Trigonometry Rules

Basic Definitions

- **Sine:** $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

- **Cosine:** $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

- **Tangent:** $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta}$

- **Cosecant:** $\csc\theta = \frac{1}{\sin\theta}$

- **Secant:** $\sec\theta = \frac{1}{\cos\theta}$

- **Cotangent:** $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$

- $1 + \tan^2\theta = \sec^2\theta$

- $1 + \cot^2\theta = \csc^2\theta$

Sum and Difference Formulas

- $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

- $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

- $\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

Double Angle Formulas

- $\sin(2\theta) = 2\sin\theta\cos\theta$

- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- $\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half Angle Formulas

- $\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2}$

- $\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$

- $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$

Product-to-Sum Formulas

- $\sin a \sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$

- $\cos a \cos b = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$

- $\sin a \cos b = \frac{1}{2}[\sin(a + b) + \sin(a - b)]$

Sum-to-Product Formulas

- $\sin a + \sin b = 2 \sin \left(\frac{a+b}{2} \right) \cos \left(\frac{a-b}{2} \right)$

- $\sin a - \sin b = 2 \cos \left(\frac{a+b}{2} \right) \sin \left(\frac{a-b}{2} \right)$

- $\cos a + \cos b = 2 \cos \left(\frac{a+b}{2} \right) \cos \left(\frac{a-b}{2} \right)$

- $\cos a - \cos b = -2 \sin \left(\frac{a+b}{2} \right) \sin \left(\frac{a-b}{2} \right)$

Law of Sines and Law of Cosines

- **Law of Sines:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

- **Law of Cosines:** $c^2 = a^2 + b^2 - 2ab \cos C$