in image due to covid.