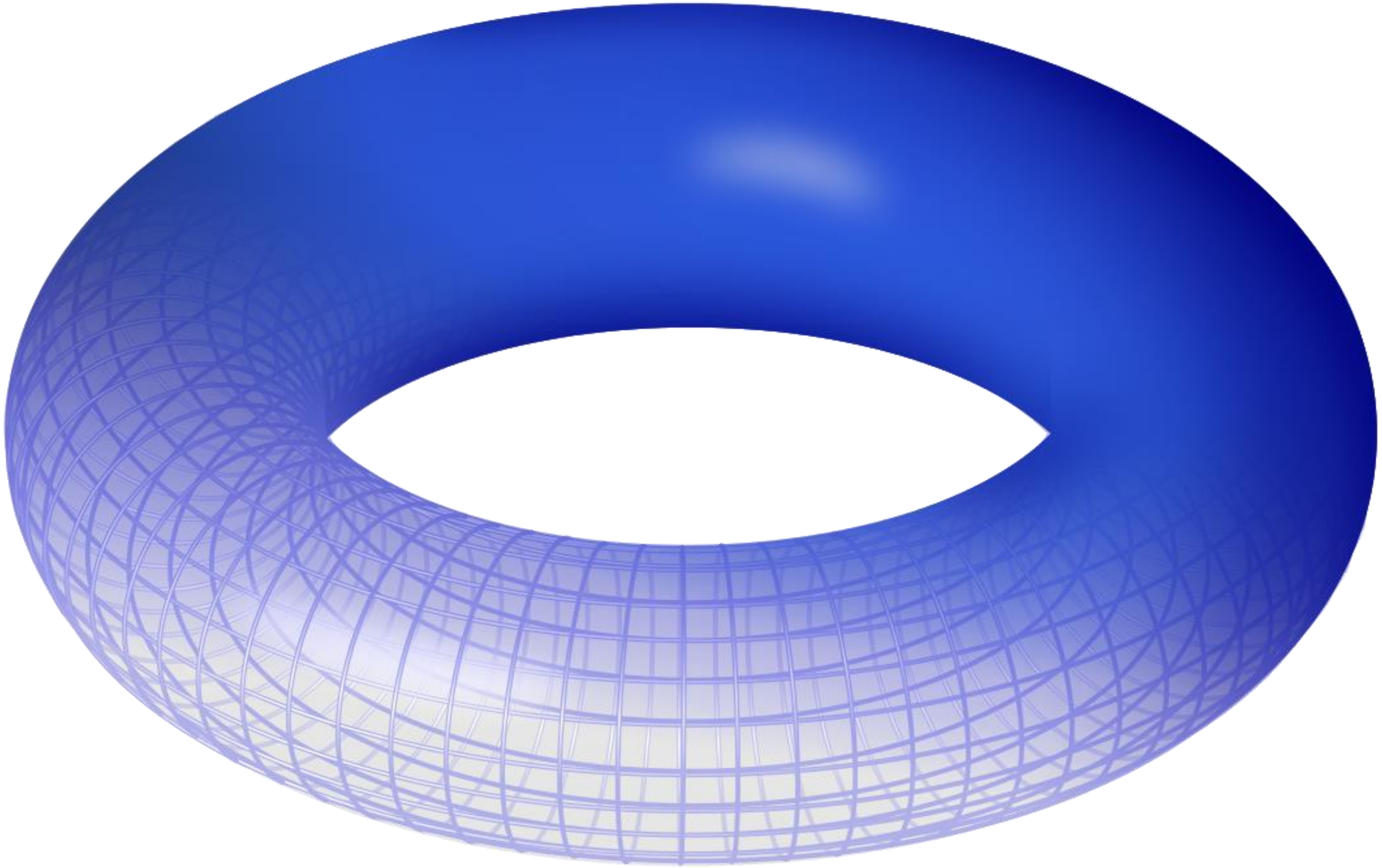
105torus

B-MAT-500

Torus



Torus

- The torus equation is:

$$(x^2 + y^2 + z^2 + R^2 - r^2)^2 = 4R^2(x^2 + y^2)$$

R = Distance between the center of the torus and the center of the tube

r = Radius of the tore's cercle

- To find the intersection between a line and a torus (cf. 104):

$$f(t) = a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 = 0$$

Solving a 4th degree equation

$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

- Direct method for 4th degree: Ferrari's method
  - Does not generalize to higher degrees…
- Iterative methods
  - Start with initial value(s): $x_0, x_1, \ldots$
  - Compute x_{n+1} from $x_n, x_{n-1}, \ldots$
  - Stop when x_n is "close enough" (or after too many iterations)

