1.)

1.1) $G(s) = \dfrac{10}{s(s+2)}$

a) Find time domain response by R(s) is Unit Step function.

$r(t) = \delta(t)$

* Unit Impulse : $R(s) = 1$ *

$r(t) = u(t)$

* Unit Step : $R(s) = \frac{1}{s}$ *

$r(t) = t$

* Unit Ramp : $R(s) = \frac{1}{s^2}$ *

* Palaboric : $R(s) = \frac{1}{s^3}$ *

↳ $r(t) = \frac{1}{2}t^2u(t)$

Transfer function : $G(s) = \dfrac{C(s)}{R(s)}$

then , $C(s) = G(s) \cdot R(s)$

$$= \frac{10}{s(s+2)} \cdot \frac{1}{s}$$

$$\therefore \quad C(s) = \frac{10}{s^2(s+2)}$$

Using Partial Fraction.

$$\frac{10}{s^2(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$$

Multiple $s^2(s+2)$ to 2 side of equation.

$$10 = A(s)(s+2) + B(s+2) + Cs^2$$

$$10 = As^2 + 2As + Bs + 2B + Cs^2$$

$$10 = (A+C)s^2 + (2A+B)s + 2B$$

Compare Coefficient

$$(A+C)s^2 = 0$$

$$A+C = 0 \quad —(1)$$

$$2A+B = 0 \quad —(2)$$

$$2B = 10$$

$B = 5$ แทนค่าใน (2) ; $2A + 5 = 0 \Rightarrow \therefore A = -5/2$ แทนค่าใน (1)

$$-\frac{5}{2} + C = 0$$

$$\therefore C = \frac{5}{2}$$

$$C(s) = -\frac{5}{2s} + \frac{5}{s^2} + \frac{5}{2(s+2)}$$

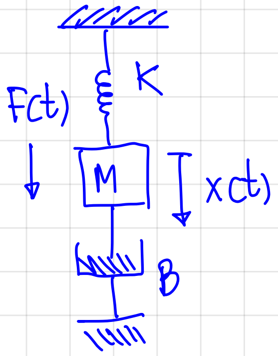
Hence , $c(t) = -\dfrac{5}{2} + 5t + \dfrac{5}{2}e^{-2t}$ ※

b) Find ω_n, ζ

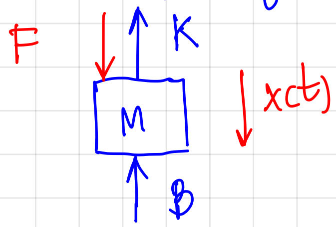
1.2) Force 8.9 N , Find M (kg.)

โจทย์กำหนด $x(t) = 0.03$ m.

$\Delta x(t) = 0.0029$ m/s

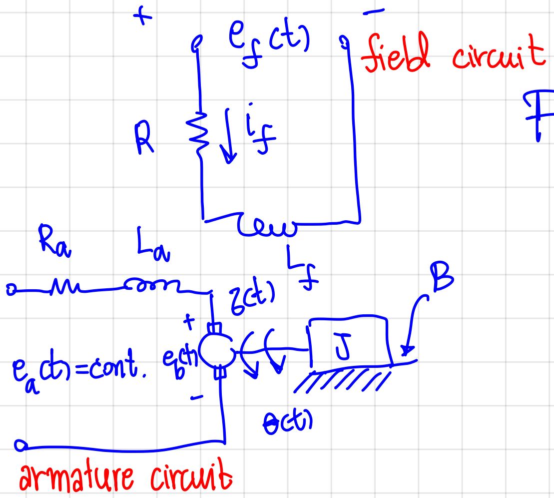


Free Body Diagram



$$F(t) - Kx - B\dot{x} = M\ddot{x}$$

$8.9 -$



Find Transfer function : $G(s) = \dfrac{E_f(s)}{\theta(s)}$

more relative

- $e_b(t) = k_m \theta(t)$ armature
- $\tau(t) = k_m i_a(t)$ armature
- $\tau(t) = k_m i_f(t)$ field

