Control Systems Engineering

Chapter 2: Modeling in the Frequency Domain

Dr.-Ing. Witthawas Pongyart

Chapter 2: Modeling in the Frequency Domain

Learning Outcomes:

- Find the Laplace transform of time functions and the inverse Laplace transform (Sections 2.1-2.2)
- Find the transfer function from a differential equation and solve the differential equation using the transfer function (Section 2.3)
- Find the transfer function for
  - linear, time-invariant electrical networks (Section 2.4)
  - linear, time-invariant translational mechanical systems
  - time-invariant rotational mechanical systems
  - gear systems with no loss and for gear systems with loss

Chapter 2: Modeling in the Frequency Domain

Learning Outcomes: (cont.)

- Find the transfer function for
  - electromechanical systems
  - Produce analogous electrical and mechanical circuits
  - Linearize a nonlinear system in order to find the transfer function

Chapter 2: Modeling in the Frequency Domain

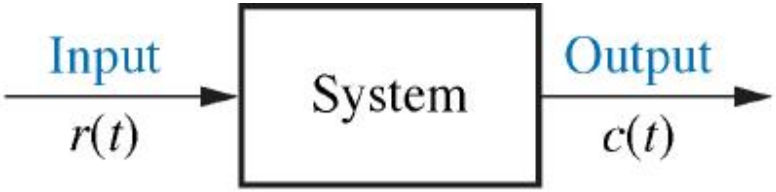
Case Study:

- Given the antenna azimuth position control system shown on the front endpapers*, you will be able to find the <u>transfer function</u> of each subsystem.*
- Given a model of a human leg or a nonlinear electrical circuit, you will be able to linearize the model and then find the transfer function.

Modeling in the Frequency Domain

What is the Transfer Function?

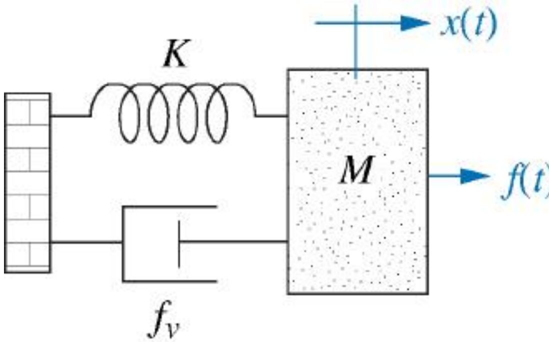
- Transfer function is the ratio of the Laplace transform of the output signal to the input signal with the initial conditions as zero.



$$\boxed{\frac{C(s)}{R(s)} = G(s)}$$

Modeling in the Frequency Domain

Transfer Function—**One Equation of Motion**



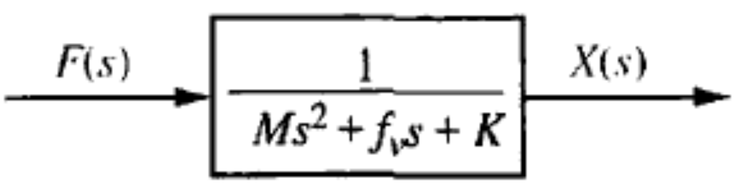
- **Example 2.11:** Find the transfer function, X(s)/F(s), for the system in the figure. (Ignore the force due to the gravity)

Solution: (cont.)

4. Taking the Laplace transform, assuming zero initial conditions
5. Solve the equation for the transfer function yields

$$\boxed{M\frac{d^2x(t)}{dt^2}+f_v\frac{dx(t)}{dt}+Kx(t)=f(t)} \Rightarrow Ms^2X(s)+f_vsX(s)+KX(s)=F(s)$$

$$(Ms^2+f_vs+K)X(s)=F(s)$$



$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{Ms^2+f_vs+K}$$

Modeling in the Frequency Domain

Concept of Mechanical Impedance

Now can we parallel our work with electrical networks by circumventing the writing of differential equations and by defining impedances for mechanical components? If so, we can apply to mechanical systems the problem-solving techniques learned in the previous section. Taking the Laplace transform of the force-displacement column in Table 2.4, we obtain for the spring,

$$\boxed{F(s) = KX(s)} \tag{2.112}$$

for the viscous damper,

$$\boxed{F(s) = f_v sX(s)} \tag{2.113}$$

and for the mass,

$$\boxed{F(s) = Ms^2X(s)} \tag{2.114}$$

If we define impedance for mechanical components as

$$\boxed{Z_M(s) = \frac{F(s)}{X(s)}} \tag{2.115}$$

Modeling in the Frequency Domain

Concept of Mechanical Impedance

If we define impedance for mechanical components as

$$\boxed{Z_M(s) = \frac{F(s)}{X(s)}} \tag{2.115}$$

and apply the definition to Eqs. (2.112) through (2.114), we arrive at the impedances of each component as summarized in Table 2.4 (*Raven, 1995*).[7]

Replacing each force in Figure 2.16(a) by its Laplace transform, which is in the format

$$\boxed{F(s) = Z_M(s)X(s)} \tag{2.116}$$

we obtain Figure 2.16(b), from which we could have obtained Eq. (2.109) immediately without writing the differential equation. From now on we use this approach.

Note:

Finally, notice that Eq. (2.110) is of the form

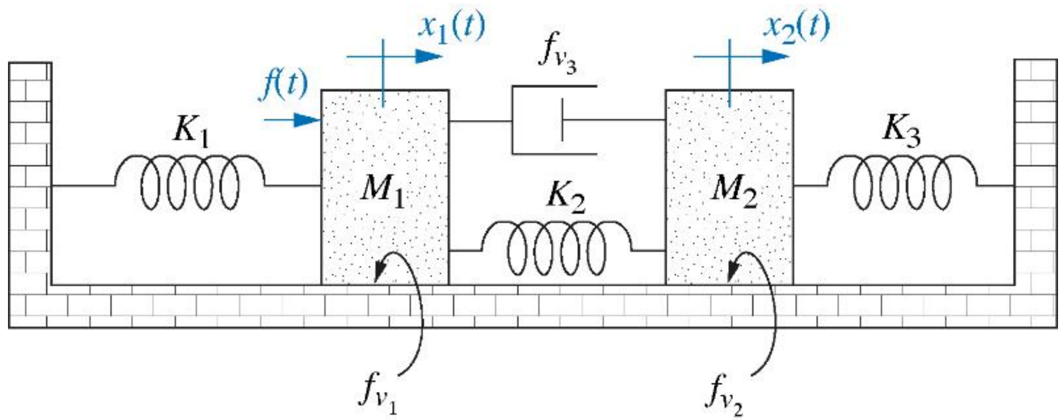
$$[\text{Sum of impedances}]X(s) = [\text{Sum of applied forces}] \tag{2.117}$$

which is similar, but not analogous, to a mesh equation (see footnote 7).

Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.



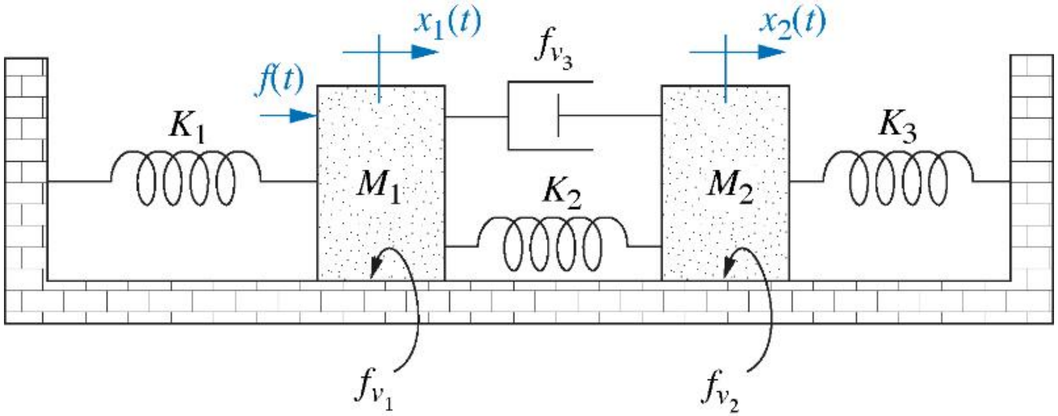
Solution:

- The system has two degrees of freedom, since each mass can be moved in the horizontal direction while the other is held still.
- Thus, two simultaneous equations of motion will be required to describe the system.

Modeling in the Frequency Domain

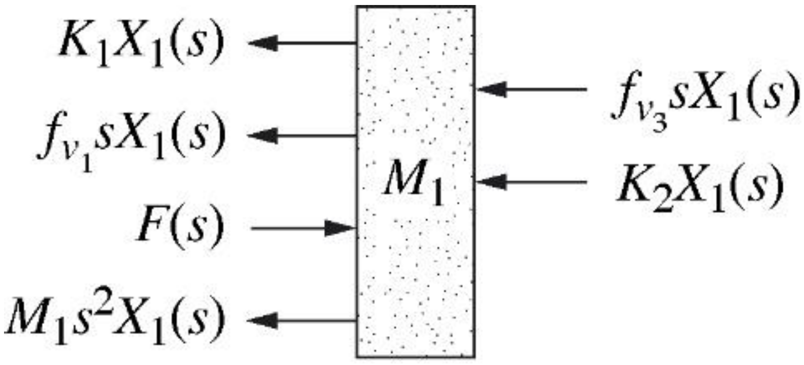
Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.

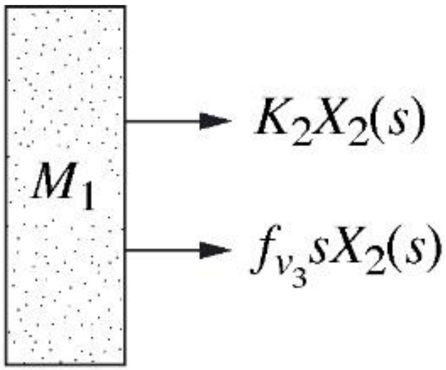


<u>Solution:</u> (For the M_1) If we hold *M_2 still and move M_1 to the right.*

Then hold *M_1 still and move M_2 to the right*



(a)



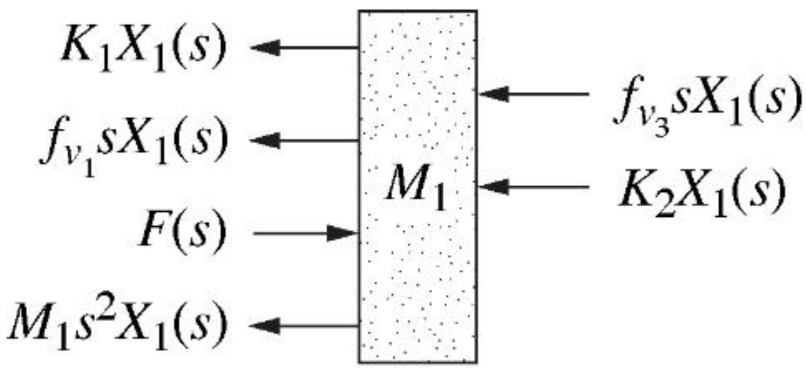
(b)

Modeling in the Frequency Domain

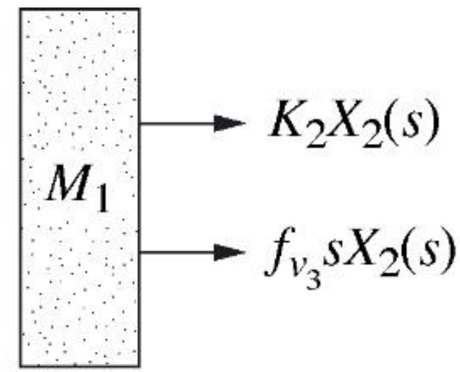
Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.

Solution: The total force on M_1 *is the superposition.*



(a)



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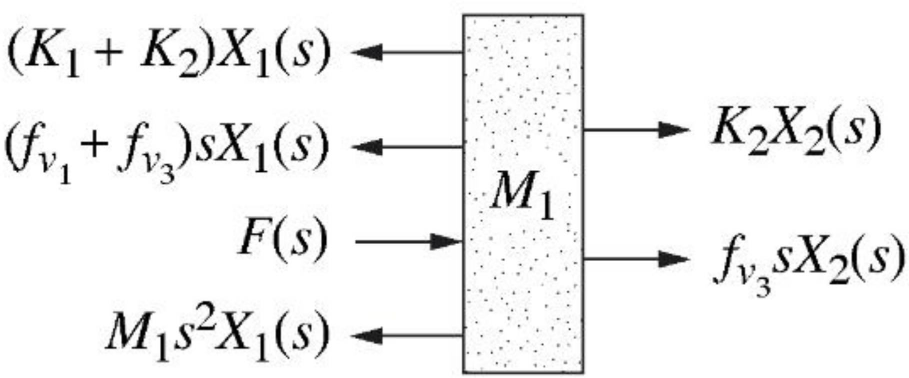

$$\boxed{[M_1s^2 + (f_{v1} + f_{v3})s + (K_1 + K_2)]X_1(s) - (f_{v3}s + K_2)X_2(s) = F(s)}$$

Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

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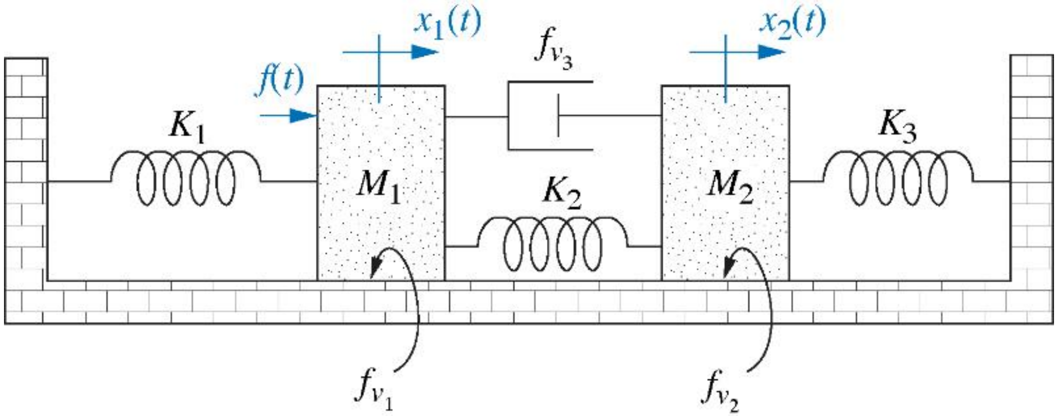
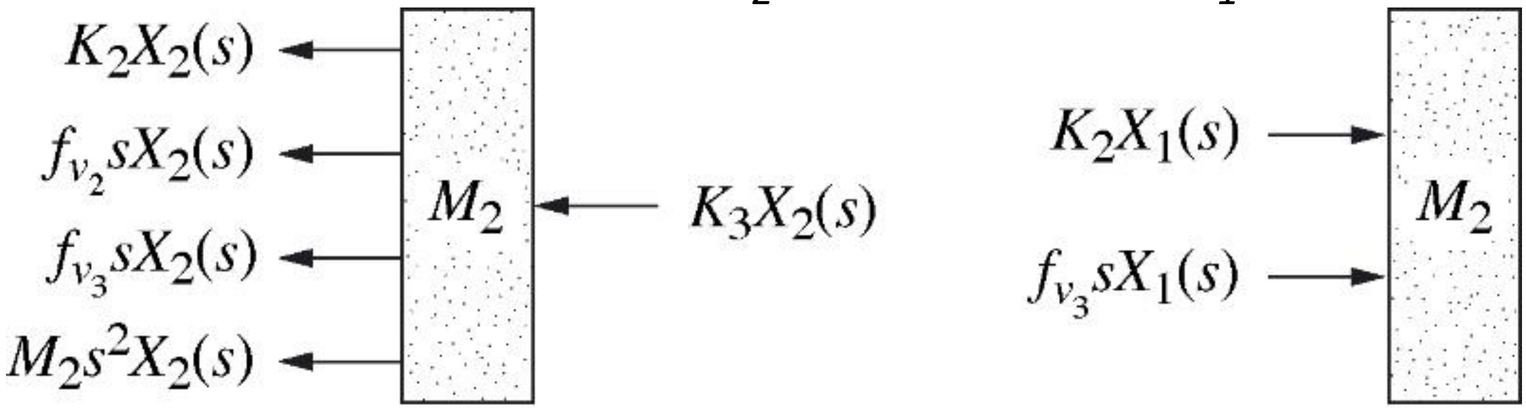


Solution: (For the M_2) If we hold *M_1 still and move M_2 to the right.*

Then hold *M_2 still and move M_1 to the right*



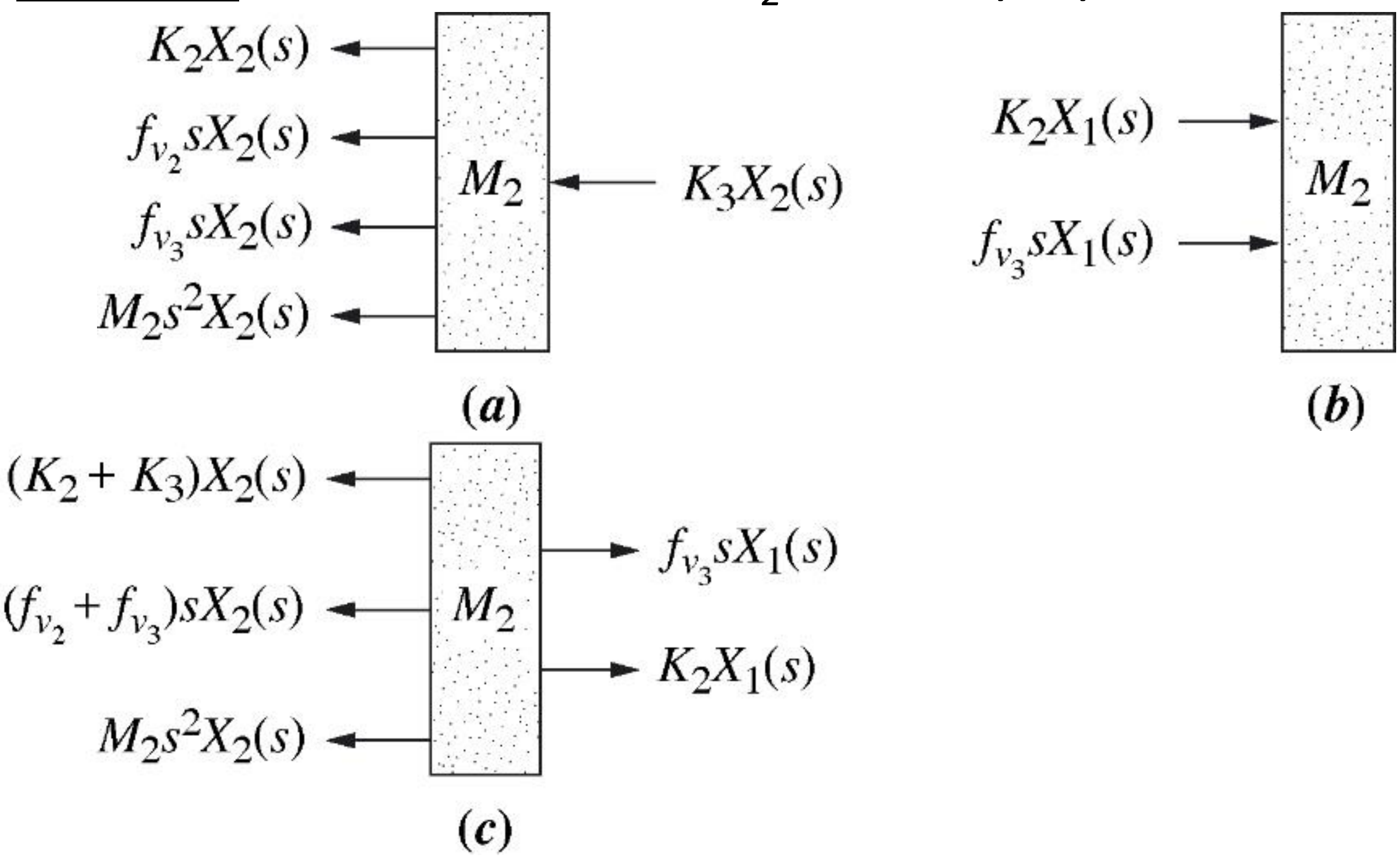
(*a*) (*b*)

Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.

<u>Solution:</u> The total force on M_2 *is the superposition.*



$$\boxed{-(f_{v_3}s+K_2)X_1(s)+\left[M_2s^2+(f_{v_2}+f_{v_3})s+(K_2+K_3)\right]X_2(s)=0}$$

Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

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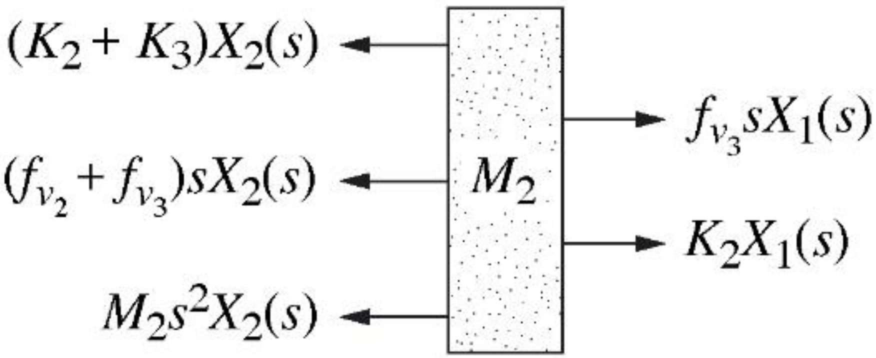
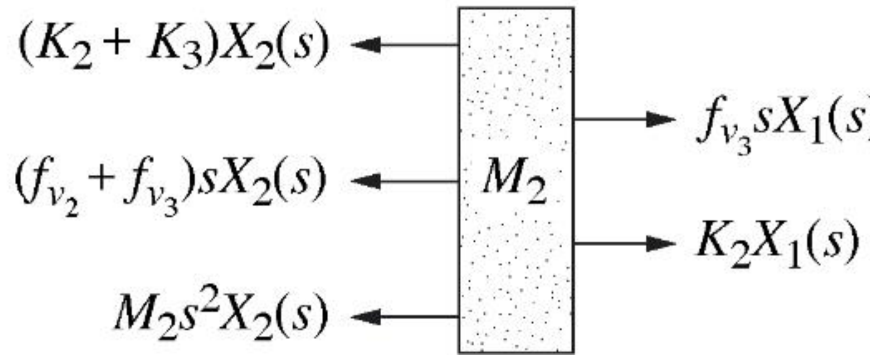
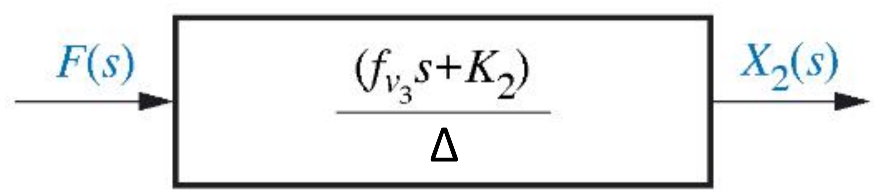




$$\boxed{[M_1s^2+(f_{v1}+f_{v3})s+(K_1+K_2)]X_1(s)-(f_{v3}s+K_2)X_2(s)=F(s)}$$

$$\boxed{-(f_{v_3}s+K_2)X_1(s)+\left[M_2s^2+(f_{v_2}+f_{v_3})s+(K_2\ +\ K_3)\right]X_2(s)=0}$$

$$\frac{X_2(s)}{F(s)}=G(s)=\frac{(f_{v_3}s+K_2)}{\Delta}$$

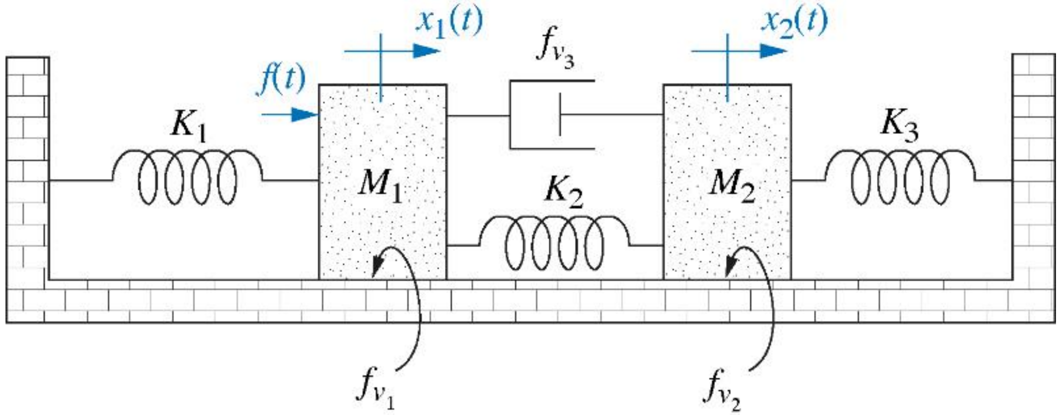


Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.

<u>Solution:</u> The total force on *Mass is the superposition.*



$$\boxed{[M_1 s^2 + (f_{v1} + f_{v3})s + (K_1 + K_2)]X_1(s) - (f_{v3}s + K_2)X_2(s) = F(s)}$$

<u>Note</u>: The form of the equations is similar to electrical mesh equations:

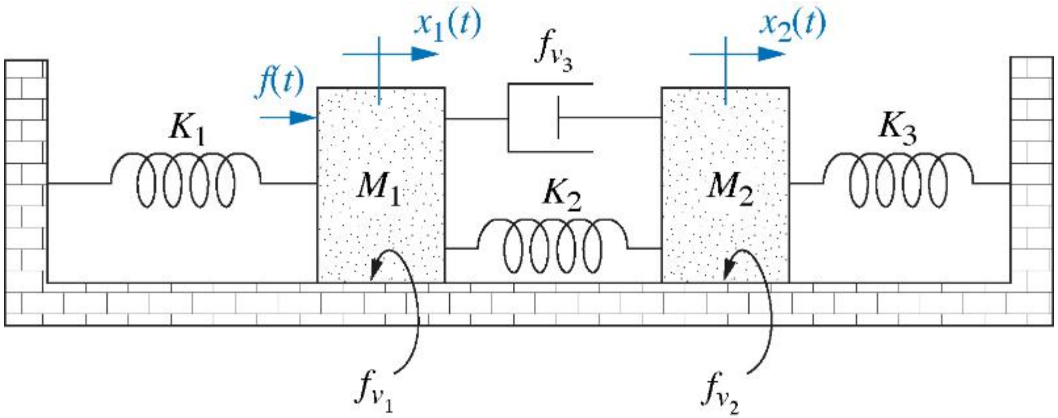
$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_1 \end{bmatrix}$$

Modeling in the Frequency Domain

Transfer Function—Two Degrees of Freedom

Example 2.17: Find the transfer function, $X_2(s)/F(s)$, for the system.

