Commu Math

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Commu math sec 1 EE(C)

2.2.4 The random variable X has PMF

$$P_X(x) = \begin{cases} c/x & x = 2, 4, 8, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[X = 4]$?

(c) What is $P[X < 4]$?

(d) What is $P[3 \le X \le 9]$?

a. c ?

$$\sum P_X(x) = 1$$

$$P_X(2) + P_X(4) + P_X(8) = 1$$

$$\frac{c}{2} + \frac{c}{4} + \frac{c}{8} = 1$$

$$\therefore \quad c = \frac{8}{7} \quad \#$$

$P_X(4) =$?

$$P_X(4) = \frac{8/7}{4}$$

$$\therefore \quad P_X(4) = \frac{2}{7} \quad \#$$

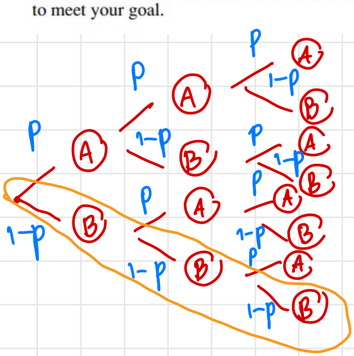
$$P[X < 4] = P_X(2) = \frac{8/7}{2}$$

$$\therefore \ P[X<4] = \frac{4}{7} \quad \#$$

$$P[3 \le X \le 9] = P_X(4) + P_X(8) = \frac{2}{7} + \frac{8/7}{8}$$

$$\therefore \ P[3 \le X \le 9] = \frac{3}{7} \quad \#$$

2.2.6 You are manager of a ticket agency that sells concert tickets. You assume that people will call three times in an attempt to buy tickets and then give up. You want to make sure that you are able to serve at least 95% of the people who want tickets. Let p be the probability that a caller gets through to your ticket agency. What is the minimum value of p necessary to meet your goal.



* ต้องให้บริการอย่างน้อย 95 %

ให้ p แทน ความน่าจะเป็นที่จะผู้โทรจะได้รับตั๋ว

ให้ A แทนรับสาย

B แทน ไม่รับสาย

* ต้องการหา p ที่น้อยที่สุด

ให้ P[รับไม่ทันทั้ง 3 ครั้ง] < 0.95

$$(1-p)^3 \le 0.05 \quad (\text{ต้องให้บริการ} > 95\%)$$

$$1 - p \le \sqrt[3]{0.05}$$

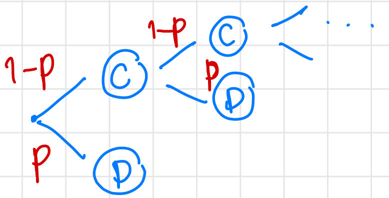
$$\therefore \ p \ge 0.6316$$

นั่นคือ ความน่าจะเป็น ที่ผู้โทรจะได้รับตั๋วอยู่ที่ 0.6316

2.3.4 Anytime a child throws a Frisbee, the child's dog catches the Frisbee with probability p, independent of whether the Frisbee is caught on any previous throw. When the dog catches the Frisbee, it runs away with the Frisbee, never to be seen again. The child continues to throw the Frisbee until the dog catches it. Let X denote the number of times the Frisbee is thrown.

(a) What is the PMF $P_X(x)$?

(b) If $p = 0.2$, what is the probability that the child will throw the Frisbee more than four times?



$P[X=1] = p$

$P[X=2] = (1-p)p$

$P[X=3] = (1-p)^2 p$

$\vdots$

$P[X=x] = (1-p)^{x-1} p$

ⓐ $P_X(x) = ?$

ให้ X แทนจำนวนครั้งที่โยนจานร่อนสำเร็จ.

C แทน สุนัขรับไม่ได้

D แทน สุนัขรับได้

$$\therefore \quad P_X(x) = \begin{cases} p(1-p)^{x-1} & , x = 1, 2, 3, \dots \\ 0 & , \text{otherwise.} \end{cases}$$

ⓑ $p = 0.2$, $X > 4$

จากข้อ ⓐ ;

$$P[1 \le X \le 4] = \sum_{x=1}^{4} P_X(x)$$

$$= (0.2)(0.8)^0 + (0.2)0.8 + (0.2)(0.8)^2 + (0.2)(0.8)^3$$

; $P[1 \le X \le 4] = 0.5904$

จาก Definition $\sum_{x \in S_X} P_X(x) = 1$;

$P[1 \le X \le 4] + P[X > 4] = 1$

; $P[X>4] = 1 - P[1 \le X \le 4]$

$\therefore \quad P[X > 4] = 0.4096$

2.3.8 In a wireless automatic meter reading system, a base station sends out a wake-up signal to nearby electric meters. On hearing the wake-up signal, a meter transmits a message indicating the electric usage. Each message is repeated eight times.