Hypothesis

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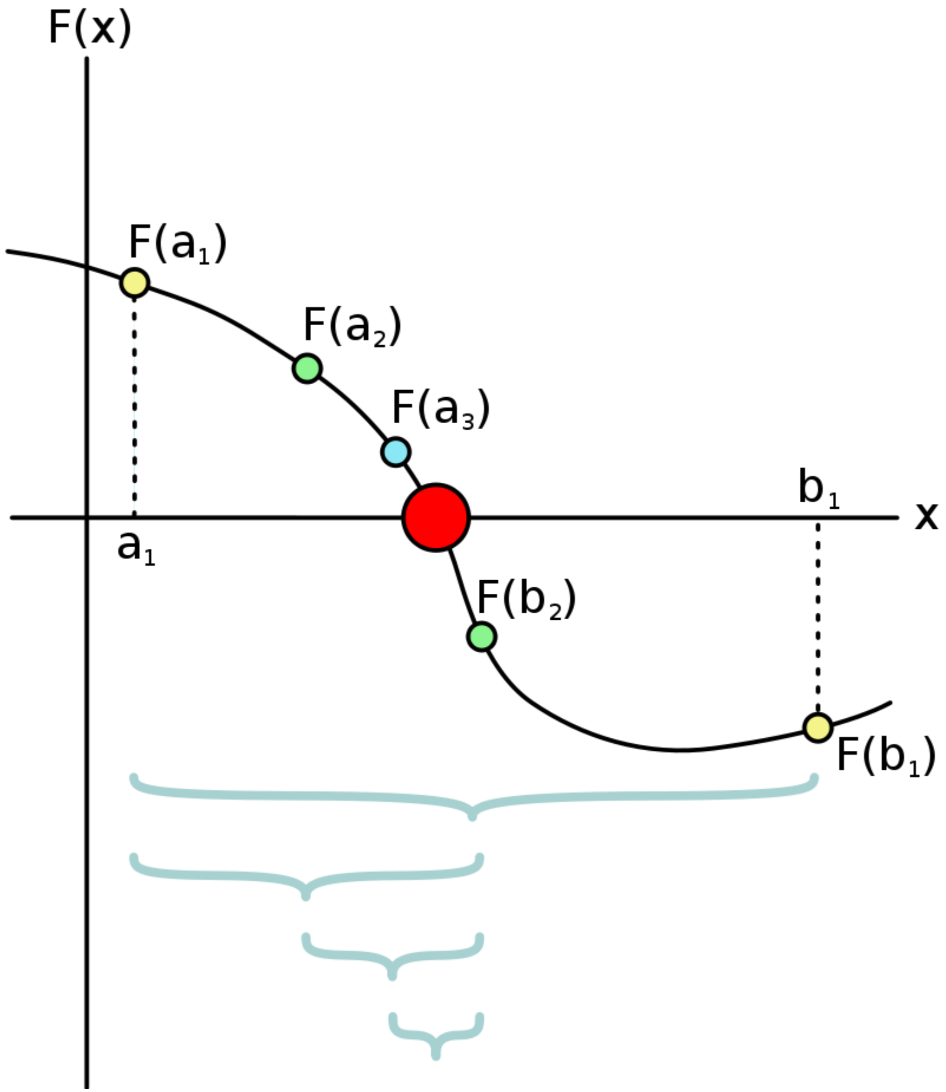
- Goal: implement three iterative methods (Bisection, Newton, Secant)
- For this project, we will solve equations where:
  - There is always one and only one x in $[0,1]$ where $f(x) = 0$
  - The sign of f changes around its zero
- We stop searching when $f(x) \leq 10^{-n}$

Bisection method

- Start with an interval $[a, b]$ that contains the solution
- Compute the midpoint $c = \frac{a+b}{2}$ of the interval
- If $f(c) \leq 10^{-n}$, stop
- If $f(a)f(c) < 0$, then the solution is in $[a, c]$
- If not, the solution is in $[c, b]$
- Start again with the new interval