พิจารณาที่ field circuit

KVL ; $i_f R_f + L\dfrac{di_f}{dt} - e_f(t) = 0$

Laplace Transform : $R_f I_f(s) + sLI_f(s) - E_f(s) = 0$

$$I_f(s) = \frac{E_f(s)}{(R_f + sL)} \quad \text{—(1)}$$

พิจารณาที่ Mechanical

$\tau(t) - B\dot{\theta}(t) = J\ddot{\theta}(t)$

Laplace Transform : $\tau(s) - sB\theta(s) = s^2 J\theta(s)$ —(2)

Relative between Electrical - Mechanical

$\tau(t) = K_m i_f(t) \xrightarrow{\mathcal{L}} \tau(s) = K_m I_f(s)$ แทนค่าใน (2);

$$K_m I_f(s) - sB\theta(s) = s^2 J\theta(s)$$

$$I_f(s) = \frac{s^2 J\theta(s) + sB\theta(s)}{K_m} \quad \text{แทนค่าใน (1);}$$

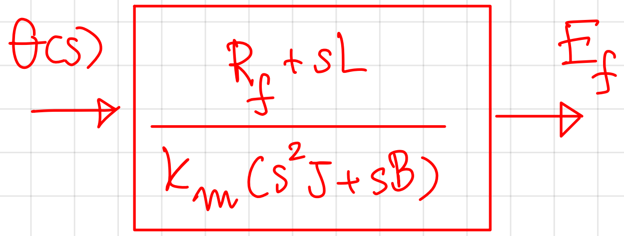
$$\frac{s^2 J\theta(s) + sB\theta(s)}{K_m} = \frac{E_f(s)}{R+sL}$$

We need transfer function : $\dfrac{E_f(s)}{\theta(s)}$

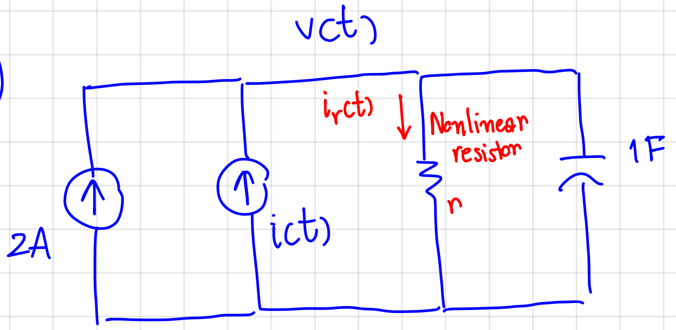
$$\frac{(s^2 J + sB)\theta(s)}{K_m} = \frac{E_f(s)}{R_f+sL}$$

$$\frac{E_f(s)}{\theta(s)} = \frac{R_f+sL}{K_m(s^2 J + sB)}$$

Block Diagram.



3.1)



let $i_r(t) = e^{v}$

Find Transfer function : $\dfrac{V(s)}{I(s)}$

Using Kirchoff's current law.

$$i_r(t) - i(t) - 2 - c\frac{dv(t)}{dt} = 0 \qquad -(1)$$

$$V = V_o + \delta V$$

$$V = 0.693 + \delta V$$

$$\delta V = V - 0.693$$

$$\frac{d\,\delta V}{dt} = \frac{dV}{dt} - \cancel{\frac{d}{dt}(0.693)}^{\,0}$$

then $\displaystyle\int \frac{d\,\delta V}{dt} = \int \frac{dV}{dt}$

$$\boxed{\delta V = V}$$

let $f(v) = e^{V_o + \delta V}$, then $f(v_o) = e^{V_o}$

Form $\quad f(v) - f(v_o) = \left.\dfrac{df}{dv}\right|_{v=v_o} (v - v_o)$

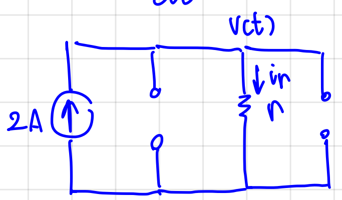
then, $\quad e^{V_o + \delta V} - e^{V_o} = \left.\dfrac{d}{dv} e^{V}\right|_{V=V_o} \delta V$

