Rules for Integrating: Indefinite Integrals

Introduction

Integration is the reverse process of differentiation and a fundamental concept in calculus. This document focuses on rules for evaluating indefinite integrals, which are written in the form:

$$\int f(x)\,dx = F(x) + C,$$

where $F(x)$ is the antiderivative of $f(x)$, and C is the constant of integration.

Basic Integration Rules

1. Constant Rule

$$\int k\,dx = kx + C, \quad \text{where } k \text{ is a constant.}$$

2. Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad \text{for } n \neq -1.$$

3. Exponential Function Rule

$$\int e^x\,dx = e^x + C,$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C, \quad \text{where } a > 0 \text{ and } a \neq 1.$$

4. Logarithmic Function Rule

$$\int \frac{1}{x}\,dx = \ln|x| + C, \quad \text{for } x \neq 0.$$

5. Trigonometric Functions

$$\int \sin x\,dx = -\cos x + C,$$

$$\int \cos x\,dx = \sin x + C,$$

$$\int \sec^2 x\,dx = \tan x + C,$$

$$\int \csc^2 x\,dx = -\cot x + C,$$

$$\int \sec x \tan x\,dx = \sec x + C,$$

$$\int \csc x \cot x\,dx = -\csc x + C.$$

6. Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C, \quad \text{for } |x| \le 1.$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + C, \quad \text{for } |x| \le 1.$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C,$$

$$\int \frac{-1}{1+x^2}\,dx = \operatorname{arccot} x + C,$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arcsec}|x| + C, \quad \text{for } |x| \ge 1.$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arccsc}|x| + C, \quad \text{for } |x| \ge 1.$$

Integration Techniques

1. Linearity of Integration

$$\int [af(x) + bg(x)]\ dx = a\int f(x)\,dx + b\int g(x)\,dx,$$

where a and b are constants.

2. Substitution Rule

If $u = g(x)$, then:

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

3. Integration by Parts

$$\int u\,dv = uv - \int v\,du,$$

where u and v are differentiable functions of x.

4. Partial Fraction Decomposition

For rational functions, decompose into partial fractions before integrating:

$$\frac{P(x)}{Q(x)} = \sum \frac{A}{(x-r)^k} + \sum \frac{Bx+C}{(x^2+px+q)^m}.$$

5. Trigonometric Substitution

Use trigonometric identities to simplify integrals involving square roots, such as:

$$x = a\sin\theta, \quad x = a\tan\theta, \quad x = a\sec\theta.$$

Conclusion

These rules and techniques form the foundation for solving indefinite integrals. Mastery of these principles will enable you to tackle a wide range of integration problems.