Solution: The total force on *Mass is the superposition.*



$$\boxed{-(f_{v_3}s + K_2)X_1(s) + \left[M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)\right]X_2(s) = 0}$$

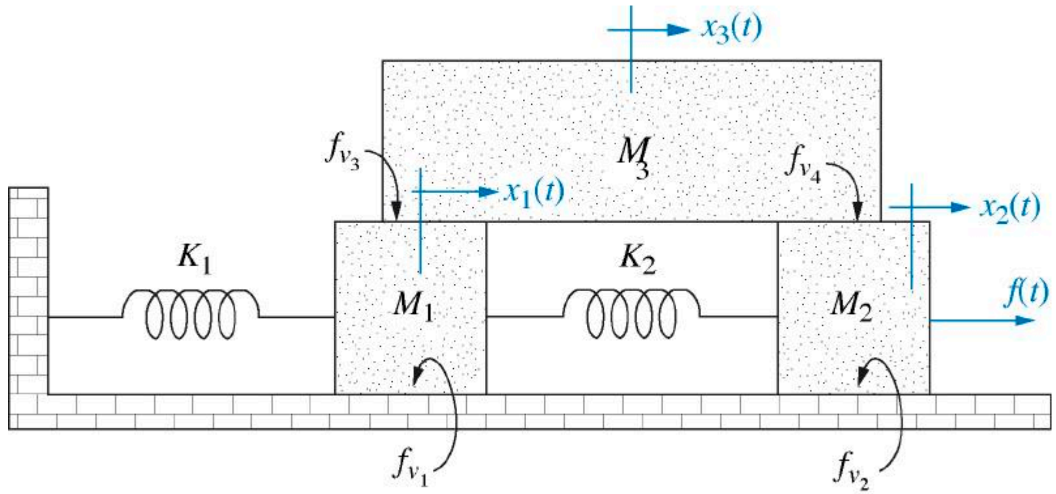
Note: The form of the equations is similar to electrical mesh equations:

$$-\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_1(s) + \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_2 \end{bmatrix}$$

Modeling in the Frequency Domain

Example 2.18 :*Equations of Motion by Inspection*

Write the equations of motion for the mechanical network.



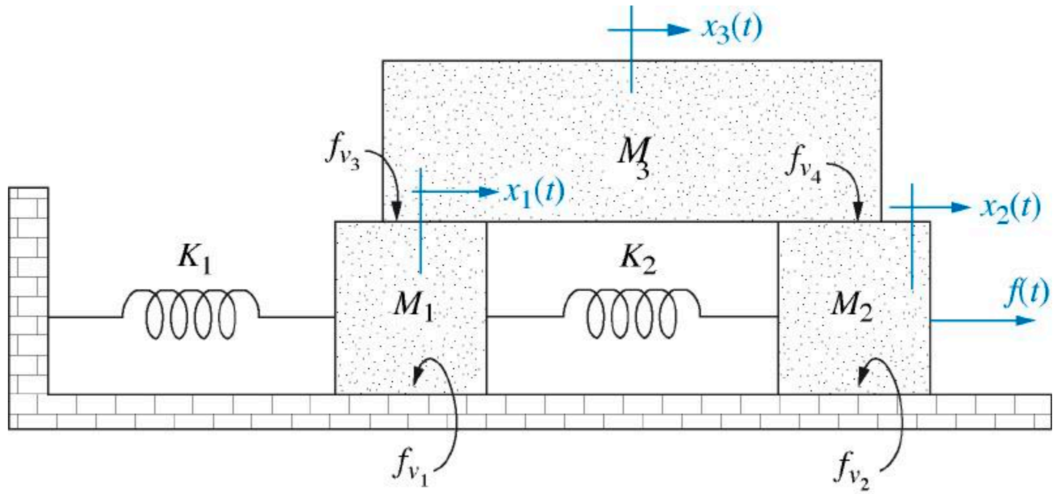
$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_2(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_3 \end{bmatrix} X_3(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_1 \end{bmatrix}$$

$$[M_1 s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)]X_1(s) - K_2 X_2(s) - f_{v_3} s X_3(s) = 0$$

Modeling in the Frequency Domain

Example 2.18 :*Equations of Motion by Inspection*

Write the equations of motion for the mechanical network.



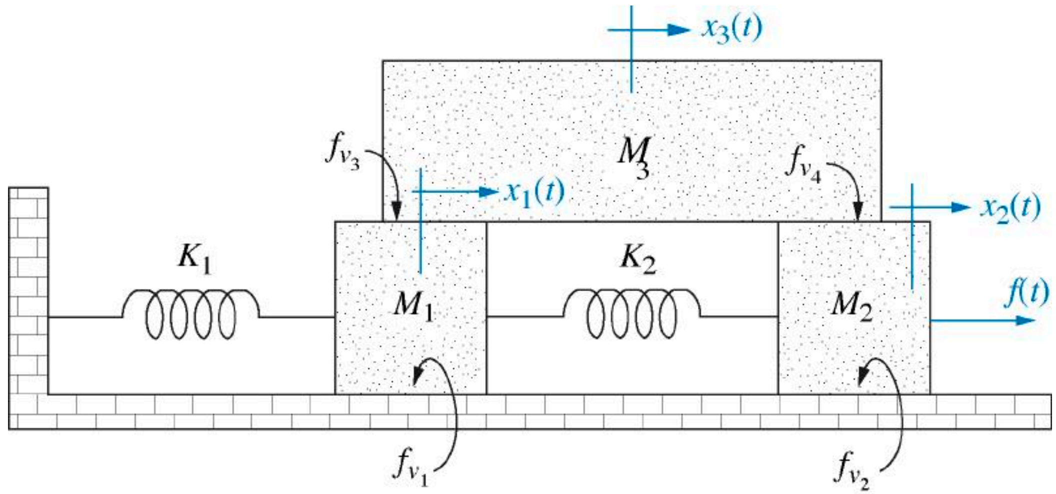
$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_2\end{bmatrix} X_2(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_2 \text{ and } x_3\end{bmatrix} X_3(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_2\end{bmatrix}$$

$$-K_2X_1(s) + [M_2s^2 + (f_{v_2} + f_{v_4})s + K_2]X_2(s) - f_{v_4}sX_3(s) = F(s)$$

Modeling in the Frequency Domain

Example 2.18 :*Equations of Motion by Inspection*

Write the equations of motion for the mechanical network.



$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_3\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_2 \text{ and } x_3\end{bmatrix} X_2(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_3\end{bmatrix} X_3(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_3\end{bmatrix}$$

$$-f_{v_3}sX_1(s) - f_{v_4}sX_2(s) + \left[M_3s^2 + (f_{v_3} + f_{v_4})s\right]X_3(s) = 0$$

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

- Rotational mechanical systems are handled the same way as translational mechanical systems, except that torque replaces force and angular displacement replaces translational displacement
- The mechanical components for rotational systems are the same as those for translational systems, except that the components undergo rotation instead of translation.

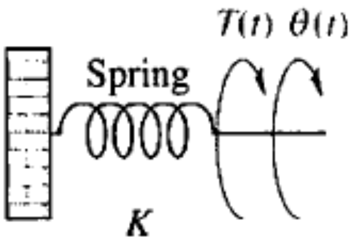
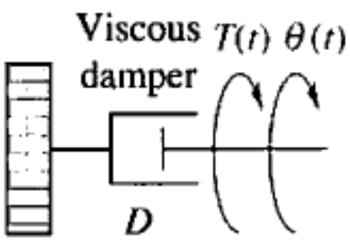
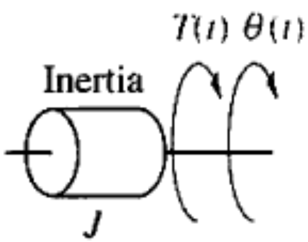
Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

- *Rotational mechanical systems are handled the same way as translational mechanical systems*, except that <u>torque replaces force and angular displacement replaces translational displacement</u>
- The mechanical components for rotational systems are the same as those for translational systems, except that the components undergo rotation instead of translation.
- Also **notice that** the term associated with the <u>mass is replaced by inertia</u>.
- Table 2.5 shows the components along with the relationships between torque and angular velocity, as well as angular displacement.

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

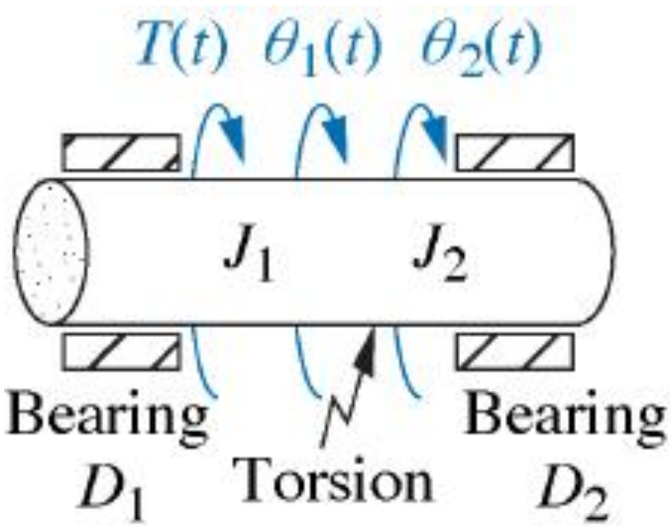
- Two examples will demonstrate the solution of rotational systems.
  - The first one uses free-body diagrams.
  - The second uses the concept of impedances to write the equations of motion by inspection.

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

- **Example 2.19 :** Find the transfer function, $\theta_2(s)/T(s)$, *for the rotational system shown* in Figure *2.22(a). The rod is supported by bearings at either end and is undergoing* torsion. A torque is applied at the left, and the displacement is measured at the right.

$$\boxed{\frac{\theta_2(s)}{T(s)} = ?}$$



(a)

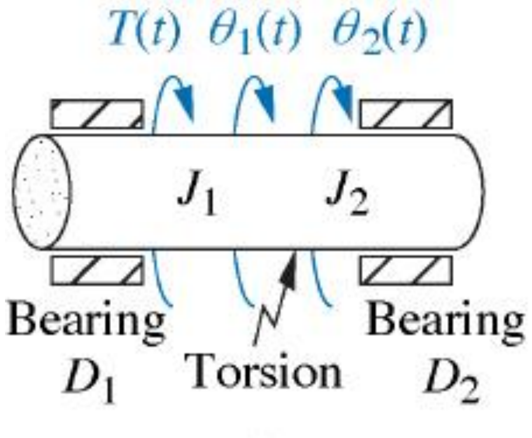
First, obtain the schematic from the physical system.

Modeling in the Frequency Domain

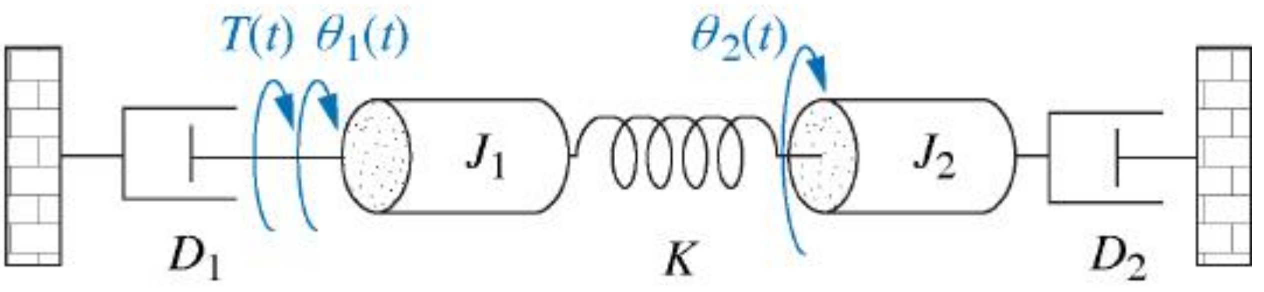
2.6 Rotational Mechanical System Transfer Functions

Example 2.19 :

- We approximate the system by assuming that the torsion acts like a spring concentrated at one particular point in the rod, with an inertia J_1 *to the left and an inertia* J_2 *to the right.*
- There are two degrees of freedom, since each inertia can be rotated while the other is held still.



(a)



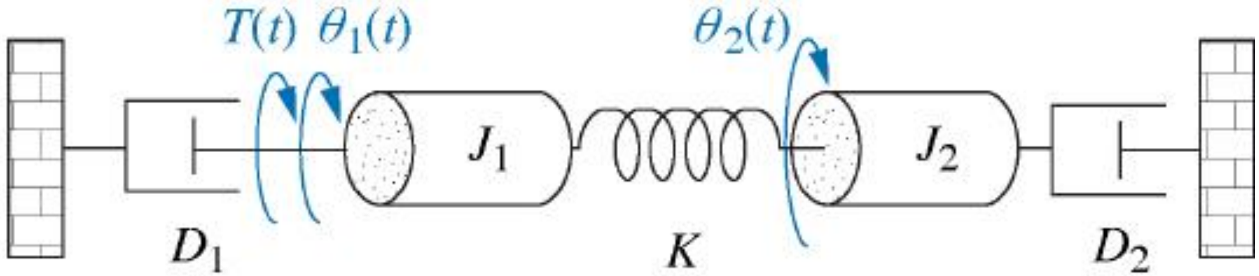
(b)

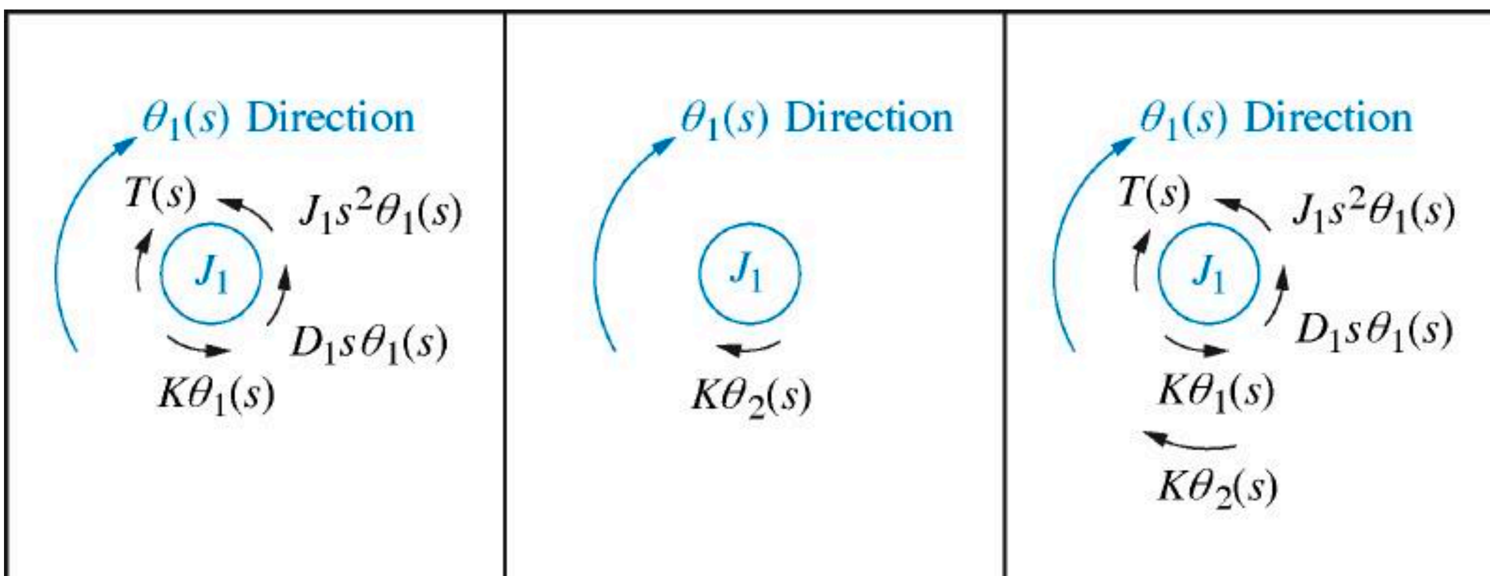
Next, draw a free-body diagram.

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Example 2.19 : Draw a free-body diagram of J_1 *and* J_2, *using superposition*.



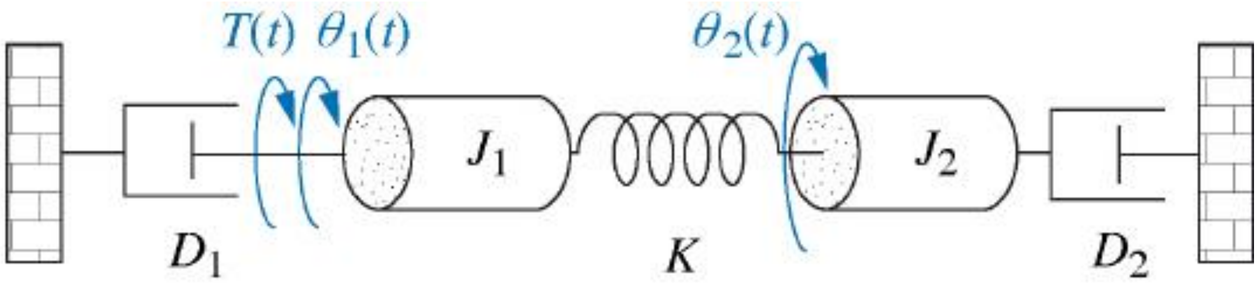


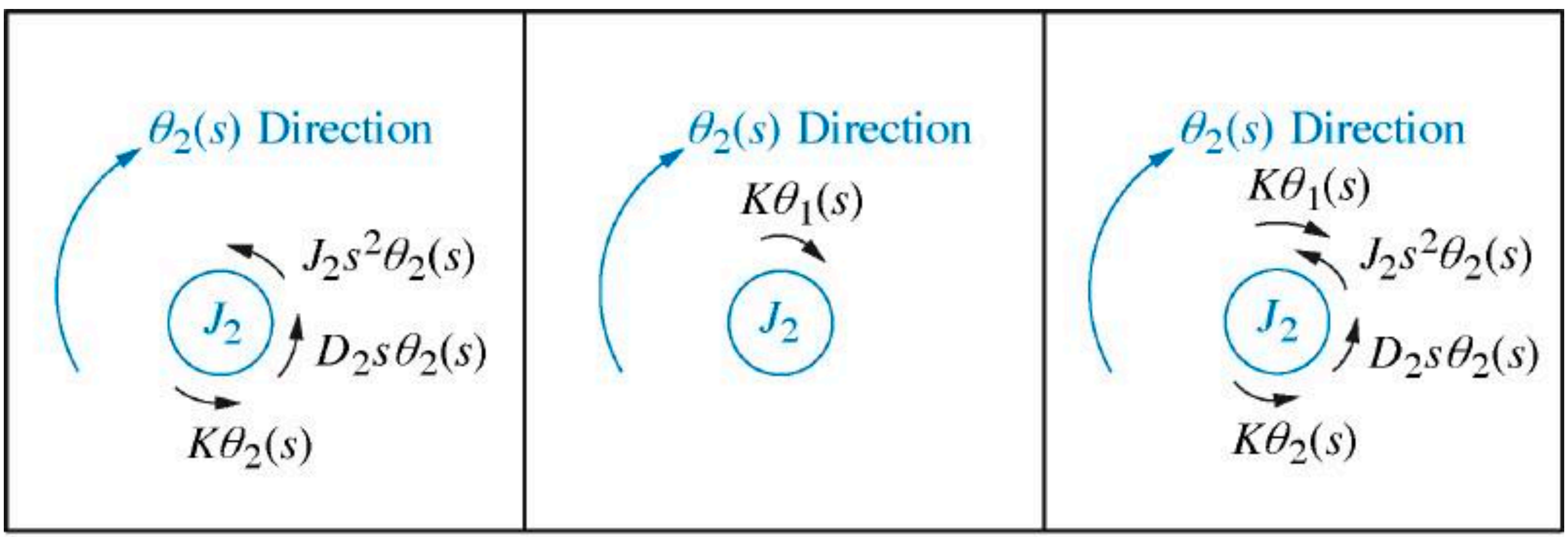
(*a*) (*b*) (*c*)

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Example 2.19 : Draw a free-body diagram of J_1 *and* J_2, <u>*using superposition*</u>.





(*a*) (*b*) (*c*)

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Example 2.19 : Summing torques respectively from Figures 2.23(c) and 2.24(c) we obtain the equations of motion

$$(J_1 s^2 + D_1 s + K)\theta_1(s) \qquad - K\theta_2(s) = T(s)$$

$$-K\theta_1(s) + (J_2 s^2 + D_2 s + K)\theta_2(s) = 0$$

$$\frac{\theta_2(s)}{T(s)} = \frac{K}{\Delta}$$

$$\Delta = \begin{vmatrix} (J_1 s^2 + D_1 s + K) & -K \\ -K & (J_2 s^2 + D_2 s + K) \end{vmatrix}$$

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Example 2.19 :

Notice that Eq. (2.127) have that now well-known form

$$(J_1s^2 + D_1s + K)\theta_1(s) \qquad - K\theta_2(s) = T(s)$$

$$-K\theta_1(s) + (J_2s^2 + D_2s + K)\theta_2(s) = 0$$

$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } \theta_1 \end{bmatrix} \theta_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ \theta_1 \text{ and } \theta_2 \end{bmatrix} \theta_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied torques} \\ \text{at } \theta_1 \end{bmatrix}$$

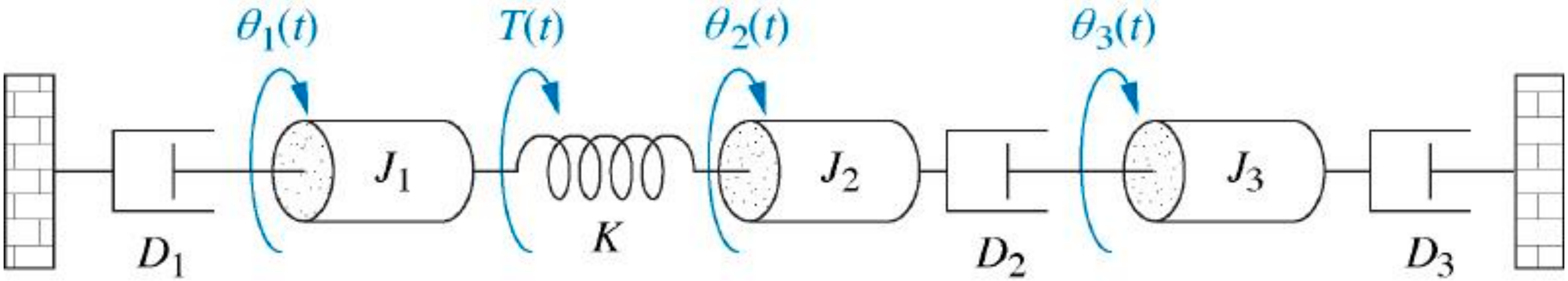
$$-\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ \theta_1 \text{ and } \theta_2 \end{bmatrix} \theta_1(s) + \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } \theta_2 \end{bmatrix} \theta_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied torques} \\ \text{at } \theta_2 \end{bmatrix}$$

Modeling in the Frequency Domain

2.6 Rotational Mechanical System Transfer Functions

Example 2.20 : Equations of Motion By Inspection

- Write, but do not solve, the Laplace transform of the equations of motion for the system

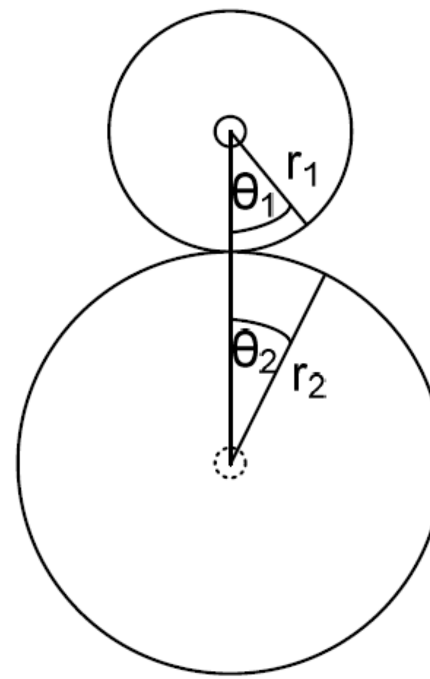


Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

- This section covers the rotational mechanical system with gear. In this section, we idealize the behavior of gears and assume that there is no backlash. As the gears turn, the distance traveled along each gear's circumference is the same.

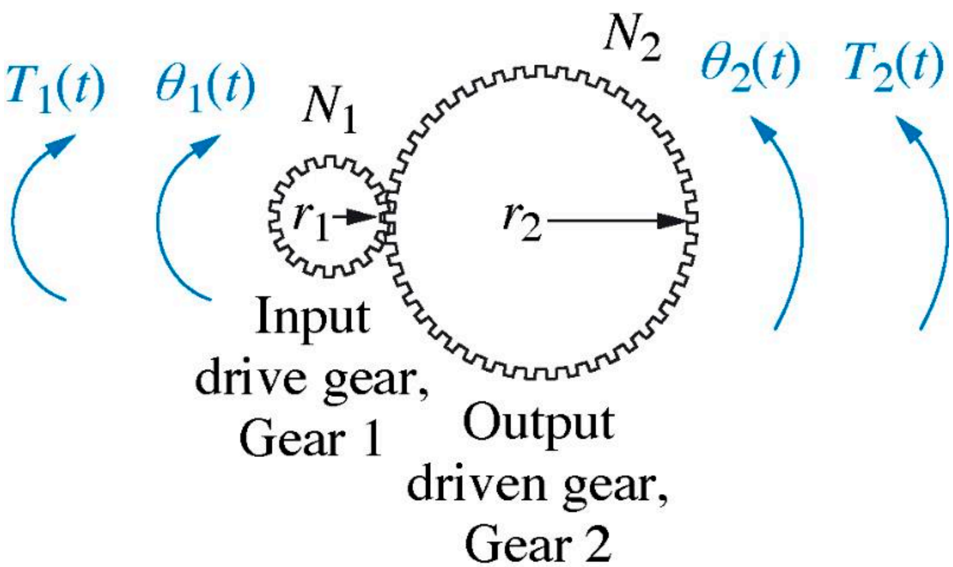
$$r_1\theta_1 = r_2\theta_2$$



Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

- Since <u>the ratio of the number of teeth</u> along the circumference is in the same proportion as the ratio of the radii.

