Advanced Kinematics

Introduction

Kinematics is the branch of mechanics that deals with the motion of objects without considering the forces that cause the motion. This document covers advanced kinematic concepts, drawing heavily from the content of Resnick and Halliday's textbook.

1 Basic Definitions and Equations

1.1 Displacement

Displacement is the vector quantity that defines the change in position of an object:

$$\vec{s} = \vec{r}_f - \vec{r}_i,$$

where $\vec{r}_f$ is the final position vector and $\vec{r}_i$ is the initial position vector.

1.2 Velocity

- **Average Velocity:**

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{s}}{\Delta t}$$

- **Instantaneous Velocity:**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

1.3 Acceleration

- **Average Acceleration:**

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$$

- **Instantaneous Acceleration:**

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{s}}{dt^2}$$

2 Equations of Motion (Constant Acceleration)

For motion with constant acceleration, the following equations apply:

$$\begin{aligned}
\vec{v} &= \vec{v}_0 + \vec{a}t, \\
\vec{s} &= \vec{v}_0 t + \frac{1}{2}\vec{a}t^2, \\
\vec{v}^2 &= \vec{v}_0^2 + 2\vec{a} \cdot \vec{s}, \\
\vec{s} &= \frac{\vec{v} + \vec{v}_0}{2} t.
\end{aligned}$$

3 Projectile Motion

Projectile motion is the motion of an object under the influence of gravity, assuming no air resistance.

3.1 Key Equations

- Horizontal motion:
$$x(t) = v_0 \cos\theta \cdot t$$
- Vertical motion:
$$y(t) = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$$
- Time of flight:
$$T = \frac{2v_0 \sin\theta}{g}$$
- Maximum height:
$$H = \frac{v_0^2 \sin^2\theta}{2g}$$
- Range:
$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

4 Relative Motion

The relative velocity of an object A with respect to another object B is given by:

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

5 Uniform Circular Motion

Uniform circular motion occurs when an object moves in a circle with constant speed.

- Centripetal acceleration:
$$a_c = \frac{v^2}{r}$$
- Angular velocity:
$$\omega = \frac{\Delta\theta}{\Delta t}$$
- Relationship between linear and angular velocity:
$$v = \omega r$$

6 Kinematics in Two Dimensions

Motion in two dimensions can be represented by breaking it into components along the x- and y-axes:

- Position vector:
$$\vec{r} = x\hat{i} + y\hat{j}$$
- Velocity vector:
$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$
- Acceleration vector:
$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Conclusion

This document provides a comprehensive overview of advanced kinematics, including key equations and concepts from Resnick and Halliday. Understanding these principles is essential for analyzing motion in various physical systems.