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Control of the Beamwidth of a Beamformer with a fixed array configuration

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ABSTRACT

The directional characteristic of the optimal beamformer of a transducer array depends not only on its hardware configuration but also on the stability factor. This parameter can be used to control the directivity of the array. In this paper, a method, which is based on the proper selection strategy of the stability factor, is suggested to control the directional characteristics of the optimal beamformer without changing the array configuration. The selection method of the stability factor was investigated considering the trade-off relation between spatial resolution and noise amplification or array gain. The suggested method was applied to the problems of both microphone- and loudspeaker arrays to obtain a specific directivity pattern of high resolution with a constant beamwidth.

1. INTRODUCTION

Beamforming is a useful spatial filtering method not only for sound source localization with a microphone array [1], but also for sound focusing with a loudspeaker array [2]. For conventional beamforming methods, such as the delay-and-sum beamformer, the directional characteristic of the beamformer is determined by its array configuration including the transducer spacing and the number of array elements. To overcome these limitations, the focus of several beamforming methods has been to develop a beamforming algorithm for a given purpose, e.g., high directivity [3~7], or constant beamwidth over a broad frequency range [8, 9].

The optimal beamformer, which is sometimes called the super-directivity beamformer, is a method employing the optimal solution, which minimizes the total signal output of the array under the constraint of unit response in the target direction [7, 10~12]. For microphone arrays this is the electrical output, for loudspeaker arrays this is the acoustical output. With this beamformer, one can achieve a higher directivity than with conventional beamforming methods. In comparison with these other methods, the optimal beamformer has an additional design parameter, called the stability factor, to suppress the noise amplification for microphone arrays and to control the minimum array gain of loudspeaker arrays. This parameter also directly affects the directional characteristics. Because the stability factor is not a design parameter determined by its hardware setting, it can be applied to the directivity control of an array system with a fixed configuration.

In this paper, a method, which is based on the proper selection strategy of the stability factor, is suggested to control the directional characteristics of the optimal beamformer without changing the array configuration. The suggested method was applied to both microphone and loudspeaker systems.

2. THEORY

2.1. Theory of the Optimal Beamformer for Microphone Arrays

The optimal beamformer is defined by the solution of an optimization problem expressed by [10, 13]

$$\min_{\mathbf{F}(\omega)} \mathbf{F}^H(\omega)\mathbf{S}_{ZZ}^T(\omega)\mathbf{F}(\omega)$$
$$\text{subject to } \mathbf{F}^T(\omega)\mathbf{W}(\omega)=1, \tag{1}$$

where **F** is the filter vector to control the signal, **W** is the propagation vector describing the delay and attenuation from the primary source direction, and $\mathbf{S}_{ZZ}$ is the coherence matrix describing the cross-spectral density of background noise as given by [13]

$$\mathbf{S}_{ZZ}(\omega)=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}W_m(\theta,\phi,\omega)\cdot W_n^*(\theta,\phi,\omega)\sin\theta d\theta d\phi \cdot \tag{2}$$

Here, the subscripts m and n denote the indices of the acoustic elements constituting an array system. If the background noise can be assumed as uniform and isotropic, and the microphones are omnidirectional, the elements of the coherence matrix can be written as [10]

$$S_{mn}=\frac{\sin(kd_{mn})}{kd_{mn}}, \tag{3}$$

where d_{mn} is the distance between the transducer elements and k denotes the wave number.

By using the Lagrange method, the solution of Eq. (1) can be estimated by [11, 13]

$$\mathbf{F}_{opt}^T(\omega)=\frac{\mathbf{W}^H(\omega)\mathbf{S}_{zz}^{-1}(\omega)}{\mathbf{W}^H(\omega)\mathbf{S}_{zz}^{-1}(\omega)\mathbf{W}(\omega)}. \tag{4}$$

The expression in Eq. (4) includes the inverse matrix, $\boldsymbol{S}_{zz}^{-1}$. If $\boldsymbol{S}_{zz}^{-1}$ has a high condition number due to the dense distribution of acoustical elements compared to the wavelength of the signal, the noise amplification becomes large. For this reason, Gilbert and Morgan [7] suggested a method to apply a stability factor β to the diagonal elements of the coherence matrix. Adopting this method, Eq. (4) can be modified to

$$\mathbf{F}_{opt}^T(\omega)=\frac{\mathbf{W}^H(\omega)\left[\mathbf{S}_{zz}+\beta(\omega)\mathbf{I}\right]^{-1}}{\mathbf{W}^H(\omega)\left[\mathbf{S}_{zz}+\beta(\omega)\mathbf{I}\right]^{-1}\mathbf{W}(\omega)}. \tag{5}$$

The basic concept of this method is similar to the regularization technique which is to obtain a stable solution in a linear equation [14].