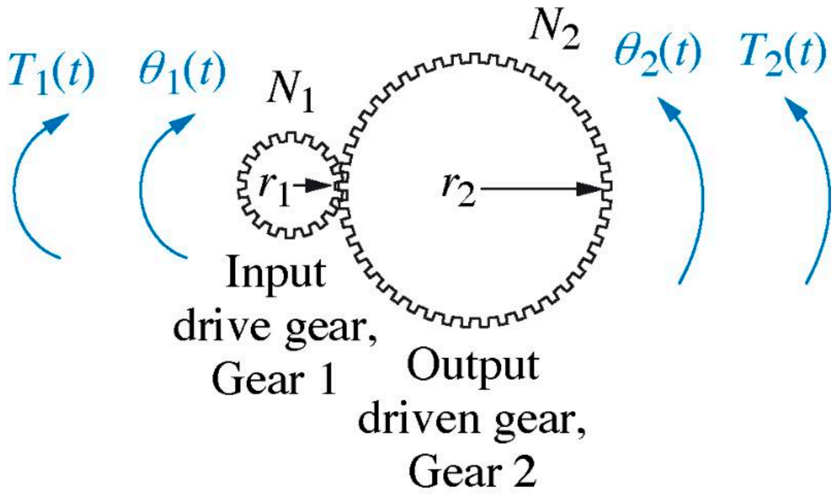


$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

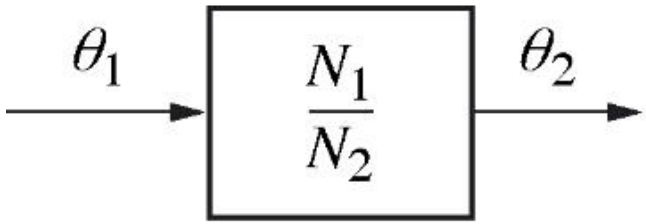
- Assume the gears are *lossless.* -> Energy into Gear 1 equals the energy out of Gear.

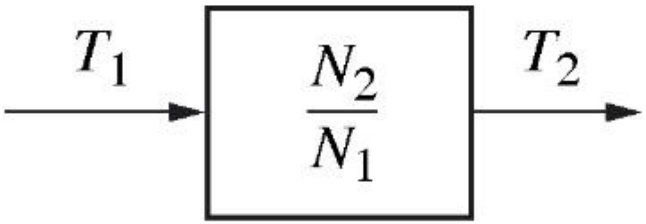


$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$T_1\theta_1 = T_2\theta_2$$

$$\boxed{\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}}$$

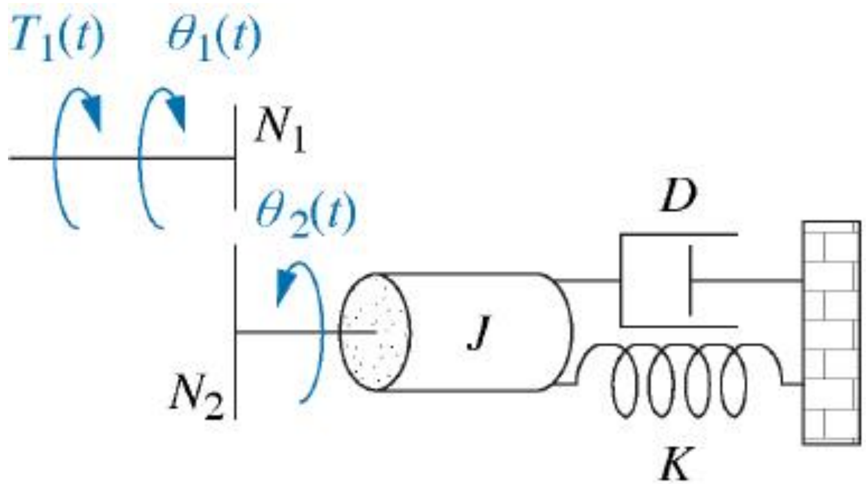




Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

- Let us see what happens to mechanical impedances that are driven by gears.
- We want to represent Figure 2.29(a) as an *equivalent system* at θ_1 without the gears.



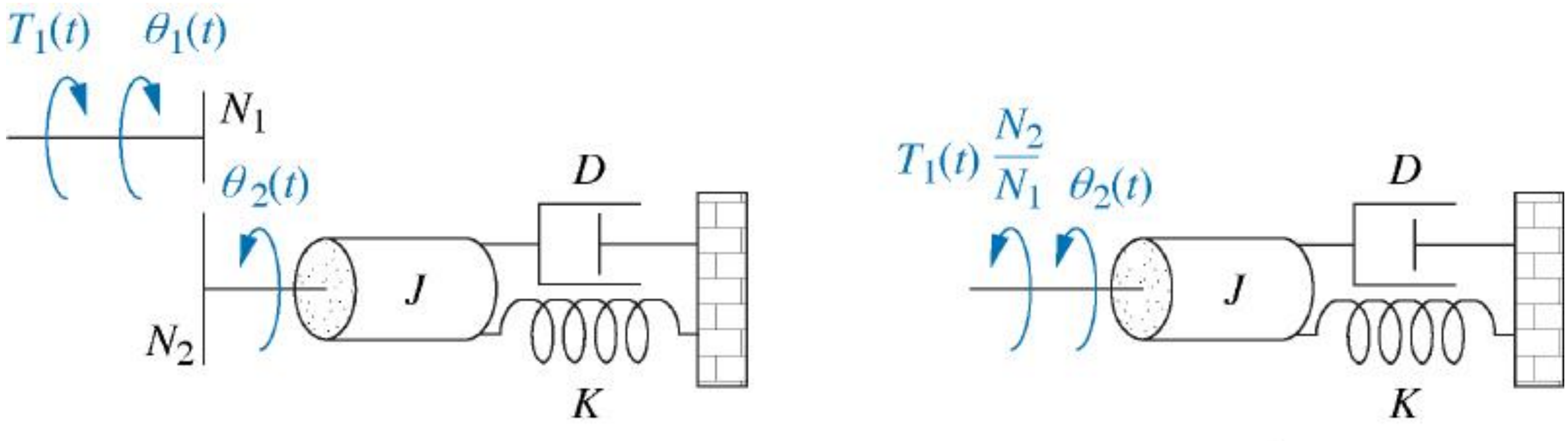
(a)

Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

- In other words, can the mechanical impedances be reflected from the output to the input, thereby eliminating the gears?
- *T_1 can be reflected to the output by multiplying by N_2/N_1.*

$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$



(a) (b)

Modeling in the Frequency Domain

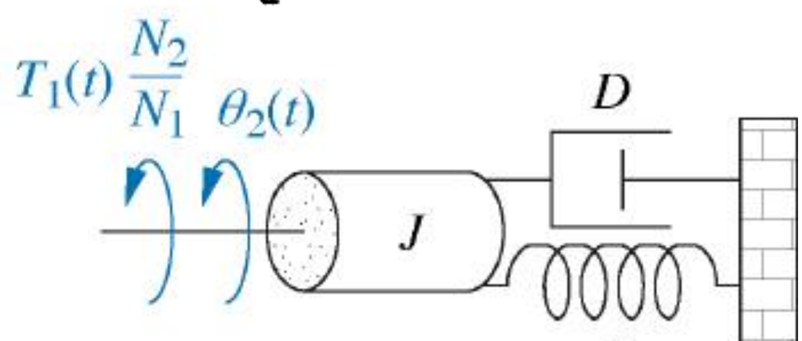
2.7 Transfer Functions for Systems with Gears

- Now convert $\theta_2(s)$ *into an equivalent* $\theta_1(s)$

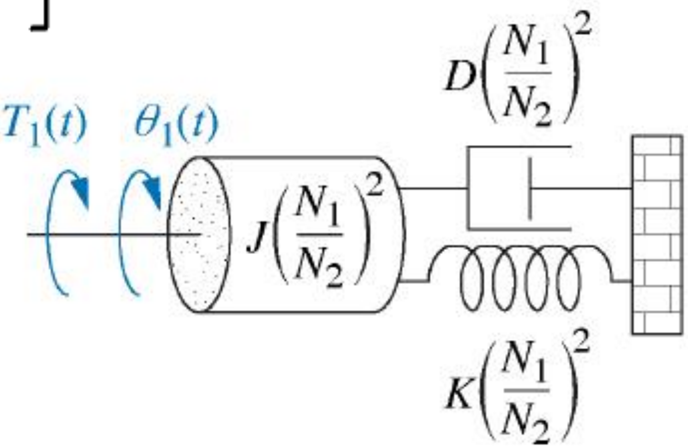
$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right]\theta_1(s) = T_1(s)$$



(b)



(c)

Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

Generalizing the results, we can make the following statement: *Rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio*

$$\boxed{\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}}\right)^2}$$

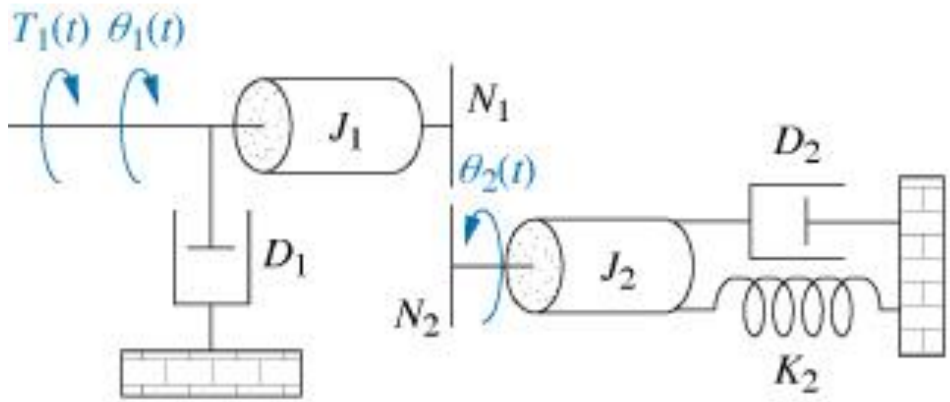
where the impedance to be reflected is attached to the source shaft and is being reflected to the destination shaft. The next example demonstrates the application of the concept of reflected impedances as we find the transfer function of a rotational mechanical system with gears.

Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

Example 2.21:System with Lossless Gears

- Find the transfer function, $\theta_2(s)/T_1(s)$, for the system.



- The inertias, however, do not undergo linearly independent motion, since they are tied together by the gears.
Thus, there is only one degree of freedom and hence one equation of motion

Modeling in the Frequency Domain

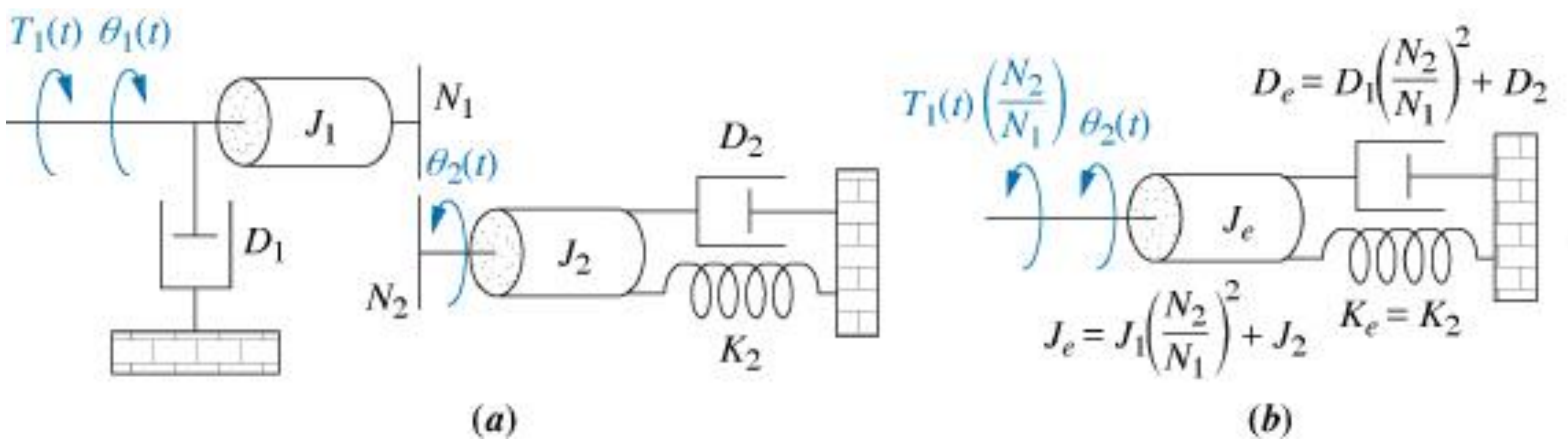
2.7 Transfer Functions for Systems with Gears

Example 2.21: Find the transfer function, $\theta_2(s)/T_1(s)$.

- Let us first reflect the impedances *(J_1 and D_1) and torque (T_1) on the input* shaft to the output

$$(J_e s^2 + D_e s + K_e)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$J_e = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2; \quad D_e = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2; \quad K_e = K_2$$



Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

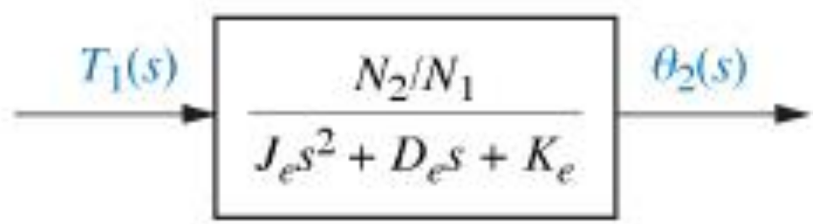
Example 2.21: Find the transfer function, $\theta_2(s)/T_1(s)$.

- Let us first reflect the impedances *(J_1 and D_1) and torque (T_1) on the input* shaft to the output

$$(J_e s^2 + D_e s + K_e)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

Solving for $\theta_2(s)/T_1(s)$, the transfer function is found to be

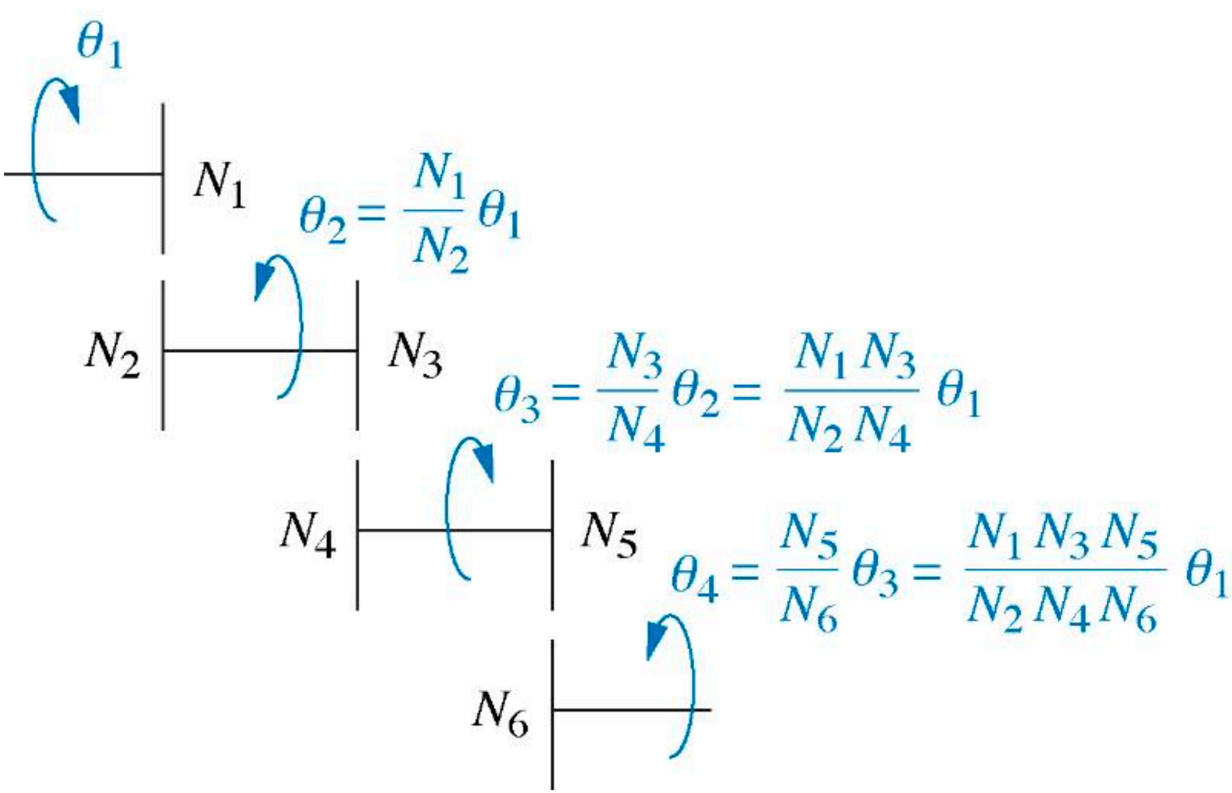
$$G(s) = \frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{J_e s^2 + D_e s + K_e}$$



Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

- For *gear trains*, we conclude that the equivalent gear ratio is the product of the individual gear ratios.



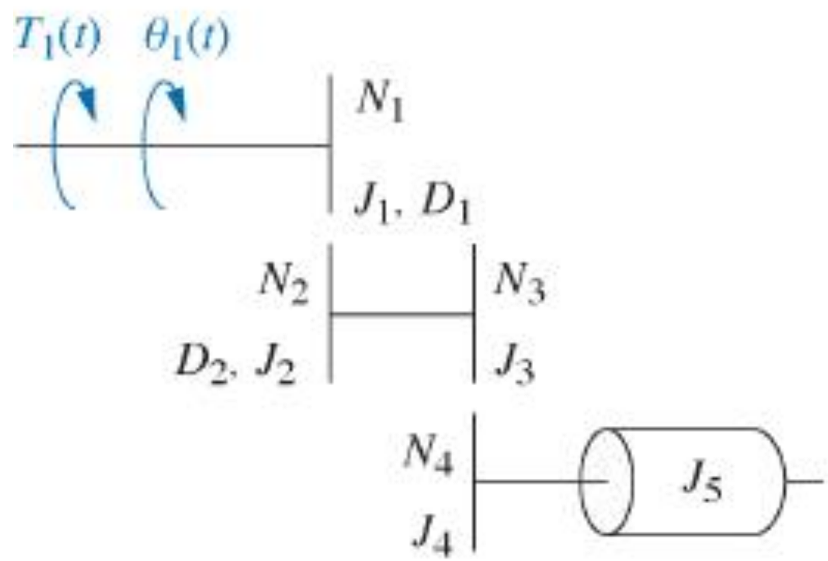
$$\theta_4 = \frac{N_1 N_3 N_5}{N_2 N_4 N_6}\theta_1$$

Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

Example 2.22: Transfer Function—Gears with Loss

- Find the transfer function, $\theta_1(s)/T_1(s)$.
- All of the gears have inertia, and for some shafts there is viscous friction.
- To solve the problem, we want to reflect all of the impedances to the input shaft, , θ_1



Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

Example 2.22: Transfer Function—Gears with Loss

- Find the transfer function, $\theta_1(s)/T_1(s)$.
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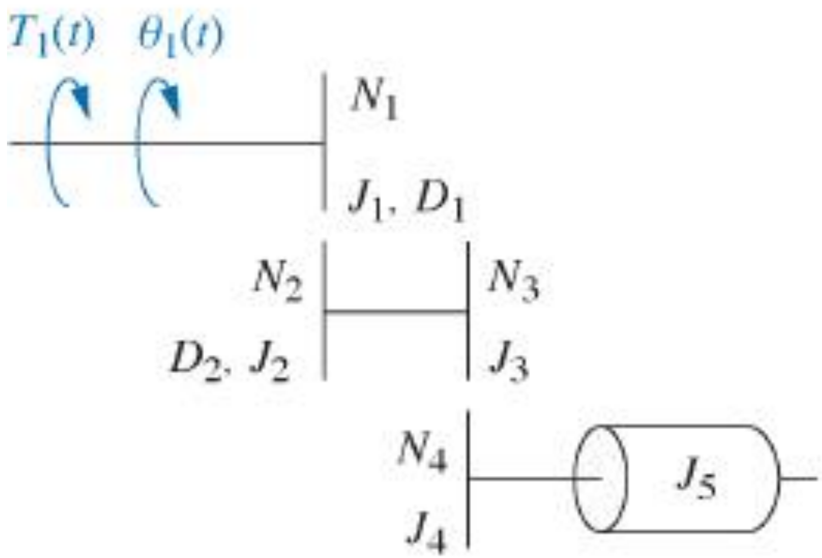


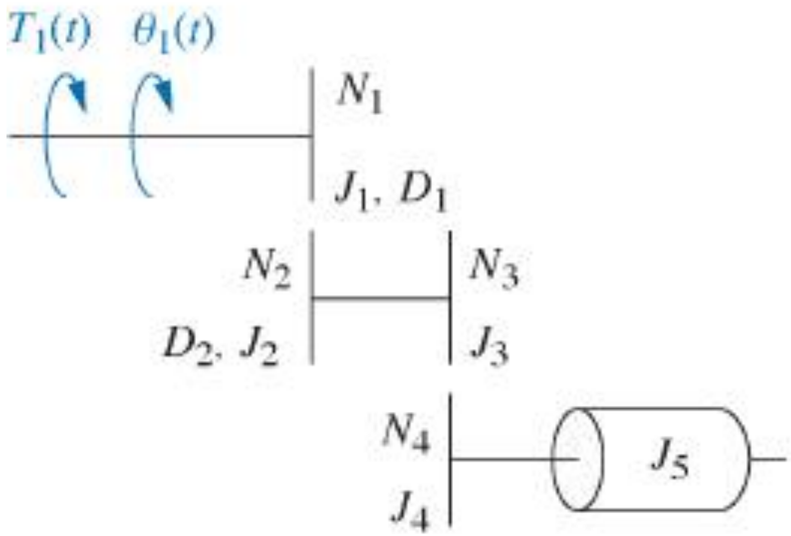
Modeling in the Frequency Domain

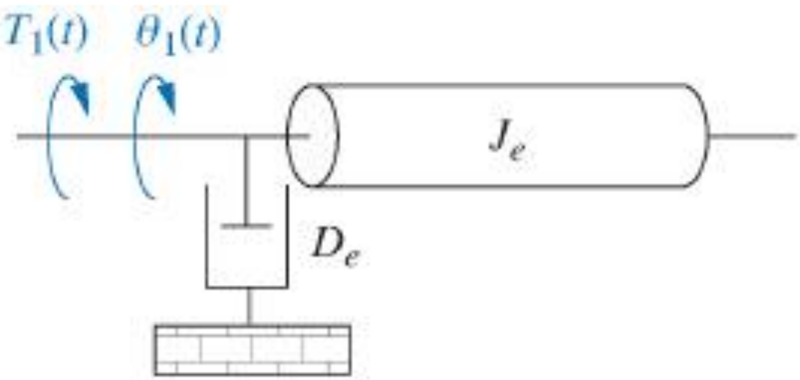
2.7 Transfer Functions for Systems with Gears

Example 2.22: Find the transfer function, $\theta_1(s)/T_1(s)$.

- To solve the problem, we want to reflect all of the impedances to the input shaft, θ_1.

$$(J_e s^2 + D_e s)\theta_1(s) = T_1(s)$$





$$J_e = J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + (J_4 + J_5)\left(\frac{N_1 N_3}{N_2 N_4}\right)^2$$

$$D_e = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$$

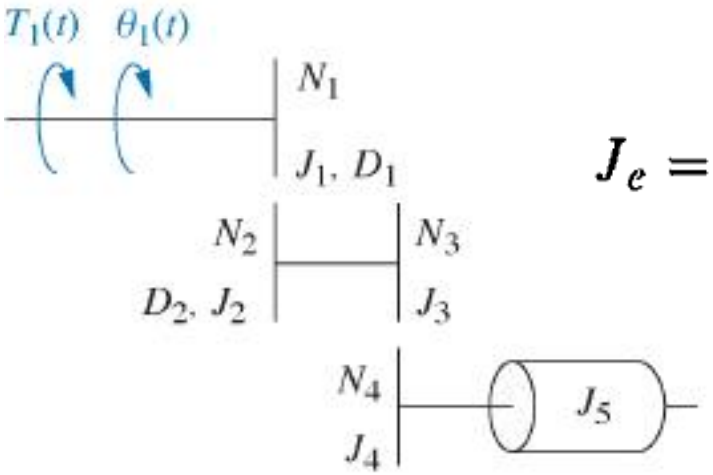
Modeling in the Frequency Domain

2.7 Transfer Functions for Systems with Gears

Example 2.22: Find the transfer function, $\theta_1(s)/T_1(s)$.

- To solve the problem, we want to reflect all of the impedances to the input shaft, θ_1.

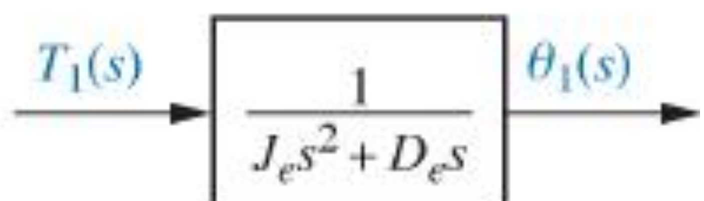
$$(J_e s^2 + D_e s)\theta_1(s) = T_1(s)$$



$$J_e = J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + (J_4 + J_5)\left(\frac{N_1 N_3}{N_2 N_4}\right)^2$$

$$D_e = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$$

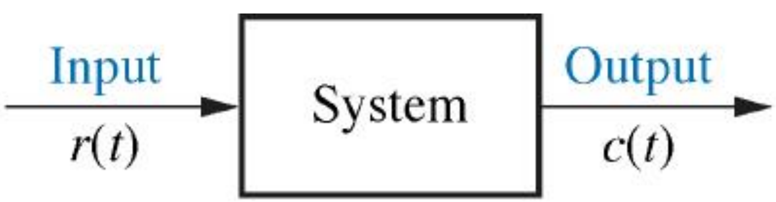
$$G(s) = \frac{\theta_1(s)}{T_1(s)} = \frac{1}{J_e s^2 + D_e s}$$



Modeling in the Frequency Domain

2.8 Electromechanical System

- Now, we move to systems that are hybrids of electrical and mechanical variables, the *electromechanical systems.*
- A **motor** is an electromechanical component that yields a displacement output for a voltage input, that is, a mechanical output generated by an electrical input.

