MAE 4345 Exercise 1

Samuel Law & Rhys Miller

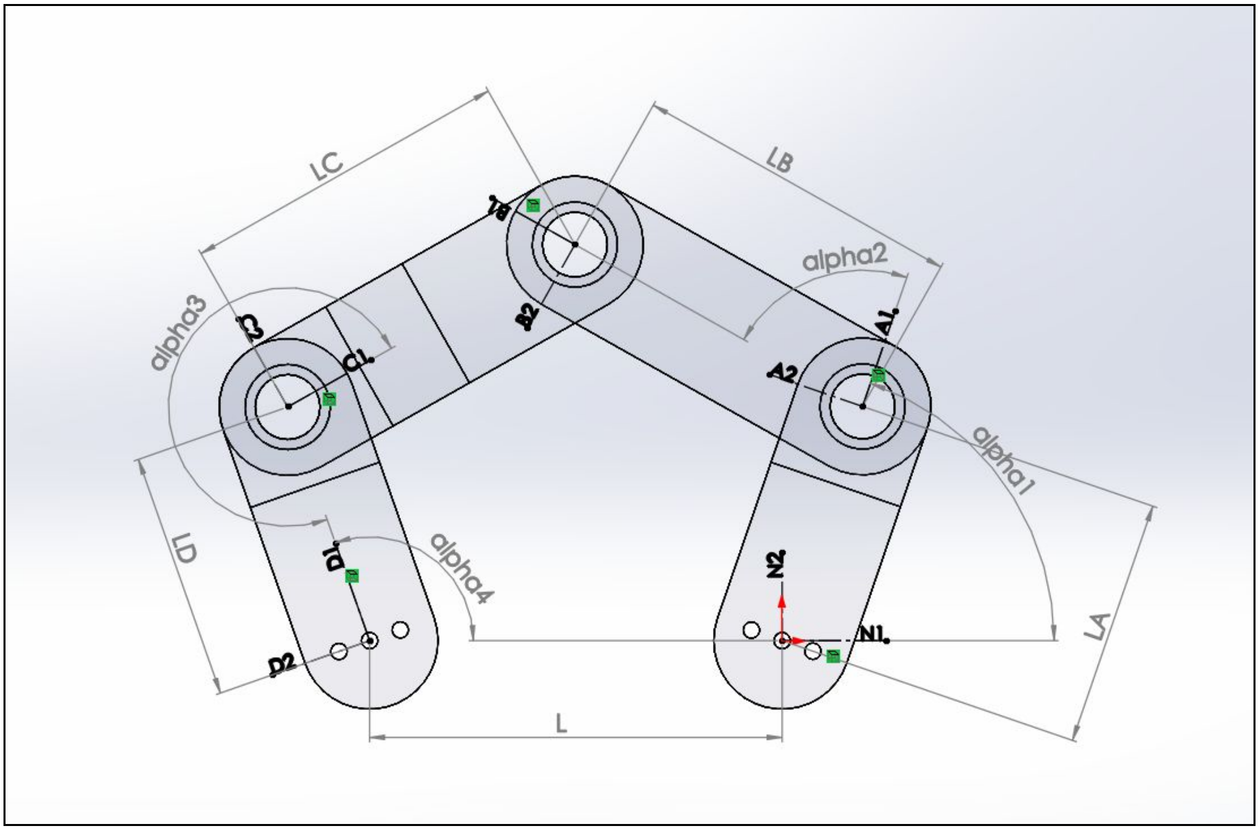


Figure 1. Linkage with Frames, Lengths, and Angles

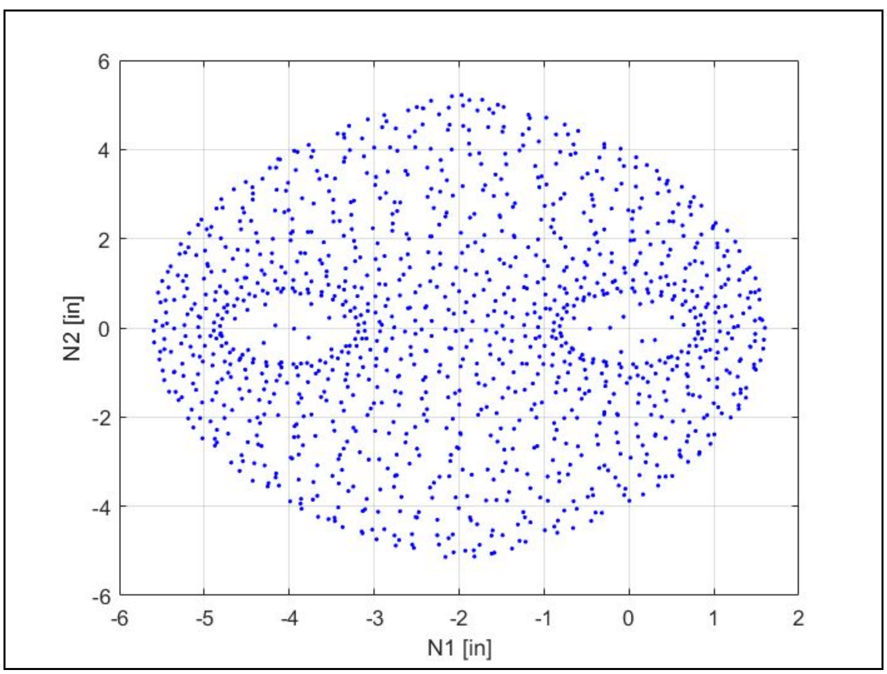
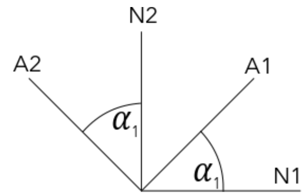


Figure 2. Point Cloud

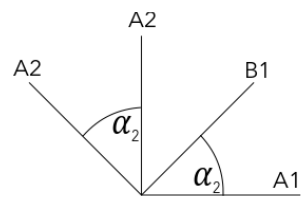
$$\mathfrak{L}_{\mathrm{A}} = \left\{P_{NA}, {}^{N}_{A}R\right\}$$



$$P_{NA} = L_A \widehat{A_1}$$

$$ {}^{N}_{A}R = \begin{bmatrix} \cos(\alpha_1) & \sin(\alpha_1) & 0 \\ -\sin(\alpha_1) & \cos(\alpha_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{N}_1 \\ \hat{N}_2 \\ \hat{N}_3 \end{bmatrix}$$

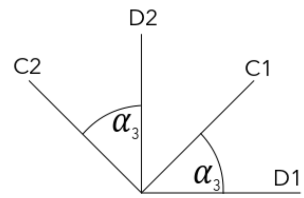
$$\mathfrak{L}_{\mathrm{B}} = \left\{P_{AB}, {}^{A}_{B}R\right\}$$



$$P_{AB} = L_B \widehat{B_1}$$

$$ {}^{A}_{B}R = \begin{bmatrix} \cos(\alpha_2) & \sin(\alpha_2) & 0 \\ -\sin(\alpha_2) & \cos(\alpha_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{A}_1 \\ \hat{A}_2 \\ \hat{A}_3 \end{bmatrix}$$

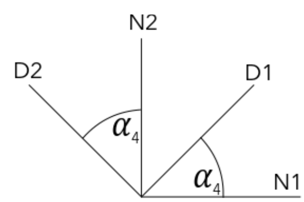
$$\mathcal{L}_{\mathrm{C}} = \{P_{DC}, {}^{D}_{C}R\}$$



$$P_{DC} = L_D \widehat{D_1}$$

$${}^{A}_{B}R = \begin{bmatrix} \cos(\alpha_3) & \sin(\alpha_3) & 0 \\ -\sin(\alpha_3) & \cos(\alpha_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{D}_1 \\ \hat{D}_2 \\ \hat{D}_3 \end{bmatrix}$$

