Definition

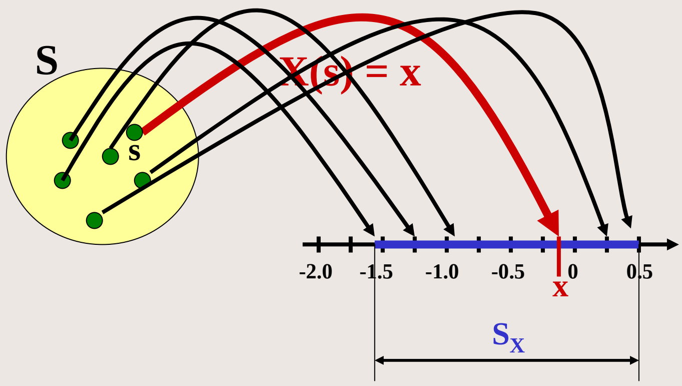
Stochastic Process

Random | **Function of Time**

Random Process

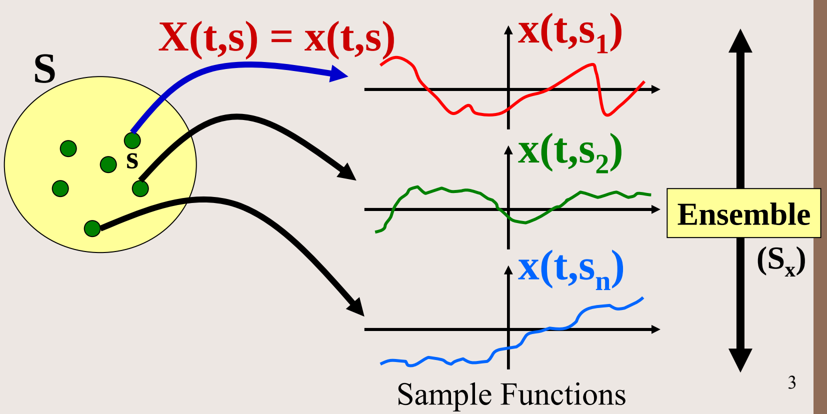
Random Variable

X is a function that maps each outcome, s, in S to a real number X(s), x



Random Process

$X(t)$ is a function that maps each outcome, s, in S to a time function $x(t,s)$



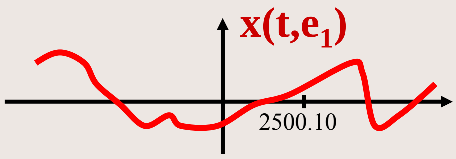
Example 1

- Taking temperature at the surface of a space shuttle
- Starting at launch time $t = 0$
- $X(t)$ = temp in degree Celsius on the surface
- Each launch, record $x(t,s)$

Source:www.inventorsmuseum.com

Example 1

Source:www.analyticalsci.com/Astronomy/ Hansen

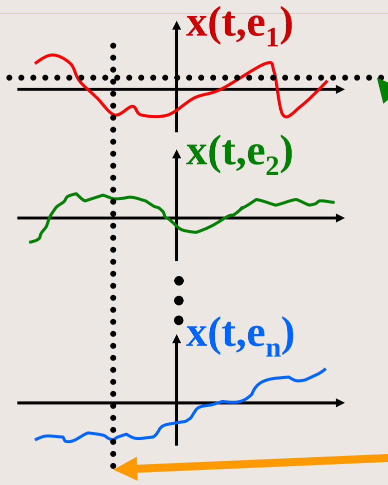


At time t = **2500.10 sec**

$x(2500.10, e_1) = 1200$ C

$e_1 = 1^{st}$ launch measuring

Example 1



Sample functions

Average temp at $\mathbf{e_1}$ for completed mission is 5000 C ➔ $\overline{[x(t, e_1)]}$

Time Average

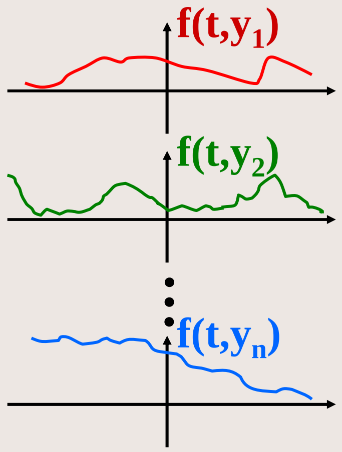
Average temp of engine at t = 2500.10 sec is 1320 C ➔ **E**[X(2500.10)]

Ensemble Average

Example 2

- Measure the rain fall in a day @Songkla province every day.
- Let F(t) = random process
- f(t,y) = a sample function for measuring at day "t" of the year "y"

Example 2



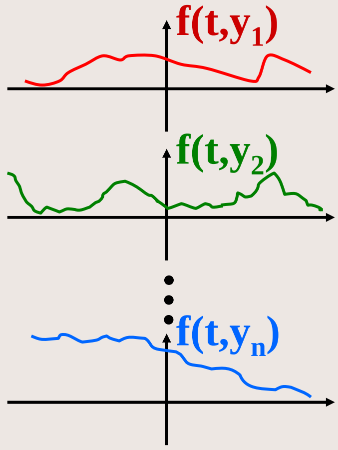
A sample function of rain fall in y_1 = year 1990 ($1 \le t \le 365$)

