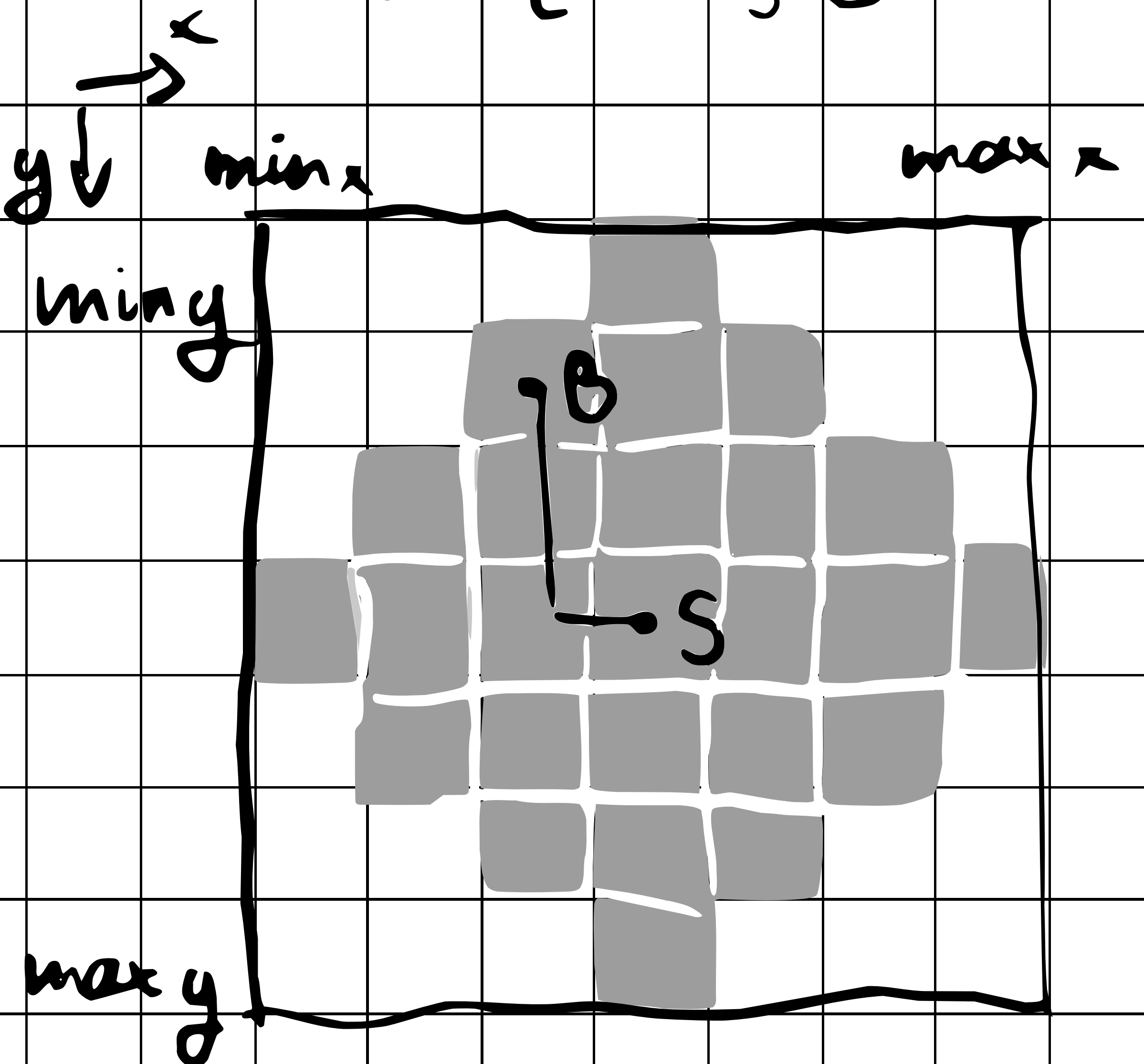
Day 15 AoC (2022)

$$dist(a, b) = |a_x - b_x| + |a_y - b_y|$$



beacon

range of sensor S is $r = dist(S, B)$

$$\left.\begin{aligned} min\ x &= S_x - r \\ max\ x &= S_x + r \\ min\ y &= S_y - r \\ max\ y &= S_y + r \end{aligned}\right\}\text{ bounding box of } S$$

Y — row (constant)

For each sensor S find interval $[x_1, x_2]$ in range of S

Interval can be non-empty only if $miny \leq Y \leq maxy$

The interval is $[minx + |Y - S_y|, maxx - |Y - S_y|]$

Now, we need to find the union of all intervals.