The optimal beamformer has a high frequency limit as given by [10, 13]

$$f_h=c/2d, \tag{6}$$

where c denotes the speed of sound and d means the spacing between the transducers. For higher frequencies grating lobes will occur as also known from the delay-and-sum beamformer [10, 13]. It can also be observed that for high frequencies the optimal beamformer tends to the delay-and-sum beamformer. The operator of the delay-and-sum beamformer is given by [15]

$$\mathbf{F}_{ds}(\omega)=\frac{\mathbf{W}^*(\omega)}{\mathbf{W}^H(\omega)\mathbf{W}(\omega)}. \tag{7}$$

Representative configurations of array systems that are widely used are the broadside and endfire configurations as shown in Fig. 1. For the broadside array, transducers are placed on an axis which is perpendicular to the primary target direction. The maximum value of the directivity index of broadside arrays is given by [16]

$$DI_{\max} = 10\log_{10} N\,, \tag{8}$$

where N is the number of transducers. For the broadside array, there is no difference of information between front side and back side, so the front-back confusion cannot be avoided.

For the endfire array system, where the transducers are on a line in the direction of the primary target direction, the front-back confusion does not occur. The maximum value of the directivity index of the endfire array is given by [16]

$$DI_{\max} = 20\log_{10} N\,. \tag{9}$$

The propagation delay vectors corresponding to the broadside and endfire array, respectively, are given by

$$\mathbf{W}(\omega) = \begin{bmatrix} e^{jkx_1 \sin\theta\cos\phi} \\ \vdots \\ e^{jkx_n \sin\theta\cos\phi} \\ \vdots \\ e^{jkx_N \sin\theta\cos\phi} \end{bmatrix},\ \mathbf{W}(\omega) = \begin{bmatrix} e^{jkz_1 \cos\theta} \\ \vdots \\ e^{jkz_n \cos\theta} \\ \vdots \\ e^{jkz_N \cos\theta} \end{bmatrix}. \tag{10 a, b}$$

As stated in Eq. (10a), the amounts of propagation delay are the same at θ and π-θ for the broadside array. In the endfire array, the propagation delay is independent of ϕ as can be noticed in Eq. (10b).

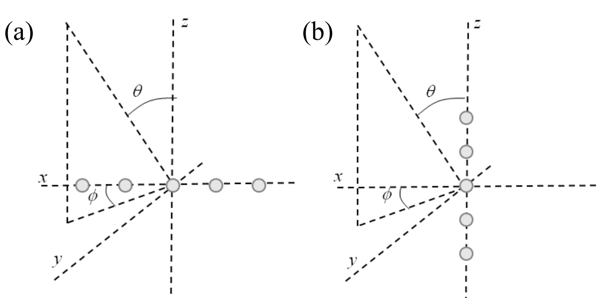


Figure 1 Configuration and coordinate system of two array systems: (a) broadside array, (b) endfire array.

2.2. Effects of the Stability Factor on the Directional Characteristics

The stability factor is suggested to suppress the noise amplification at the low frequency range; however, it also directly affects the directional characteristics. If the stability factor is set to zero, the highest directivity can be achieved. However, the sensitivity against noise becomes large when the coherence matrix has a high condition number. This is because the inversion of the coherence matrix is included in the optimal solution. The stability factor can be used like the regularization used in image enhancement.

Figure 2 shows a comparison of directivity indices of several types of endfire beamformers in a conceptual manner. As the stability factor increases, the resulting directivity moves in the direction of the delay-and-sum beamformer. Hence, the optimal beamformer becomes more stable as shown in Figure 3 in a conceptual comparison of the noise sensitivity of the beamformers. It is shown that for high stability factors the noise sensitivity is also moving to the value of the delay-and-sum beamformer which is low and steady for the whole frequency range. In case of an array with two transducers, the estimated filter coefficients are shown in Fig. 4. With the increase of the stability factor, the estimated filter shape of the optimal beamformer becomes similar to that of the delay-and-sum beamformer.

