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Influence of components precision on characteristics of dual microphone arrays

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ABSTRACT

Microphone arrays have great potential in practical applications due to their ability for significant improvement in speech quality and signal to noise ratio in noisy environments. Large amount of scientific papers and patents have been devoted to different algorithmic techniques for producing optimal output of microphone array using different optimization criteria. However, in practice performance of microphone arrays in a large extent depend on the quality of their components such as amplitude matching, phase matching, error in distance between microphones and etc. This paper analyses dependence of a dual microphone array characteristics on the above factors.

1. INTRODUCTION

Microphone arrays have great potential in practical applications due to their ability for significant improvement in speech quality and signal to noise ratio in noisy environments.

The fundamental theory of beamforming is well established, and there are plenty of beamforming algorithms that perfectly work in theory. However, in practice many of these beamforming techniques are very sensitive to microphone amplitude and phase matching as well other factors reflecting the quality of microphone array components and its design. Influence of these factors on microphone arrays performance is not well addressed in scientific literature, patents or patent applications.

The present paper investigates influence of different factors and parameters of a dual microphone array on its characteristics. Factors such as amplitude matching, phase matching, error in distance between microphones are analyzed and reported.

2. THEORETICAL GRADIENT DUAL MICROPHONE ARRAYS

First of all let's consider theoretical output of a first order gradient dual microphone array. Figure 1 show a schematic of such array.

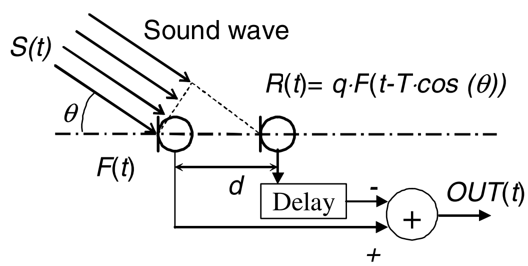

Figure 1 Theoretical gradient dual microphone array

For a plane wave case of angular frequency ω the signals on the front and rear microphones can be represented as

$$M_F(t) = e^{j\omega t}, \quad M_R(t,\varphi) = e^{j(\omega t - \varphi - \varphi_\tau)},$$

where $\varphi = \omega T \cdot \cos(\theta)$ is phase difference between front and rear microphone signals, θ is the angle of incidence relative to the microphone axis, $T = d/V_S$ is sound propagation time, d is distance between microphones, V_S is sound velocity, $\varphi_\tau = \omega \cdot \tau$ is phase difference produced by additional delay element and τ is the additional delay.

The output signal is than given as:

$$M_{OUT}(t,\varphi) = M_F(t) - M_R(t,\varphi) = e^{j\omega t}(1 - e^{-j(\varphi + \varphi_\tau)}) \tag{1}$$

Taking the magnitude of the output signal (1) gives:

$$A_{OUT}(\varphi) = \left| M_{OUT}(t,\varphi) \right| = 2\left| \sin\left(\frac{\varphi + \varphi_\tau}{2}\right) \right| \tag{2}$$

For small distances d (relative to wave length) $\varphi + \varphi_\tau << 1$ so that expression (2) can be simplified as

$$2\left| \sin\left(\frac{\varphi + \varphi_\tau}{2}\right) \right| \approx 2\left| \frac{\varphi + \varphi_\tau}{2} \right| = \omega(T\left|\cos(\theta)\right| + \tau) \tag{3}$$

Varying the delay τ between 0 and T, it is possible to get different polar patterns of the resulting directional microphone. For example, $\tau=0$ leads to a bi-directional (figure eight) pattern, $\tau=T$ leads to a cardioid pattern, $\tau=0.5T$ leads to a super-cardioid pattern [1].

According to (3) array sensitivity is proportional to frequency. Hence if a flat frequency response is required, proper compensation is to be done [2].

