

Example

$P_{S,T}(s,t)$	$t=40$	$t=60$
$s=1$ sheet	0.15	0.1
$s=2$ sheets	0.3	0.2
$s=3$ sheets	0.15	0.1

Quiz 4.7 Find
(1) $E[S]$ and $\text{Var}[S]$
(2) $E[T]$ and $\text{Var}[T]$
(3) $r_{S,T} = E[ST]$
(4) $\text{Cov}[S,T]$
(5) $\rho_{S,T}$

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$$\textcircled{1} E[S] = \sum_{s=1}^3 s P_S(s)$$

$$= 1(0.15+0.1) + 2(0.3+0.2) + 3(0.15+0.1)$$

$$\therefore E[S] = 2 \quad \times$$

$$\text{Var}[S] = E[S^2] - (E[S])^2$$

$$= \sum_{s=1}^3 s^2 P_S(s) - (E[S])^2$$

$$= 1^2(0.15+0.1) + 2^2(0.3+0.2) + 3^2(0.15+0.1) - 2^2$$

$$= 4.5 - 4$$

$$\therefore \text{Var}[S] = 0.5 \quad \times$$

$$\textcircled{2} E[T] = \sum_{t=40,60} t P_T(t)$$

$$= 40(0.15+0.3+0.15) + 60(0.1+0.2+0.1)$$

$$= 24 + 24$$

$$\therefore E[T] = 48 \quad \times$$

$$\text{Var}[T] = E[T^2] - (E[T])^2$$

$$= \sum_{t=40,60} t^2 P_T(t) - (E[T])^2$$

$$= 40^2(0.6) + 60^2(0.4) - 48^2$$

$$P_S(s) = \begin{cases} 0.15+0.1 & , s=1 \\ 0.3+0.2 & , s=2 \\ 0.15+0.1 & , s=3 \\ 0 & , \text{otherwise} \end{cases}$$

$$P_T(t) = \begin{cases} 0.1+0.1+0.2 & ; t=60 \\ 0.15+0.3+0.15 & ; t=40 \\ 0 & , \text{otherwise} \end{cases}$$

$$\therefore \text{Var}[T] = 96 \quad \times$$

$$\begin{aligned}
 \textcircled{3} \quad r_{S,T} &= E[ST] \\
 &= E[S]E[T] \\
 &= 2 \times 48 \\
 \therefore r_{S,T} &= 96 \quad \#
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad \text{Cov}[S,T] &= E[ST] - E[S]E[T] && \text{multiplication} \\
 &= 96 - 96 = 0 \\
 \therefore \text{Cov}[S,T] &= 0 \quad \# && \text{(Independent experiment.)}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad \rho_{S,T} &= \frac{\text{Cov}[S,T]}{\sqrt{\text{Var}[S] \text{Var}[T]}} \\
 &= \frac{0}{\sqrt{0.5 \times 96}} \\
 \therefore \rho_{S,T} &= 0 \quad \#
 \end{aligned}$$