Basic Rules of Matrix Manipulation

Introduction

Matrices are fundamental tools in mathematics, physics, engineering, and computer science. This document provides an overview of the basic rules of matrix manipulation.

Matrix Addition and Subtraction

- Matrices can be added or subtracted only if they have the same dimensions.
- If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two $m \times n$ matrices, their sum $C = A + B$ is defined as:
$$C = [c_{ij}], \quad c_{ij} = a_{ij} + b_{ij}.$$

Scalar Multiplication

- A matrix $A = [a_{ij}]$ can be multiplied by a scalar k. The result is:
$$kA = [k \cdot a_{ij}].$$

Matrix Multiplication

- The product of two matrices A and B is defined only if the number of columns in A equals the number of rows in B.
- If A is an $m \times n$ matrix and B is an $n \times p$ matrix, the product $C = AB$ is an $m \times p$ matrix, where:
$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

Transpose of a Matrix

- The transpose of a matrix A, denoted by $A^\top$, is obtained by interchanging its rows and columns.
- If $A = [a_{ij}]$, then:
$$A^\top = [a_{ji}].$$

Identity Matrix

- The identity matrix I_n is a square matrix of size $n \times n$ with ones on the diagonal and zeros elsewhere:
$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}.$$
- $AI_n = I_n A = A$ for any $n \times n$ matrix A.

Matrix Inversion

- A square matrix A is invertible (or non-singular) if there exists a matrix A^{-1} such that:
$$AA^{-1} = A^{-1}A = I_n.$$
- Only square matrices can have inverses, and not all square matrices are invertible.

Properties of Matrix Operations

1. **Associativity of Addition:** $A + (B + C) = (A + B) + C$.
2. **Commutativity of Addition:** $A + B = B + A$.
3. **Distributive Property:** $A(B + C) = AB + AC$ and $(A + B)C = AC + BC$.
4. **Associativity of Multiplication:** $A(BC) = (AB)C$.
5. **Transpose Properties:**
$$(A + B)^\top = A^\top + B^\top, \quad (AB)^\top = B^\top A^\top.$$

Conclusion

These basic rules of matrix manipulation are foundational for understanding advanced topics in linear algebra. Mastery of these operations will enable you to solve complex problems in various disciplines.