A sample function of rain fall in y_2 = year 1991 ($1 \le t \le 365$)

A sample function of rain fall in y_n = year 2001 ($1 \le t \le 365$)

Sample functions

Example 2

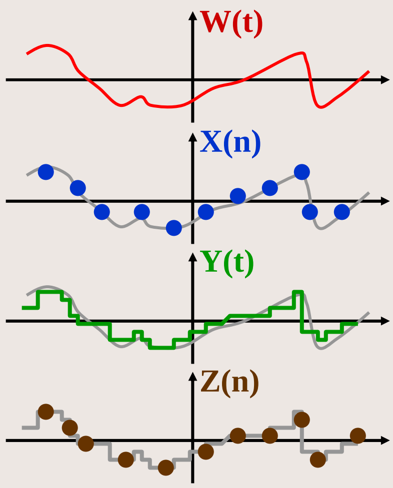


Sample functions

Therefore, we might want to know

- The average rain fall in year 2001
- The average rain fall for Sep. 3rd

Types of Stochastic Process



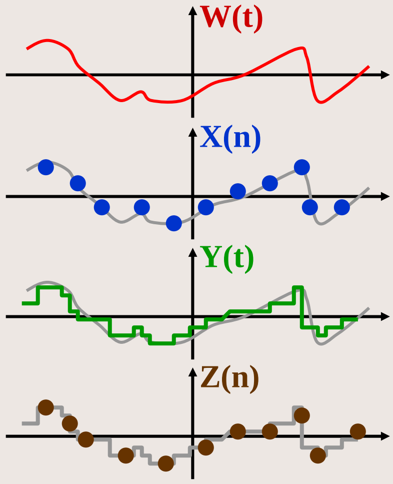
Continuous Time, Continuous value Process

Discrete Time, Continuous value Process

Continuous Time, Discrete value Process

Discrete Time, Discrete value Process

Stochastic Process Examples



Record temperature as a continuous time

Record temperature as a discrete time

Record round (temperature) every T seconds

Record round (temperature) every T seconds

IID Random Sequence

- Independent, Identically Distributed (IID) Random Sequence
- Independent trials of an experiment at a constant rate
- Discrete / Continuous

Theorem:

$$P_{X_{n_1}\dots X_{n_k}}(x_1,\dots,x_k) = P_X(x_1)\dots P_X(x_k) = \prod_{i=1}^{k} P_X(x_i)$$

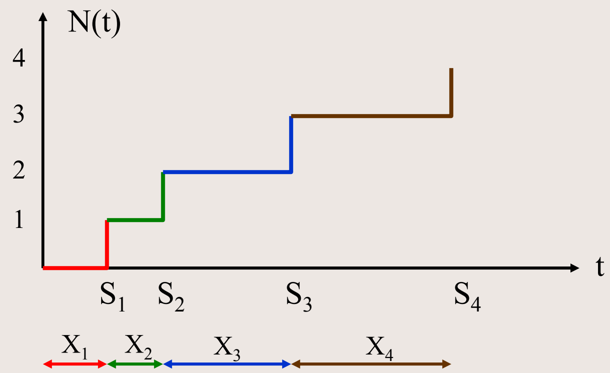
Counting Process

Definition: A Stochastic Process is a Counting Process $N(t)$ if

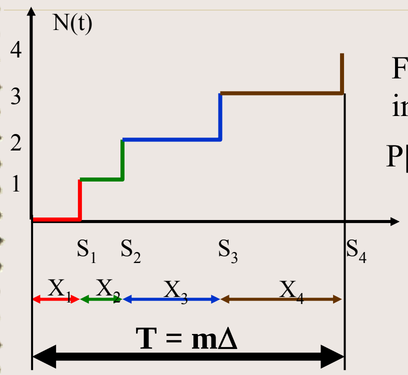
- $n(t,s) = 0$ for $t < 0$
- $n(t,s)$ = integer valued and non-decreasing

Counting Process

of customers **arrive** at (0,t]



Counting Process



For a small step Δ
in which only one may arrival,

$$P[N(t+\Delta) - N(t) = 1] = \lambda\Delta = \lambda T/m$$

Binomial PMF

$$P_{Nm}(n) = \begin{cases} \binom{m}{n}(\lambda T/m)^n(1-\lambda T/m)^{m-n} & n = 0,1,2,.. \\ 0 & \text{Otherwise} \end{cases}$$

Counting Process

Binomial Process

$$P_{Nm}(n) = \begin{cases} \binom{m}{n}(\lambda T/m)^n(1-\lambda T/m)^{m-n} & n = 0,1,2,.. \\ 0 & \text{Otherwise} \end{cases}$$

$m \rightarrow \infty$ or $\Delta \rightarrow 0$

Poisson Process

$$P_{N(T)}(n) = \begin{cases} \dfrac{(\lambda T)^n e^{-\lambda T}}{n!} & n = 0,1,2,.. \\ 0 & \text{Otherwise} \end{cases}$$

Poisson Process

- Poisson Process is

  a Counting Process that the # of **Arrival** during any interval is Poisson RV
- An arrival during any instant is **independent** of the past history of the process → **Memoryless**
- X_n is called **Interarrival Time**

Counting Process