Problem : Root Locus Techniques.

3. Sketch the root locus for the unity feedback system shown in Figure P8.3 for the following transfer functions: [Section: 8.4]

a. $G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25}$

b. $G(s) = \dfrac{K(s^2+4)}{(s^2+1)}$

c. $G(s) = \dfrac{K(s^2+1)}{s^2}$

d. $G(s) = \dfrac{K}{(s+1)^3(s+4)}$

a. Transfer function

$G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25}$

Using Quadratic's formula

$s = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(25)}}{2(1)}$

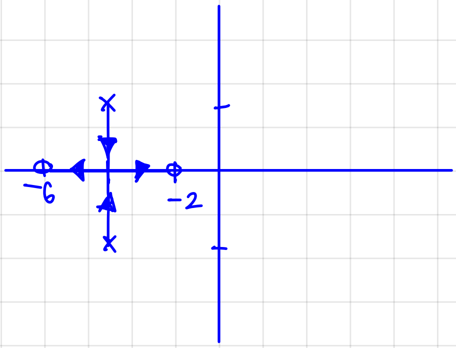
$= -4 \pm j3$

$\therefore$ poles is $-4+j3$ and $-4-j3$

zeroes is -2 and -6

- $\alpha = n - m = 2 - 2 = 0 \rightarrow$ asymptote line is 0

- Centroid is none

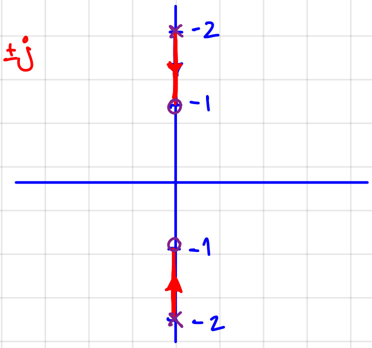


b. Transfer function

$G(s) = \dfrac{K(s^2+4)}{(s^2+1)}$ $\;s = \pm j2$; $s = \pm j$

Poles = $+j2$, $-j2$

Zeroes = $+j$, $-j$

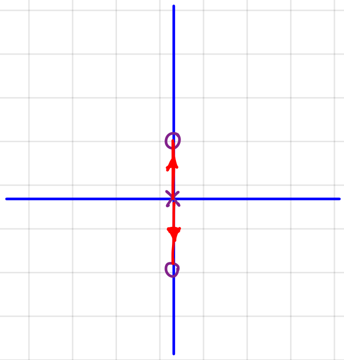


c. Transfer function

$G(s) = \dfrac{K(s^2+1)}{s^2}$

Poles = 0, 0

Zeroes = $-j$, $+j$



d. Transfer function

$G(s) = \dfrac{K}{(s+1)^3(s+4)}$

Poles is $-1, -1, -1, -4$

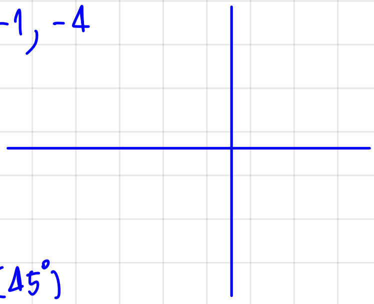
zeroes is none

$\alpha = n - m = 4$

$\theta = \dfrac{r(180^\circ)}{4} = r(45^\circ)$

$\therefore \theta = +45^\circ, -45^\circ, +135^\circ$ and -135°

$\sigma_a = \dfrac{\Sigma p - \Sigma z}{\alpha} = \dfrac{-1-1-1-4}{4} = -1.75$



Breakaway Point

$\dfrac{dK}{ds} = 0 \Rightarrow \dfrac{d}{ds}\left[(s+1)^3(s+4)\right] = 0$

$(s+1)^3 \dfrac{d}{ds}(s+4) + (s+4)\dfrac{d}{ds}(s+1)^3 = 0$

$(s+1)^3 + 3(s+4)(s+1)^2 = 0$

$s^2+4s+s+4 = s^2+5s+4$

$(s+1)^2 + 3(s+4)(s+1) = 0$

$s^2+2s+1+3s^2+15s+12 = 0$

$4s^2+17s+13 = 0$

$s = -1, -3.25$