$$e^{V_o + \delta V} = e^{V_o}\delta V + e^{V_o} \quad \text{แทนใน (2)}$$

จาก (1) ; $\quad e^{V_o + \delta V} - i(t) - 2 - c\dfrac{d}{dt}(V_o + \delta V) = 0 \qquad -(2)$

$$e^{V_o}\delta V + e^{V_o} - i(t) - 2 - c\frac{d\,\delta V}{dt} = 0$$

พิจารณาวงจรที่สภาวะ: Steady-State



$$-2A + i_r = 0$$

$$i_r = 2 \text{ A}$$

เนื่องจาก $i_r = e^{V_o}$

$$e^{V_o} = 2 \Rightarrow V_o = \ln 2$$

then, $\quad 2\delta V + \cancel{2} - i(t) - \cancel{2} - c\dfrac{d\,\delta V}{dt} = 0$

Using Laplace Transform ;

$$2\delta V - I(s) - sC\,\delta V(s) = 0$$

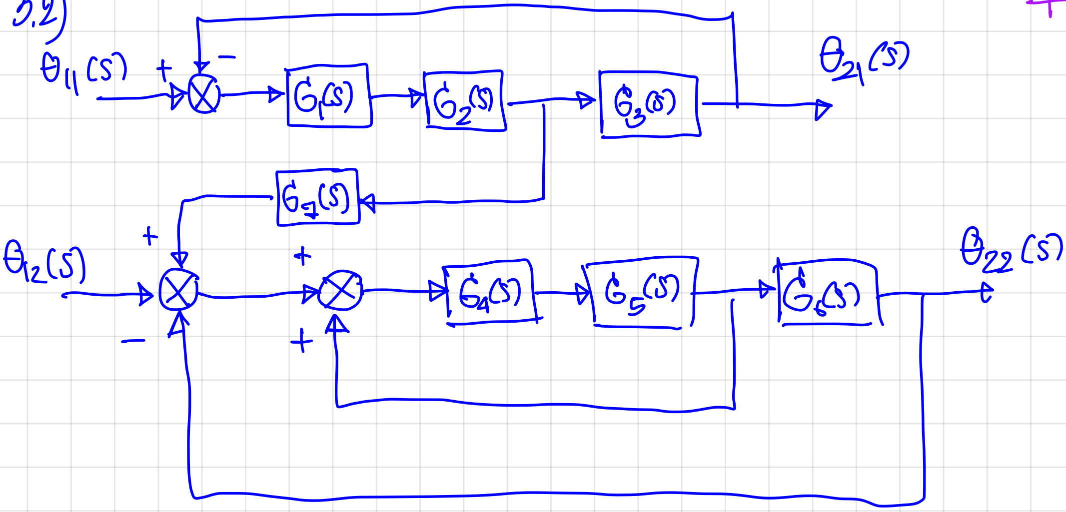
$$(2 + sC)\,\delta V(s) = I(s)$$

We need tranfer function : $\dfrac{V(s)}{I(s)}$, then.

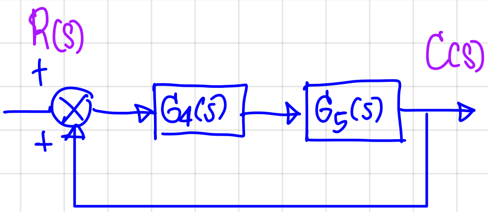
Hence, $\quad \boxed{\dfrac{\delta V(s)}{I(s)} = \dfrac{1}{2 + sC}}$ $\quad$ by $\delta V(s) = V(s)$ #

3.2)