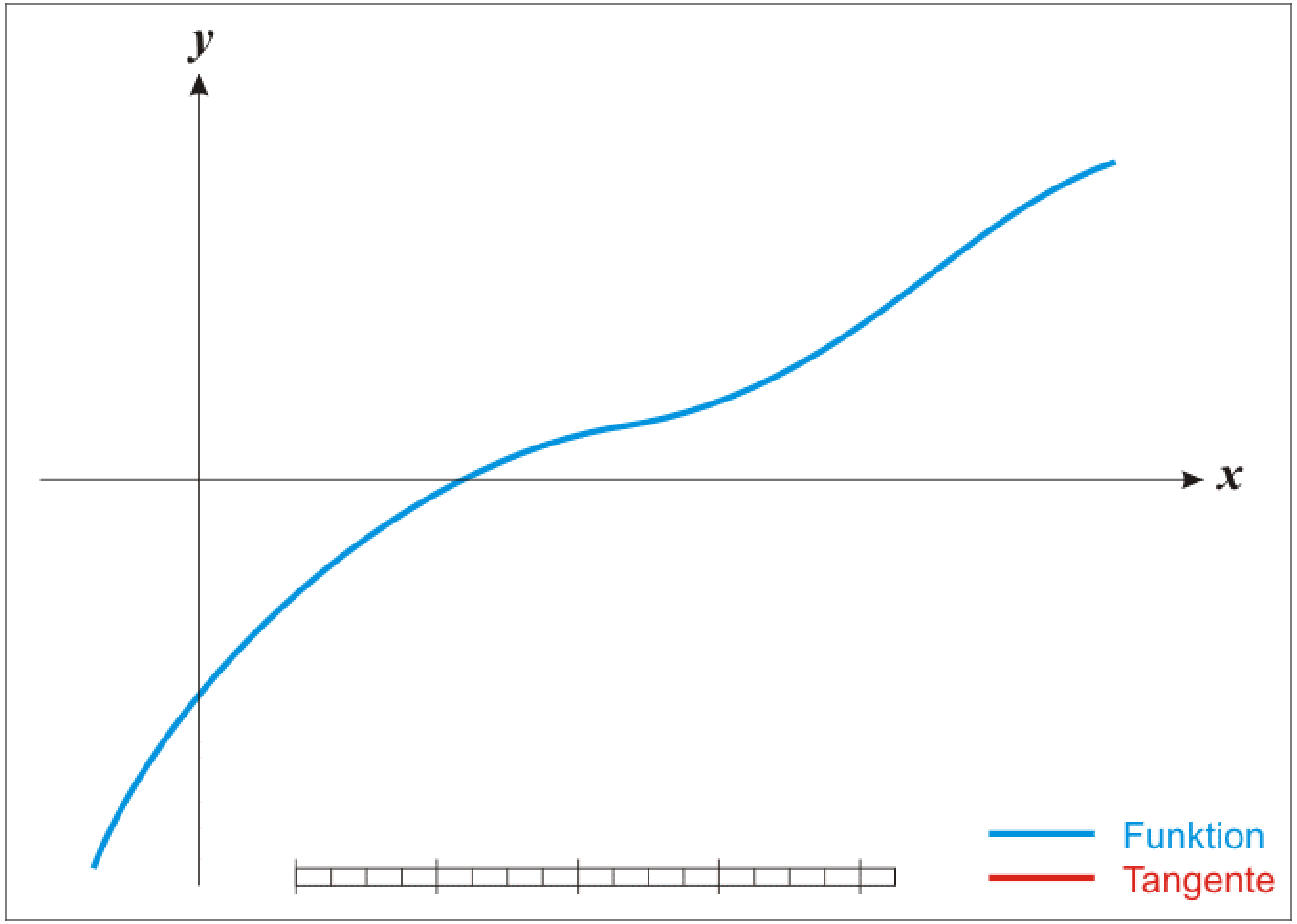
Bisection method



Newton method

- Uses the tangent to the graph of the function to find a better approximation
- Start with the initial value x_0
- Consider the tangent to the graph at the point $(x_n, f(x_n))$
- This tangent intercept the x-axis in $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- If $f(x_{n+1}) \leq 10^{-n}$, stop