(a) If a single transmission of a message is successful with probability p, what is the PMF of N, the number of successful message transmissions?

Binomial

ให้ N แทน จำนวนของความสำเร็จในการส่งข้อความ 8 ครั้ง

(a)

จาก Definition ของ Binomial Random Variable

$$P_N(n) = \begin{cases} \binom{8}{n} p^n (1-p)^{8-n}, & n = 0, 1, 2, \ldots, 8 \\ 0, & \text{otherwise} \end{cases}$$

#

(b) I is an indicator random variable such that $I = 1$ if at least one message is transmitted successfully; otherwise $I = 0$. Find the PMF of I.

$I = 1$ เมื่อ $n = 1, 2, 3, \ldots, 8$

$P[1 \leq N \leq 8]$

กรณีที่ $I = 1$; $P_I(1) = 1 - P_N(0)$

$$= 1 - \binom{8}{0} p^0 (1-p)^8$$

$$\therefore P_I(1) = 1 - (1-p)^8$$

กรณีที่ $I = 0$ เมื่อ $n = 0$;

$$P_I(0) = P_N(0)$$

$$\therefore P_I(0) = (1-p)^8$$

ดังนั้น

$$P_I(i) = \begin{cases} (1-p)^8, & i = 1 \\ 1-(1-p)^8, & i = 0 \\ 0, & \text{otherwise} \end{cases}$$

#

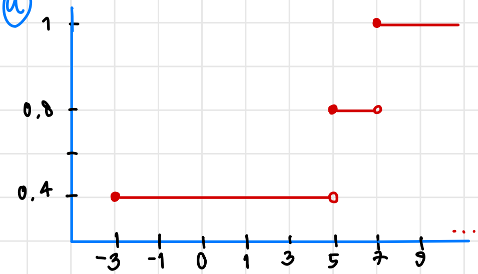
2.4.3 The random variable X has CDF

$$F_X(x) = \begin{cases} 0 & x < -3, \\ 0.4 & -3 \le x < 5, \\ 0.8 & 5 \le x < 7, \\ 1 & x \ge 7. \end{cases}$$

(a) Draw a graph of the CDF.

(b) Write $P_X(x)$, the PMF of X.

ⓐ



ⓑ PMF of X

$$P_X(x) = \begin{cases} 0.4 & , x = -3 \\ 0.4 & , x = 5 \\ 0.2 & , x = 7 \\ 0 & , \text{otherwise} \end{cases}$$

2.5.2 Voice calls cost 20 cents each and data calls cost 30 cents each. C is the cost of one telephone call. The probability that a call is a voice call is $P[V] = 0.6$. The probability of a data call is $P[D] = 0.4$.

(a) Find $P_C(c)$, the PMF of C.

(b) What is $E[C]$, the expected value of C?

Voice call 20 cents/times

Data call 30 cents/times.

Let C is the cost of one telphone call.

, V is voice calls , $P[V] = 0.6$

; D is Data calls. , $P[D] = 0.4$

ⓐ

$$P_C(C) = \begin{cases} 0.6 & , C = 20 \\ 0.4 & , C = 30 \\ 0 & , \text{otherwise} \end{cases}$$

ⓑ Expected Value

$$E[C] = \sum_{c \in S_C} c P_C(c)$$

$$= 20(0.6) + 30(0.4)$$

$\therefore E[C] = 24$ cents

2.5.9 Suppose you go to a casino with exactly \$63. At this casino, the only game is roulette and the only bets allowed are red and green. In addition, the wheel is fair so that $P[\text{red}] = P[\text{green}] = 1/2$. You have the following strategy: First, you bet \$1. If you win the bet, you quit and leave the casino with \$64. If you lose, you then bet \$2. If you win, you quit and go home. If you lose, you bet \$4. In fact, whenever you lose, you double your bet until either you win a bet or you lose all of your money. However, as soon as you win a bet, you quit and go home. Let Y equal the amount of money that you take home. Find $P_Y(y)$ and $E[Y]$. Would you like to play this game every day?

Wheel is fair,

let R = red

G = green

; $P[R] = P[G] = 0.5$

let Y = the amount of money that you take home.

ชนะ:จบ เท่าเงิน bet

$$P_Y(y) = \begin{cases} 0.5^6 & , y=0 \\ 1-(0.5)^6 & , y=64 \\ 0 & , \text{otherwise} \end{cases}$$

$$E[Y] = \sum_{y=0}^{64} y P_Y(y)$$

$$= 0(0.5)^6 + 64(1-0.5^6)$$

$$\therefore E[Y] = 63 \ \$$$

2.6.6 Suppose that a cellular phone costs \$20 per month with 30 minutes of use included and that each additional minute of use costs \$0.50. If the number of minutes you use in a month is a geometric random variable M with expected value of $E[M] = 1/p = 30$ minutes, what is the PMF of C, the cost of the phone for one month?