$$\mathcal{L}_{\mathrm{D}} = \{P_{ND}, {}^{N}_{D}R\}$$



$$P_{ND} = L\left(-\widehat{N_1}\right)$$

$${}^{N}_{D}R = \begin{bmatrix} \cos(\alpha_4) & \sin(\alpha_4) & 0 \\ -\sin(\alpha_4) & \cos(\alpha_4) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{N}_1 \\ \hat{N}_2 \\ \hat{N}_3 \end{bmatrix}$$

$${}^{N}_{B}R = {}^{N}_{A}R\,{}^{A}_{B}R$$
$$P_{NB} = P_{NA} + P_{AB} = P_{ND} + P_{DC} + P_{CB}$$

Title: Exercise 1

Authors: Samuel Law & Rhys Miller

Define rotation matrix function

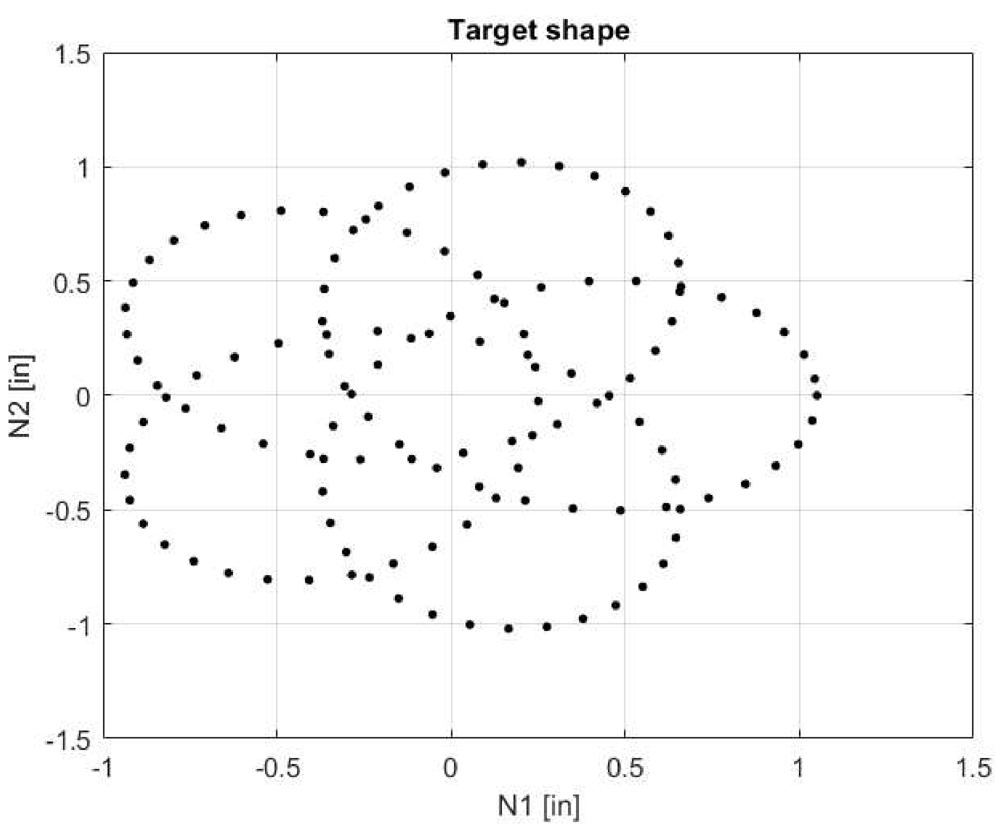
```
rot = @(radians) [cos(radians), sin(radians);
                  -sin(radians), cos(radians)];
```

Define hypotrochoid equations

```
R = 0.5; r = 0.1; d = 0.65;
xh = @(theta) (((R-r)*cos(theta)) + (d*cos((R-r)*(theta/r))));
yh = @(theta) (((R-r)*sin(theta)) - (d*sin((R-r)*(theta/r))));
```

