Rules for Definite Integrals

Introduction

Definite integrals are a fundamental concept in calculus, representing the area under a curve over a specific interval. This document outlines the key rules and properties of definite integrals.

1. Definition of a Definite Integral

The definite integral of a function $f(x)$ from a to b is given by:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and x_i^* is a point in the ith subinterval.

2. Linearity of Integration

For any two functions $f(x)$ and $g(x)$, and constants c_1 and c_2:

$$\int_a^b (c_1 f(x) + c_2 g(x))\,dx = c_1 \int_a^b f(x)\,dx + c_2 \int_a^b g(x)\,dx$$

3. Additivity over Intervals

If c is a point in the interval $[a, b]$, then:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

4. Reversal of Limits

Reversing the limits of integration changes the sign of the integral:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

5. Zero Interval Property

If the limits of integration are the same, the integral evaluates to zero:

$$\int_a^a f(x)\,dx = 0$$

6. Comparison Property

If $f(x) \leq g(x)$ for all $x \in [a, b]$, then:

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

7. Integration of an Absolute Value

For a function $f(x)$, the integral of its absolute value satisfies:

$$\left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f(x)|\,dx$$

8. Fundamental Theorem of Calculus

Part 1: Derivative of an Integral

If $F(x)$ is defined as:

$$F(x) = \int_a^x f(t)\,dt,$$

then $F'(x) = f(x)$.

Part 2: Evaluation of a Definite Integral

If $F(x)$ is an antiderivative of $f(x)$, then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Conclusion

These rules provide the foundation for solving problems involving definite integrals and understanding their applications in mathematics and science.