ให้ C แทนค่าโทรศัพท์สำหรับ 1 เดือน

M แทน เวลาที่ใช้ในการโทรศัพท์

$$P_M(m) = \begin{cases} p(1-p)^{m-1} & , m = 1,2,3,\ldots \\ 0 & , \text{otherwise} \end{cases}$$

$$P_C(20) = P_M[M \leq 30]$$

$$= \sum_{m=1}^{30} p(1-p)^{m-1}$$

$$\therefore P_C(20) = 1-(1-p)^{30}$$

ถ้า $M \geq 30$, $C = 20 + \frac{(M-30)}{2} \rightarrow M = 2C-10$;

$$P_C(c) = P_M(2c-10) \quad , c = 20.5, 21, 21.5, \ldots$$

$\therefore$

$$P_C(c) = \begin{cases} 1-(1-p)^{30} & , c = 20 \\ (1-p)^{2c-10-1} p & , c = 20.5, 21, 21.5, \ldots \\ 0 & , \text{otherwise} \end{cases}$$

2.7.7 A particular circuit works if all 10 of its component devices work. Each circuit is tested before leaving the factory. Each working circuit can be sold for k dollars, but each nonworking circuit is worthless and must be thrown away. Each circuit can be built with either ordinary devices or ultrareliable devices. An ordinary device has a failure probability of $q = 0.1$ while an ultrareliable device has a failure probability of $q/2$, independent of any other device. However, each ordinary device costs \$1 while an ultrareliable device costs \$3. Should you build your circuit with ordinary devices or ultrareliable devices in order to maximize your expected profit $E[R]$? Keep in mind that your answer will depend on k.

- วงจรจะทำงาน เมื่ออุปกรณ์ 10 ตัวทำงาน
- วงจรแต่ละวงจร ขายได้ K Dollar
- อุปกรณ์ธรรมดา มีค่า $q = 0.1$
- อุปกรณ์ที่เชื่อถือได้ มีค่า $q = 0.05$
- อุปกรณ์ธรรมดามีราคา \$1
- อุปกรณ์ที่เชื่อถือได้ มีราคา \$3

ให้ A แทน วงจรที่ทำงาน

$$E[R] = P[A]E[R|A] + P[A']E[R|A']$$

Case 1 Standard Divice $(q=0.1)$

work $P[A] = (1-q)^{10}$, $E[R|A] = k-10$

non-work $P[A'] = 1-(1-q)^{10}$, $P[R|A'] = -10$

$$; \quad E_1[R] = (1-q)^{10}(k-10) + (1-(1-q)^{10})(-10)$$

$$\therefore E_1[R] = (0.9)^{10}k - 10 \quad \text{—(1)}$$

Case 2 Ultra reliable device $(q = 0.05)$

$P[A] = (1-q)^{10}$, $E[R|A] = k-30$

$P[A'] = 1-(1-q)^{10}$, $E[R|A'] = -30$

$$; \quad E_2[R] = (1-q)^{10}(k-10) + (1-(1-q)^{10})(-30)$$

$$\therefore E_2 = (0.95)^{10}k - 30 \quad \text{—(2)}$$

สมมติการขายด้วย Standard device ได้กำไรสูงกว่า Ultra device ;

$$E_1[R] \geq E_2[R]$$

$$(0.9)^{10}k - 10 \geq (0.95)^{10}k - 30$$

$$(0.95)^{10}k - (0.9)^{10}k \leq 20$$

$$k \leq 79.981 \ \$$$

หรือ $E_1[R] \leq E_2[R]$

$$; \quad k \geq 79.981 \ \$$$

ดังนั้น ราคา สูงสุด คือ 79.981 \$ ถ้าเลือกขายด้วย Standard device
ราคา ต่ำสุด คือ 79.981 \$ สำหรับขายด้วย Ultra device.

2.8.1 In an experiment to monitor two calls, the PMF of N, the number of voice calls, is

$$P_N(n) = \begin{cases} 0.2 & n = 0, \\ 0.7 & n = 1, \\ 0.1 & n = 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $E[N]$, the expected number of voice calls.

(b) Find $E[N^2]$, the second moment of N.

(c) Find $\mathrm{Var}[N]$, the variance of N.

(d) Find σ_N, the standard deviation of N.

