Comprehensive Guide to Logic Basics, Quantifiers, and Set Theory

Contents

1 Introduction

Logic, quantifiers, and set theory form the foundational pillars of mathematics and computer science. This guide provides an introduction to these concepts, their notation, and usage.

2 Logic Basics

Logic is the study of reasoning and principles of valid inference. It provides tools to evaluate the truth of statements.

2.1 Propositional Logic

Propositional logic deals with propositions, which are statements that are either true (T) or false (F).

- **Logical Operators:**
  - **Negation ($\neg$):** $\neg P$ is true if P is false.
  - **Conjunction ($\wedge$):** $P \wedge Q$ is true if both P and Q are true.
  - **Disjunction ($\vee$):** $P \vee Q$ is true if at least one of P or Q is true.
  - **Implication ($\implies$):** $P \implies Q$ is false only if P is true and Q is false.
  - **Biconditional ($\iff$):** $P \iff Q$ is true if P and Q have the same truth value.

2.2 Truth Tables

Truth tables are used to systematically explore all possible truth values of logical expressions. Example:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

2.3 Logical Equivalence

Two statements are logically equivalent if they have the same truth table. For example:

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q \text{ (De Morgan's Law)}$$

3 Quantifiers

Quantifiers extend logic to statements about collections of objects, often expressed as variables.

3.1 Universal Quantifier ($\forall$)

$\forall x \in S, P(x)$: The statement is true if $P(x)$ is true for all elements x in the set S.

- Example: $\forall x \in \mathbb{R}, x^2 \geq 0$.

3.2 Existential Quantifier ($\exists$)

$\exists x \in S, P(x)$: The statement is true if there exists at least one element x in S such that $P(x)$ is true.

- Example: $\exists x \in \mathbb{R}, x^2 = 4$.

3.3 Negating Quantifiers

Negating statements with quantifiers involves switching between $\forall$ and $\exists$:

$$\neg(\forall x, P(x)) \equiv \exists x, \neg P(x)$$

$$\neg(\exists x, P(x)) \equiv \forall x, \neg P(x)$$

4 Set Theory

Set theory studies collections of objects, called sets. Sets are fundamental in mathematics.

4.1 Basic Concepts

- **Set:** A collection of distinct elements. Example: $A = \{1, 2, 3\}$.
- **Element:** If x is in A, we write $x \in A$.
- **Empty Set:** The set with no elements, denoted $\emptyset$.
- **Subset:** $A \subseteq B$ if every element of A is also in B.

4.2 Set Operations

- **Union ($\cup$):** $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
- **Intersection ($\cap$):** $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
- **Difference ($\setminus$):** $A \setminus B = \{x : x \in A \text{ and } x \notin B\}$.
- **Complement (A^c):** $A^c = \{x : x \notin A\}$.

4.3 Power Set

The power set of A, denoted $\mathcal{P}(A)$, is the set of all subsets of A.

$$\mathcal{P}(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

4.4 Cartesian Product

The Cartesian product of A and B, denoted $A \times B$, is the set of ordered pairs:

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

4.5 Venn Diagrams

Venn diagrams visually represent set operations and relationships.

5 Conclusion

This guide introduces the basics of logic, quantifiers, and set theory. Mastery of these topics provides a strong foundation for advanced studies in mathematics, computer science, and related fields.