Since sensitivity to the front direction ($\theta=0$) need to be compensated, the compensated output signal is given as:

$$A^N_{OUT}(\varphi) = \frac{A_{OUT}(\varphi)}{A_{OUT}(\omega T)} = \left| \frac{\sin(0.5 \cdot (\varphi + \varphi_\tau))}{\sin(0.5 \cdot \omega(T + \tau))} \right| = K \cdot 2\left| \sin\left(\frac{\varphi + \varphi_\tau}{2}\right) \right|$$

where K is compensation coefficient:

$$K = \frac{1}{2\left|\sin(0.5 \cdot \omega(T + \tau))\right|} \tag{4}$$

Figure 2 plots compensation coefficient as a function of frequency for two distances between the microphones.

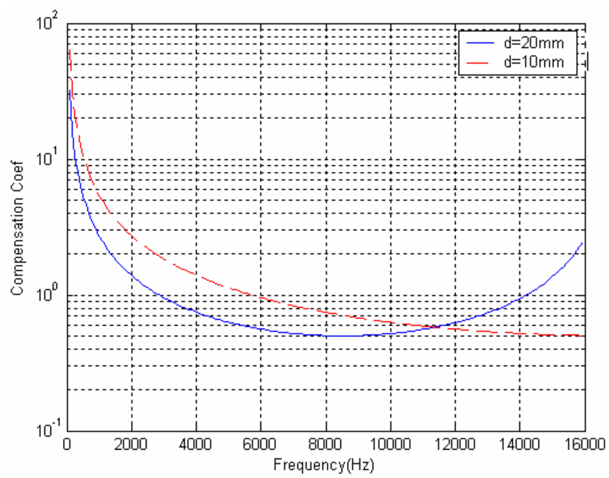

Figure 2 Compensation coefficient ($\tau=0$, $d=20mm$ – dashed, $d=10mm$ – lined)

Above some frequency ($\omega(T+\tau_{MAX})=\pi$) the desired signal from the front direction is attenuated. As such the frequency range of a dual microphone array is limited by $f_{MAX} < 0.5/(T+\tau_{MAX})$ or, equivalently, the distance between microphones is limited as $d < (0.5-f_{MAX}\cdot\tau_{MAX})/f_{MAX}$ so that $\omega_{MAX}(T+\tau_{MAX})<\pi$. This condition corresponds to a well-known spatial sampling theorem, which says that in order to prevent the occurrence of spatial aliasing in the directivity pattern of sensor array distance between sensors must be lower, then half of minimal wavelength in the signal of interest [3].

According to the above analysis of the array shown on Figure 1, varying τ between 0 and T and using equation (4) for compensating frequency response, it is possible to build all types of perfect first order polar patterns from bi-direction to cardioids with flat frequency response. In practice, inevitable mismatch in dual microphone array components does not allow achieving such characteristics.

We'll now consider the most significant sources of errors and their influences on the output characteristics:

- Mismatch in microphone sensitivities;
- Mismatch in microphone phase responses;
- Distance error between the microphones;

For analysis simplification we will use bi-directional microphone array in the far filed assumption. Extension of the analysis to other directivity patterns is straightforward.

3. MISMATCH IN MICROPHONE SENSITIVITIES

In a far field case and in presence of microphone sensitivity mismatch normalized input amplitudes are given as:

$$M_F^N(t) = e^{j\omega t}\,,\; M_R^N(t,\varphi) = q\cdot e^{j(\omega t-\varphi)}\,,$$

where q is sensitivity ratio between the two microphones. The output signal will be:

$$M_{OUT}(t,\varphi) = M_F^N(t) - M_R^N(t,\varphi) = e^{j\omega t}(1-q\cdot e^{-j\varphi})$$

Taking the magnitude of output signal gives:

$$A_{OUT}(\varphi) = \left|M_{OUT}(t,\varphi)\right| = \sqrt{q^2-2q\cos(\varphi)+1} =$$

$$= \sqrt{(q-1)^2+4q\sin^2(\frac{\varphi}{2})}$$

Amplitude of the normalized output signal:

$$A_{OUT}^N(\varphi) = \frac{\sqrt{(q-1)^2+4q\sin^2(\frac{\varphi}{2})}}{2\left|\sin(0.5\cdot\omega T)\right|}\,. \qquad (5\,)$$

Analyzing the last expression leads to conclusion that sensitivity mismatch does not let directivity pattern to approach zero even if phase difference is equal to zero. For such case ($\varphi=0$) the output signal is given by value:

$$A_{OUT}^N(0) = \frac{(q-1)}{2\left|\sin(0.5\cdot\omega T)\right|}\,.$$

Polar patterns for different q and different frequencies are shown on Figure 3 ($q=0.9$) and Figure 4 ($q=0.8$).