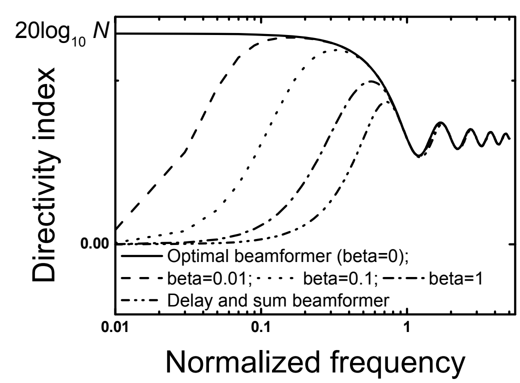


Figure 2 A comparison of directivity indices of various beamformers.

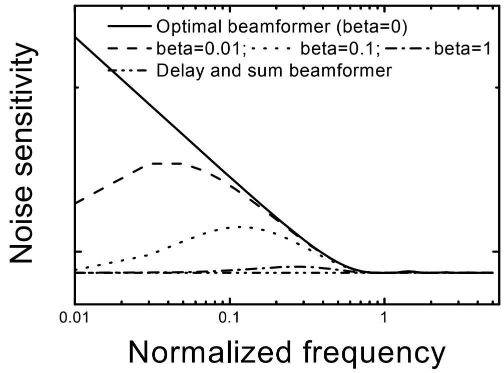


Figure 3 A comparison of noise sensitivity of beamformers.

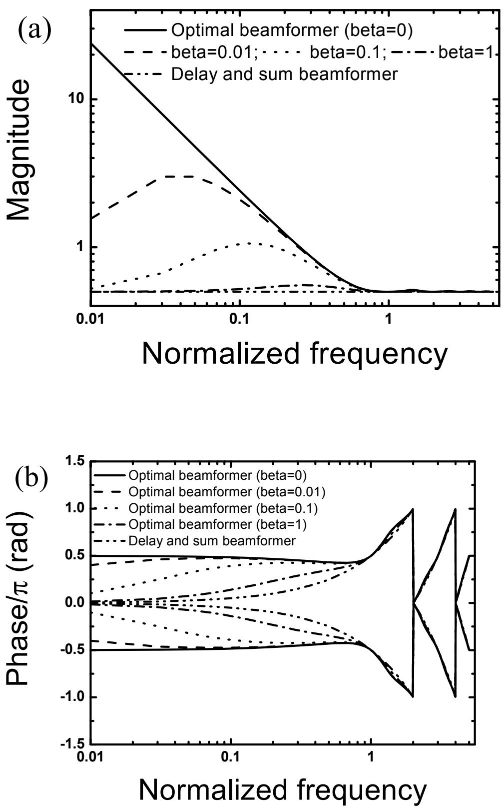


Figure 4 A comparison of filter coefficients estimated by different beamformers: (a) magnitude, (b) phase.

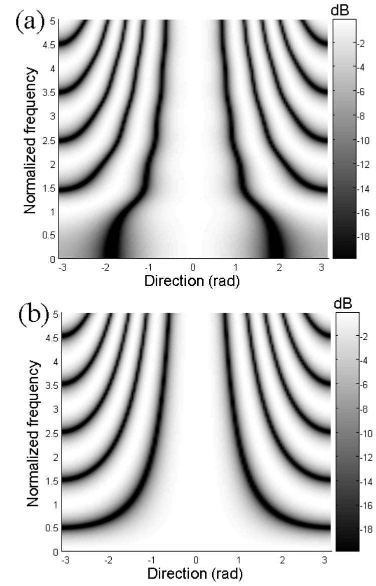


Figure 5 Directivity pattern of beamformers: (a) optimal beamformer (β=0), (b) dealy-and-sum beamformer.