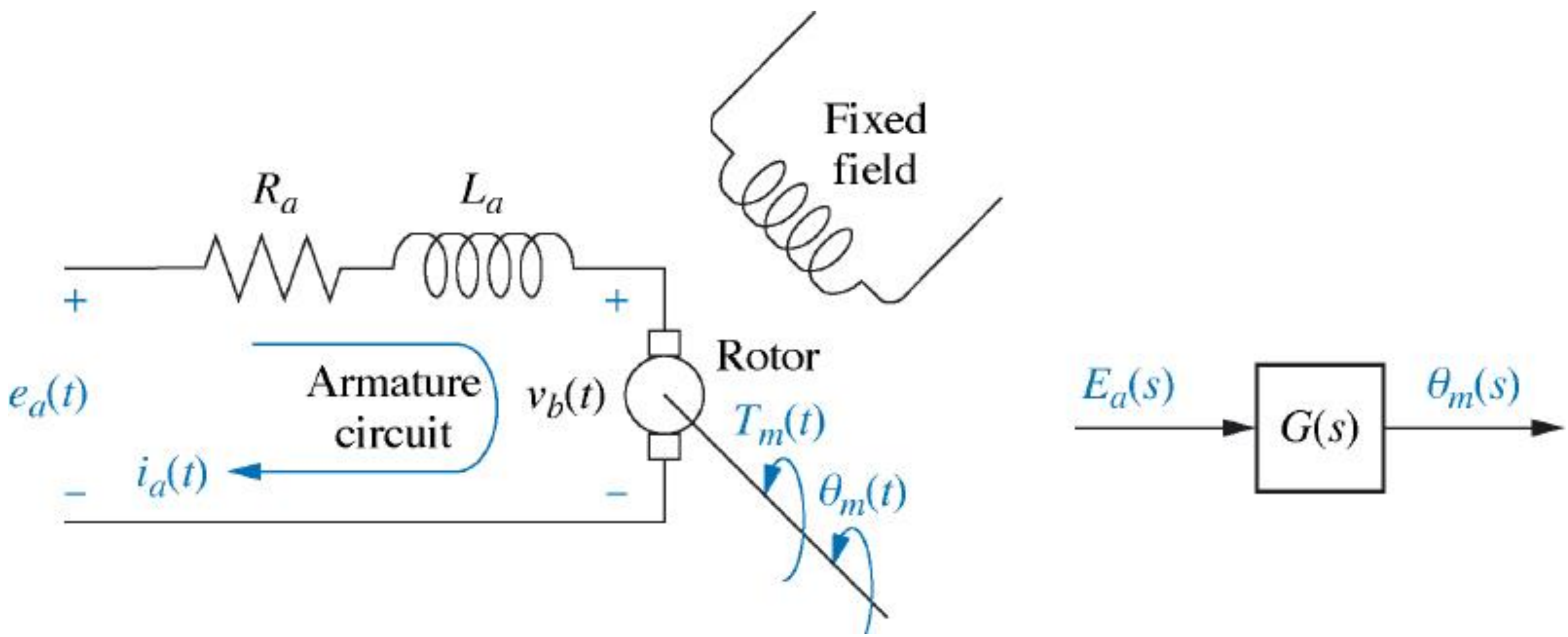


$$\boxed{\frac{C(s)}{R(s)} = G(s)}$$

Modeling in the Frequency Domain

2.8 Electromechanical System

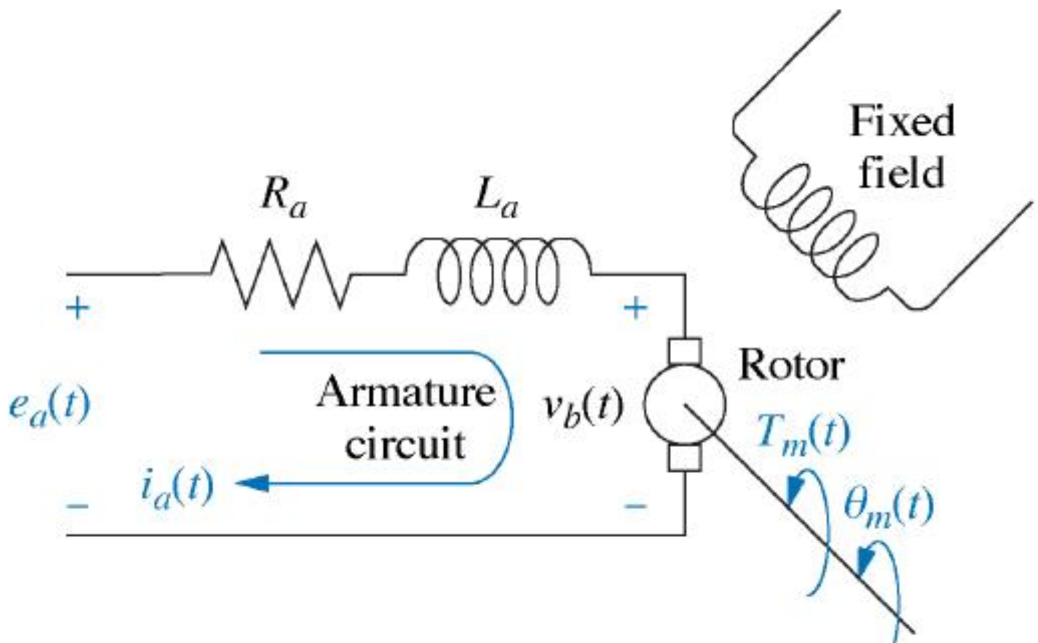
- We will derive the transfer function for one particular kind of electromechanical system, the armature-controlled dc servomotor.
- The motor's schematic is shown in Figure 2.35(a).

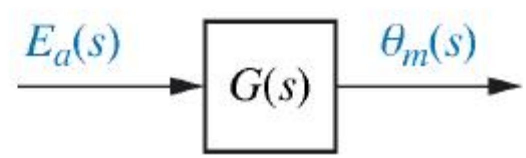


Modeling in the Frequency Domain

2.8 Electromechanical System

- The magnetic field is developed by stationary permanent magnets or a stationary electromagnet called the *__fixed field__.*
- Rotating circuit called the *armature,* through which current $i_a(t)$ flows, passes through this magnetic field.



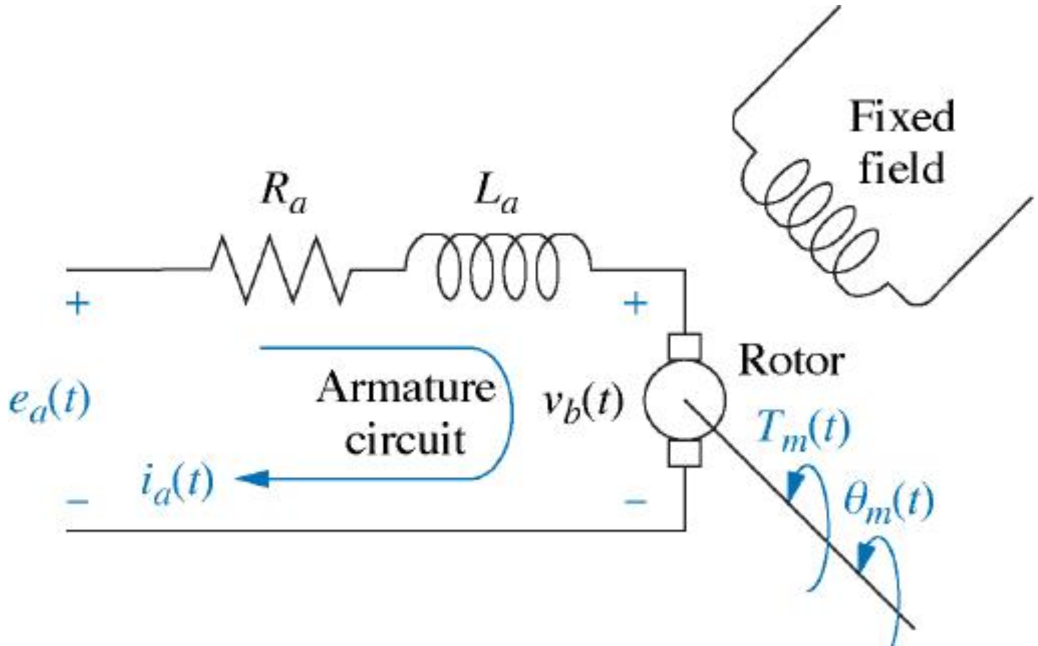


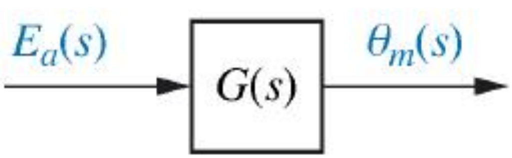
Modeling in the Frequency Domain

2.8 Electromechanical System

- <u>There is another phenomenon that occurs in the motor</u>: A conductor moving at right angles to a magnetic field generates a voltage at the terminals of the conductor equal to *e = Blv.*

$$v_b(t) = K_b \frac{d\theta_m(t)}{dt}$$





Modeling in the Frequency Domain

2.8 Electromechanical System

- We call $v_b(t)$ *the back electromotive force (back emf);* K_b *is a constant of* proportionality called the back emf constant; and $d\theta_m(t)/dt = \omega_m(t)$ *is the angular* velocity of the motor. Taking the Laplace transform, we get

$$v_b(t) = K_b \frac{d\theta_m(t)}{dt} \Rightarrow V_b(s) = K_b s\theta_m(s)$$

- The relationship between the armature current, $i_a(t)$, *the applied armature* voltage, $e_a(t)$, *and the back emf,* $v_b(t)$, *is found by writing a loop equation around the* armature circuit.

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

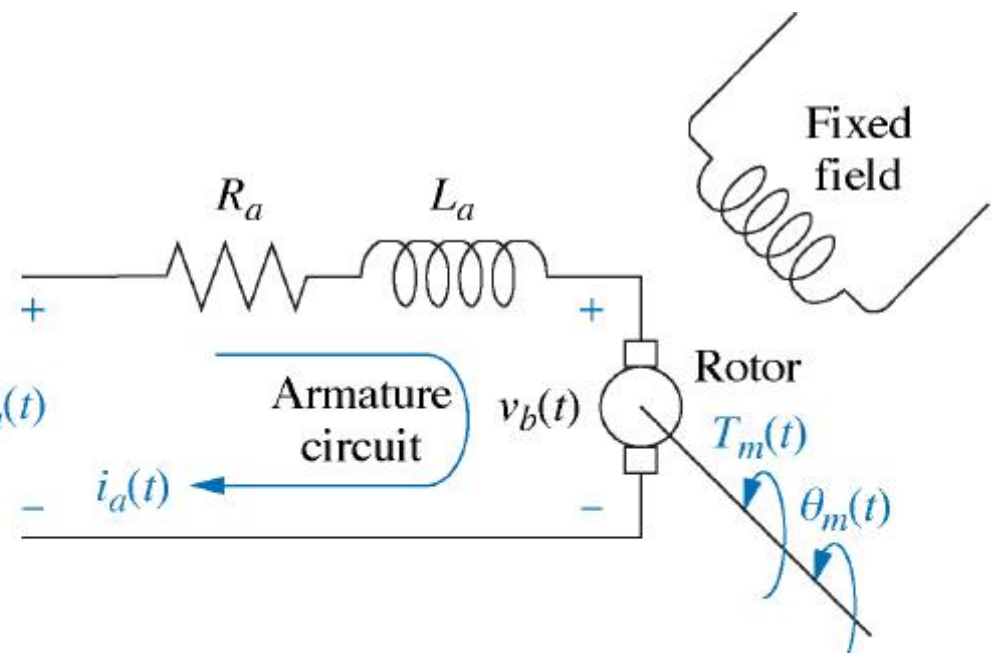
Modeling in the Frequency Domain

2.8 Electromechanical System

- The torque developed by the motor is proportional to the armature current.

$$T_m(s) = K_t I_a(s) \quad \Rightarrow \quad I_a(s) = \frac{1}{K_t} T_m(s)$$

- In a consistent set of units, the value of K_t *is* <u>*equal*</u> *to the value of* K_b.



Modeling in the Frequency Domain

2.8 Electromechanical System

- To find the transfer function, we substitute the equations..

$$I_a(s) = \frac{1}{K_t} T_m(s) \qquad V_b(s) = K_b s\theta_m(s)$$

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

$$\Downarrow$$

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$

- Now we must find *$T_m(s)$ in terms of $\theta_m(s)$, then we can separate the input and* output variables and obtain the transfer function, $\theta_m(s)/E_a(s)$.

Modeling in the Frequency Domain

2.8 Electromechanical System

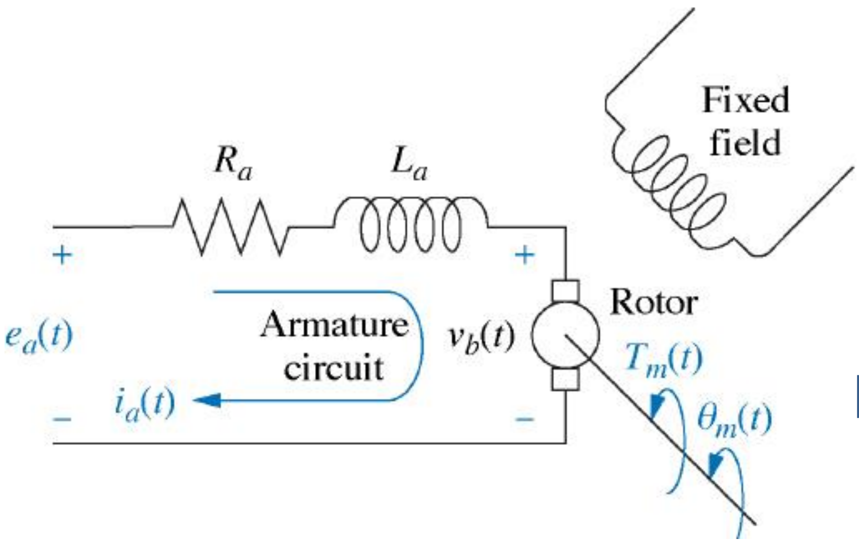
- To find the transfer function, the *$T_m(s)$ must be written in terms of $\theta_m(s)$.*

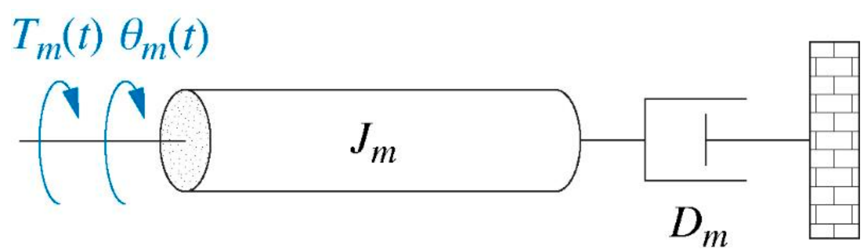
$$\boxed{\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)}$$

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s)$$

J_m is the equivalent inertia

D_m is the equivalent viscous damping





Modeling in the Frequency Domain

2.8 Electromechanical System

- To find the transfer function, the *$T_m(s)$ must be written in terms of $\theta_m(s)$.*

$$\boxed{\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)}$$

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s)$$

J_m is the equivalent inertia

D_m is the equivalent viscous damping

$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$

Assume: $L_a \ll R_a$

$$\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right] s\theta_m(s) = E_a(s)$$

$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}}$$

Modeling in the Frequency Domain

2.8 Electromechanical System

- This simplified transfer function has only two parameters.

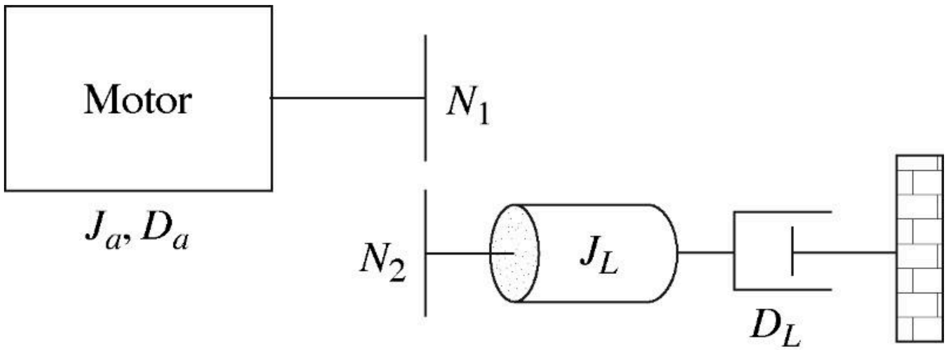
$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}} \Rightarrow \frac{\theta_m(s)}{E_a(s)} = \frac{K}{s(s+\alpha)}$$

- However, to determine the K and α, the mechanical and electrical parameters must be identified.

Modeling in the Frequency Domain

2.8 Electromechanical System

- Consider the mechanical parameters.



$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_tK_b}{R_a}\right)\right]}$$

- When the motor parameters - J_a and D_a - and the mechanical load parameters - turn ratio, J_L and D_L - are given, the J_m and D_m can be determined.

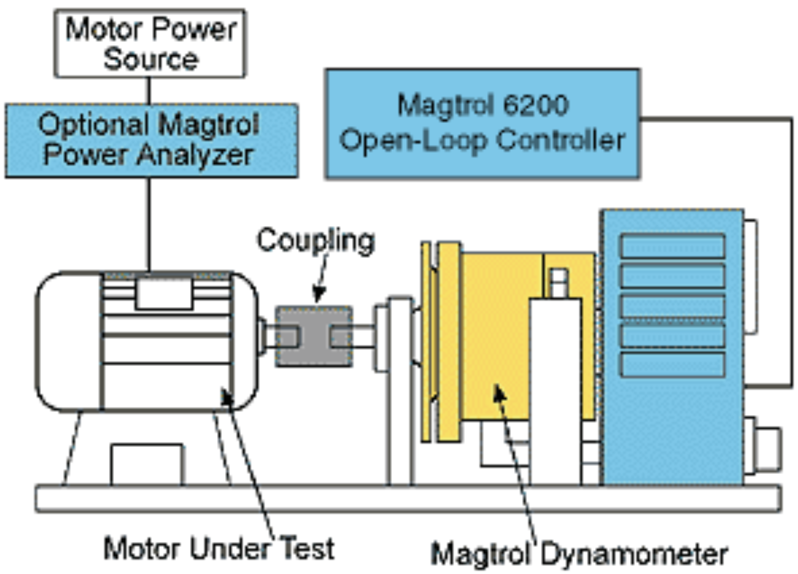
$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2;\ D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

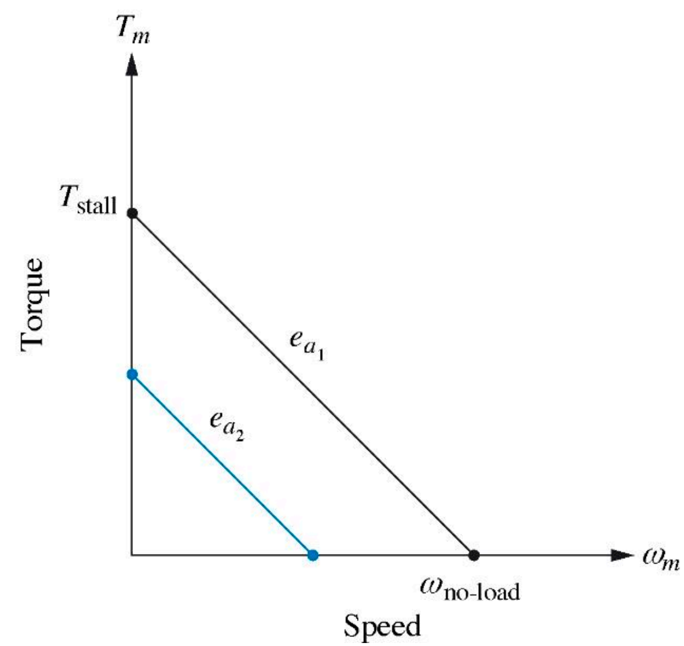
Modeling in the Frequency Domain

2.8 Electromechanical System

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

- Consider the electrical parameters.
- The ***motor constants***, K_t/R_a and K_b, can be obtained through a *dynamometer test of the motor,* where a dynamometer measures the torque and speed of a motor under the condition of a constant applied voltage.





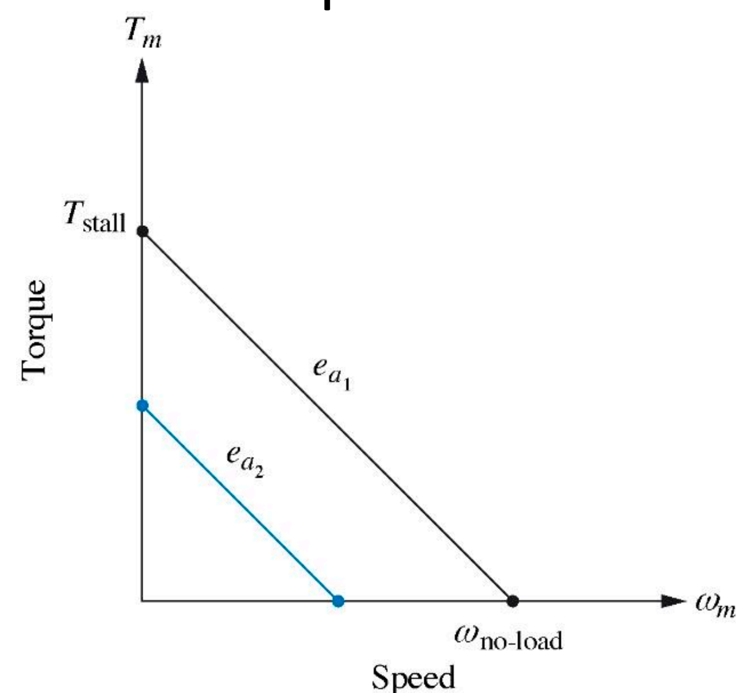
Modeling in the Frequency Domain

2.8 Electromechanical System

- The *torque-speed curve* can be used to determine the parameter K_t/R_a and K_b .
- The torque axis intercept occurring when the angular velocity reaches zero is called the *stall torque, T_{stall}*.
- The angular velocity occurring when the torque is zero is called the *no-load speed, $\omega_{no\text{-}load}$*.

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a}$$

$$K_b = \frac{e_a}{\omega_{\text{no-load}}}$$



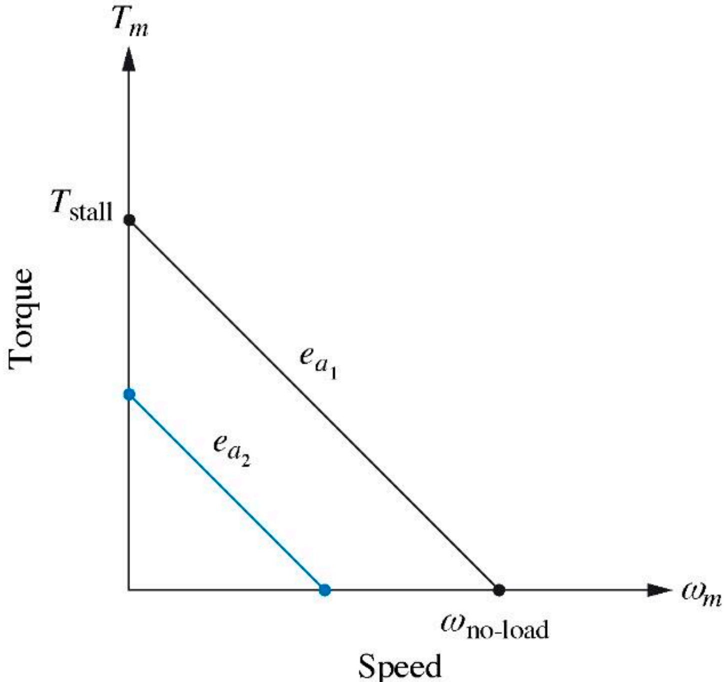
Modeling in the Frequency Domain

2.8 Electromechanical System

- The *torque-speed curve* can be used to determine the parameter K_t/R_a and K_b .
- The torque axis intercept occurring when the angular velocity reaches zero is called the *stall torque, T_{stall}*.
- The angular velocity occurring when the torque is zero is called the *no-load speed, $\omega_{no\text{-}load}$*.

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a}$$

$$K_b = \frac{e_a}{\omega_{\text{no-load}}}$$



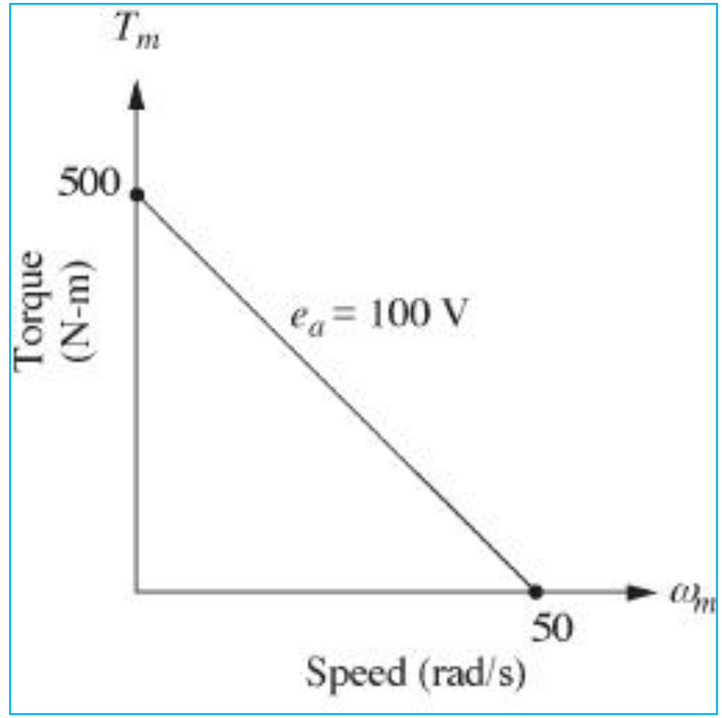
Note: There are other methods for determining the motor constants.