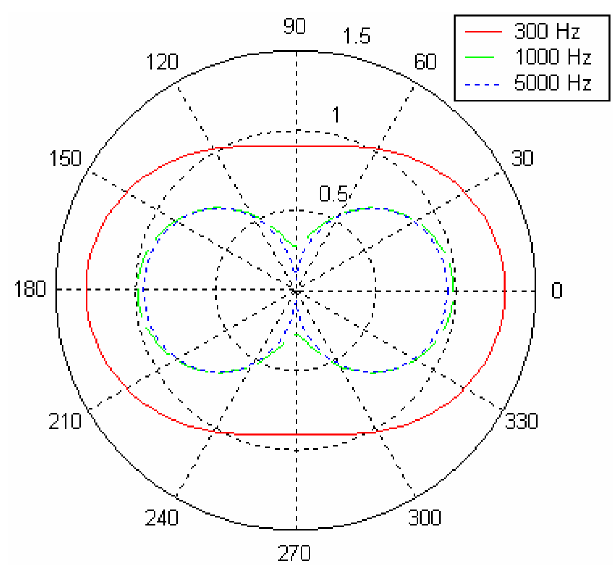

Figure 3 Directivity pattern ($d=20$mm, $q=0.9$)

In addition to spoiled directivity, sensitivity mismatch also leads to excessive amplification of low frequencies. Figure 5 illustrates dependence of such excessive amplification on frequency for $q=1$, $q=0.9$ and $q=0.8$.

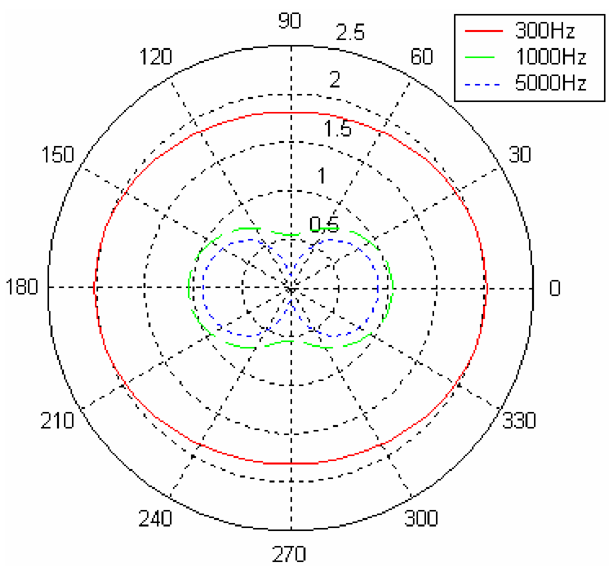

Figure 4 Directivity pattern ($d=20$mm, $q=0.8$)

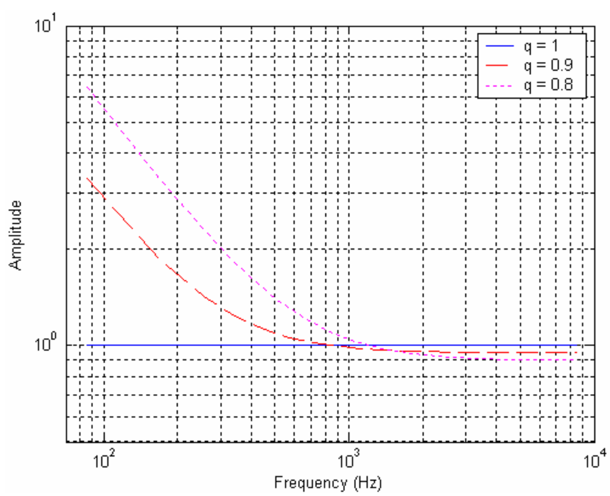

Figure 5 Frequency response ($d=20$mm, $q=1$ – solid, $q=0.9$ – dashed, $q=0.8$ – dotted)