ⓐ $E[N] = \sum_{n \in S_N} n P_N(n)$

$= 0(0.2) + 1(0.7) + 2(0.1)$

$\therefore E[N] = 0.9$ #

ⓑ $E[N^2] = \sum_{n \in S_N} n^2 P_N(n)$

$= 0^2(0.2) + 1^2(0.7) + 2^2(0.1)$

$\therefore E[N^2] = 1.1$ #

ⓒ $\mathrm{Var}[N] = E[X^2] - (E[X])^2$

$= 1.1 - 0.9^2$

$\therefore \mathrm{Var}[N] = 0.29$ #

ⓓ $\sigma = \sqrt{\mathrm{Var}[X]}$

$= \sqrt{0.29}$

$\therefore \sigma = 0.53851$ #

2.9.3 In Problem 2.4.3, find $P_{X|B}(x)$, where the condition $B = \{X > 0\}$. What are $E[X|B]$ and $\mathrm{Var}[X|B]$?

$$P_X = \begin{cases} 0.4 & , x = -3 \\ 0.4 & , x = 5 \\ 0.2 & , x = 7 \\ 0 & , \text{otherwise} \end{cases}$$

$P[B] = P_X(5) + P_X(7) = 0.6$

$$P_{X|B}(x) = \begin{cases} \dfrac{P_X(x)}{P[B]} & , x \in B \\ 0 & , \text{otherwise} \end{cases}$$

$$= \begin{cases} 2/3 & , x = 5 \\ 1/3 & , x = 7 \\ 0 & , \text{otherwise} \end{cases}$$ #

$E[X|B] = \sum_{x|B \in S_B} x P_{X|B}(x)$

$= 5\left(\frac{2}{3}\right) + 7\left(\frac{1}{3}\right)$

$= \frac{10}{3} + \frac{7}{3}$

$\therefore E[X|B] = \frac{17}{3}$ #

$\mathrm{Var}[X|B] = E[X^2|B] - (E[X|B])^2$

$= \frac{50}{3} + \frac{49}{3} - \frac{289}{9} \Rightarrow \therefore \mathrm{Var}[X|B] = \frac{8}{9}$ #

2.9.7 Every day you consider going jogging. Before each mile, including the first, you will quit with probability q, independent of the number of miles you have already run. However, you are sufficiently decisive that you never run a fraction of a mile. Also, we say you have run a marathon whenever you run at least 26 miles.

(a) Let M equal the number of miles that you run on an arbitrary day. What is $P[M > 0]$? Find the PMF $P_M(m)$.

(b) Let r be the probability that you run a marathon on an arbitrary day. Find r.

(c) Let J be the number of days in one year (not a leap year) in which you run a marathon. Find the PMF $P_J(j)$. This answer may be expressed in terms of r found in part (b).

(d) Define $K = M - 26$. Let A be the event that you have run a marathon. Find $P_{K|A}(k)$.

- Geometric RV
- Marathon when run at least 26 miles.
- Prob of quit is q

ให้ M แทน จำนวนไมล์ในการวิ่งของแต่ละวัน.

(a)

$$P_M(m) = \begin{cases} q(1-q)^m & , m = 0, 1, 2, \dots \\ 0 & , \text{otherwise} \end{cases}$$

$$P[M>0] = 1 - P_M(0)$$

$$= 1 - q(1-q)^0$$

$$\therefore P[M>0] = 1 - q$$ ※

(b) ให้ r แทน Prob ที่คุณวิ่งมาราธอนในแต่ละวัน หา r

$$r = P[M \geq 26] = \sum_{m=26}^{\infty} q(1-q)^m$$

$$= q \sum_{n=0}^{\infty} q(1-q)^{n+26} \times \frac{(1-q)^{26}}{(1-q)^{26}}$$

$$= q(1-q)^{26} \sum_{n=0}^{\infty} (1-q)^n$$

$$= \frac{q(1-q)^{26}}{1-(1-q)}$$

$$\therefore r = (1-q)^{26}$$ ※

(c) Binomial

ให้ J แทน จำนวนวันใน 1 ปีที่คุณวิ่งมาราธอน, $1-r$ คือวันที่ไม่ได้วิ่งมาราธอน.

$$P_J(j) = \begin{cases} \binom{365}{j} r (1-r)^{365-j} & , j = 0, 1, 2, 3, \dots 365 \\ 0 & , \text{otherwise.} \end{cases}$$ ※

d) $K = M - 26$, ให้ A แทนเหตุการที่ดูหลังจากมาราธอน

Find $P_{K|A}(k)$

$$P_{K|A}(k) = \frac{P[K=k, A]}{P[A]} = \frac{P[M=26+k]}{P[A]}$$

ซึ่ง $P[A] = \gamma$, for $k = 0, 1, 2, \ldots$

$$P_{K|A}(k) = \frac{q(1-q)^{\cancel{26}+k}}{\cancel{(1-q)^{26}}} = q(1-q)^k$$

$$\therefore \quad P_{K|A}(k) = \begin{cases} q(1-q)^k & , k = 0, 1, 2, \ldots \\ 0 & , \text{otherwise.} \end{cases}$$