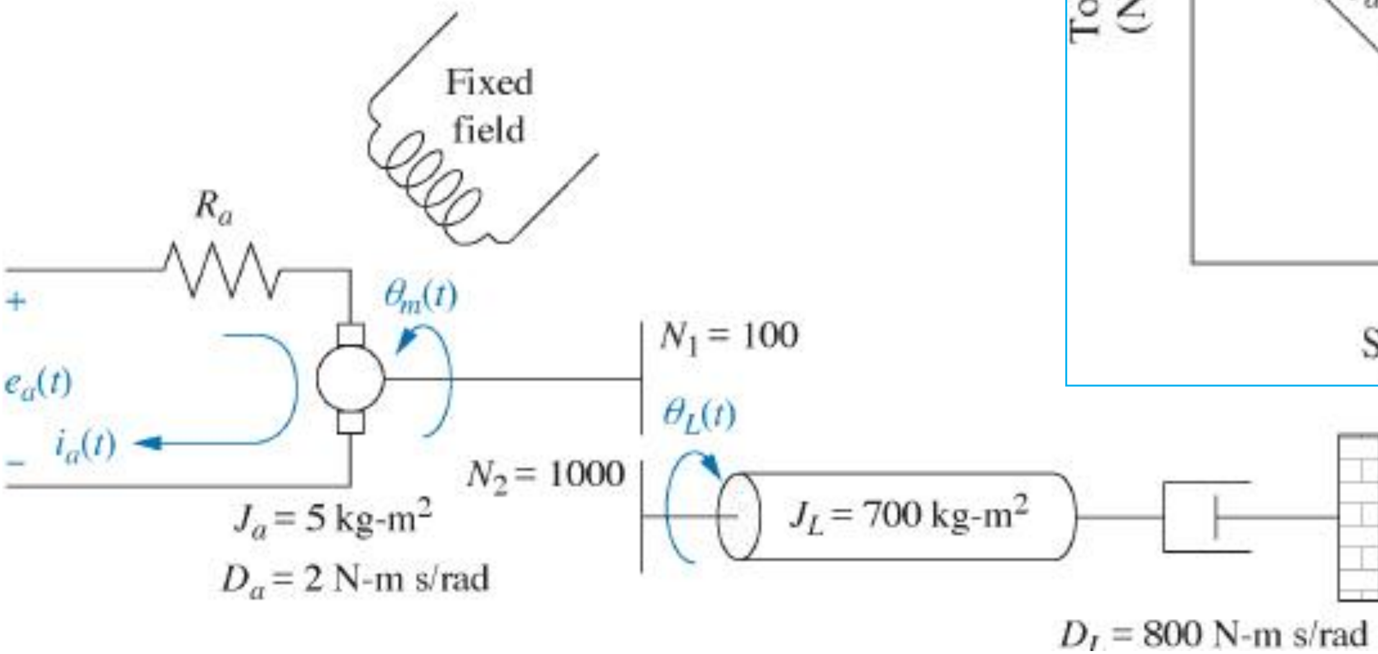
Modeling in the Frequency Domain

2.8 Electromechanical System

PROBLEM: Given the system and torque-speed curve of Figure 2.39(*a*) and (*b*), find the transfer function, $\theta_L(s)/E_a(s)$.

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$





Modeling in the Frequency Domain

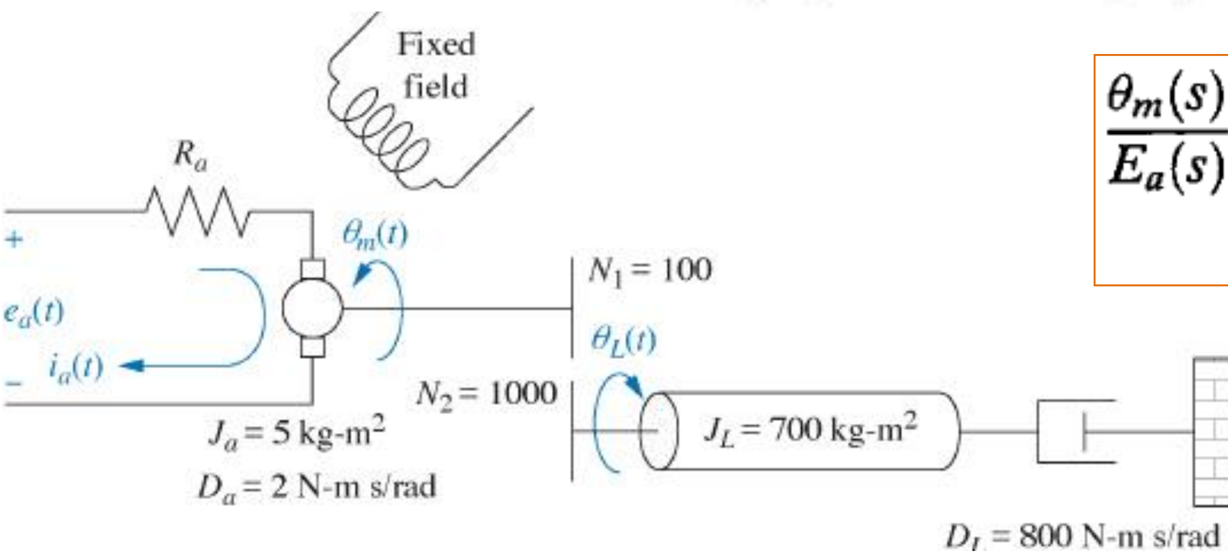
2.8 Electromechanical System

SOLUTION: Begin by finding the mechanical constants, J_m and D_m, in Eq. (2.153). From Eq. (2.155), the total inertia at the armature of the motor is

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12 \tag{2.164}$$

and the total damping at the armature of the motor is

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10 \tag{2.165}$$



$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_tK_b}{R_a}\right)\right]}}$$

Modeling in the Frequency Domain

2.8 Electromechanical System

Now we will find the electrical constants, K_t/R_a and K_b. From the torque-speed curve of Figure 2.39(b),

$$T_{\text{stall}} = 500 \tag{2.166}$$

$$\omega_{\text{no-load}} = 50 \tag{2.167}$$

$$e_a = 100 \tag{2.168}$$

Hence the electrical constants are

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a} = \frac{500}{100} = 5$$

$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}}$$

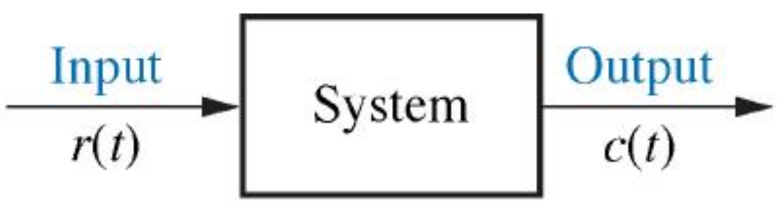
$$K_b = \frac{e_a}{\omega_{\text{no-load}}} = \frac{100}{50} = 2$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{5/12}{s\left\{s + \frac{1}{12}[10 + (5)(2)]\right\}} = \frac{0.417}{s(s + 1.667)}$$

Modeling in the Frequency Domain

2.10 Nonlinearities

- The systems can be roughly classified in 2 groups namely: **Linear** and **Nonlinear.** If a system does not belong to the first group, it will be a nonlinear system.
- So far we have studied only about the linear time invariant system (LTI), and we loosely called it linear system.



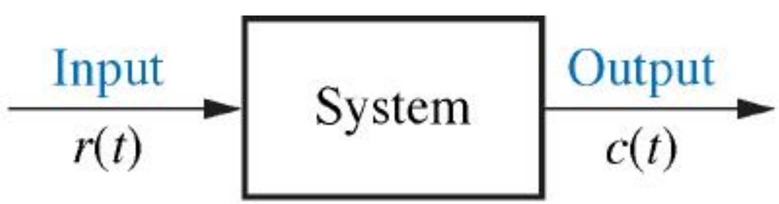
$$\boxed{\frac{C(s)}{R(s)} = G(s)}$$

Note: A *linear time invariant system* can be modeled by a transfer function.

Modeling in the Frequency Domain

2.10 Nonlinearities

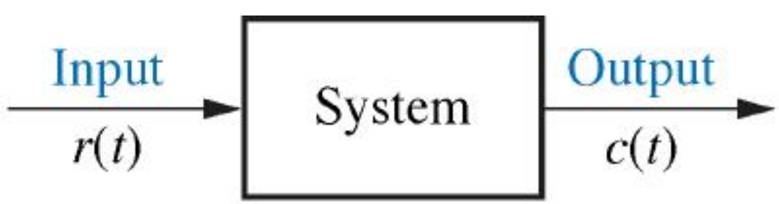
- *A linear system possesses two properties:* ***superposition*** *and* ***homogeneity****.*
- *Superposition means that the output response of a system to the sum of* inputs is the sum of the responses to the individual inputs. Thus, if an input of $r_1(t)$ yields an output of $c_1(t)$ *and an input of* $r_2(t)$ *yields an output of* $c_2(t)$, *then an input of* $r_1(t) + r_2(t)$ *yields an output of* $c_1(t) + c_2(t)$.



Modeling in the Frequency Domain

2.10 Nonlinearities

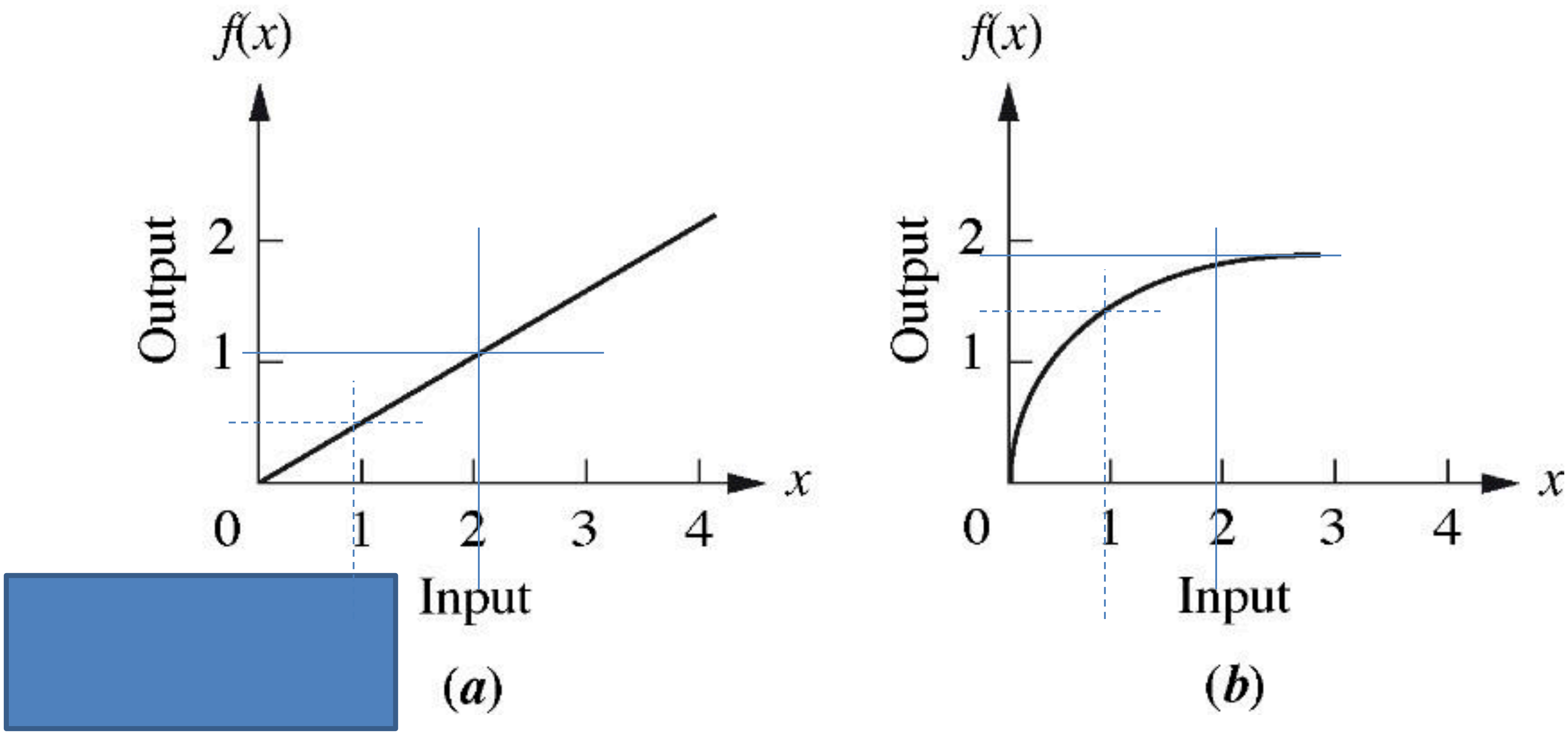
- *A linear system possesses two properties:* ***superposition*** *and* ***homogeneity****.*
- ***Homogeneity*** *describes* the response of the system to a multiplication of the input by a scalar.
- A system is said to be a homogeneity system, if for an input of $r_1(t)$ that yields an output of $c_1(t)$, *and input of* $Ar_1(t)$ *yields an output of* $Ac_1(t)$.
- *That means,* multiplication of an input by a scalar yields a response that is multiplied by the same scalar.



Modeling in the Frequency Domain

2.10 Nonlinearities

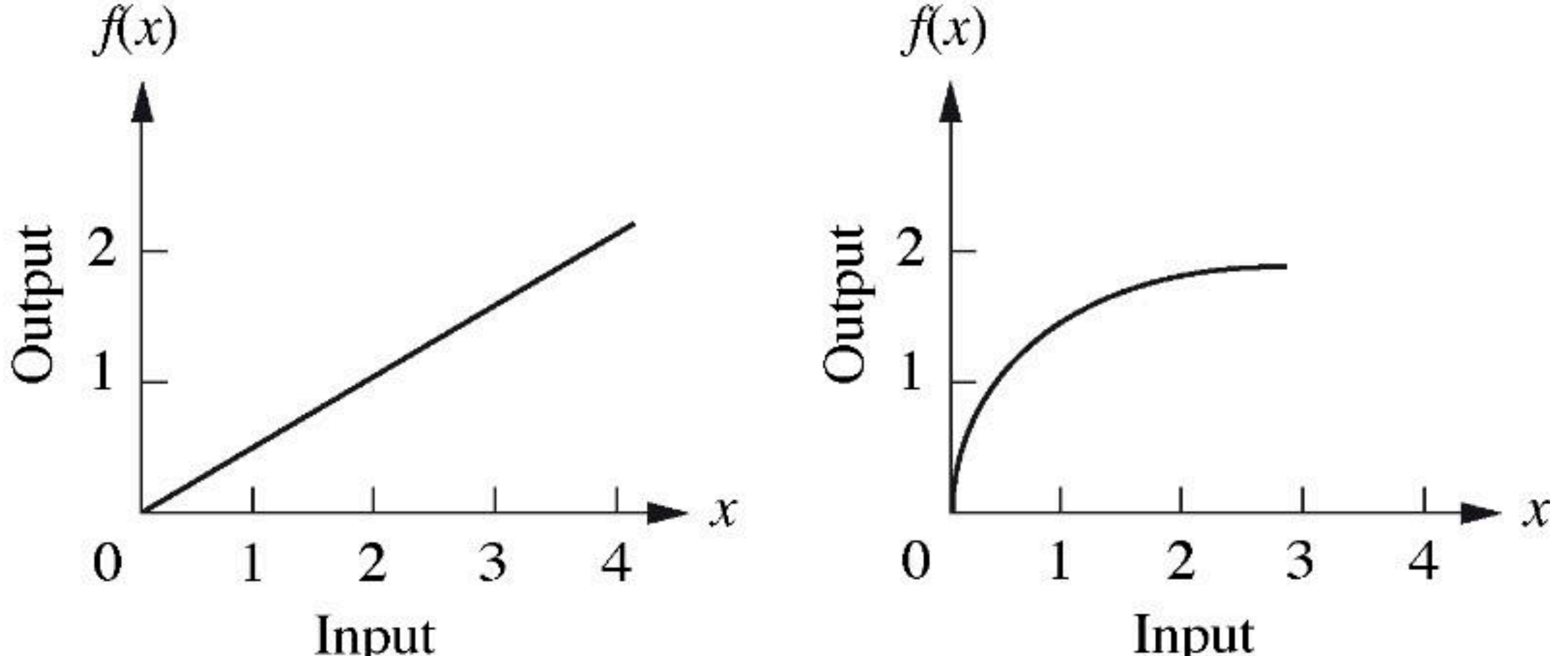
- *A <u>linear system possesses</u> two properties:* ***superposition*** *and* ***homogeneity****.*



Modeling in the Frequency Domain

2.10 Nonlinearities

- *A linear system possesses two properties:* ***superposition*** *and* ***homogeneity****.*



$f(x) = 0.5\,x$

(*a*)

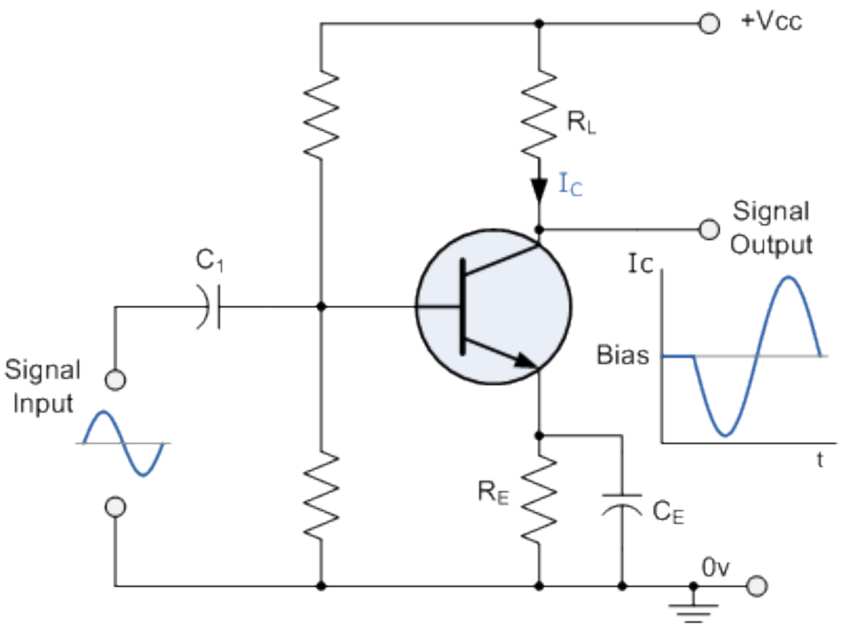
(*b*)

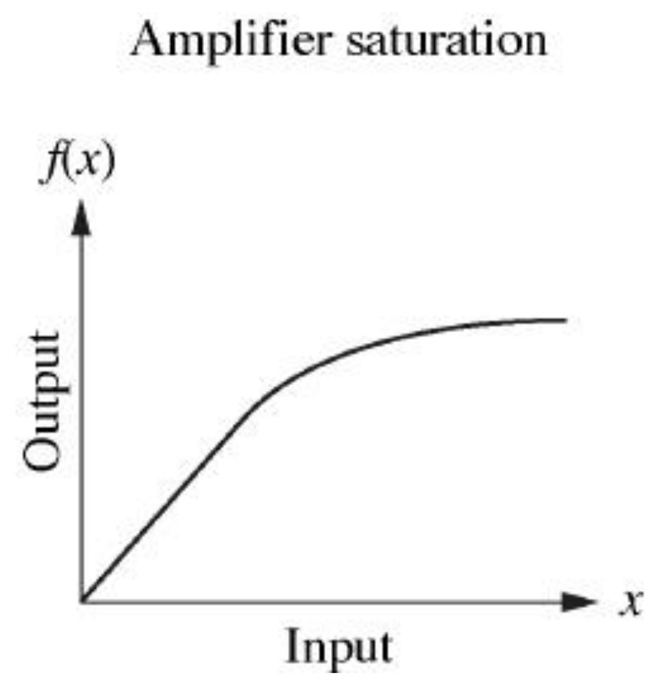
Modeling in the Frequency Domain

2.10 Nonlinearities

<u>Examples of physical nonlinearities</u>:

- An electronic amplifier is linear over a specific range but exhibits the nonlinearity called <u>*saturation*</u> at high input voltages.





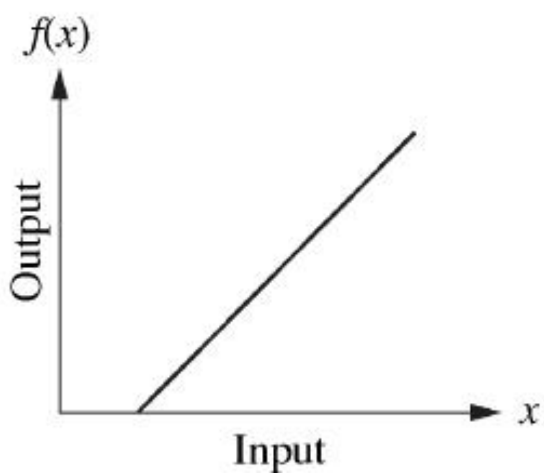
Modeling in the Frequency Domain

2.10 Nonlinearities

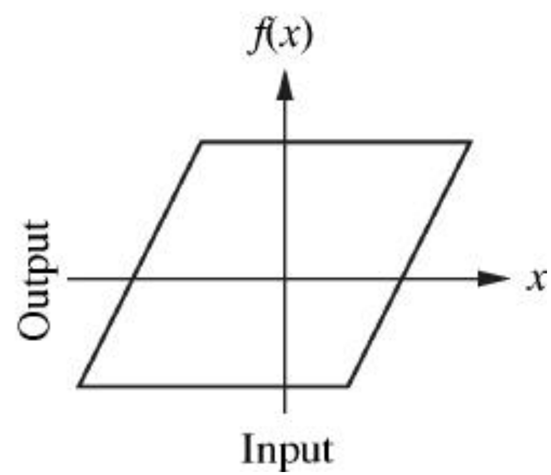
Examples of physical nonlinearities:

- A motor that does not respond at very low input voltages due to frictional forces exhibits a nonlinearity called *dead zone.*
- *Gears that do not fit* tightly exhibit a nonlinearity called *backlash.*

Motor dead zone



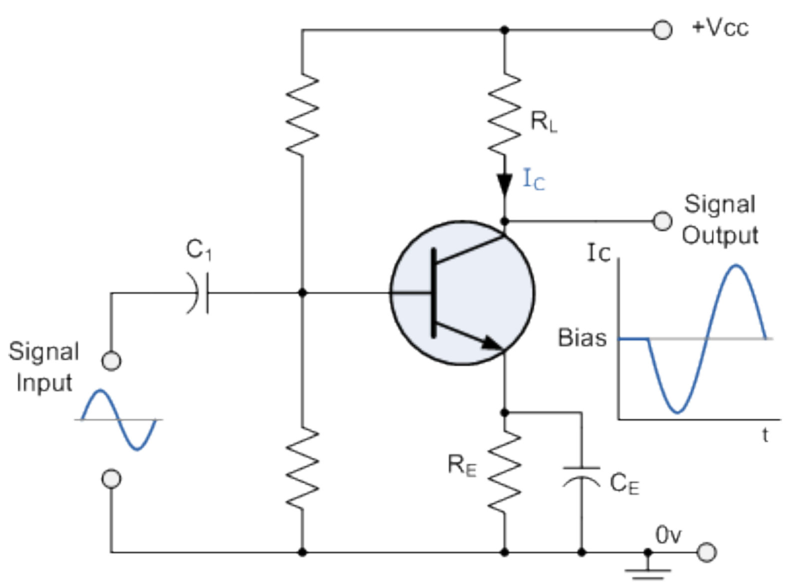
Backlash in gears

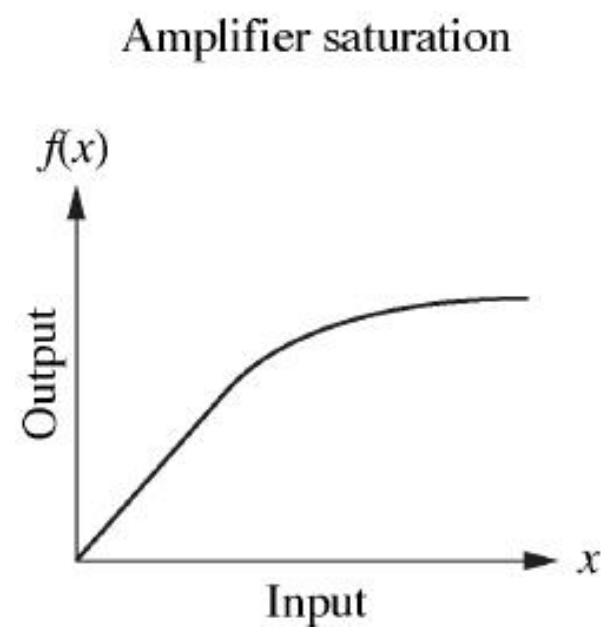


Modeling in the Frequency Domain

2.10 Nonlinearities

- Actually most of the physical systems are nonlinear!
- But a designer can often make a linear approximation to a nonlinear system. Linear approximations simplify the analysis and design of a system and are used as long as the results yield a good approximation to reality.

