Basic Derivative Formulas

Introduction

This document provides a comprehensive list of basic derivative formulas useful for calculus. Each formula is presented in a simple and clear manner.

Basic Rules of Differentiation

1. **Constant:**
$$\frac{d}{dx}[c] = 0$$
where c is a constant.

2. **Power:**
$$\frac{d}{dx}[x^n] = nx^{n-1}$$
where n is any real number.

3. **Derivative of a Constant Multiple:**
$$\frac{d}{dx}[a \cdot f(x)] = a \cdot f'(x)$$

4. **Sum/Difference Rule:**
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

5. **Product Rule:**
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

6. **Quotient Rule:**
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

7. **Chain Rule:**
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Derivatives of Common Functions

1. **Square Root Function:**
$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

2. **Reciprocal Function:**
$$\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

3. **Exponential Functions:**
$$\frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[a^x] = a^x \ln(a), \quad a > 0$$

4. **Logarithmic Functions:**
$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}, \quad x > 0$$
$$\frac{d}{dx}[\log_a(x)] = \frac{1}{x\ln(a)}, \quad x > 0,\ a > 0$$

5. **Trigonometric Functions:**
$$\frac{d}{dx}[\sin(x)] = \cos(x)$$
$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$
$$\frac{d}{dx}[\tan(x)] = \sec^2(x), \quad x \neq \frac{\pi}{2} + n\pi$$

6. **Inverse Trigonometric Functions:**
$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}, \quad |x| < 1$$
$$\frac{d}{dx}[\arccos(x)] = \frac{-1}{\sqrt{1-x^2}}, \quad |x| < 1$$
$$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$$

7. **Hyperbolic Functions:**
$$\frac{d}{dx}[\sinh(x)] = \cosh(x)$$
$$\frac{d}{dx}[\cosh(x)] = \sinh(x)$$
$$\frac{d}{dx}[\tanh(x)] = \operatorname{sech}^2(x)$$

8. **Inverse Hyperbolic Functions:**

$$\frac{d}{dx}[\text{arsinh}(x)] = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}[\text{arcosh}(x)] = \frac{1}{\sqrt{x^2-1}}, \quad x > 1$$

$$\frac{d}{dx}[\text{artanh}(x)] = \frac{1}{1-x^2}, \quad |x| < 1$$

These formulas form the foundation of differential calculus and are essential tools for solving a wide variety of problems in mathematics, physics, and engineering.