Nonlinearization.

Answer $\dfrac{6.3245}{s+6.3245} = \dfrac{\delta V_r(s)}{V(s)}$

for small excursions about the equilibrium point, $\theta = 0$.

CHALLENGE: We now introduce a case study challenge to test your knowledge of this chapter's objectives. Although the physical system is different from a human leg, the problem demonstrates the same principles: linearization followed by transfer function evaluation.

Given the nonlinear electrical network shown in Figure 2.53, find the transfer function relating the output nonlinear resistor voltage, $V_r(s)$, to the input source voltage, $V(s)$.

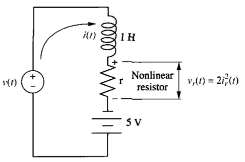

FIGURE 2.53 Nonlinear electric circuit

Using Kirchoff's Voltage law

$$-v(t) + L\frac{di(t)}{dt} + v_r(t) - 5 = 0$$

$$-v(t) + L\frac{di(t)}{dt} + 2i_r^2(t) - 5 = 0 \quad —(1)$$

let $i_r = i_0 + \delta i$ แทนใน (1);

$$-v(t) + L\frac{d(i_0+\delta i)}{dt} + 2(i_0+\delta i)^2 - 5 = 0 \quad —(2)$$

Linearization

From : $f(x) - f(x_0) = \left.\dfrac{df}{dx}\right|_{x=x_0}(x - x_0)$

then ; $(i_0+\delta i)^2 - i_0^2 = \left.\dfrac{d\, i^2}{di}\right|_{i=i_0} \delta i$

$\therefore (i_0+\delta i)^2 = 2i_0\delta i + i_0^2$ แทนใน (2)

$$-v(t) + L\frac{d\,\delta i}{dt} + 2(2i_0\delta i + i_0^2) - 5 = 0$$

$$-v(t) + L\frac{d\,\delta i}{dt} + \cancel{5} + 4\sqrt{2.5}\,\delta i - \cancel{5} = 0$$

Laplace Transform

$$-V(s) + sL\,\delta I(s) + 4\sqrt{2.5}\,\delta I(s) = 0$$

$$(4\sqrt{2.5} + sL)\,\delta I(s) = V(s)$$

$$\frac{sV_r(s) - 5}{4\sqrt{2.5}\,s}(4\sqrt{2.5} + s) = V(s)$$

$$(sV_r(s) - 5)\left(\frac{4\sqrt{2.5} + s}{4\sqrt{2.5}\,s}\right) = V(s)$$

พิจารณาในสภาวะ: Steady-State.

$V = V_r = 5\text{ V}$

$5 = 2i_r^2$

$i_r = \sqrt{2.5}$

$V_r(t) = 2i_r^2$

$V_r(t) = 2(i_0 + \delta i_r)^2$

$V_r(t) = 2(2i_0\delta i_r + i_0^2)$

$V_r(t) = 4\sqrt{2.5}\,\delta i_r + 5$

$V_r(t) = 4\sqrt{2.5}\,\delta i_r + 5$

$V_r(s) = 4\sqrt{2.5}\,\delta I_r(s) + \dfrac{5}{s}$

$sV_r(s) - 5 = 4\sqrt{2.5}\,\delta s I_r(s)$

$\dfrac{sV_r(s) - 5}{4\sqrt{2.5}\,s} = \delta I_r(s)$

PROBLEM: Linearize Eq. (2.184) for small excursions about $x = \pi/4$.

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + \cos x = 0 \quad \text{— (1)}$$

let $x = x_0 + \delta x$ when $x_0 = \pi/4$ แทนค่าใน (1) ;

$$\frac{d^2}{dt^2}\left(\pi/4 + \delta x\right) + 2\frac{d}{dt}\left(\pi/4 + \delta x\right) + \cos\left(\pi/4 + \delta x\right) = 0$$

$$\frac{d^2}{dt^2}(\delta x) + 2\frac{d}{dt}(\delta x) + \cos\left(\pi/4 + \delta x\right) = 0 \quad \text{— (1)}$$

let $f(x) = \cos\left(\pi/4 + \delta x\right)$

From : $f(x) - f(x_0) = \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0)$ — (2)

$$\cos\left(\pi/4 + \delta x\right) - \cos\left(\pi/4\right) = \frac{d}{dx}\cos x \cdot \delta x$$

$$= -\sin(x)\Big|_{x=\pi/4} \cdot \delta x$$

$$\therefore \cos\left(\pi/4 + \delta x\right) = \cos\pi/4 - \delta x \sin\pi/4$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\delta x$$ แทนค่าใน (1)

$$\frac{d^2}{dt^2}\delta x + 2\frac{d}{dt}\delta x + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\delta x = 0$$