Generate points from hypotrochoid equations

```
theta = [0:0.05:2*pi];
x_points = xh(theta);
y_points = yh(theta);
figure; hold off;
plot(x_points, y_points, '.k', 'MarkerSize', 10);
grid;
title("Target shape");
xlabel("N1 [in]");
ylabel("N2 [in]");
```



Define link lengths

```
L = 4;   % in
LA = 2.4;  % in
LB = 3.2;  % in
LC = 3.2;  % in
LD = 2.4;  % in
```

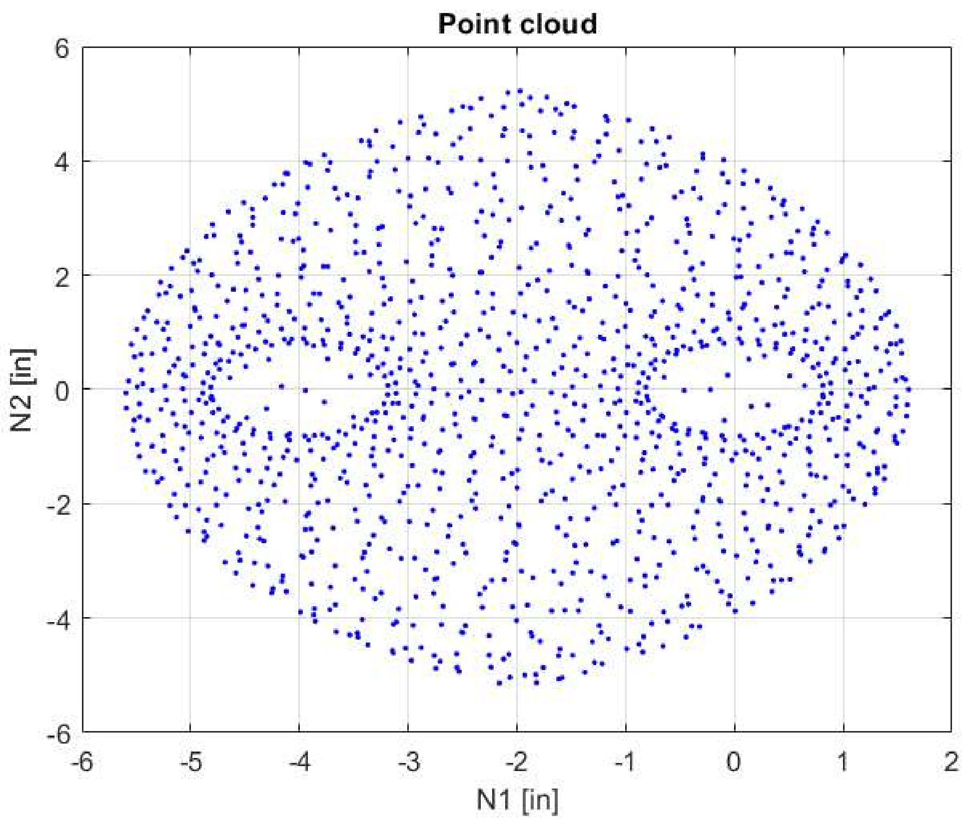
Define shape offset

```
off_set_N1 = -L/2;  % in
off_set_N2 = 4;  % in
```

