

CSS57 Lab 1 - Logic Revision

1. Warming Up

3 hats : 2 red & 1 blue
2 w/ feathers & 1 w/o

Mary: red & feather

John:

1 hidden hat

1 red hat has a feather - Mary

1 blue hat w/o feather - John

1 red hat w/ feather - Hidden

2. The Connectives

1. Not more than one of them is dangerous.
 $\neg (A \wedge B)$

2. Both of them are dangerous.
 $(A \wedge B)$

3. Neither of them are dangerous.
 $\neg A \wedge \neg B$

4. At least one of them is dangerous
 $A \vee B \vee (A \wedge B)$

5. Either both or neither of them is dangerous
 $(A \wedge B) \vee (\neg A \wedge \neg B)$

6. Exactly one of them is dangerous.

$$\cancel{(A \leftrightarrow B)} \wedge \neg (A \leftrightarrow B)$$

$$\rightarrow (A \wedge \neg B) \vee (\neg A \wedge B)$$

3. Conditions

1. Either John doesn't play well or he does & he wins

$$(\neg P) \vee (P \wedge W)$$

P	$\neg P$	\vee	(P	\wedge	W)	$P \rightarrow W$
0	1	1	0	0	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	0	1	1	1	1	1

2. This will not happen: John plays well & doesn't win.

$$\neg (P \wedge \neg W)$$

\neg	(P	\wedge	$\neg W$)	$P \rightarrow W$
1	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1

4. Logic Exercises

D = Demand has remained constant.

P = Prices have increased.

T = Turnover has decreased.

1. If demand has remained constant & prices have increased, then turnover must have decreased.

$$(D \wedge P) \rightarrow T$$

2. The sum of two sums is even if and only if either or both numbers are even or both numbers are odd.

S = sum of two numbers is even

E = both numbers are even.

O = both numbers are odd.

$$S \leftrightarrow ((E \wedge O) \vee (E) \vee (O))$$

3. If y is an integer then z is not real, provided that x is a rational number.

Y = y is an integer.

Z = z is not real.

X = x is a rational number.

$$(Y \rightarrow Z) \leftrightarrow X$$

2.2 Logical Statistics

Exercise 5

A	B	$(A \wedge B)$	$\neg A \vee (A \wedge B)$	$(A \wedge \neg B)$	\neg
0	0	0	1 1 0	0 0 1	1
0	1	0	1 1 0	0 0 0	1
1	0	0	0 0 0	1 1 1	0
1	1	1	0 1 1	1 0 0	1

Exercise 6

$E(S,A)$	$E(S,P)$	$E(P,S)$	$\neg E(S,A)$	$E(S,P) \vee E(P,S)$	$\neg E(S,P)$
F	F	F	T	F	T
F	F	T	T	T	T
F	T	F	T	T	F
F	T	T	T	T	F
T	F	F	F	F	T
T	F	T	F	T	T
T	T	F	F	T	F
T	T	T	F	T	F

3. Predicates & Quantifiers

Exercise 7.

1. Every cat washes itself
 $\forall x : Cx \cdot Wx$

2. If all dogs are mortal, Fido is ~~normal~~ ^{mortal}.
 $Dx = x$ is a dog.
 $Mx = x$ is mortal.
 $Fx = x$ is called Fido.

$$\forall x : Dx \cdot Mx$$

$$\exists x : Dx \cdot Fx$$

$$\exists x : Fx \cdot Mx$$

3. Not every scorpion is lethal.

$$\neg \forall x : Sx \cdot Lx$$

$$Sx = x \text{ is a scorpion.}$$

$$Lx = x \text{ is lethal.}$$

$$\neg (\forall x : Sx \cdot Lx)$$

4. Every scorpion is non-lethal.

$S(x)$ = x is a scorpion.

$L(x)$ = x is lethal.

$$\forall x: S(x) \rightarrow \neg L(x)$$

5. At least one satellite orbits Mars

$S(x)$ = x is a satellite.

$O(x)$ = x orbits Mars.

$$\exists x: S(x) \wedge O(x)$$

6. There is a planet larger than Neptune, but cooler than it.

$P(x)$ = x is a planet. ~~larger than Neptune~~

$C(x)$ = x is cooler than Neptune.

$L(x)$ = x is larger than Neptune.

$$\exists x: P(x) \rightarrow (L(x) \wedge C(x))$$

7. Every village has at least one church

$V(x)$ = x is a village.

$C(y)$ = y is a church

$$\forall x: V(x) \rightarrow \exists y: C(y)$$

Exercise 8

1. Not all birds can fly.

$B(x)$ = x is a bird

$F(x)$ = x can fly.

$$\exists x: B(x) \wedge \neg F(x)$$

2. Anyone can do that

$A(x)$: x is someone

$D(x)$: x can do that.

$$\forall x : A(x) \cdot D(x)$$

3. Some people are stupid.

$P(x)$: x is a person.

$S(x)$: x is stupid.

$\exists x : P(x) \wedge S(x)$.

4. There's an integer that is greater than every other integer.

$I(x)$: x is an integer.

$G(x)$: x is greater than every other integer.

$$\forall x : I(x) \rightarrow \exists y : G(y)$$