Approximate PDFs with Parzen window density estimation

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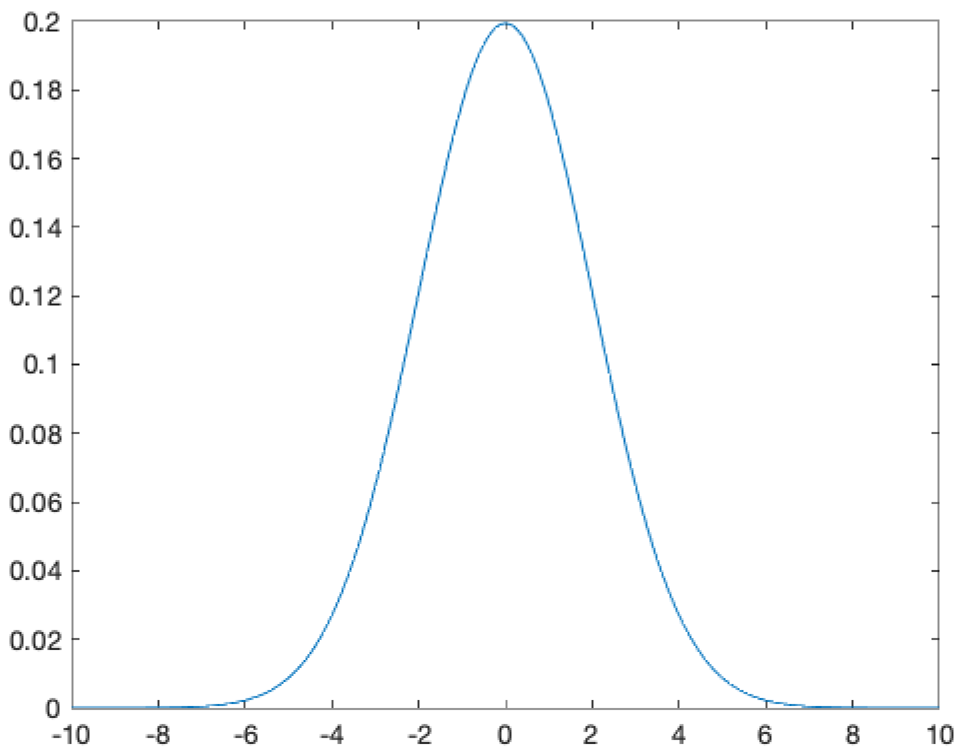
1 Approximate PDFs with Parzen window density estimation

1.1 Get a sample of a distribution

Given a usually unknown probability density function PDF that we aim to approximate.

We use a Normal distribution $\mathcal{N}(\mu, \sigma)$ in the example below.

[29]:
```
mu = 0;
sigma=2;
dist = makedist('Normal', 'mu',mu,'sigma',sigma);
x = -10:.1:10;
plot(x,pdf(dist,x));
```



We generate some values of this distribution. These values are observed and make up the training data.

[30]:
```
n=100;
for i=1:n
    X(i) = random(dist);
end
```

2 Approximate the sample

We create the approximate PDF `F` in two steps. First, we compute a numeric mapping `f` of a sample of the value range `x`. to their probability (density). Then we interpolate `f` with a function `F` so that we can get the probability density even between the sample values.

We start with initializing the approximation mapping `f`.

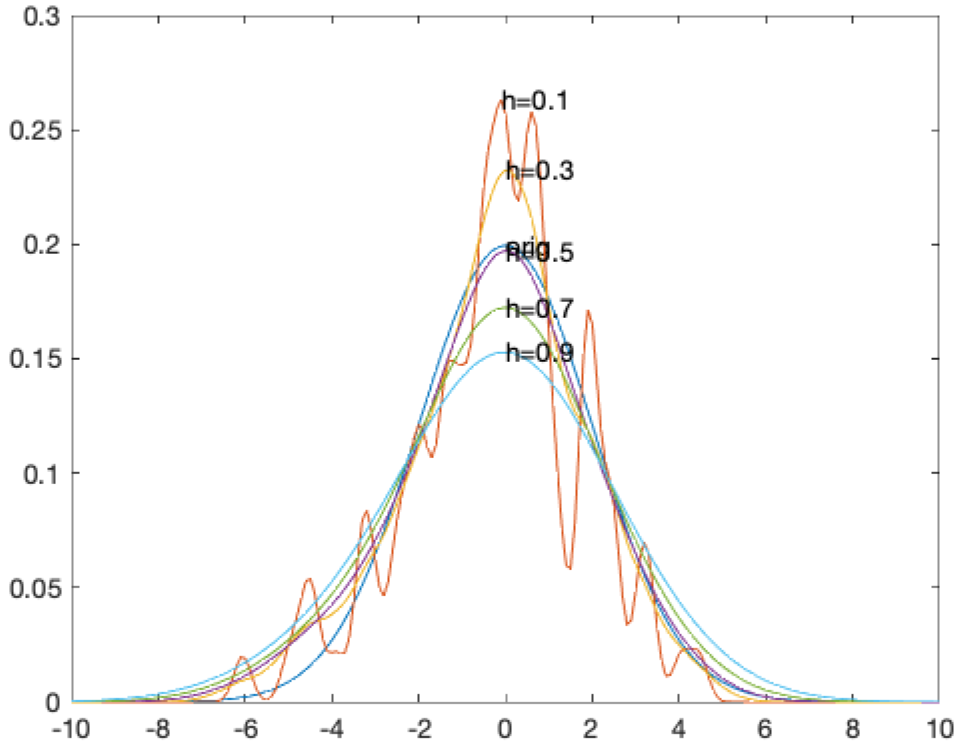
[31]:
```
len_x = length(x);
f = zeros([1 len_x]);
```