Consider intervals $[a, b]$, $[c, d]$.

If they overlap, the union is $[min(a, c), max(b, d)]$

If they don't, the union must be array $\{[a, b], [c, d]\}$.

How to find the union efficiently?

Bruteforce: check each interval with other interals, merge them when possible $O(n^2)$, n – # intervals (sensors)

x-sweep: walk along the x axis, keeping track of the current interval, make it as long as possible each time a new interval is encountered

1° Make event points {x position, start/end, id of interval}

2° Sort lexicographically (position first, type second), starts at the same position must be before ends.

3° Start walking (ending points can be ignored)

At each point we have current interval $[a, b]$.

Consider point P from range $[c, d]$

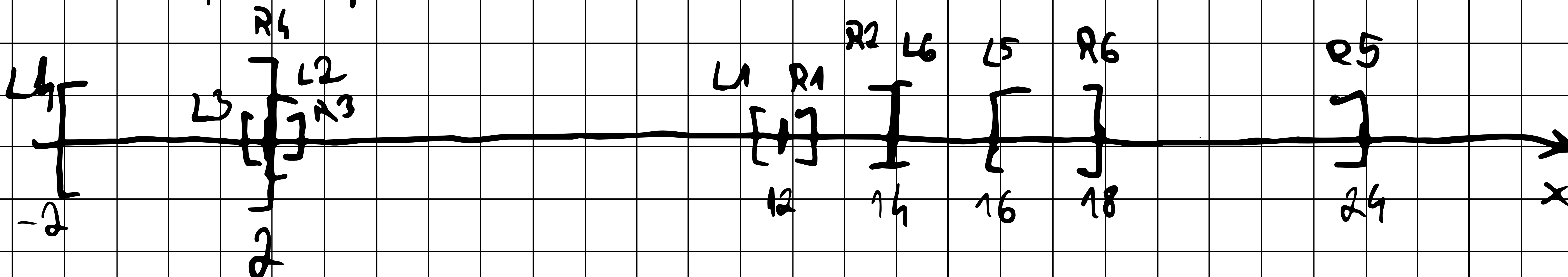
1° $c \in [a, b] \rightarrow$ current $= [a, \max(b, d)]$

2° $c \notin [a, b] \rightarrow$ save $[a, b]$, current $= [c, d]$

$O(n)$ walk, but $O(n \log n)$ sort beforehand

$O(n \log n)$ overall

Example input.



L4, L2, L3, R4, R3, L1, R1, L6, R2, L5, R6, R5

start at x = -2, L4 → start range [-2, 2]

x = 2, L2 → overlapping, range is now [-2, 14]

x = 2, L3 → overlapping, range is now [-2, 14]

x = 2, R4 → ignore

x = 2, R3 → ignore

x = 12, L1 → overlapping, range is now [-2, 14]

x = 12, R1 → ignore

x = 14, L6 → overlapping, range is now [-2, 18]