Find transfer function $\dfrac{\theta_{22}(s)}{\theta_{12}(s)}$



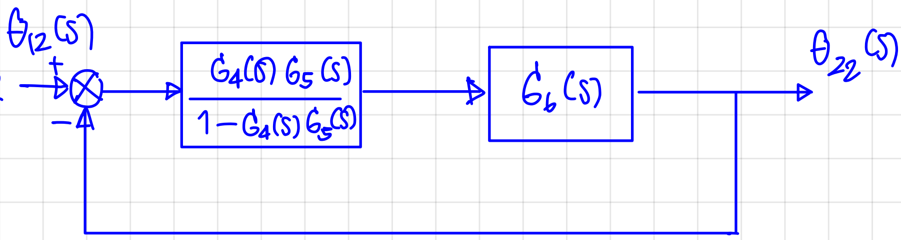
Step 1



Feedback

$$G(s) = \frac{C(s)}{R(s)} = \frac{G_4 G_5}{1 - G_4 G_5}$$

Step 2

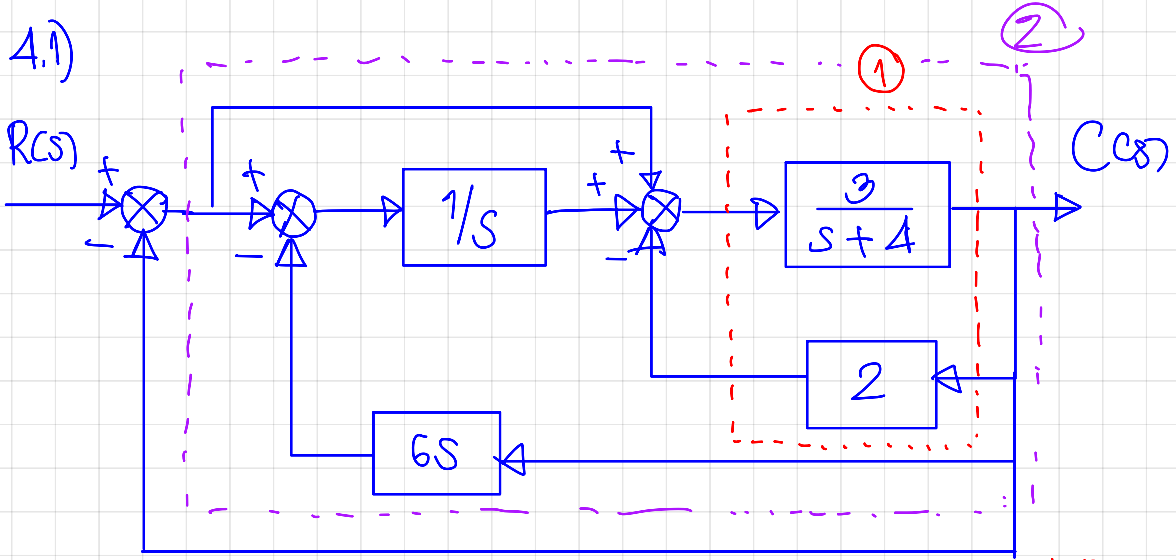


$$G(s) = \frac{\theta_{22}(s)}{\theta_{12}(s)} = \frac{\dfrac{G_4(s)\,G_5(s)\,G_6(s)}{1 - G_4(s)\,G_5(s)}}{1 + \dfrac{G_4(s)\,G_5(s)\,G_6(s)}{1 - G_4(s)\,G_5(s)}}$$

$$= \frac{G_4(s)\,G_5(s)\,G_6(s)}{1 - G_4(s)\,G_5(s) + G_4(s)\,G_5(s)\,G_6(s)}$$

Hence, Transfer function $\dfrac{\theta_{22}(s)}{\theta_{12}(s)} = \dfrac{G_4(s)\,G_5(s)\,G_6(s)}{1 - G_4(s)\,G_5(s) + G_4(s)\,G_5(s)\,G_6(s)}$

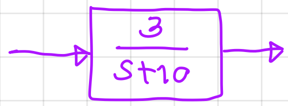
4.1)



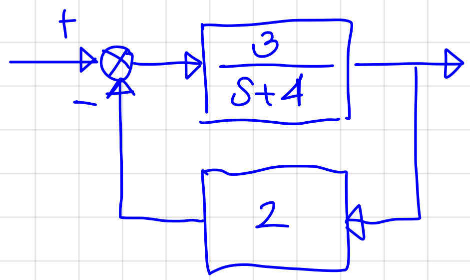
Unit Step, Unit Ramp, Palaboric

Find Steady-State Error of $10u(t)$, $10tu(t)$, $10t^2u(t)$

Step 1 Find equivalent Block Diagram.