Generate point cloud

```
PND = [-L, 0];  % offset for the left motor;
results = [];

% right arm through a full rotation
for alpha_1 = [0:0.2:2*pi]
    for alpha_2 = [0:0.2:2*pi]
        % solve for PNA
        RNA = rot(alpha_1);
        A1 = RNA(1,:);
        PNA = LA*A1;

        % solve for PAB
        RAB = rot(alpha_2)*RNA;
        B1 = RAB(1,:);
        PAB = LB*B1;

        % solve for PNB
        PNB = PNA + PAB;

        % check to see if PDB is
        % longer than the arms
        % could possibly reach
        if norm(PNB - PND) <= (LD+LC)
            results = [results; PNB];
        end
    end
end

% left arm through a full rotation
for alpha_4 = [0:0.2:2*pi]
    for alpha_3 = [0:0.2:2*pi]
        % solve for PNC
        RND = rot(alpha_4);
        D1 = RND(1,:);
        PDC = LD*D1;

        % solve for PCB
        RDC = rot(alpha_3)*RND;
        C1 = RDC(1,:);
        PCB = LC*C1;
```

```
        % solve for PDB & PNB
        PDB = PDC + PCB;
        PNB = PND + PDC + PCB;

        % check to see if PDB is
        % longer than the arms
        % could possibly reach
        if norm(PNB) <= (LA+LB)
            results = [results; PNB];
        end
    end
end

% plot the output
figure; hold off;
plot(results(:,1), results(:,2), '.b')
title("Point cloud")
xlabel("N1 [in]");
ylabel("N2 [in]");
grid;
```



Solve inverse kinematics

```
PND = [-L, 0];  % offset for the left motor;

figure; hold on;
for i = [1:1:length(x_points)]
    % define target position
    x = x_points(i) + off_set_N1;
    y = y_points(i) + off_set_N2;
```

```
% solve for alpha_2 to achive target radius
PXY = [x, y];
R = norm(PXY);
gamma = acos(((abs(R^2)) - (abs(LA^2)) - (abs(LB^2)))/(-2*LA*LB));
alpha_2 = pi - gamma;  % alpha_2 > 0;

% solve for alpha_1 from alpha 2
PNA = [LA, 0];  % N1 direction
RAB = rot(alpha_2);
B1 = RAB(1,:);
PAB = LB*B1;
Shape = PNA + PAB;
desired = atan2(PXY(2), PXY(1));
current = atan2(Shape(2), Shape(1));
alpha_1 = desired - current;

% solve for alpha_3 to achive target radius
R = norm(PXY - PND);
gamma = acos(((R^2) - (LD^2) - (LC^2))/(-2*LD*LC));
alpha_3 = -(pi - gamma);  % alpha_3 < 0;

% solve for alpha_4 from alpha_3
PDC = [LD, 0];  % N1 direction
RCB = rot(alpha_3);
C1 = RCB(1,:);
PCB = LC*C1;
Shape = PDC + PCB;  % the shape, not actual position
desired = atan2(PXY(2) - PND(2), PXY(1) - PND(1));
current = atan2(Shape(2), Shape(1));
alpha_4 = desired - current;

% define rotation matricies
RNA = rot(alpha_1);
RAB = rot(alpha_2)*RNA;
RND = rot(alpha_4);
RDC = rot(alpha_3)*RND;

% define direction vector;
A1 = RNA(1,:);
A2 = RNA(2,:);
B1 = RAB(1,:);
B2 = RAB(2,:);
D1 = RND(1,:);
D2 = RND(2,:);
C1 = RDC(1,:);
C2 = RDC(2,:);

% define position vectors
PNA = LA*A1;
PAB = LB*B1;
PDC = LD*D1;
PCB = LC*C1;
```

```
    % define points to plot
    Pa = PNA;
    Pb = PNA + PAB;
    Pc = PND + PDC;
    Pd = PND;

    % plot line approximation of bodies
    plot([0, Pa(1)],[0, Pa(2)], 'k');
    plot([Pa(1), Pb(1)],[Pa(2), Pb(2)], 'b');
    plot([PND(1), Pc(1)], [PND(2), Pc(2)], 'Y');
    plot([Pc(1), Pb(1)],[Pc(2), Pb(2)], 'g');
    plot(Pb(1), Pb(2), '.r', 'MarkerSize', 10);

end
hold off;

% format the plot
grid;
title("Inverse kinematics point cloud");
xlabel("N1 [in]");
ylabel("N2 [in]");
xlim([-6, 2]);
ylim([-2, 8]);
```