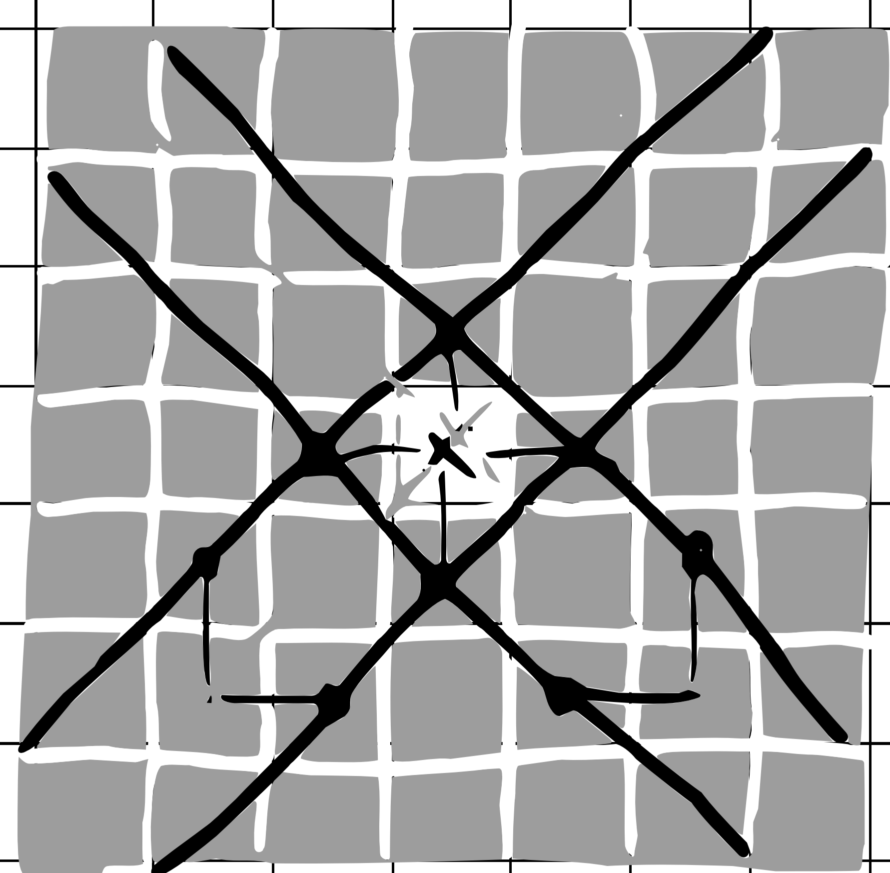
x = 14, R2 → ignore

x = 16, L5 → overlapping, range is now [-2, 24]

x = 18, R6 → ignore

x = 24, R5 → ignore

return [-2, 24]