For the approximation, we use a Gaussian Kernal function.

We plot the original distribution again. Then we iterate over different values of the h parameter of the approximation to see its effect by ploting the approximation in the same figure as the distribution.

[32]:
```
% plot the distribution
plot(x,pdf(dist,x));
text(mu,pdf(dist,mu),'orig');
hold on;
for h = .1:.2:1
    for j=1:len_x
        xx=x(j);
        % Use a Gaussian Kernal function
        for i=1:n
            f(j) = f(j) + normpdf((xx-X(i))/h,mu,sigma);
        end
        f(j) = f(j) /(n*h);
    end
    % plot the approximation in the same figure as the distribution
    plot(x,f);
    [argv,argmax] = max(f);
    i=argmax;
    text(x(i),f(i),['h=',num2str(h)]);
```

```
end
```

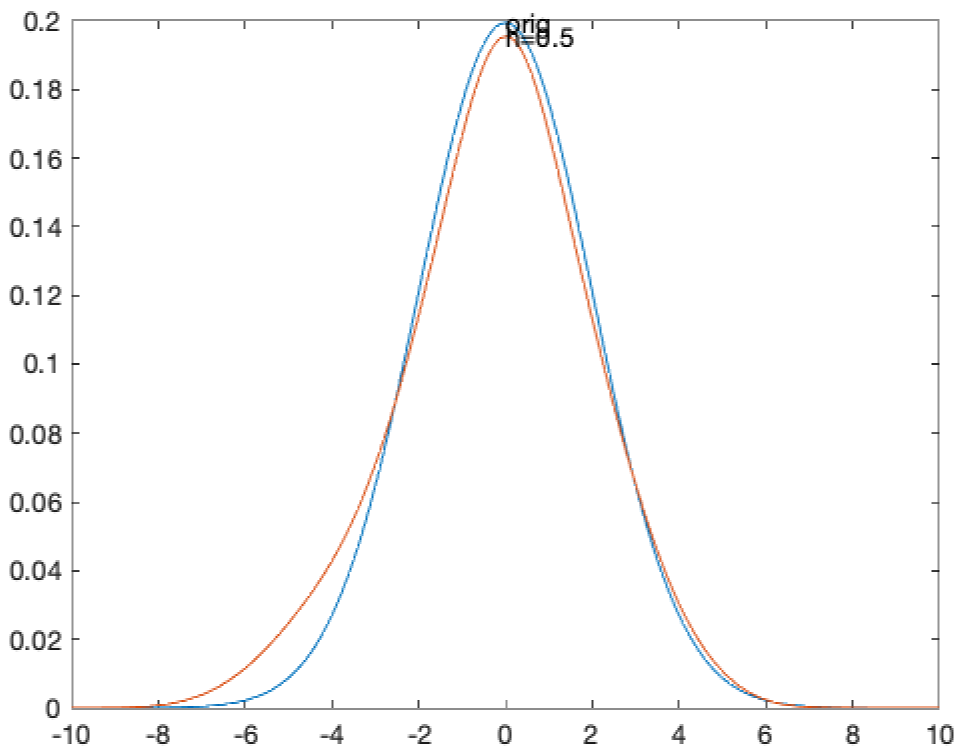


We choose $h = 0.5$ as a best parameter and repeat the approximation.

Then we interpolate the approximated samples `f(x)` to get the probability density functopn `F` applicable to **any** argument in the value range `x`.

[33]:
```
% plot the distribution
plot(x,pdf(dist,x));
text(mu,pdf(dist,mu),'orig');
hold on;
h = .5;
for j=1:len_x
    xx=x(j);
    % Use a Gaussian Kernal function
    for i=1:n
        f(j) = f(j) + normpdf((xx-X(i))/h,mu,sigma);
    end
    f(j) = f(j) /(n*h);
end
```

```
% interpolate the approximated function
F = griddedInterpolant(x,f);
% plot the interpolated function in the same figure as the distribution
plot(x,F(x));
[argv,argmax] = max(F(x));
i=argmax;
text(x(i),F(x(i)),['h=',num2str(h)]);
```



The PDF approximation is now usable in between and outside the range of the sample points, e.g.

[34]:
```
p1=F(1.23456)
p2=F(-11)
```

```
p1 =

    0.1549


p2 =
```

-9.6955e-06