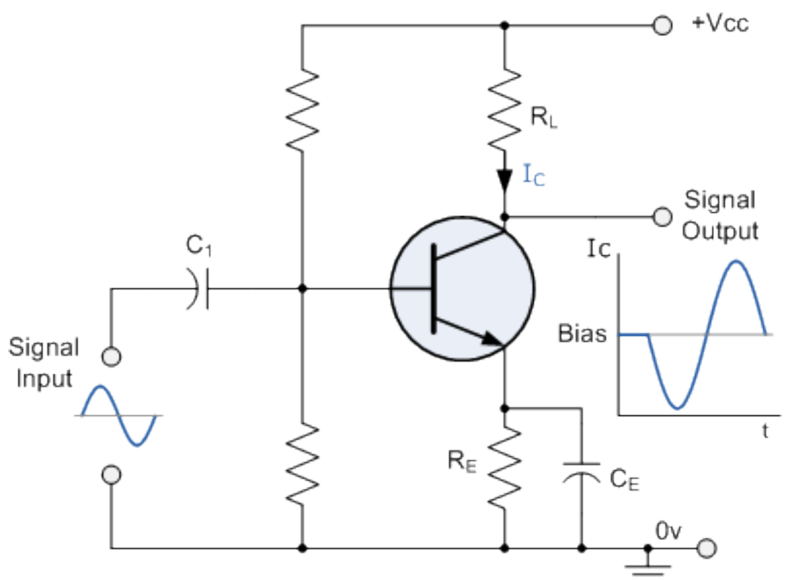


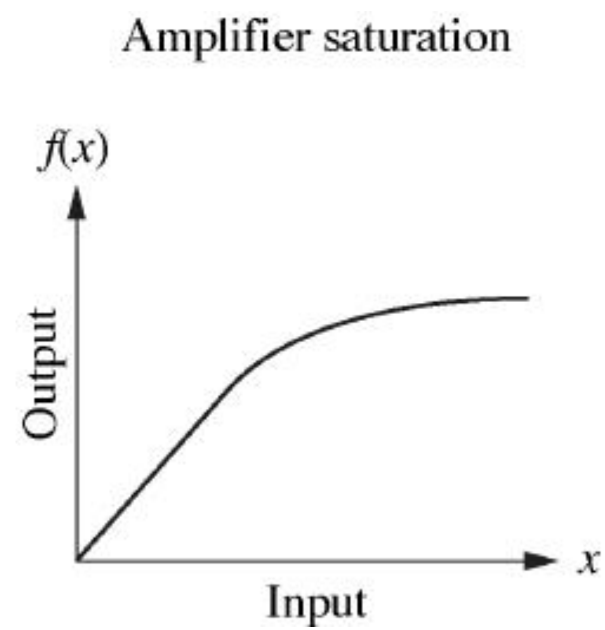


Modeling in the Frequency Domain

2.10 Nonlinearities

- For example, a linear relationship can be established at a point on the nonlinear curve if the range of input values about that point is small and the origin is translated to that point. Electronic amplifiers are an example of physical devices that perform linear amplification with small excursions about a point.





Modeling in the Frequency Domain

2.11 Linearization

- The electrical and mechanical systems covered thus far were assumed to be linear. However, if any nonlinear components are present, <u>we must **linearize the system** before we can find the transfer function</u>

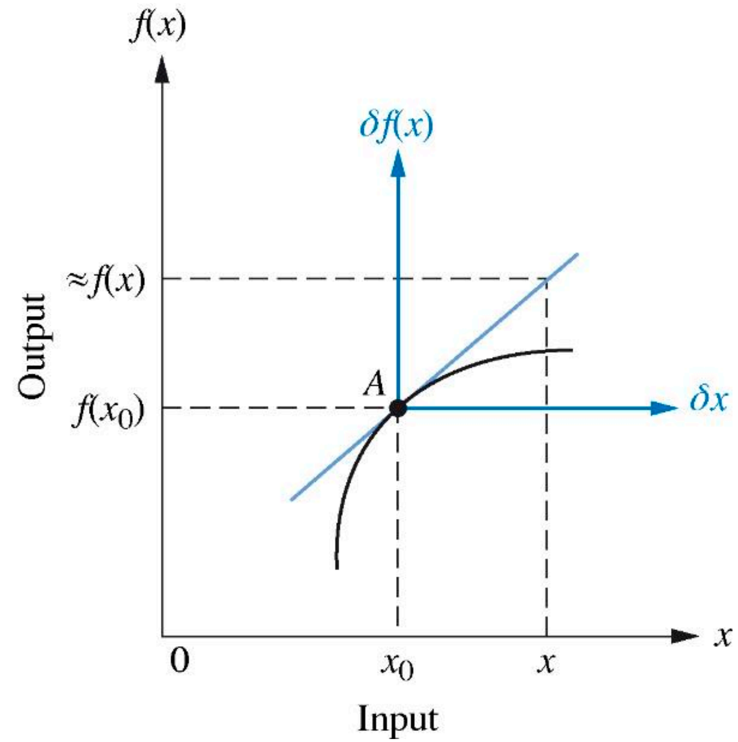
$$\boxed{\frac{C(s)}{R(s)} = G(s)}$$

- In this section, we show how to obtain linear approximations to nonlinear systems in order to obtain transfer functions.

Modeling in the Frequency Domain

2.11 Linearization

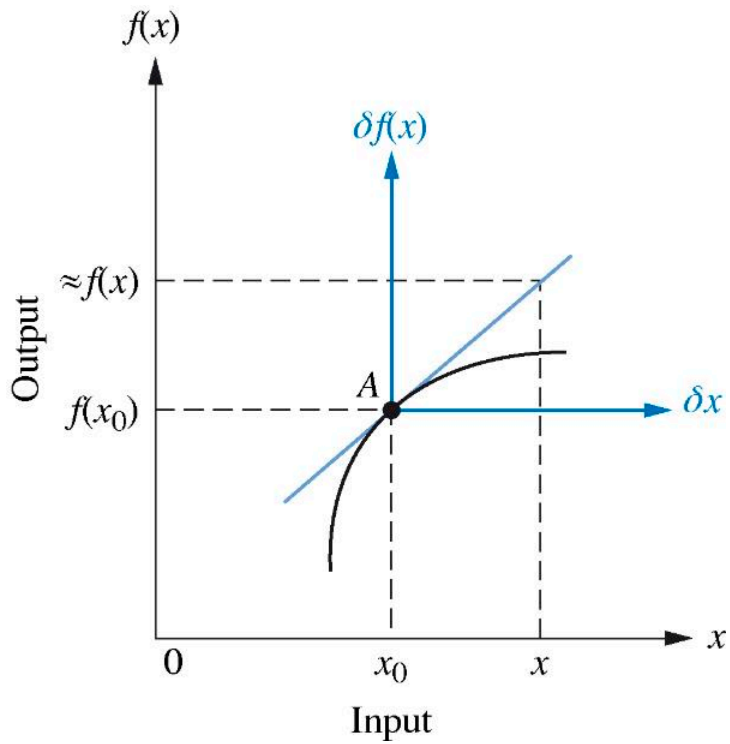
- The **first step** is to recognize the nonlinear component and write the nonlinear differential equation.
- When we linearize a nonlinear differential equation, we linearize it for small-signal inputs about the steady-state solution when the small-signal input is equal to zero. This steady-state solution is called *equilibrium* and is selected as the **second step** in the linearization process.



Modeling in the Frequency Domain

2.11 Linearization

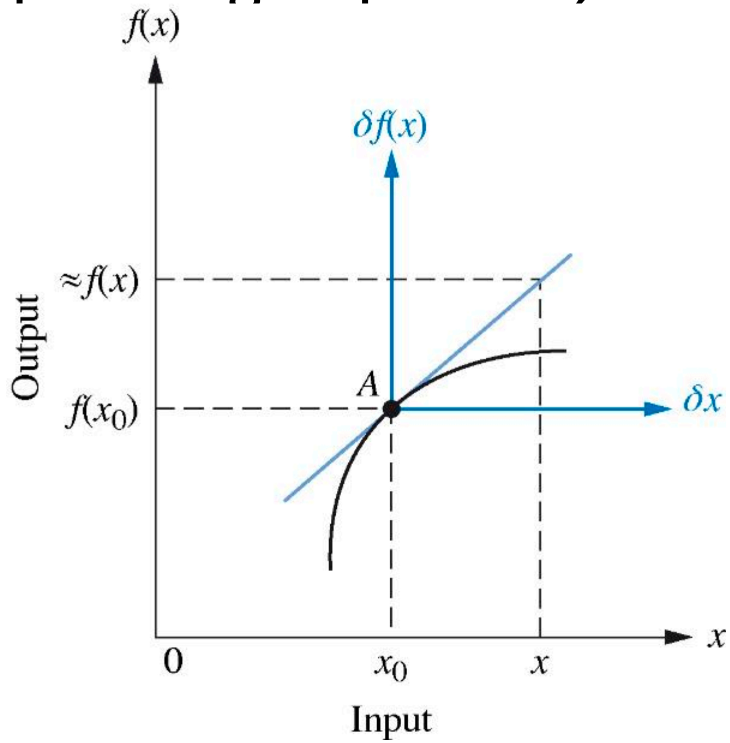
- Next we linearize the nonlinear differential equation, and then we take the Laplace transform of the linearized differential equation, assuming zero initial conditions.
- Finally, we separate input and output variables and form the transfer function.



Modeling in the Frequency Domain

2.11 Linearization

- Let us first see how to linearize a function; later, we will apply the method to the linearization of a differential equation.
- If we assume a nonlinear system operating at point *A*, [x_0, $f(x_0)$] *in* Figure 2.47, small changes in the input can be related to changes in the output about the point by way of the slope of the curve at the point *A*.



Modeling in the Frequency Domain

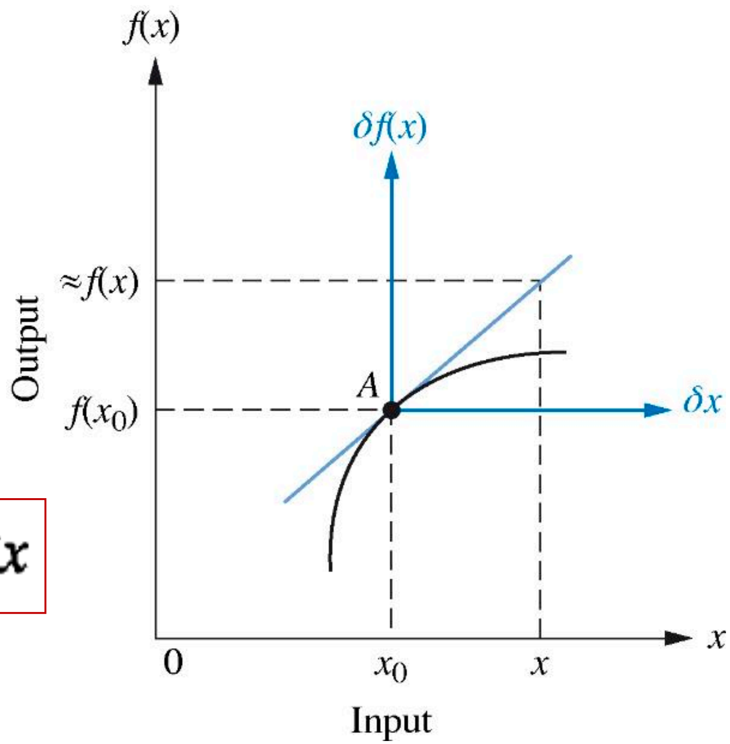
2.11 Linearization: function

- Thus, if the slope of the curve at point A is m_a, *then small excursions of* the input about point *A, σx, yield small changes in the output, σ f(x),* related by the slope at point *A. Thus,*

$$[f(x) - f(x_0)] \approx m_a(x - x_0)$$

$$\delta f(x) \approx m_a \delta x$$

$$\boxed{f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a \delta x}$$



Modeling in the Frequency Domain

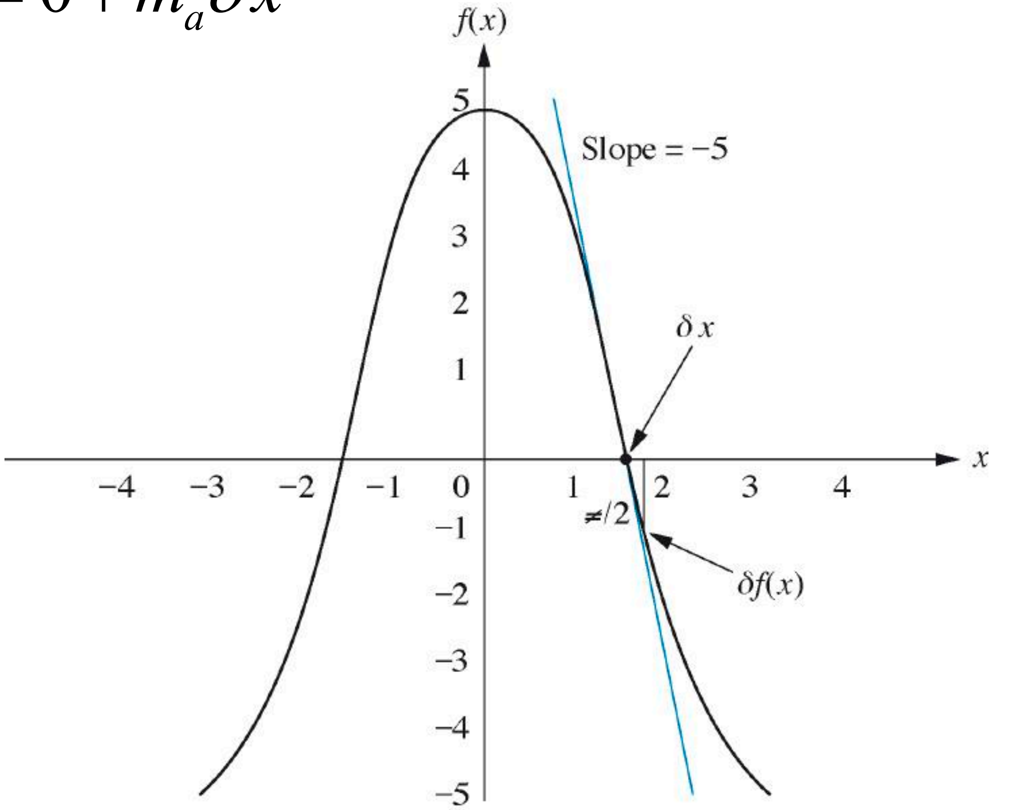
Example 2.26 Linearize *f(x) = 5 cos*(*x*) *about x = π/2.*

$$\boxed{f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a\delta x}$$

$$\boxed{f(x_0) = 5\cos(\frac{\pi}{2}) = 0} \Rightarrow \quad f(x) = 0 + m_a\delta x$$

$$\boxed{m_a = \left.\frac{df}{dx}\right|_{x=\frac{\pi}{2}} = -5\sin(x)\Big|_{x=\frac{\pi}{2}} = -5}$$

$$f(x) = -5\delta x$$



Modeling in the Frequency Domain

2.11 Linearization: function

- The previous discussion can be formalized using the Taylor series expansion.

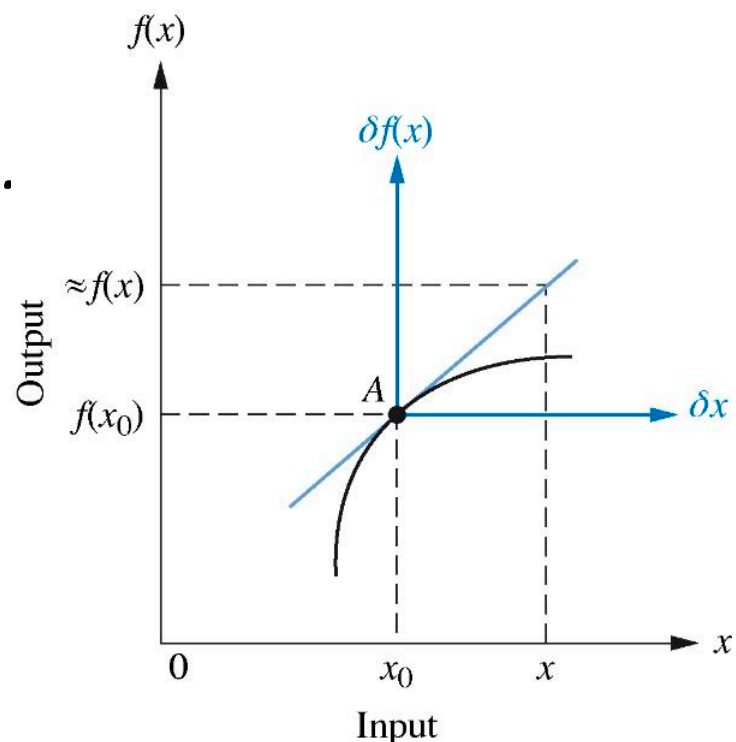
$$f(x) = f(x_0) + \frac{df}{dx}\bigg|_{x=x_0} \frac{(x - x_0)}{1!} + \frac{d^2f}{dx^2}\bigg|_{x=x_0} \frac{(x - x_0)^2}{2!} + \cdots$$

- For small excursions of x *from* x_0, *we can neglect higher-order terms.*

$$\boxed{f(x) - f(x_0) \approx \frac{df}{dx}\bigg|_{x=x_0} (x - x_0)}$$

or

$$\boxed{\delta f(x) \approx m|_{x=x_0} \delta x}$$



Modeling in the Frequency Domain

2.11 Linearization: differential equation

PROBLEM: Linearize Eq. (2.184) for small excursions about $x = \pi/4$.

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + \cos x = 0 \tag{2.184}$$

SOLUTION: The presence of the term $\cos x$ makes this equation nonlinear. Since we want to linearize the equation about $x = \pi/4$, we let $x = \delta x + \pi/4$, where δx is the small excursion about $\pi/4$, and substitute x into Eq. (2.184):

$$\frac{d^2\left(\delta x + \frac{\pi}{4}\right)}{dt^2} + 2\frac{d\left(\delta x + \frac{\pi}{4}\right)}{dt} + \cos\left(\delta x + \frac{\pi}{4}\right) = 0 \tag{2.185}$$

But

$$\frac{d^2\left(\delta x + \frac{\pi}{4}\right)}{dt^2} = \frac{d^2\delta x}{dt^2} \tag{2.186}$$

and

$$\frac{d\left(\delta x + \frac{\pi}{4}\right)}{dt} = \frac{d\delta x}{dt} \tag{2.187}$$

Modeling in the Frequency Domain

2.11 Linearization: differential equation

Finally, the term $\cos(\delta x + (\pi/4))$ can be linearized with the truncated Taylor series. Substituting $f(x) = \cos(\delta x + (\pi/4))$, $f(x_0) = f(\pi/4) = \cos(\pi/4)$, and $(x - x_0) = \delta x$ into Eq. (2.182) yields

$$\cos\left(\delta x + \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = \left.\frac{d\cos x}{dx}\right|_{x=\frac{\pi}{4}} \delta x = -\sin\left(\frac{\pi}{4}\right)\delta x \tag{2.188}$$

Solving Eq. (2.188) for $\cos(\delta x + (\pi/4))$, we get

$$\cos\left(\delta x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\delta x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\delta x \tag{2.189}$$

Substituting Eqs. (2.186), (2.187), and (2.189) into Eq. (2.185) yields the following linearized differential equation:

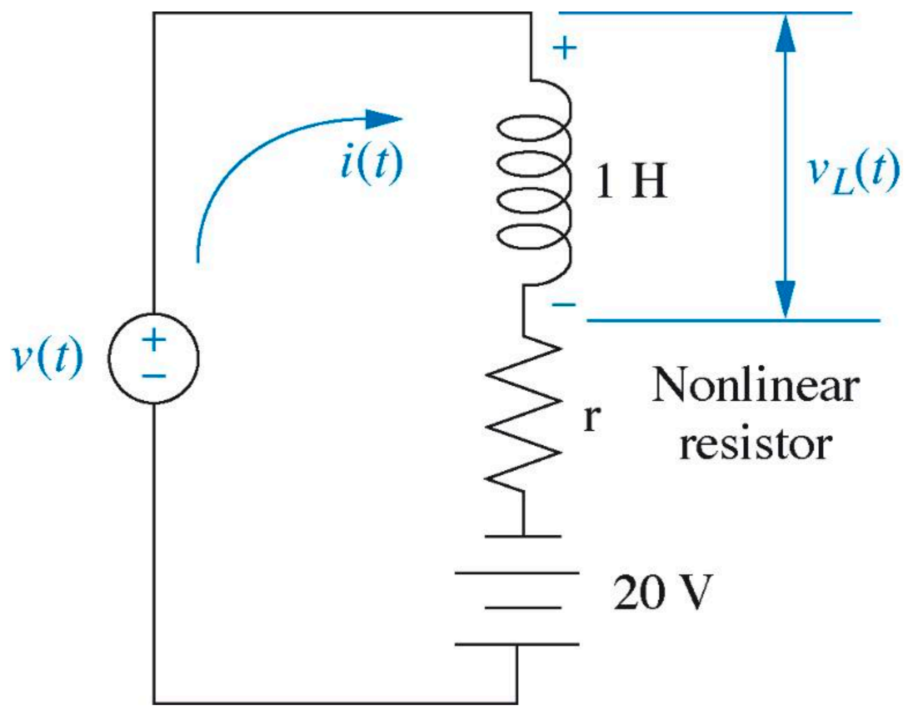
$$\frac{d^2\delta x}{dt^2} + 2\frac{d\delta x}{dt} - \frac{\sqrt{2}}{2}\delta x = -\frac{\sqrt{2}}{2} \tag{2.190}$$

This equation can now be solved for δx, from which we can obtain $x = \delta x + (\pi/4)$.

Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

PROBLEM: Find the transfer function, $V_L(s)/V(s)$, for the electrical network shown in Figure 2.49, which contains a nonlinear resistor whose voltage-current relationship is defined by $i_r = 2e^{0.1v_r}$, where i_r and v_r are the resistor current and voltage, respectively. Also, $v(t)$ in Figure 2.49 is a small-signal source.



Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

Solve:

- Applying KVL around the loop, where $i_r = i$, *yields,*

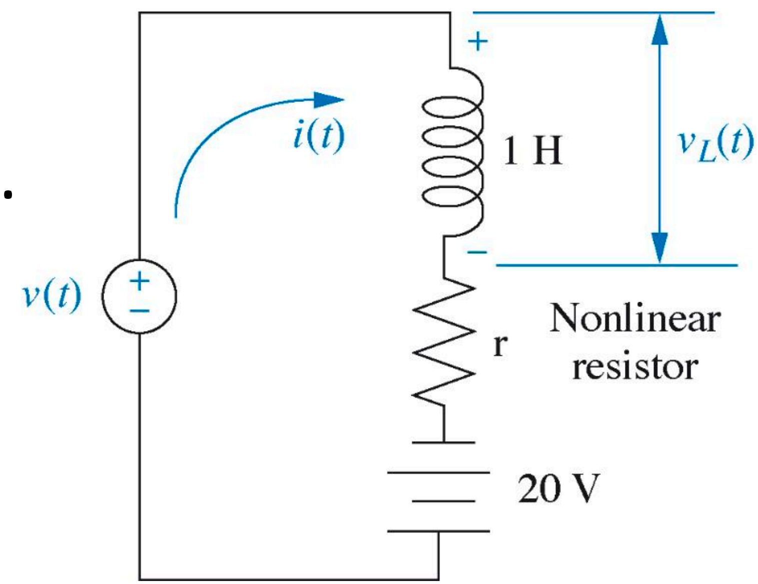
$$L\frac{di}{dt} + 10\ln\frac{1}{2}i - 20 = v(t)$$

- Next, let us evaluate the equilibrium solution.

  Set the small-signal v(t) = 0.

  Evaluate the steady-state current.

$$i_r = i = 14.78 \text{ amps}$$

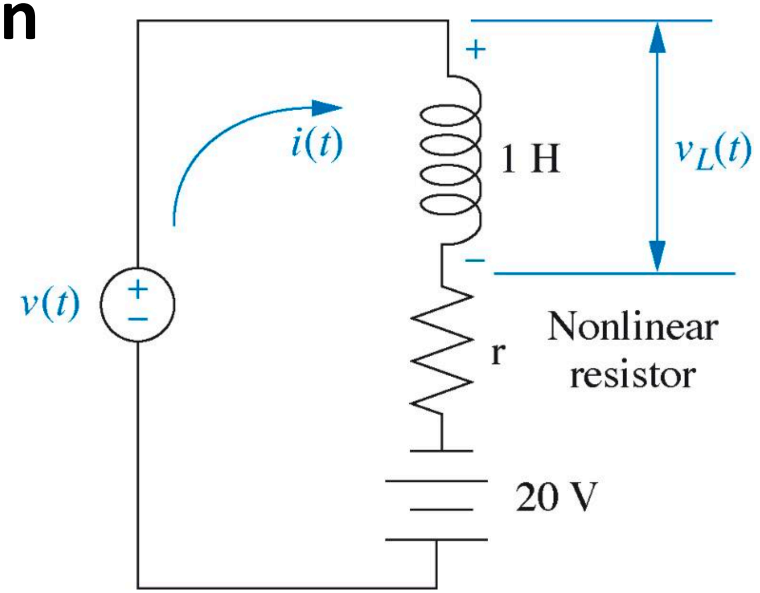


- This current, i_0, *is the* equilibrium value of the network current.

Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

Solve:



- Since $i = i_0 + \delta i$

$$L\frac{di}{dt} + 10\ln\frac{1}{2}i - 20 = v(t)$$

$$L\frac{d(i_0+\delta i)}{dt} + 10\ln\frac{1}{2}(i_0+\delta i) - 20 = v(t)$$

linearize $\ln\frac{1}{2}(i_0+\delta i)$, we get

$$\ln\frac{1}{2}(i_0+\delta i) - \ln\frac{1}{2}i_0 = \frac{d\left(\ln\frac{1}{2}i\right)}{di}\bigg|_{i=i_0}\delta i = \frac{1}{i}\bigg|_{i=i_0}\delta i = \frac{1}{i_0}\delta i$$

$$\ln\frac{1}{2}(i_0+\delta i) = \ln\frac{i_0}{2} + \frac{1}{i_0}\delta i$$

Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

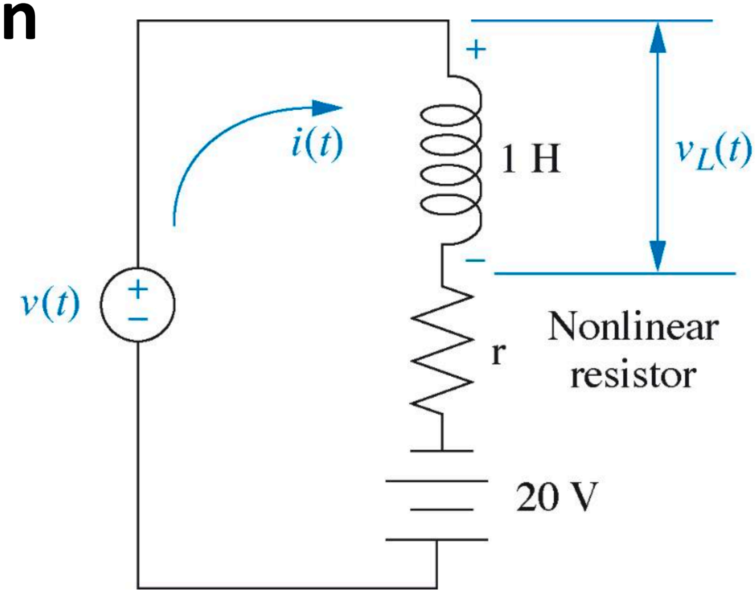
Solve:

- Since $i = i_0 + \delta i$

$$L\frac{di}{dt} + 10\ln\frac{1}{2}i - 20 = v(t)$$

$$\Downarrow$$

$$L\frac{d(i_0+\delta i)}{dt} + 10\ln\frac{1}{2}(i_0+\delta i) - 20 = v(t)$$



$\boxed{\text{linearize } \ln\frac{1}{2}(i_0+\delta i)\text{, we get}}$

$$\ln\frac{1}{2}(i_0+\delta i) = \ln\frac{i_0}{2} + \frac{1}{i_0}\delta i$$

$$\boxed{L\frac{d\delta i}{dt} + 10\left(\ln\frac{i_0}{2} + \frac{1}{i_0}\delta i\right) - 20 = v(t)}$$

Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

Solve:

$$\boxed{L\frac{d\delta i}{dt} + 10\left(\ln\frac{i_0}{2} + \frac{1}{i_0}\delta i\right) - 20 = v(t)}$$

Letting $L = 1$ and $i_0 = 14.78$, the final linearized differential equation is

$$\frac{d\delta i}{dt} + 0.677\delta i = v(t) \quad \Rightarrow \quad \delta i(s) = \frac{V(s)}{s + 0.677}$$

But the voltage across the inductor about the equilibrium point is

$$v_L(t) = L\frac{d}{dt}(i_0 + \delta i) = L\frac{d\delta i}{dt} \quad \Rightarrow \quad V_L(s) = Ls\delta i(s) = s\delta i(s)$$

Modeling in the Frequency Domain

2.11 Linearization: Transfer Function

Solve:

Letting $L = 1$ and $i_0 = 14.78$, the final linearized differential equation is

$$\frac{d\delta i}{dt} + 0.677\delta i = v(t) \quad \Rightarrow \quad \delta i(s) = \frac{V(s)}{s + 0.677}$$

But the voltage across the inductor about the equilibrium point is

$$v_L(t) = L\frac{d}{dt}(i_0 + \delta i) = L\frac{d\delta i}{dt} \quad \Rightarrow \quad V_L(s) = Ls\delta i(s) = s\delta i(s)$$

$$V_L(s) = s\frac{V(s)}{s + 0.677}$$

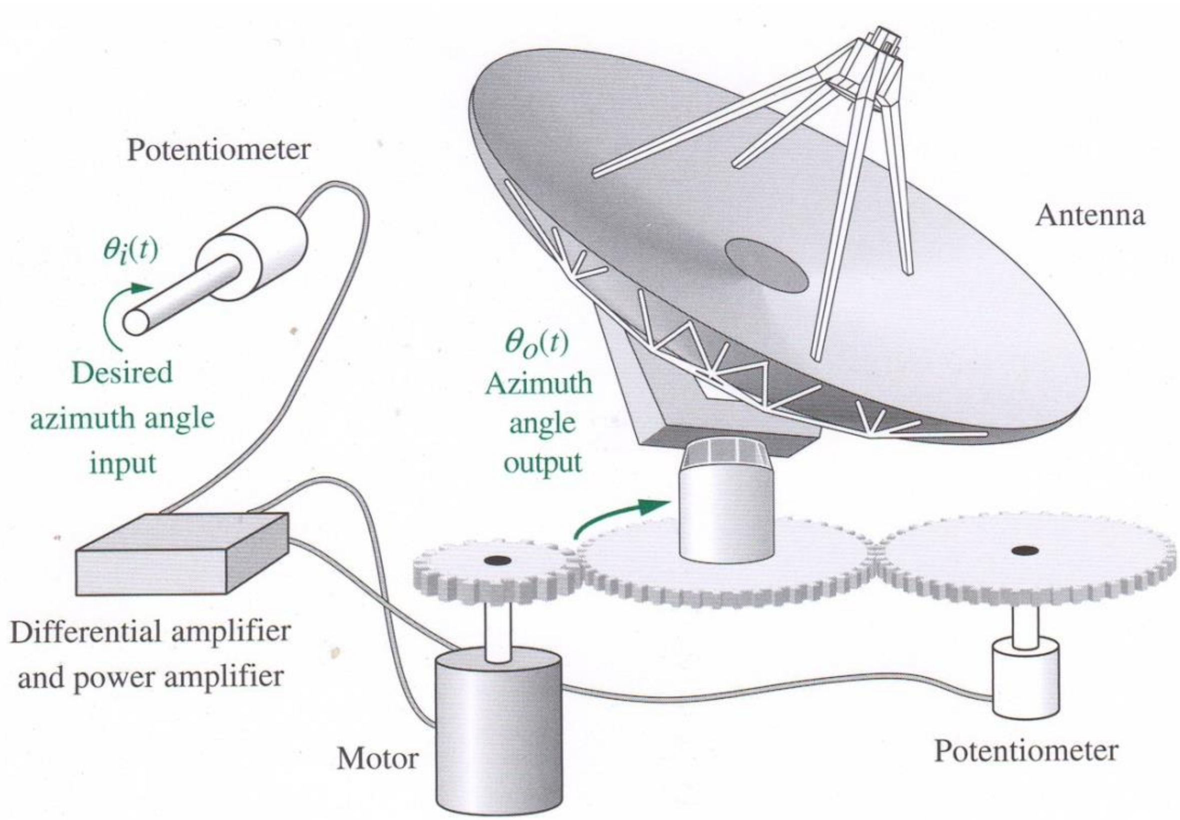
$$\boxed{\frac{V_L(s)}{V(s)} = \frac{s}{s + 0.677}}$$

for small excursions about $i = 14.78$ or, equivalently, about $v(t) = 0$.

Modeling in the Frequency Domain

Case Study: Antenna Control

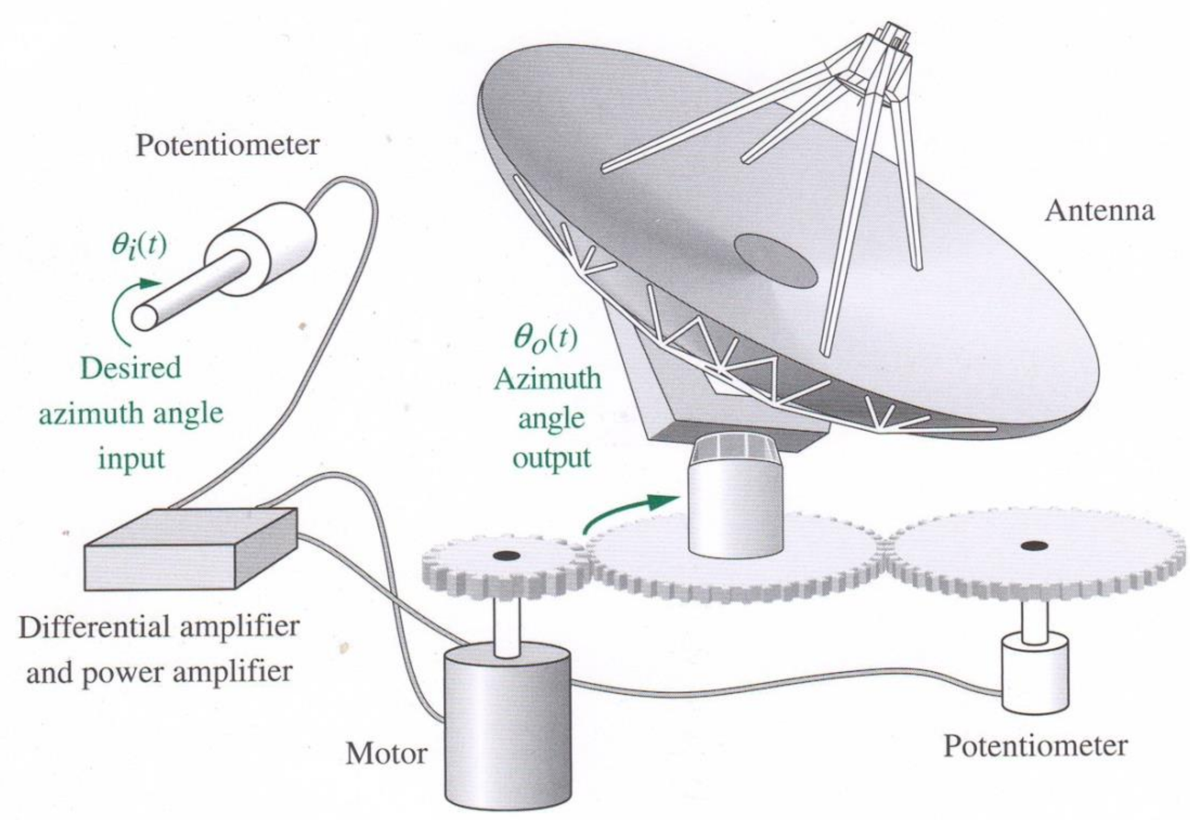
- This chapter showed that physical systems can be modeled mathematically with transfer function. (Typically, systems are composed of subsystems of different types, such as electrical, mechanical, and electromechanical.)



Modeling in the Frequency Domain

Case Study: Antenna Control

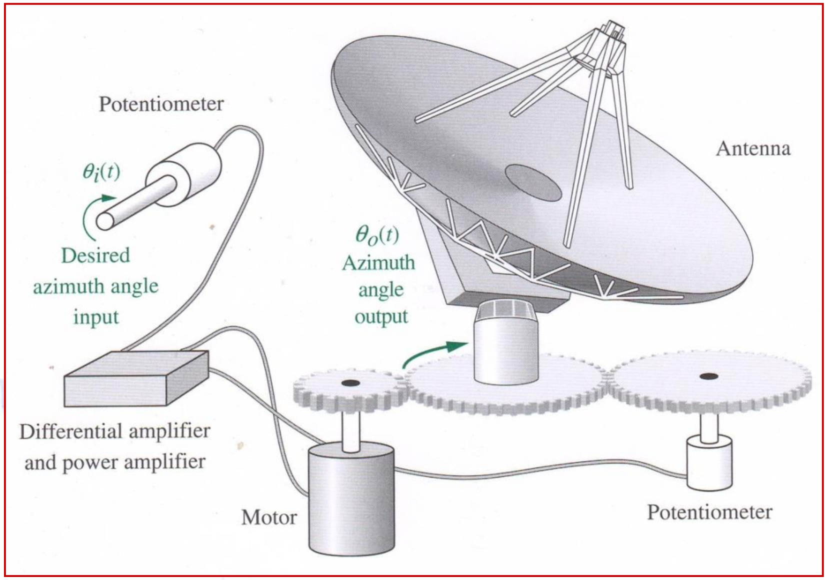
- PROBLEM: Find the transfer function for each subsystem of the antenna azimuth position control system schematic shown on the front endpapers. Use Configuration 1.



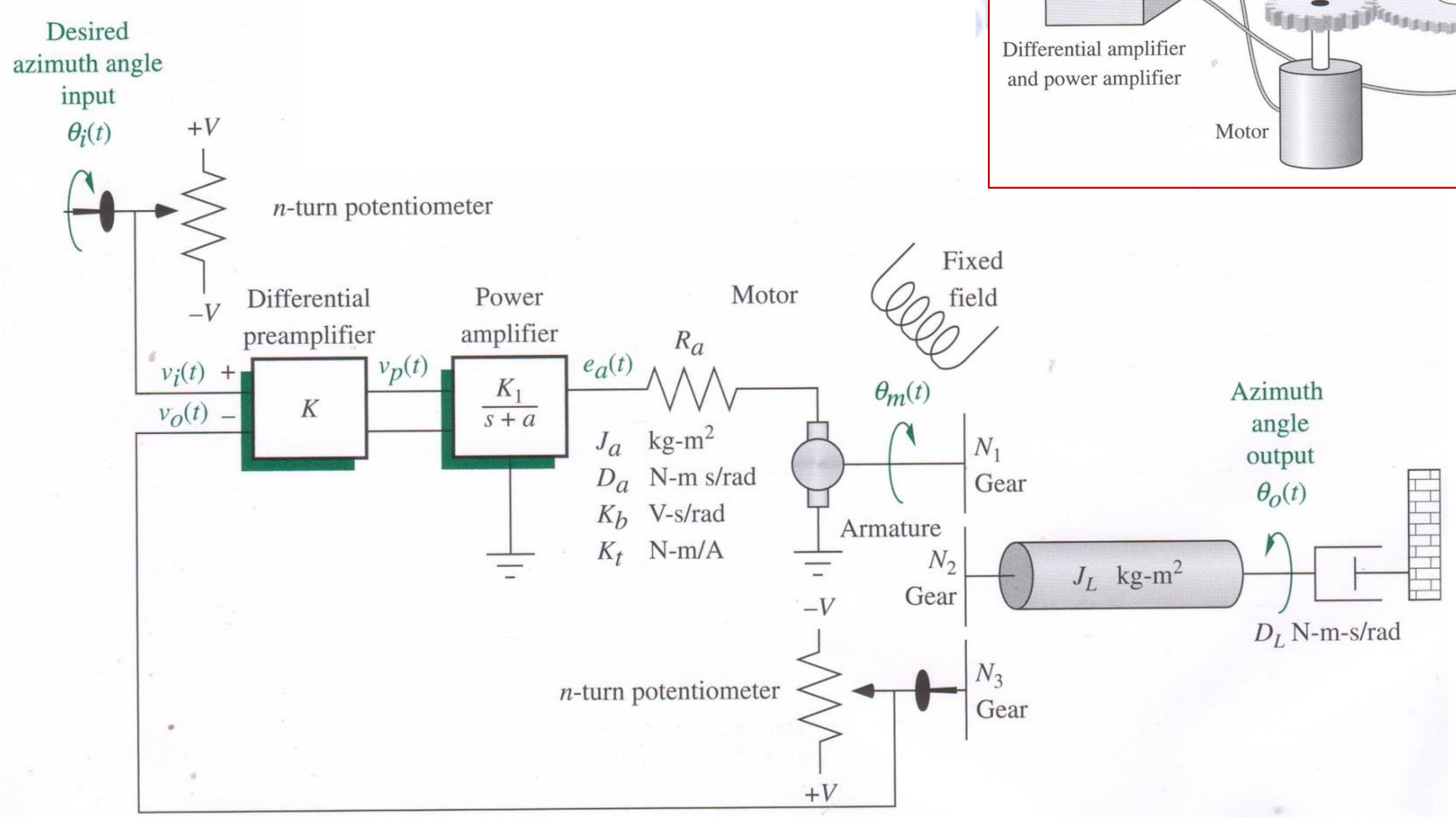
Modeling in the Frequency Domain

Case Study: Antenna Control

1. Identify the individual subsystems.



Schematic

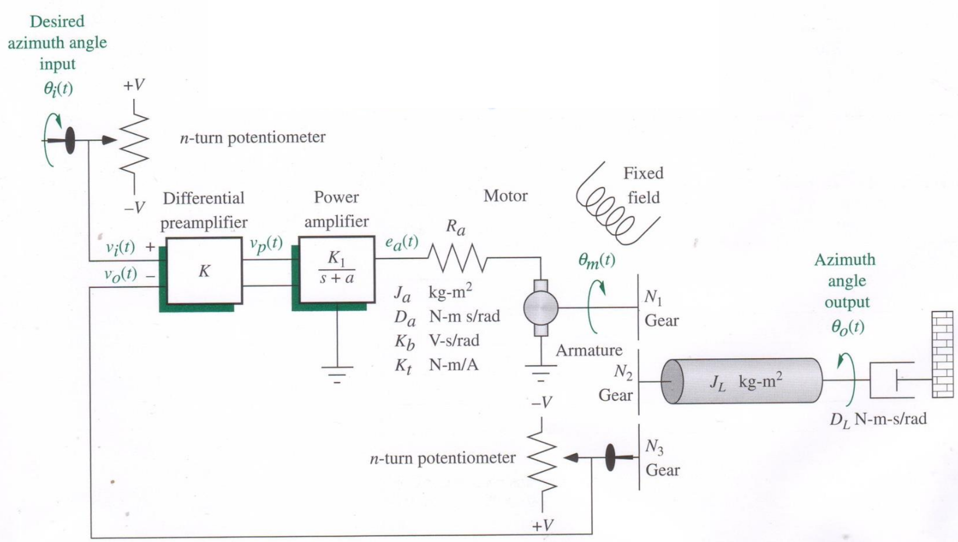


Modeling in the Frequency Domain

Case Study: Antenna Control

2. Identify the transfer functions.
3. Determine the parameters.

Schematic



Schematic Parameters

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	—	—	—
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_t	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250
J_L	1	5	5
D_L	1	3	3

Block Diagram Parameters

Parameter	Configuration 1	Configuration 2	Configuration 3
K_{pot}	0.318		
K	—		
K_1	100		
a	100		
K_m	2.083		
a_m	1.71		
K_g	0.1		

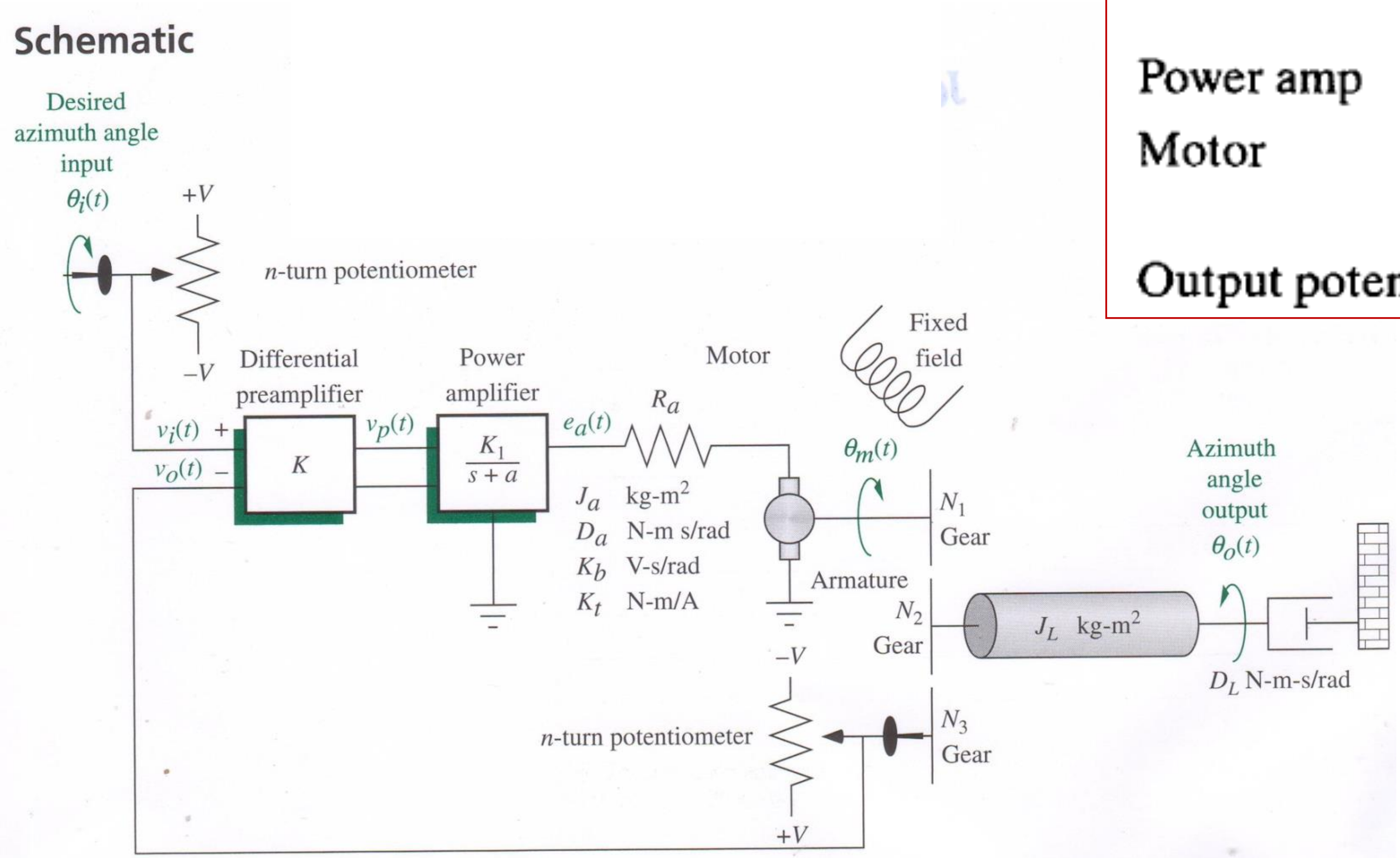
Note: reader may fill in Configuration 2 and Configuration 3 columns after completing the antenna control Case Study challenge problems in Chapters 2 and 10, respectively.

Modeling in the Frequency Domain

Case Study: Antenna Control

1. Identify the individual subsystems.

Subsystem
Input potentiometer
Preamp
Power amp
Motor
Output potentiometer

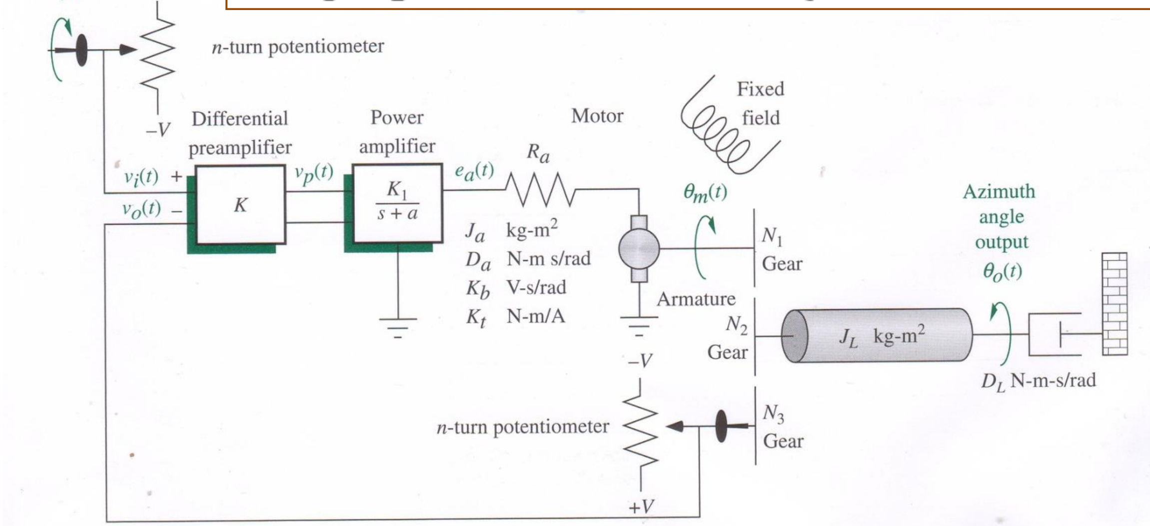


Modeling in the Frequency Domain

Case Study: Antenna Control

Subsystem	Input	Output
Input potentiometer	Angular rotation from user, $\theta_i(t)$	Voltage to preamp, $v_i(t)$
Preamp	Voltage from potentiometers, $v_e(t) = v_i(t) - v_0(t)$	Voltage to power amp, $v_p(t)$
Power amp	Voltage from preamp, $v_p(t)$	Voltage to motor, $e_a(t)$
Motor	Voltage from power amp, $e_a(t)$	Angular rotation to load, $\theta_0(t)$
Output potentiometer	Angular rotation from load, $\theta_0(t)$	Voltage to preamp, $v_0(t)$

Schematic



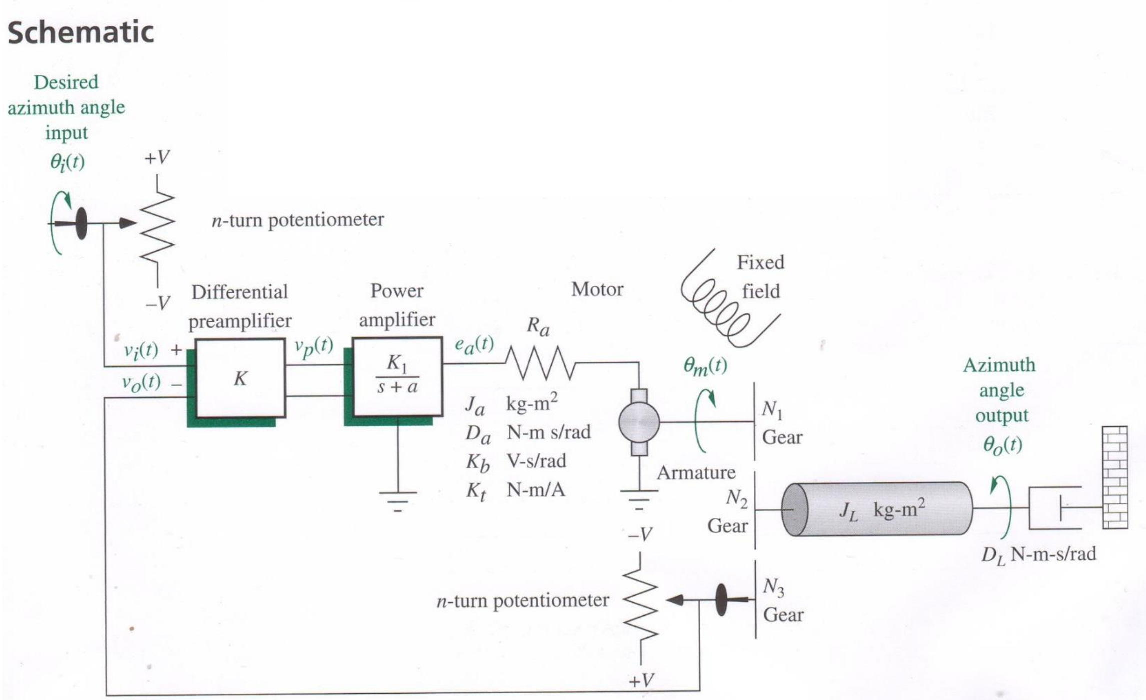
Schematic Parameters

Parameter	Configuration 1
V	10
n	10
K	—
K_1	100
a	100
R_a	8
J_a	0.02
D_a	0.01
K_b	0.5

Modeling in the Frequency Domain

Case Study: Antenna Control

- **Potentiometer :** Since the input and output potentiometers are configured in the same way, their transfer functions will be the same. We *neglect the dynamics for the potentiometers* and simply find the relationship between the output voltage and the input angular displacement.