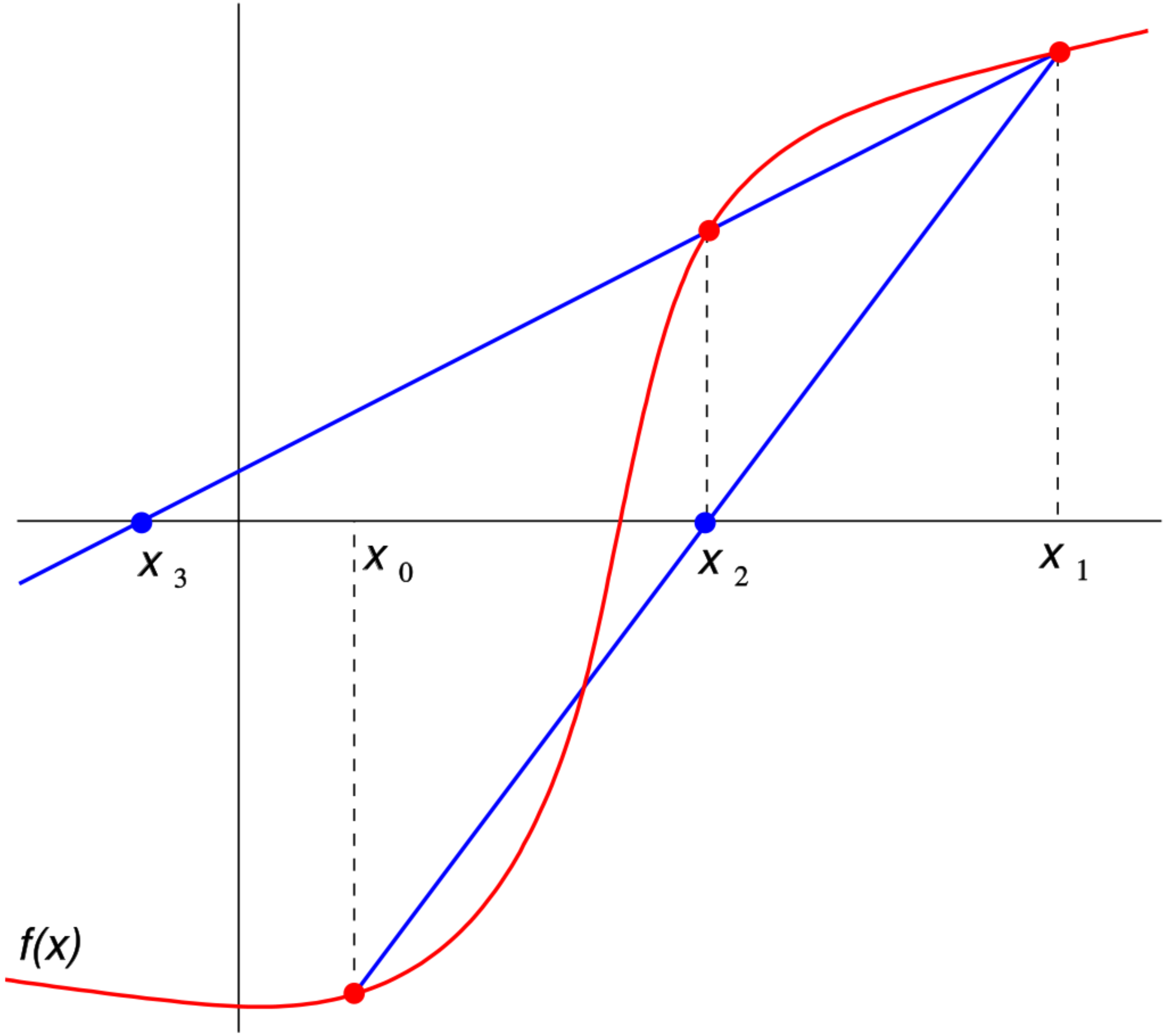
Bisection method



Secant method

- Approximation of Newton's method when f is not derivable
  - Replaces the tangent with a line between two point on the graph
- Start with the initial values x_0 and x_1
- Consider the line passing through $(x_{n-1}, f(x_{n-1}))$ and $(x_n, f(x_n))$
- This line intercept the x-axis in $x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$
- If $f(x_{n+1}) \leq 10^{-n}$, stop

Bisection method



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- Solving 4th degree equations
- Inputs
  - Method (bisection, Newton or secant)
  - Coefficients of the equation: a_0, a_1, a_2, a_3, a_4
  - Precision (stops when $f(x) \leq 10^{-n}$)
- Outputs
  - Iterations

Points of attention

- Initial values:
  - 0 and 1 for bisection and secant methods
  - 0.5 for Newton
- Don’t forget to handle cases where there are no solutions
- Beware of dividing by 0
- Some methods may not always converge…

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- Graphical interface to compare the rates of convergence
- Solving higher degree equations
- Ferrari’s method