3. APPLICATION OF THE STABILITY FACTOR TO MICROPHONE ARRAYS

3.1. Control of Resolution and Noise Effect in Source Localization

As afore mentioned in Sec. 2.2, the directivity can be controlled between the delay-and-sum beamformer using a high stability factor and the optimal beamformer using a zero stability factor. However, the available range of directivity should be determined from the given condition because the directivity of the optimal beamformer has a trade-off relation with the noise amplification. Also, its effective frequency range is limited by the fact that, above a certain frequency, the optimized beamformer works as a delay-and-sum beamformer. Consequently, the beamwidth can no longer be controlled with the stability factor. To select the optimal value of the stability factor, a graphical method was suggested [2]. This method is useful when the characteristics are changing without change of its configuration. This is because the index describing the target performance is expressed as a function of the stability factor.

As an example, three different types of arrays, i.e., a broadside, endfire, and planar array, each with 16 transducers were considered. The sensors were distributed with equal spacing *d*. The planar array was set as a 4x4 square. Figures 6 and 7 show the contour maps of the estimated directivity index and the noise sensitivity, respectively. One can observe that the endfire array has a wide range of directivity index variations and that the change of noise sensitivity is small for different frequencies with a fixed value of the stability factor. Therefore, if a high variability is required for an array system, the endfire configuration would be useful. However, the broadside array has an advantage that the reduction of directivity with the increase of the stability factor is small. In Fig. 8, the directional response at $0.1f_h$ is presented. In this simulation uncorrelated white noise was added with a level that was 30 dB lower than the signal detected by the sensors. The change of DI and NS according to the applied β value is summarized in Table 1.

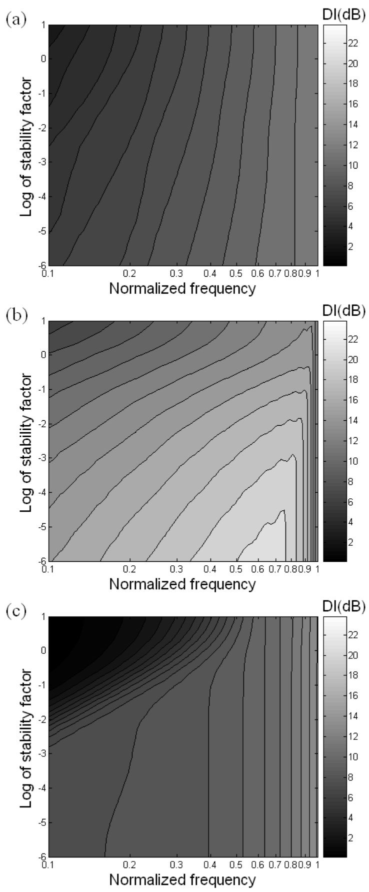


Figure 6 Contour map of the directivity index as a function of the stability factor and normalized frequency: (a) broadside array with 16 equally spaced microphones, (b) endfire array with 16 equally spaced microphones, (c) 4x4 planar array.

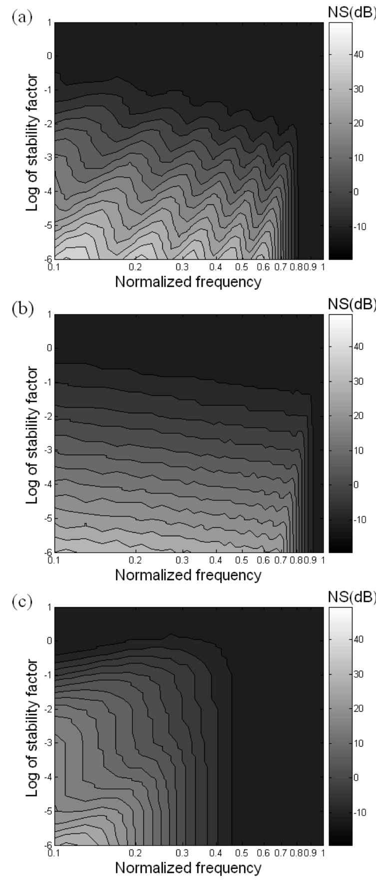
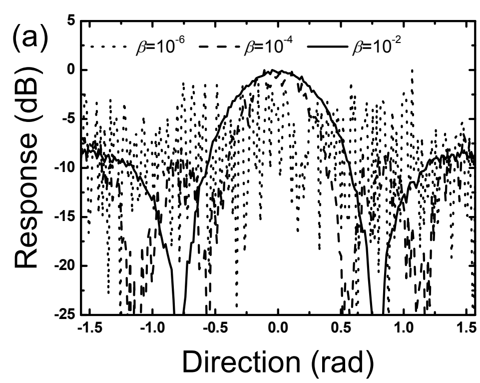


Figure 7 Contour map of the noise sensitivity as a function of the stability factor and normalized frequency: (a) broadside array with 16 equally spaced microphones, (b) endfire array with 16 equally spaced microphones, (c) 4x4 planar array.



