Lecture #17 Stochastic Process (3)

Stationary Process

- For a random process X(t), normally, at t_1 : $X(t_1)$ has $pdf = f_{X(t_1)}(x)$ [depends on t_1]
- For a random process X(t), at t1: $X(t_1)$ has $pdf = f_{X(t_1)}(x)$ [not depend on t_1]

Stationary Process

- = same random variable at all time
- = no statistical properties change with time

$$f_{X(t_1)}(x) = f_{X(t_1 + \tau)}(x) = f_X(x)$$

Stationary Process

Definition: A stochastic process X(t) is stationary iif for all sets of time $t_1, ..., t_m$ and any time different τ ,

$$\begin{split} f_{X(t_1),...,X(t_m)}(x_1,...,x_m) &= \\ f_{X(t_1+\tau),...,X(t_m+\tau)}(x_1,...,x_m) \end{split}$$

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Stationary Random Sequence

Definition: A random sequence X_n is stationary iif for any finite sets of time instants $n_1, ..., n_m$ and any time different k,

$$f_{X(n_1,...,X(n_m)}(x_1,...,x_m) = f_{X(n_1+k),...,X(n_m+k)}(x_1,...,x_m)$$



Theorem: A stationary process X(t),

$$\begin{split} \mu_X(t) &= \mu_X \\ R_X(t,\tau) &= R_X(0,\tau) = R_X(\tau) \\ C_X(t,\tau) &= R_X(\tau) - \mu^2_X = C_X(\tau) \end{split}$$

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Stationary Random Sequence

Theorem: A stationary random sequence X_n , for all m

$$\begin{split} E[X_m] &= \mu_X \\ R_X[m.k] &= R_X[0,k] = R_X[k] \\ C_X[m,k] &= R_X[k] - \mu^2_X = C_X[k] \end{split}$$



Definition: X(t) is a wide sense stationary random process iff for all t,

$$\begin{split} E[X(t)] &= \mu_X \\ R_X(t,\tau) &= R_X(0,\tau) = R_X(\tau) \end{split}$$

Definition: X_n is a wide sense stationary random sequence iff for all n,

$$E[X_n] = \mu_X$$

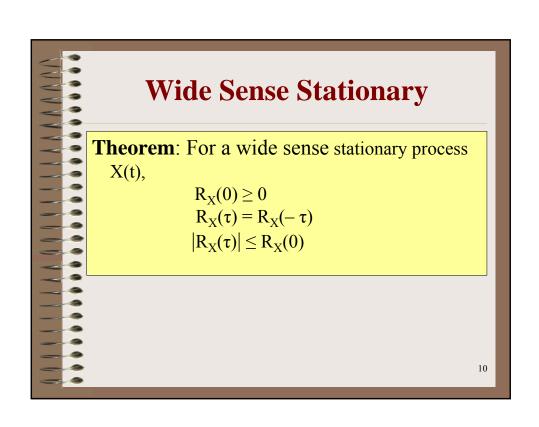
$$R_X[n,k] = R_X[0,k] = R_X[k]$$

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Wide Sense Stationary

- For every stationary process or sequence, it is also wide sense stationary.
- However, if it is a wide sense stationary it may or may not be stationary.

Example • Let $X_n = \pm 1$ with prob = $\frac{1}{2}$ (n = even) • For n = odd $X_n = -\frac{1}{3}$ with prob = $\frac{9}{10}$ $X_n = 3$ with prob = $\frac{1}{10}$ • Stationary? - No • Wide sense stationary? - Mean = 0 for all n - $C_X(t,\tau) = 0$ for $\tau > 0$ - $C_X(t,\tau) = 1$ for $\tau = 0$ - Yes, it's wide sense stationary



Wide Sense Stationary

Theorem: For a wide sense stationary sequence X_n ,

$$\begin{aligned} R_X[0] &\geq 0 \\ R_X[k] &= R_X[-k] \\ |R_X[k]| &\leq R_X[0] \end{aligned}$$

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Average Power

- From Ohm's Law: V = IR
- For v(t), i(t), R Ω, the instantaneous power dissipated, P(t),

$$P(t) = v^2(t)/R = i^2(t)R$$

- For $R = 1 \Omega$, $P(t) = v^2(t) = i^2(t)$
- For a voltage or current is a sample function of random process, x(t,s)
 - → P across 1 Ω resistor = $x^2(t,s)$