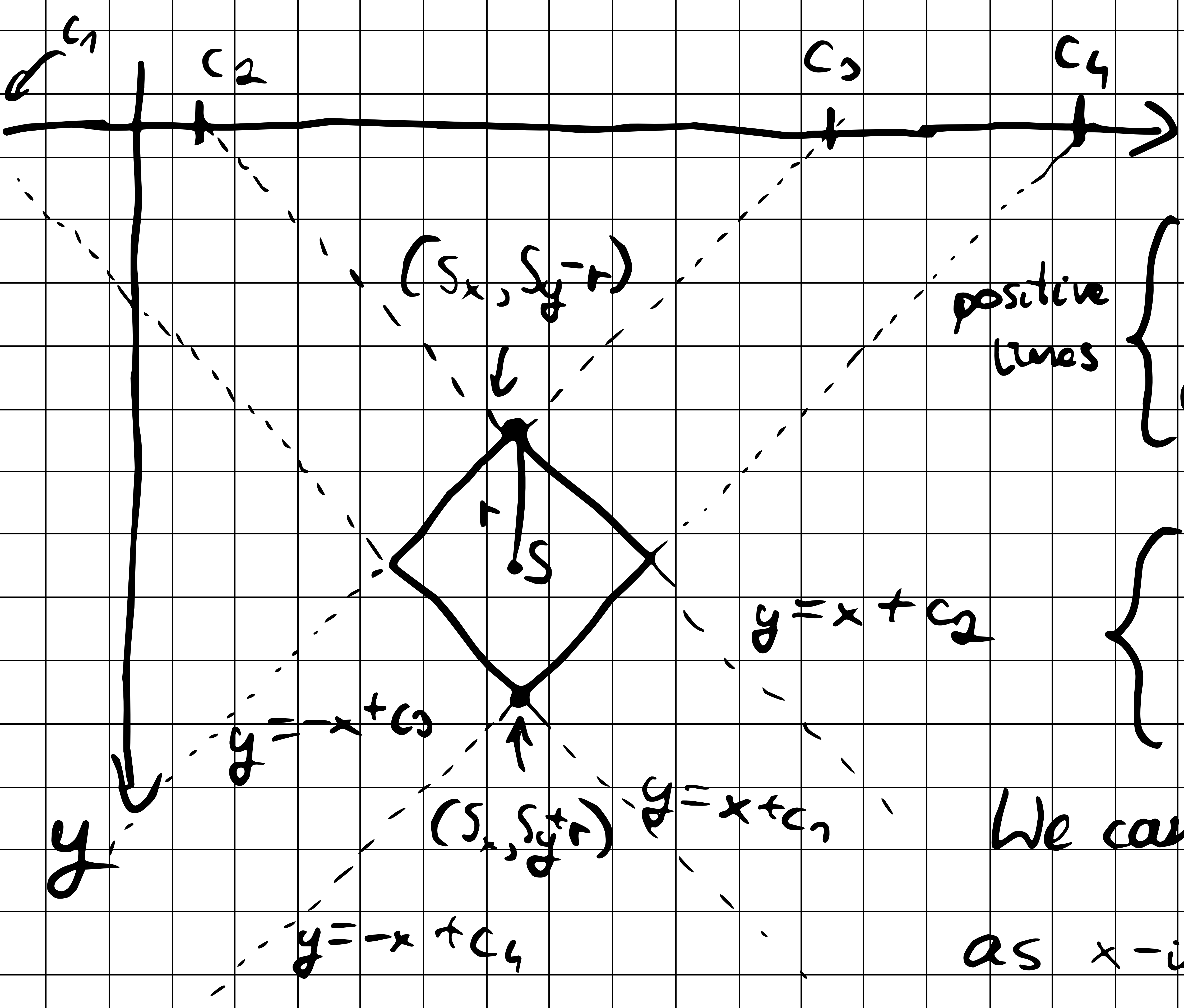


Only one such position.
Hence, it must be 1 unit away from every edge around it.

Find 4 lines for each sensor. $O(n)$ Find pairs which are 2 units apart. $O(n^2)$
Map pairs to lines in-between. Find intersections. $O(n^2)$ Find the one not in range of any of n sensors. $O(n^2) \cdot O(n) \to O(n^3)$

n is small $n = 27$, $n^3 = 19683 < 4\,000\,000$ rows $\cdot\, O(n \log n)$

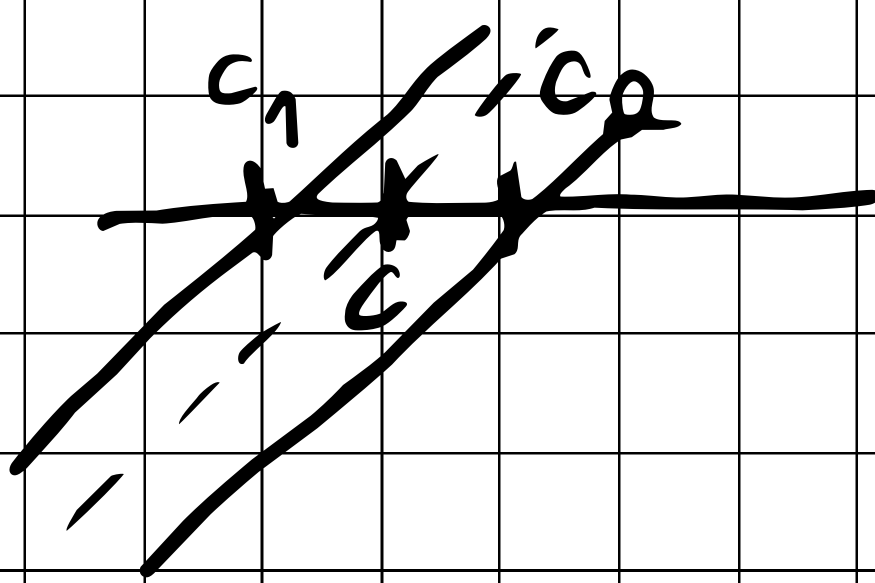
How to find the lines?