4. MISMATCH IN MICROPHONE PHASE RESPONSES

With existence of phase mismatch the outputs of the front and rear microphone signals are given as

$$M_F(t) = e^{j\omega t} \quad , M_R(t,\varphi') = e^{j(\omega t - \varphi')} ,$$

where $\varphi' = \varphi - \Delta\varphi$ and $\Delta\varphi$ is phase mismatch. The magnitude of the output signal is then given as

$$A_{OUT}^{N}(\varphi) = \frac{2\left|\sin(\frac{\varphi'}{2})\right|}{2\left|\sin(0.5\cdot\omega T)\right|} =$$

$$= \left|\frac{\sin(0.5\cdot\varphi')}{\sin(0.5\cdot\omega T)}\right| = \left|\frac{\sin(0.5\cdot(\varphi-\Delta\varphi))}{\sin(0.5\cdot\omega T)}\right|$$

From the above equation it follows that phase mismatch steers null position of bi-directional directivity pattern from *90* degrees to some angle α which is solution of the equation $\omega\cdot T\cdot\cos(\alpha)=\Delta\varphi$ give as $\alpha=\arccos(\Delta\varphi/\omega\cdot T)$. Figure 6 illustrates such steering for constant frequency and variable phase mismatch for $d=20mm$ and $f=300Hz$. It is seen that it may result in significant suppression of desired signal ($\theta=0$) while noise components in the output signal are preserved.

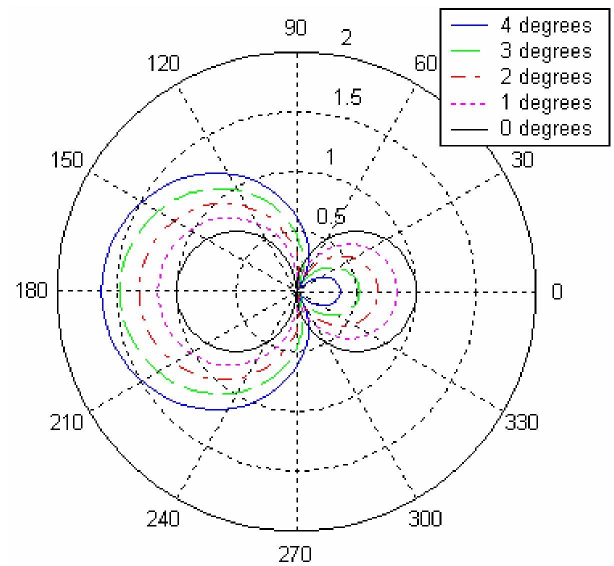

Figure 6 Phase error Null steering ($d=20mm$, $f=300Hz$, $\Delta\varphi=4$ *degrees* – solid, $\Delta\varphi=3$ *degrees* – dashed, $\Delta\varphi=2$ *degrees* – dash-dot, $\Delta\varphi=1$ *degrees* – dash-dot, $\Delta\varphi=0$ *degrees* – solid).

For $\Delta\varphi > \omega\cdot T$ the equation for α has no solution and there is no null on directivity pattern. If the microphone phase mismatch $\Delta\varphi$ does not depend on frequency, the situation occurs at low frequencies where $\omega\cdot T$ is smaller then $\Delta\varphi$. Figure 7 shows the null steering angle α as a function of frequency for $d=20mm$, $d=10mm$ and two values of $\Delta\varphi$.