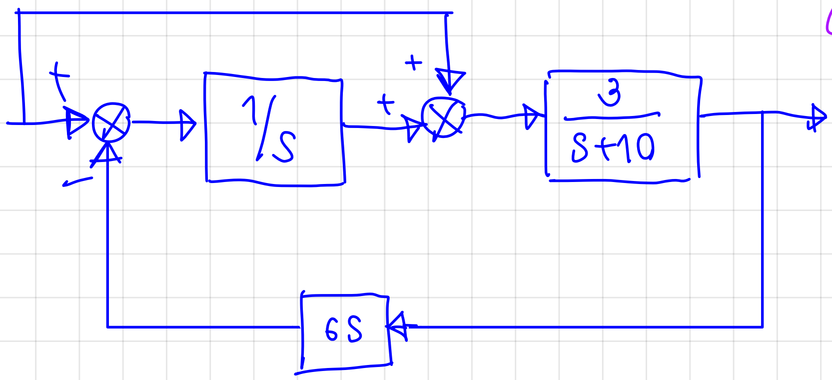


①

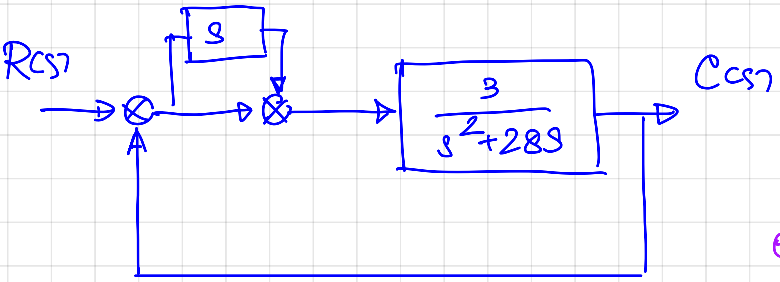


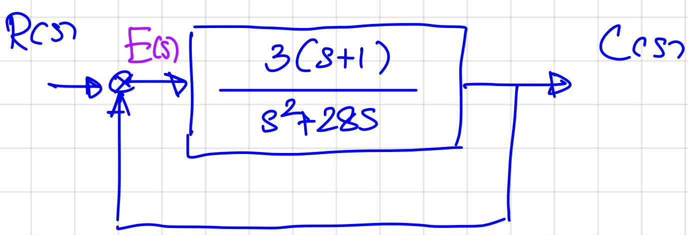
$$G(s) = \frac{\frac{3}{s+4}}{1+\frac{6}{s+4}} = \frac{3}{s+4+6} = \frac{3}{s+10}$$

②



$$G(s) = \frac{\left(\frac{1}{s}\right)\left(\frac{3}{s+10}\right)}{1+\left(\frac{1}{s}\right)\left(\frac{3}{s+10}\right)6s} = \frac{3}{(s)(s+10)+18s} = \frac{3}{s^2+28s}$$



