

$$④ \quad P_{X,Y}(x,y) = \begin{cases} k 2^{-(x+|y|)} & , x=0,1,2,\dots; y=-x, -x+2, -x+4, \dots \\ 0 & , \text{otherwise} \end{cases}$$

4.1) k

จาก Theorem : $\sum_{x \in S_X} \sum_{y \in S_Y} P_{X,Y}(x,y) = 1$

วิธีที่ 1 $\sum_{x=0}^{\infty} \sum_{y \in \{-x, -x+2, -x+4, \dots\}} k 2^{-(x+|y|)} = 1$

$$k \sum_{x=0}^{\infty} 2^{-x} \sum_{y \in \{-x, -x+2, -x+4, \dots\}} 2^{-|y|} = 1$$

ถ้า $y = -x + 2n$;

$$k \sum_{x=0}^{\infty} 2^{-x} \sum_{n=0}^{\infty} 2^{-|-x+2n|} = 1$$

จากข้อ 1.1) abs

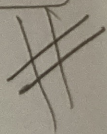
$$-|-x+2n| = \begin{cases} x-2n & ; -x+2n \geq 0 \\ & \underline{x \leq 2n} \\ -x+2n & ; -x+2n < 0 \\ & \underline{x \geq 2n} \end{cases}$$

$$k \sum_{x=0}^{\infty} 2^{-x} \sum_{n=0}^{\infty} 2^{x-2n} = 1$$

$$k \sum_{x=0}^{\infty} 2^{-x+x} \sum_{n=0}^{\infty} 2^{-2n} = 1$$

$\therefore k = \frac{3}{4}$

$$k \sum_{x=0}^{\infty} 1 \cdot \left(\frac{1}{1 - \left(\frac{1}{4}\right)} \right) = 1$$



4.2) Marginal PMF $P_X(x)$

$$\begin{aligned} P_X(x) &= \sum_y P_{X,Y}(x,y) \\ &= \sum_{n=0}^{\infty} \frac{3}{4} \cdot (2^{-x+x+2n}) \\ &= \frac{3}{4} \sum_{n=0}^{\infty} 2^{-2n} \\ &= \frac{3}{4} \cdot \frac{1}{1 - \frac{1}{4}} \end{aligned}$$

$$\therefore P_X(x) = 1$$

4.3 Marginal PMF $P_Y(y)$

$$\begin{aligned} P_Y(y) &= \sum_x P_{X,Y}(x,y) \\ &= \frac{3}{4} \sum_{x=0}^{\infty} 2^{-x-|y|} \\ &= \frac{3}{4} 2^{-|y|} \left(\frac{1}{1 - \frac{1}{2}} \right) \end{aligned}$$