# Lecture #17
# Stochastic Process (3)

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## Stationary Process

- For a random process X(t), normally,
  at $t_1$: $X(t_1)$ has pdf = $f_{X(t_1)}(x)$ [depends on $t_1$]
- For a random process X(t),
  at t1: $X(t_1)$ has pdf = $f_{X(t_1)}(x)$ [not depend on $t_1$]

**Stationary Process**

= same random variable at all time

= no statistical properties change with time

$$f_{X(t_1)}(x) = f_{X(t_1+\tau)}(x) = f_X(x)$$

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# Stationary Process

**Definition**: A stochastic process X(t) is stationary iif for all sets of time $t_1,\ldots,t_m$ and any time different $\tau$,

$$f_{X(t_1),\ldots,X(t_m)}(x_1,\ldots,x_m) = f_{X(t_1+\tau),\ldots,X(t_m+\tau)}(x_1,\ldots,x_m)$$



# Stationary Random Sequence

**Definition**: A random sequence $X_n$ is stationary iif for any finite sets of time instants $n_1,\ldots,n_m$ and any time different k,

$$f_{X(n_1),\ldots,X(n_m)}(x_1,\ldots,x_m) = f_{X(n_1+k),\ldots,X(n_m+k)}(x_1,\ldots,x_m)$$

## Stationary Process

**Theorem**: A stationary process X(t),

$$\mu_X(t) = \mu_X$$
$$R_X(t,\tau) = R_X(0,\tau) = R_X(\tau)$$
$$C_X(t,\tau) = R_X(\tau) - \mu^2{}_X = C_X(\tau)$$

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## Stationary Random Sequence

**Theorem**: A stationary random sequence $X_n$, for all m

$$E[X_m] = \mu_X$$
$$R_X[m.k] = R_X[0,k] = R_X[k]$$
$$C_X[m,k] = R_X[k] - \mu^2{}_X = C_X[k]$$

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## Wide Sense Stationary

**Definition**: X(t) is a wide sense stationary random process iff for all t,

$$E[X(t)] = \mu_X$$
$$R_X(t,\tau) = R_X(0,\tau) = R_X(\tau)$$

**Definition**: $X_n$ is a wide sense stationary random sequence iff for all n,

$$E[X_n] = \mu_X$$
$$R_X[n,k] = R_X[0,k] = R_X[k]$$

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## Wide Sense Stationary

- For every **stationary** process or sequence, it is also **wide sense stationary**.
- However, if it is a **wide sense stationary** it may or may not be stationary.

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## Example

- Let $X_n = \pm 1$ with prob = ½ (n = even)
- For n = odd

  $X_n = -1/3$ with prob = 9/10

  $X_n = 3$ with prob = 1/10
- Stationary ?
  - No
- Wide sense stationary ?
  - Mean = 0 for all n
  - $C_X(t,\tau) = 0$ for $\tau > 0$
  - $C_X(t,\tau) = 1$ for $\tau = 0$
  - Yes , it's wide sense stationary

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## Wide Sense Stationary

**Theorem**: For a wide sense stationary process X(t),

$$R_X(0) \geq 0$$
$$R_X(\tau) = R_X(-\tau)$$
$$|R_X(\tau)| \leq R_X(0)$$

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## Wide Sense Stationary

**Theorem**: For a wide sense stationary sequence $X_n$,

$$R_X[0] \geq 0$$
$$R_X[k] = R_X[-k]$$
$$|R_X[k]| \leq R_X[0]$$

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## Average Power

- From Ohm's Law : V = IR
- For v(t), i(t), R Ω, the instantaneous power dissipated, P(t),

$$P(t) = v^2(t)/R = i^2(t)R$$

- For R = 1 Ω, $P(t) = v^2(t) = i^2(t)$
- For a voltage or current is a sample function of random process, x(t,s)

➔ P across 1 Ω resistor = $x^2(t,s)$

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# Average Power

- Define $x^2(t,s)$ as the instantaneous power of $x(t,s)$
- For a $X(t)$, $X^2(t)$ is the instantaneous of power $X(t)$

**Definition**:

For a wide sense stationary process $X(t)$,

$$R_X(0) = E[X^2(t)]$$

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# Homework

- 6.2.2
- 6.3.3
- 6.4.1
- 6.5.3
- 6.7.2
- 6.8.2

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