positive lines
$$\begin{cases} c_1 = y - x = S_y + r - S_x \\ c_2 = y - x = S_y - r - S_x \end{cases}$$

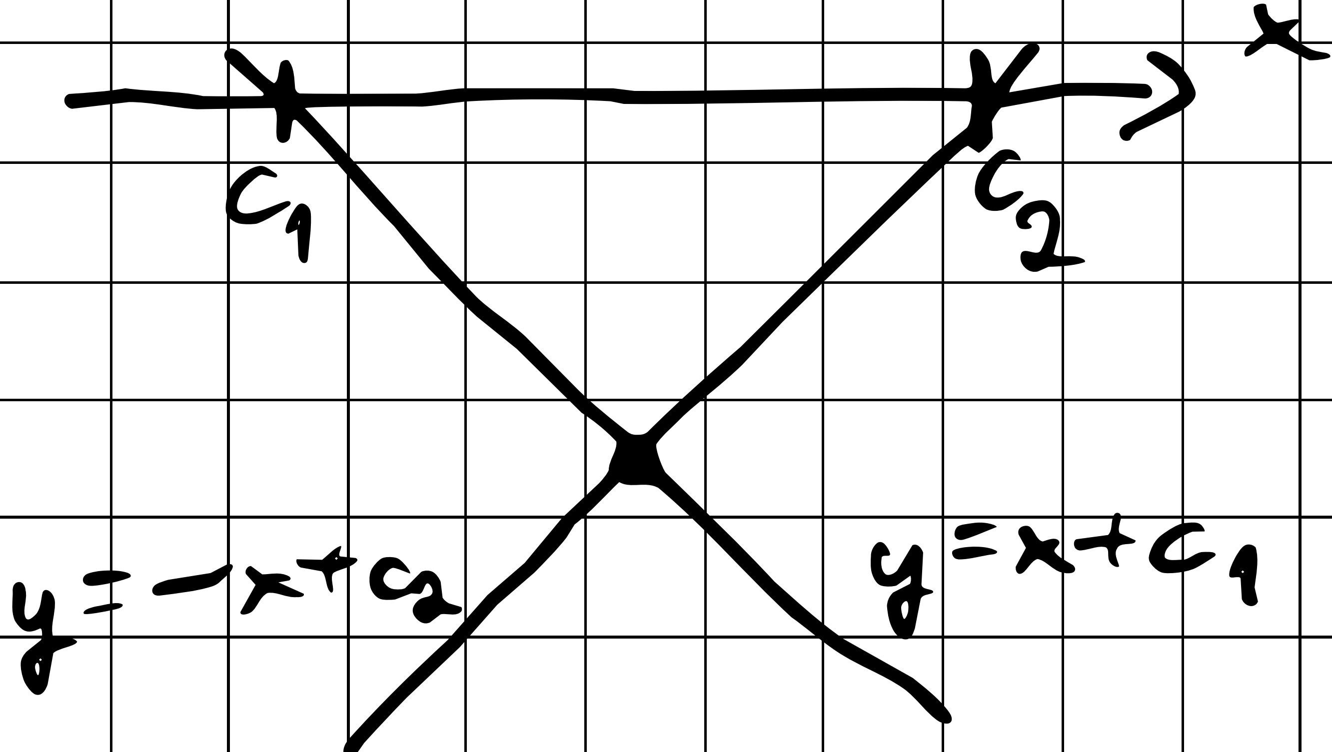
$$\begin{cases} c_3 = y + x = S_y - r + S_x \\ c_4 = y + x = S_y + r + S_x \end{cases}$$

We can store them as x-intercept (c_1, c_2, c_3, c_4)



Line in between lines $c_1, c_2 = \frac{c_1+c_2}{2}$

Intersection



$$\begin{cases} y = -x + c_2 \\ y = x + c_1 \end{cases} +$$

$$2y = c_1 + c_2$$

$$y = \frac{c_1+c_2}{2}$$

$$\frac{c_1+c_2}{2} = x + c_1$$

$$x = \frac{c_2 - c_1}{2}$$