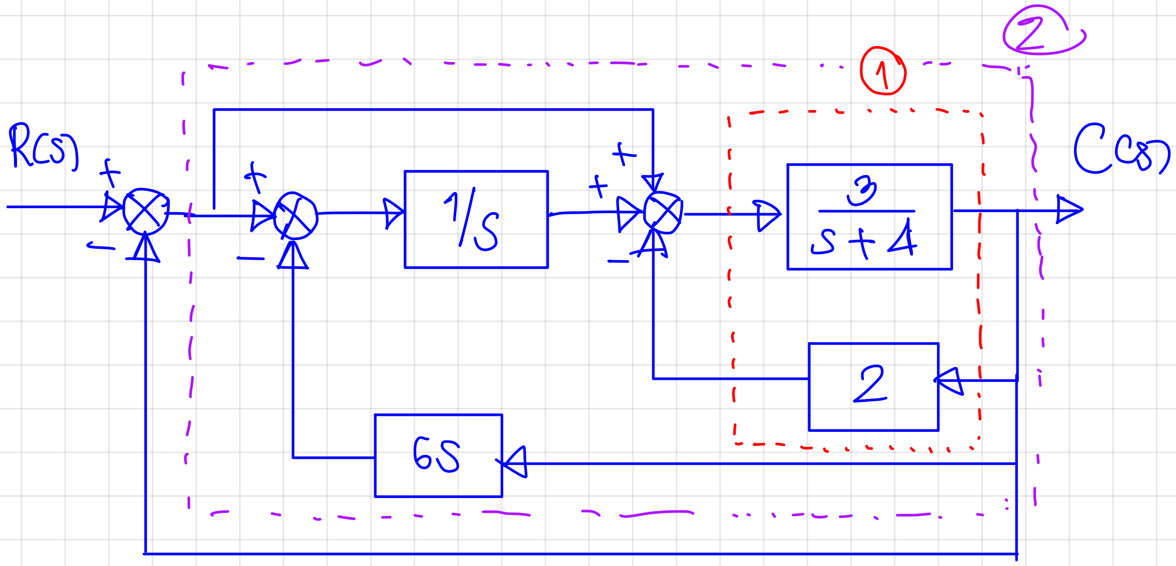


$\boxed{N = 1}$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{sR(s)}{1+\frac{3(s+1)}{s^2+28s}}$$

$$= \lim_{s\to 0} \frac{s(s^2+28s)R(s)}{s^2+28s+3s+3}$$



$$\frac{\frac{3}{s+4}}{1+8\cdot\frac{3}{s+4}} = \frac{3}{s+4+24} = \frac{3}{s+28}$$

$$\left(\frac{1}{s}+1\right)\left(\frac{3}{s+28}\right) = \left(\frac{s+1}{s}\right)\left(\frac{3}{s+28}\right) = \frac{3(s+1)}{s(s+28)}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} = \cancel{s}\left(\frac{10}{\cancel{s}}\right)\cdot\left(\frac{1}{1+\frac{3(s+1)}{s(s+28)}}\right)$$

$$= \frac{10}{s(s+28)+(3s+3)} = \frac{10}{s^2+28s+3s+3}$$

$$= \frac{10\,(s\,(s+28))}{s^2+28s+3s+3}$$

$$= 0$$

$$e_{ss} = \lim_{s\to 0}$$

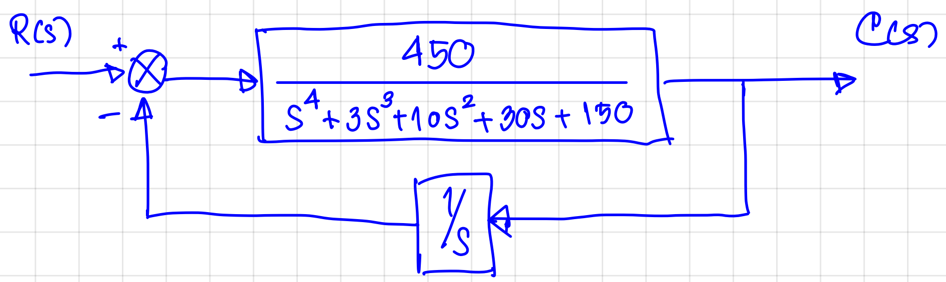
an $e_{ss} = \lim_{s \to 0} \frac{s(s^2+28s)R(s)}{s^2+28s+3}$

let $R(s) = \frac{10}{s}$; $e_{ss} = \lim_{s \to 0} \frac{10(s^2+28s)}{s^2+28s+3} = \lim_{s \to 0} \frac{s+28}{s+28+3/s} = \frac{0+28}{0+28+\infty} = 0$

let $R(s) = \frac{10}{s^2}$; $e_{ss} = \lim_{s \to 0} \frac{s(s^2+28s) \times 10}{(s^2+28s+3) \cdot s^2} = \lim_{s \to 0} \frac{s+28}{s^2+28s+3} \cdot 10 = \frac{0+28}{0+0+3} \cdot 10 = 93.33$

let $R(s) = \frac{10}{s^3}$; $e_{ss} = \lim_{s \to 0} \frac{s(s^2+28s)}{s^2+28s+3} \cdot \frac{10}{s^{3}} = \lim_{s \to 0} \frac{s+28}{s^2+28s+3} \cdot \frac{10}{s} = \frac{280}{0} = \infty$

4.2)



$G(s) = \frac{C(s)}{R(s)} = \frac{450}{s^4+3s^3+1}$