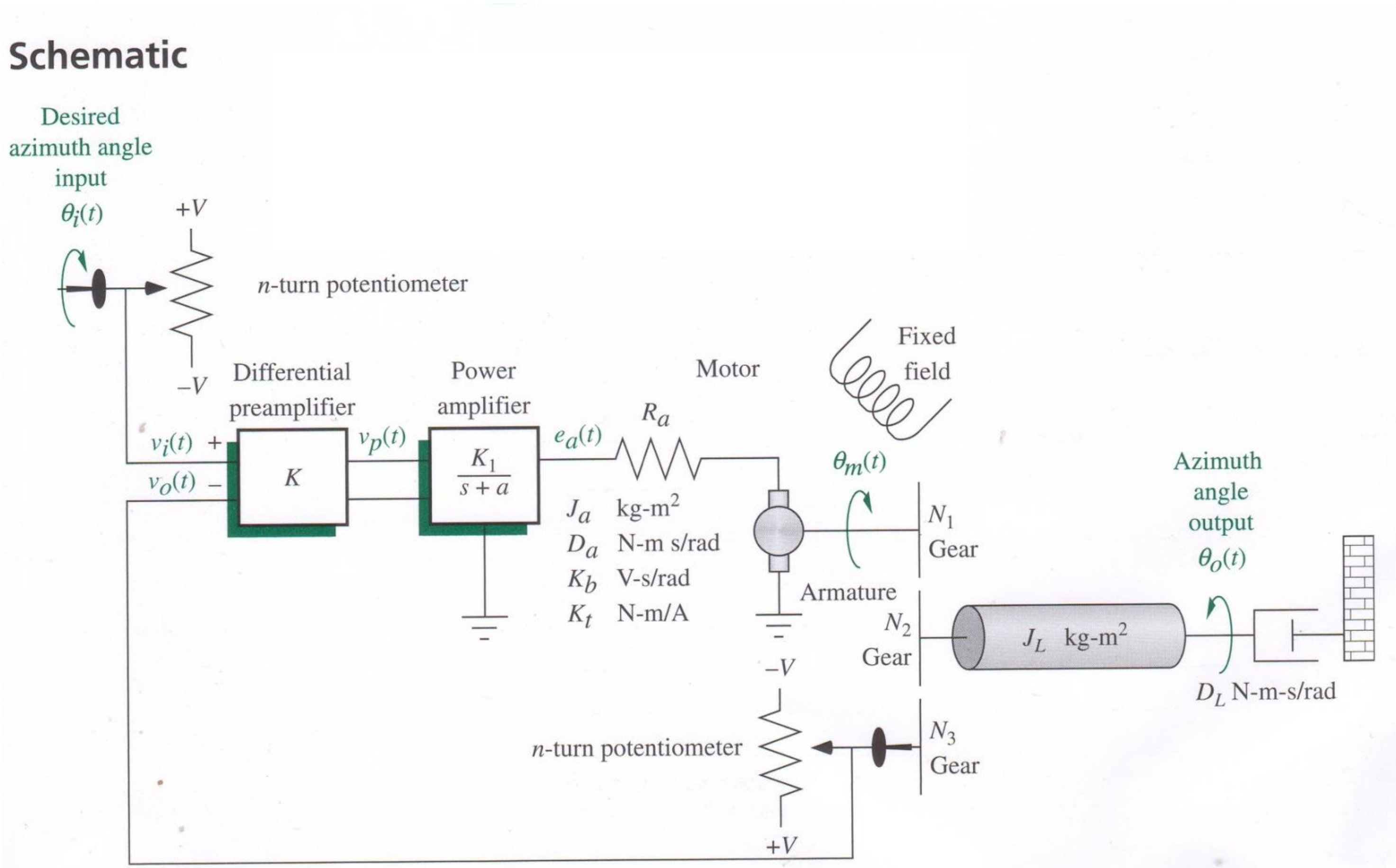


$$\frac{V_i(s)}{\theta_i(s)} = \frac{10}{10\pi} = \frac{1}{\pi}$$

Modeling in the Frequency Domain

Case Study: Antenna Control

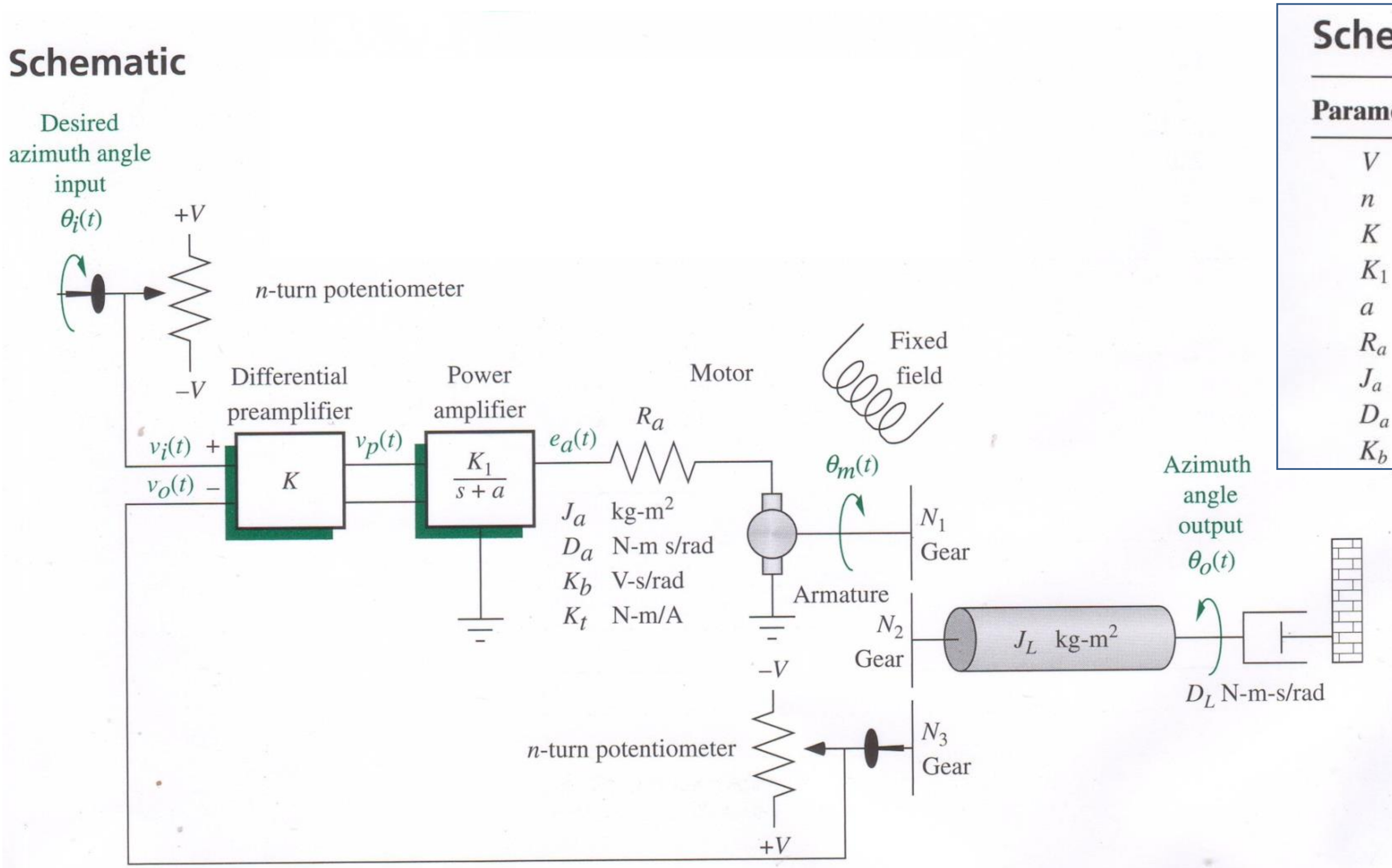
- **Preamplifier; Power Amplifier:** The transfer functions of the amplifiers are given in the problem statement.



Modeling in the Frequency Domain

Case Study: Antenna Control

- **Preamplifier; Power Amplifier:** The transfer functions of the amplifiers are given in the problem statement.



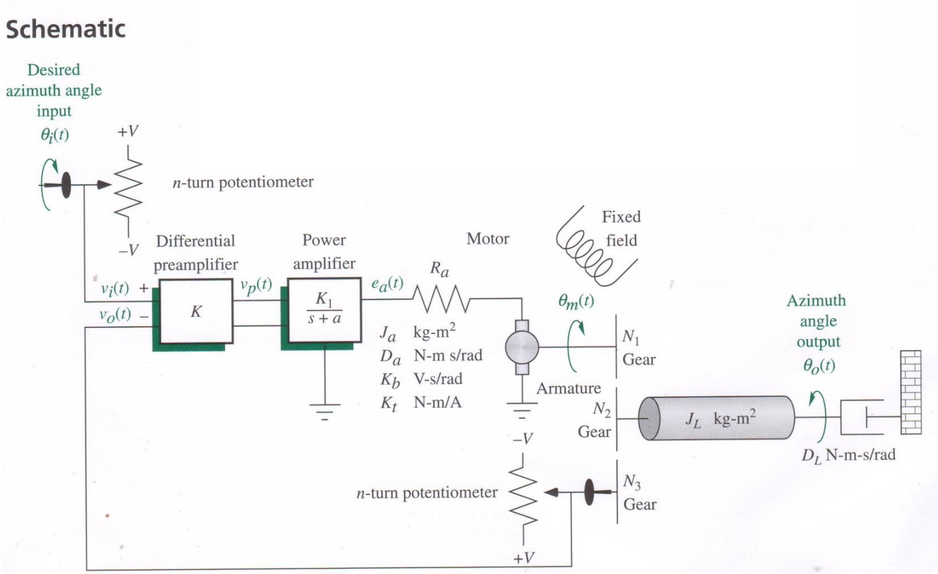
Schematic Parameters

Parameter	Configuration 1
V	10
n	10
K	—
K_1	100
a	100
R_a	8
J_a	0.02
D_a	0.01
K_b	0.5

Modeling in the Frequency Domain

Case Study: Antenna Control

- **Preamplifier; Power Amplifier:** The transfer functions of the amplifiers are given in the problem statement.
- Two phenomena are *neglected.* First, we assume that saturation is never reached. Second, the dynamics of the preamplifier are neglected, since its speed of response is typically much greater than that of the power amplifier.



$$\frac{V_p(s)}{V_e(s)} = K$$

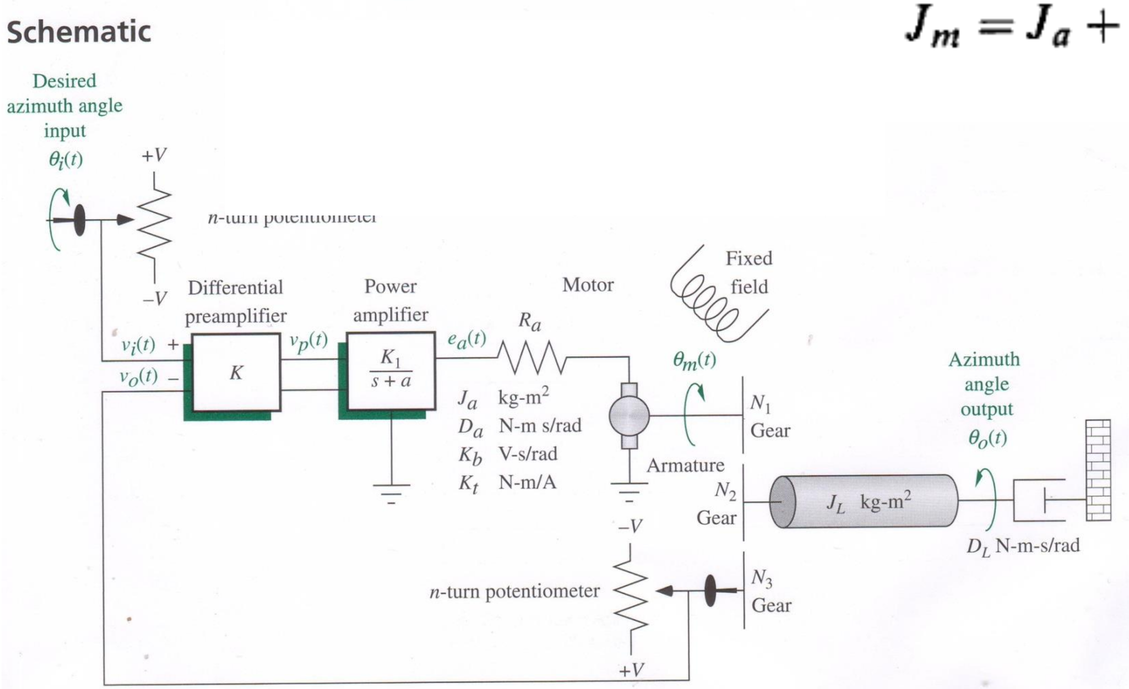
$$\frac{E_a(s)}{V_p(s)} = \frac{100}{s+100}$$

Modeling in the Frequency Domain

Case Study: Antenna Control

- **Motor and Load :**
- The transfer function relating the armature displacement to the armature voltage is given in Eq. (2.153).
- The equivalent inertia, J_m,is

$$J_m = J_a + J_L\left(\frac{25}{250}\right)^2 = 0.02 + 1\frac{1}{100} = 0.03$$



$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_tK_b}{R_a}\right)\right]}}$$

Modeling in the Frequency Domain

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

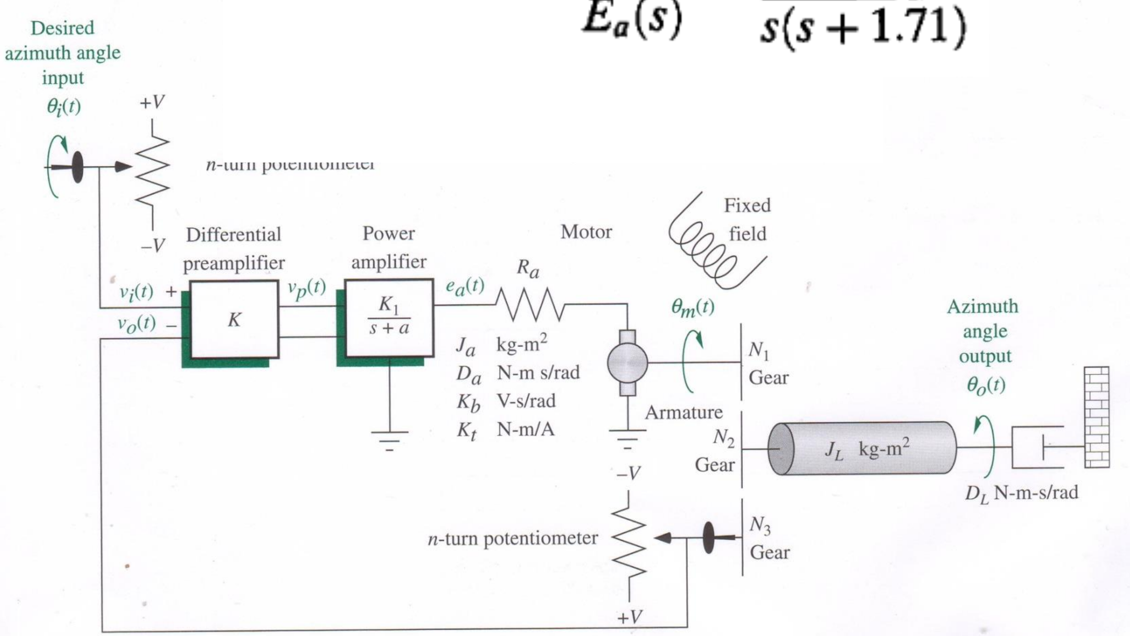
Case Study: Antenna Control

- **Motor and Load :**
- The equivalent viscous damping, Dm, at the armature is

$$D_m = D_a + D_L\left(\frac{25}{250}\right)^2 = 0.01 + 1\frac{1}{100} = 0.02$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2.083}{s(s+1.71)}$$

Schematic



Schematic Parameters

Parameter	Configuration 1
V	10
n	10
K	—
K_1	100
a	100
R_a	8
J_a	0.02
D_a	0.01
K_b	0.5
K_t	0.5
N_1	25
N_2	250
N_3	250
J_L	1
D_L	1

Modeling in the Frequency Domain

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

Case Study: Antenna Control

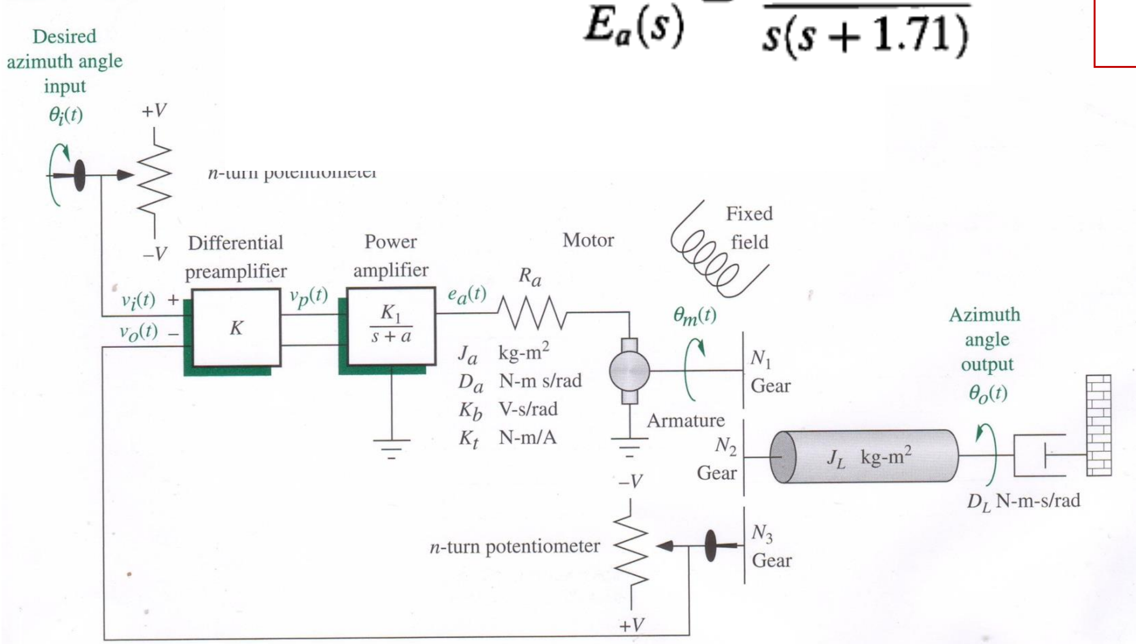
- **Motor and Load :**
- The equivalent viscous damping, Dm, at the armature is

$$D_m = D_a + D_L\left(\frac{25}{250}\right)^2 = 0.01 + 1\frac{1}{100} = 0.02$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2.083}{s(s+1.71)}$$

$$\frac{\theta_0(s)}{E_a(s)} = 0.1\frac{\theta_m(s)}{E_a(s)} = \frac{0.2083}{s(s+1.71)}$$

Schematic



Modeling in the Frequency Domain

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_tK_b}{R_a}\right)\right]}$$

Case Study: Antenna Control

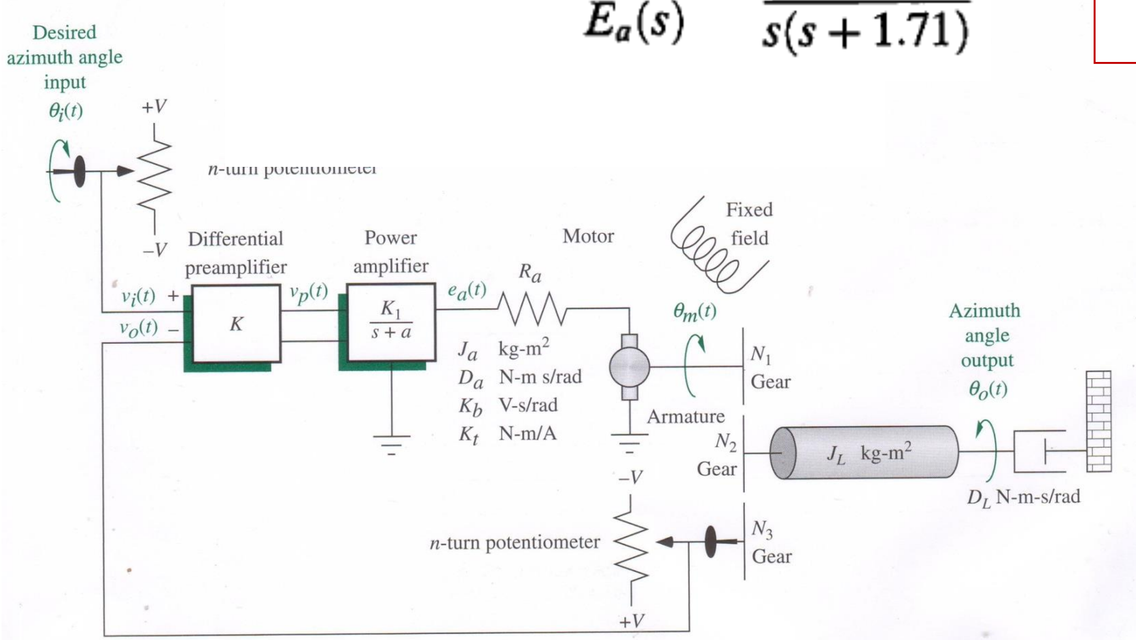
- **Motor and Load :**
- The equivalent viscous damping, Dm, at the armature is

$$D_m = D_a + D_L\left(\frac{25}{250}\right)^2 = 0.01 + 1\frac{1}{100} = 0.02$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2.083}{s(s+1.71)}$$

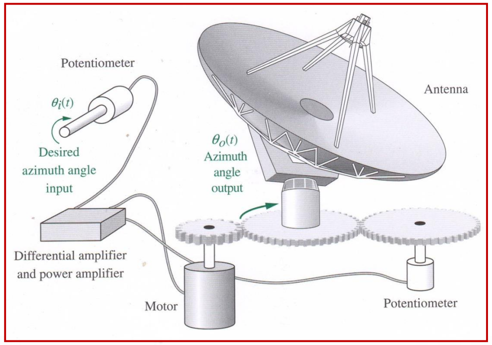
$$\frac{\theta_0(s)}{E_a(s)} = 0.1\frac{\theta_m(s)}{E_a(s)} = \frac{0.2083}{s(s+1.71)}$$

Schematic



Modeling in the Frequency Domain

Case Study: Antenna Control

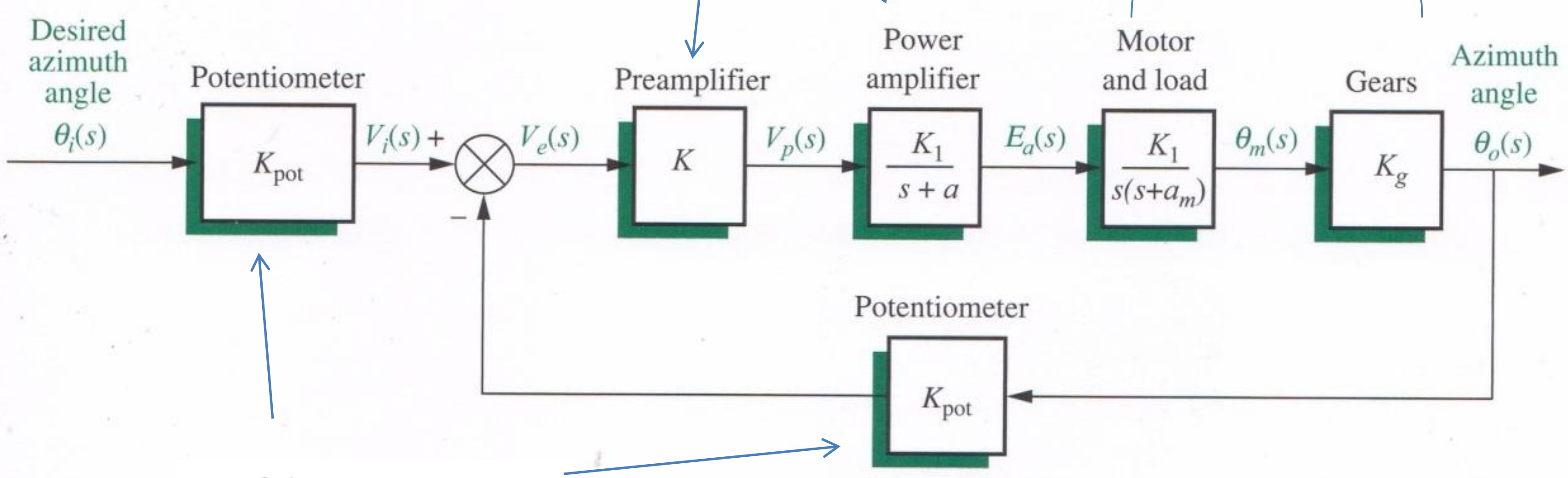


$$\frac{E_a(s)}{V_p(s)} = \frac{100}{s+100}$$

$$\frac{\theta_0(s)}{E_a(s)} = 0.1\,\frac{\theta_m(s)}{E_a(s)} = \frac{0.2083}{s(s+1.71)}$$

$$\frac{V_p(s)}{V_e(s)} = K$$

Block Diagram



$$\frac{V_i(s)}{\theta_i(s)} = \frac{10}{10\pi} = \frac{1}{\pi}$$