② Open loop function is $\frac{3}{s(s+2)}$, Use Root Locus for Design controller that Percent Overshoot is 0%, settling time < 0.8 s

① Find Region

Formula: $PO = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

EQN ; $\zeta = 0.0975$

and $\cos\alpha = \zeta$

$\alpha = \cos^{-1}\zeta$

$= 4.0523^\circ$

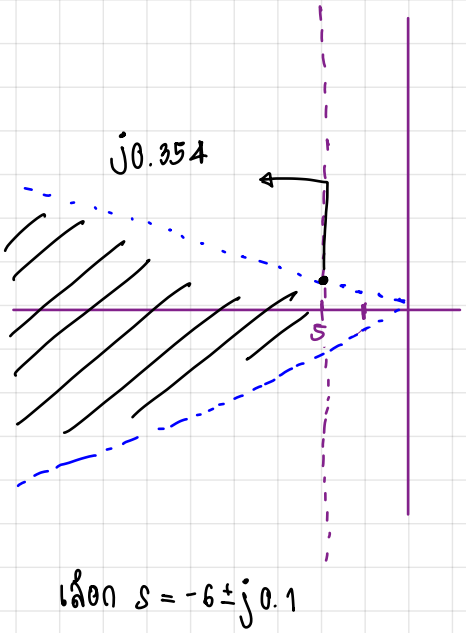
Formula : $T_s < 0.8$

$4\tau < 0.8$

$\therefore \tau < 0.2$ s

$\frac{\tau}{0.2} < 1$

$\frac{1}{0.2} < \frac{1}{\tau}$



เลือก $s = -6 \pm j0.1$

② Plot Root Locus.

Poles is 0 and -2

Zero is none

Centroid is $\frac{\Sigma p - \Sigma z}{\alpha} = \frac{-2-0}{2} = -1$

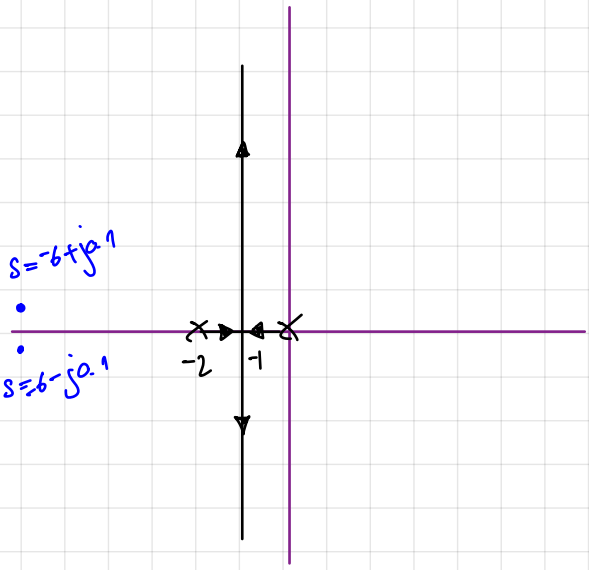
Breakaway Point is $\frac{dK}{ds} = 0$

$$\frac{d}{ds}\left(\frac{s(s+2)}{3}\right) = 0$$

$$s(1) + (s+2)(1) = 0$$

$$2s + 2 = 0$$

$$\boxed{s = -1}$$



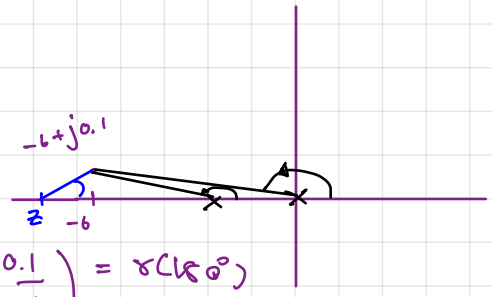
เลือก PD to Shift Root Locus.

③ Design Controller to Control system

$G_{c_1}(s) = K_p + sK_D = K(s+z)$ → PD-Controller to lead system.

Since $G_p(s) = \frac{3}{s(s+2)}$, then $G_c(s)G_p(s) = \frac{3K(s+z)}{s(s+2)}$

From Angle Criterian * let $s = -6 + j0.1$



$$\angle G_p(s) G_c(s) = \angle(s+z) - \angle(s) - \angle(s+2) = \pm(180^\circ)$$

$$= \tan^{-1}\left(\frac{0.1}{z-6}\right) - 90^\circ - \tan^{-1}\left(\frac{0.1}{6}\right) - 90^\circ - \tan^{-1}\left(\frac{0.1}{4}\right) = \pm(180^\circ)$$

