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PSV Commu. Math

④ $$P_{X,Y}(x,y) = \begin{cases} k2^{-(x+|y|)} & , x=0,1,2,\dots ; \ y=-x,-x+2,-x+4,\dots \\ 0 & , \text{otherwise} \end{cases}$$

4.1) k

จาก Theorem : $\sum_{x\in S_X}\sum_{y\in S_y} P_{X,Y}(x,y) = 1$

จะได้ $$\sum_{x=0}^{\infty}\sum_{y\in\{x,-x+2,-x+4,\dots\}} k2^{-(x+|y|)} = 1$$

$$k\sum_{x=0}^{\infty} 2^{-x} \sum_{y\in\{-x,-x+2,-x+4,\dots\}} 2^{-|y|} = 1$$

ให้ $y=-x+2n$ ;

$$k\sum_{x=0}^{\infty} 2^{-x}\sum_{n=0}^{\infty} 2^{-|-x+2n|} = 1$$

จากเงื่อนไขของ abs

$$-|-x+2n| = \begin{cases} x-2n & ; -x+2n \geq 0 \\ & \ \ \underline{\underline{x \leq 2n}} \\ -x+2n & ; -x+2n < 0 \\ & \ \ \underline{\underline{x \geq 2n}} \end{cases}$$

$$k\sum_{x=0}^{\infty} 2^{-x}\sum_{n=0}^{\infty} 2^{x-2n} = 1$$

$$k\sum_{x=0}^{\infty} 2^{-x+x}\sum_{n=0}^{\infty} 2^{-2n} = 1$$

$$k \cancel{\sum_{x=0}^{\infty}}^{1} 1\cdot\left(\frac{1}{1-\left(\frac{1}{4}\right)}\right) = 1$$

$$\boxed{\therefore k = \frac{3}{4}}$$

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4.2) Marginal PMF $P_X(x)$

$$P_X(x) = \sum_{y} P_{X,Y}(x,y)$$

$$= \sum_{n=0}^{\infty} \frac{3}{4} \cdot (2^{-x+x-2n})$$

$$= \frac{3}{4} \sum_{n=0}^{\infty} 2^{-2n}$$

$$= \frac{3}{4} \cdot \frac{1}{1-\frac{1}{4}}$$

$\therefore P_X(x) = 1$

4.3 Marginal PMF $P_Y(y)$

$$P_Y(y) = \sum_{x} P_{X,Y}(x,y)$$

$$= \frac{3}{4} \sum_{x=0}^{\infty} 2^{-x-|y|}$$

$$= \frac{3}{4} 2^{-|y|} \left( \frac{1}{1-} \right.$$