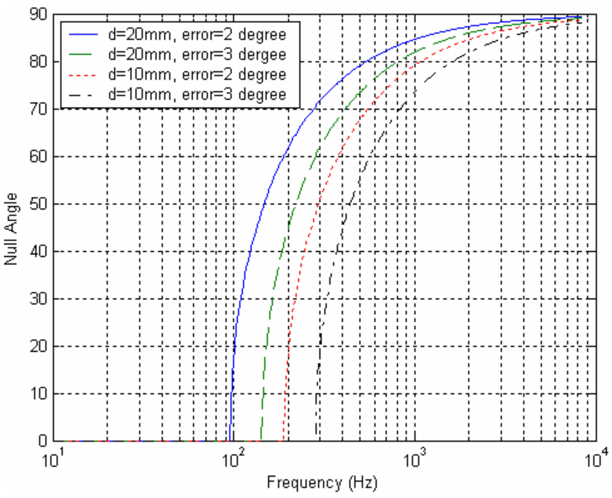

Figure 7 Null angle steering ($d=20mm$, $\Delta\varphi=2$ *degrees* – solid, $d=20mm$, $\Delta\varphi=3$ *degrees* – dashed, $d=10mm$, $\Delta\varphi=2$ *degrees* – dotted, $d=10mm$, $\Delta\varphi=3$ *degrees* – dash-dot)

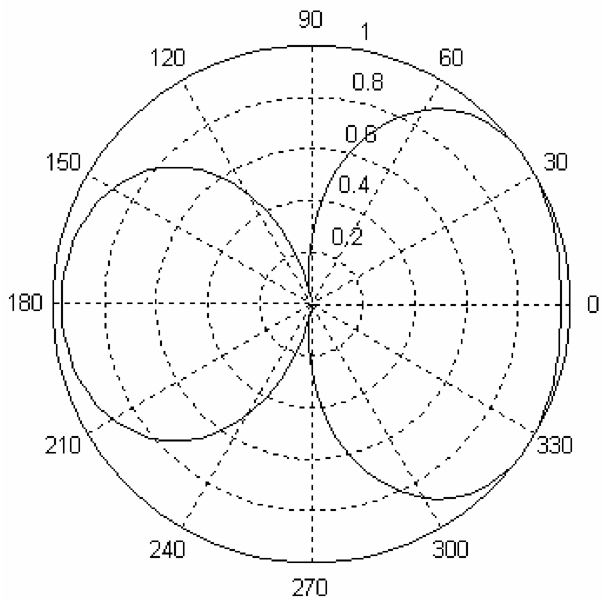

Figure 8 Spatial aliasing caused by phase error ($d=20mm$, $\Delta\varphi=15$ *degrees*, $f=8525Hz$)

At high frequencies phase error can lead to the beginning of spatial aliasing effect so that the desired signal coming from $\theta=0$ is attenuated. Spatial aliasing appears when $\varphi'=\varphi-\Delta\varphi>\pi$ and $\Delta\varphi<0$. Such effect can be observed on Figure 8.

Besides distorted directivity pattern, in presence of microphone phase mismatch, using of compensation factor given by equation (4) doesn't provide flat frequency response. Figure 9 illustrates the effect.

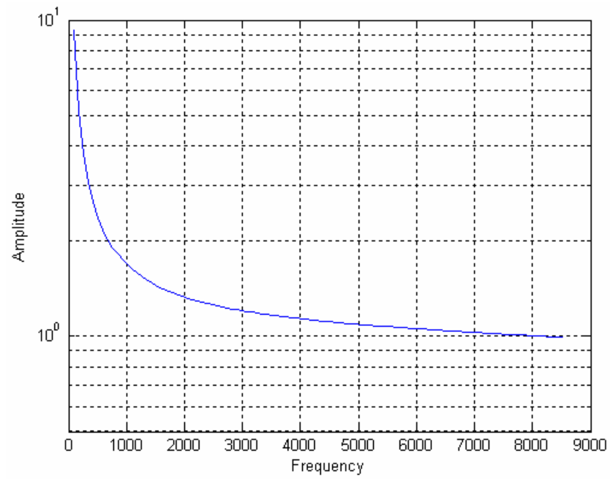

Figure 9 Frequency response in presence of phase error ($d=20mm$, $\Delta\varphi=-15$ *degrees*)

5. MICROPHONES SPACING ERROR

Assuming that error in distance between microphones is equal to Δd, the corresponding phase difference between signals is $\varphi'=\varphi+\Delta\varphi=\omega(T+\Delta T)\cos(\theta)$, where $\Delta T=\Delta d/V_S$ and $\Delta\varphi=\omega\cdot\Delta T\cdot\cos(\theta)$. Amplitude of normalized output signal is:

$$A_{OUT}^{N}(\varphi)=\left|\frac{\sin(0.5\cdot\varphi')}{\sin(0.5\cdot\omega T)}\right|.$$