$$= \tan^{-1}\left(\frac{0.1}{z-6}\right) - 0.95484^\circ - 1.4321^\circ = 0$$

$$= \tan^{-1}\left(\frac{0.1}{z-6}\right) = 2.38694^\circ$$

$$\frac{0.1}{z-6} = 0.04168$$

$$\therefore z = 8.399$$

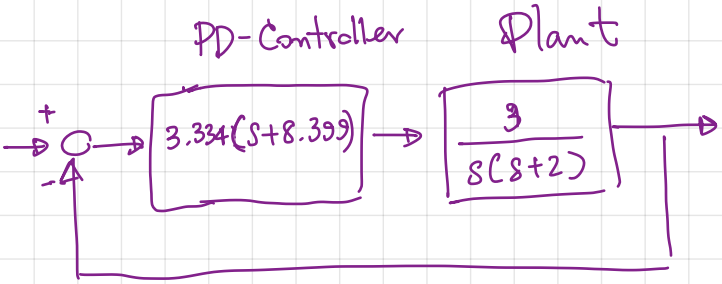
④ Find K

Magnitude Criterian.

$$K = \frac{|s||s+2|}{3|s+8.399|} = \frac{|-6+j0.1||-6+j0.1+2|}{3|-6+j0.1+8.3991|}$$

$$= \frac{\sqrt{(-6)^2+0.1^2}\cdot\sqrt{(-4)^2+0.1^2}}{3\left(\sqrt{2.399^2+0.1^2}\right)}$$

$$\therefore K = 3.334$$



ดังนั้น $G_p(s) G_c(s) = \frac{10(s+8.399)}{s(s+2)}$

System Type is 1

$$e_{ss} = \frac{1}{K_v} \quad ; \quad K_v = \lim_{s \to 0} s\frac{(10)(s+8.399)}{s(s+2)}$$

$$K_v = \frac{10(8.399)}{2} = 41.995$$

$$\therefore e_{ss} = 0.0238$$

③ Open-loop function is $\frac{K(s+15)}{(s+12)(s+6)}$, Use Root Locus to control system when dumpling ratio < 0.707, time constant $< 0.1\,s$ and $\frac{1}{1+k_p} = 0$

① Find Region

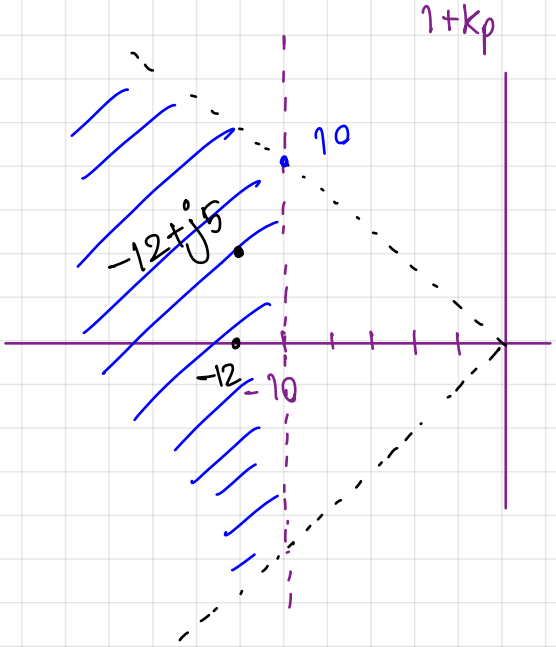
$\cos\alpha = \zeta$

$\alpha = \cos^{-1}(0.707)$

$\therefore \alpha = 45^\circ$

$\zeta < 0.15$

$\tau_0 < \frac{1}{\zeta}$

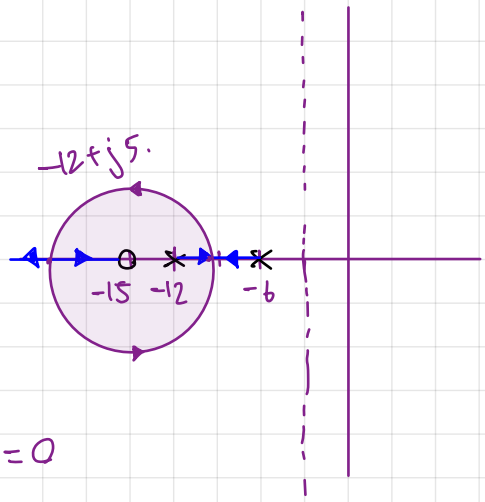


② Find Root Locus.

Poles is -12 and -6

Zero is -15.

Centroid is $\frac{\Sigma P - \Sigma Z}{a} = \frac{-12-6+15}{1} = -3$



Breakaway Point $\rightarrow s^2+18s+72$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{d}{ds}\left[\frac{(s+6)(s+12)}{(s+15)}\right] = 0$$

$$(s+15)\frac{d}{ds}[s+6][s+12] - [s+6][s+12]\frac{d}{ds}[s+15] = 0$$

$$(s+15)(2s+18) - (s^2+18s+72)(1) = 0$$

$$2s^2+18s+30s+270-s^2-18s-72=0$$

$$s^2+30s+198=0$$

$$s = -9.8, -20.2$$