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sec.1 Comm Math (PSU)

(5) กำหนดให้ $X(t)$ เป็น Random Process ที่มี Joint pdf

$X(t_1)$ & $X(t_2)$ เมื่อ $t_1 \neq t_2$ ดังนี้

$$f_{X(t_1)X(t_2)}(x_1,x_2) = \begin{cases} c \; ; \; t_1 \le x_1 \le t_1+4 \; ; \; t_2 \le x_2 \le t_2+4 \\ 0 \; ; \; \text{otherwise} \end{cases}$$

5.1) จงหาค่า c

$$\int_{t_1}^{t_1+4}\int_{t_2}^{t_2+4} f_{X(t_1)X(t_2)}(x_1,x_2)\,dx_2\,dx_1 = 1$$

$$\int_{t_1}^{t_1+4}\int_{t_2}^{t_2+4} c\,dx_2\,dx_1 = 1$$

$$c(t_2+4-t_2)(t_1+4-t_1) = 1$$

$$\therefore c = \frac{1}{16}$$

5.2) $\mu_X(t) = E_{t_1+4}[X(t_1)]$

$$= \int_{t_1} x\frac{1}{16}\,dx_1$$

$$= \frac{1}{16}x^2\Big|_{t_1}^{t_1+4}$$

$$\frac{1}{16}\left[(t_1+4)^2 - t_1^2\right]$$

5.3 auto correlation

$$R_X(t,\tau) = E[X(t)\,X(t+\tau)]$$

$$R_X(t,\tau) = \begin{cases} E[X(t)]\,E[X(t+\tau)] & ;\ \tau \neq 0 \quad ,\tau > -t \\ E[X^2(t_1)] & ;\ \tau = 0 \end{cases}$$

$$= \begin{cases} \left(\int_{t_1}^{t_1+4} \frac{1}{16} x\, dx_1\right)\left(\int_{t_2}^{t_2+4} x \frac{1}{16}\, dx_2\right) & ;\ \tau \neq c,\ \tau > t \\ \frac{1}{16}\left[(t_1+4)^2 - t_1^2\right] & ;\ \tau = 0 \end{cases}$$

5.4 auto covariance

$$C_X(t,\tau) = R_X(t,\tau) - \mu_X(t)\,\mu_X(t+\tau)$$

$$=$$