Rewritten as a function of the incidence angle θ

$$A_{OUT}^{N}(\theta)=\left|\frac{\sin(0.5\cdot\omega(T+\Delta T)\cos(\theta))}{\sin(0.5\cdot\omega T)}\right|.$$

The worst things caused by presence of distance error Δd are minimal effects of spatial aliasing and incorrect frequency response compensation. The first occurs when $\omega(T+\Delta T)\cos(\theta)>\pi$ and the result of this is similar to the cases shown on Figure 8.

Incorrect frequency response compensation is also caused by microphone spacing error. If coefficient calculated by expression (4) is used to obtain flat frequency response in presence of spacing error, such a flat frequency response will not be achieved. This illustrated at figure 10 for positive and negative ΔT and for two different distances. Figure 10 shows that positive ΔT corresponds to overcompensation in low frequencies, negative to undercompensation in low frequencies. For constant relative position error ($\Delta T/T$ - const) and for the same frequency, array with bigger size has smaller deviation of frequency response from uniform, another words, has smaller compensation error.

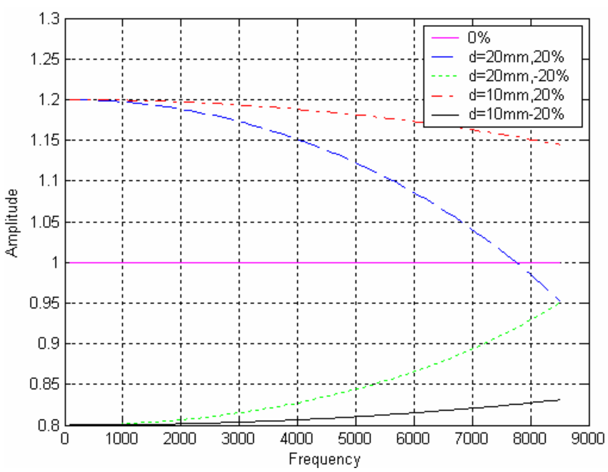

Figure 10 Impulse response in presence of position error (ΔT is given in % from T, $\Delta T=0\%$ - solid, $d=20mm$, $\Delta T=+20\%$ - dashed, $d=20mm$, $\Delta T=-20\%$ - dotted, $d=10mm$, $\Delta T=+20\%$ - dash-dot, $d=20mm$, $\Delta T=-20\%$ - solid)

6. SUMMARY AND CONCLUSION

Summarizing the analyses given above into one expression, the output sensitivity pattern is given as:

$$A_{OUT}^{N}(\theta)=\frac{\sqrt{(q-1)^2+4q\sin^2(\frac{\varphi-\Delta\varphi+\varphi_T}{2})}}{2\left|\sin(0.5\cdot\omega T)\right|},$$

where $\Delta\varphi$ is phase difference caused by microphone phase error, $\varphi_{T=}\omega\cdot\Delta T\cdot\cos(\theta)$ is phase difference caused by microphone position error and q is ratio of microphone sensitivities .

In practical microphone array applications, the far field assumption is not always valid. The above equation may be useful for consideration of near field cases as far field source with presence of amplitude, phase and position errors.

The conducted analysis has shown significant influence of component precision on directivity pattern. It is seen that most problems lie in low frequency range where the phase difference between microphone signals is small. Good beamforming strategy must take such errors into account; otherwise the results may be far from expected.

7. REFERENCES

[1] Alexander Goldin, “Autodirective Dual Microphone”, presented at the 114th Convention of Audio Engineering Society, 2003.

[2] Alexander Goldin, “Close-talking Autodirective Dual Microphone”, presented at the 116th Convention of Audio Engineering Society, 2004.

[3] I.A. McCowan. “Robust Speech Recognition using Microphone Array”, PhD Thesis, Queensland University of Technology, Australia, 2001.