(Figure 8. Continued)

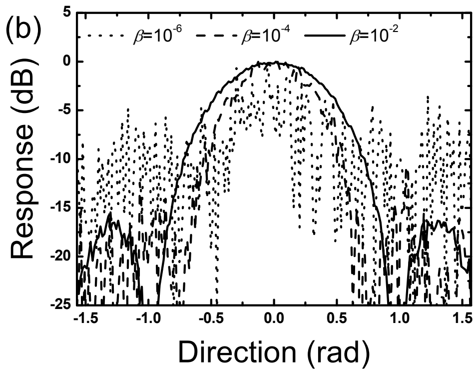


Figure 8 Directional response of beamformer consisting of equally spaced 16 sensors at $0.1f_h$, with noise: (a) broadside array, (b) endfire array.

Table 1 Comparison of performance index changes with β between broadside and endfire array, consisted of equally spaced 16 sensors at $0.1f_h$, with noise.

β value	Directivity index		Noise sensitivity	
	broadside	endfire	broadside	Endfire
10^{-6}	6.0 dB	15 dB	40 dB	33 dB
10^{-5}	5.7 dB	15 dB	23 dB	25 dB
10^{-4}	5.7 dB	14 dB	18 dB	16 dB
10^{-3}	5.4 dB	13 dB	14 dB	8.8 dB
10^{-2}	4.4 dB	11 dB	2.7 dB	0.5 dB

3.2. Constant Beamwidth Array Using the Optimal Beamformer

The constant beamwidth array (CBA) is a useful system because it can provide the same spatial resolution over a broad frequency range. As an application of directivity control with the stability factor to the microphone array system, a beamformer having a constant beamwidth can be formed with an optimal beamforming technique by a proper selection of the stability factor [2].

The available range of directivity for constant beamwidth is limited by several factors. The controllable range of frequencies is limited to a value lower than the high frequency limit of the optimal beamformer. The lower limit of the available frequency range is determined by the allowable noise sensitivity. Also, the available range of directivities is between the value of the delay-and-sum beamformer and the optimal beamformer with β=0. For the widest frequency range of the CBA, the DI will be chosen as the highest value of the delay-and-sum beamformer. Another point of concern is the effect of noise. At the low frequency side, the beamformer has relatively high noise sensitivity. Therefore, the actual resolution at the low frequency side will be restricted.

As an example, a constant beamwidth beamformer was constructed with 16 sensors. As shown in Fig. 6, a constant beamwidth beamformer having a wider range can be achieved with the endfire configuration. From Fig. 6, a constant beamwidth with a DI of 15 dB can be achieved and the highest NS in this case is 25 dB. Figure 9 depicts the selected β values for this purpose. The directivity pattern of the designed beamformer in the noise field is illustrated in Fig. 10. If the system NS is close to or higher than the signal-to-noise ratio, the response is distorted. Therefore, the lower bound of the applicable frequency range for a constant beamwidth is determined by the noise level.

3.3. Combination with harmonic-nesting based CDB

The method using the optimal beamformer can achieve a higher directivity, but the applicable frequency range is narrower than that by the harmonic-nesting based CDB. On the other hand, it is possible to extend the range of a fixed-length harmonic-nested CDB at the low frequency side by application of the optimized beamforming processing [17].

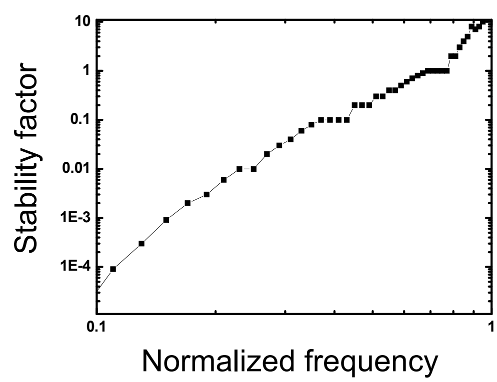


Figure 9 Selected stability factors for a constant beamwidth array.

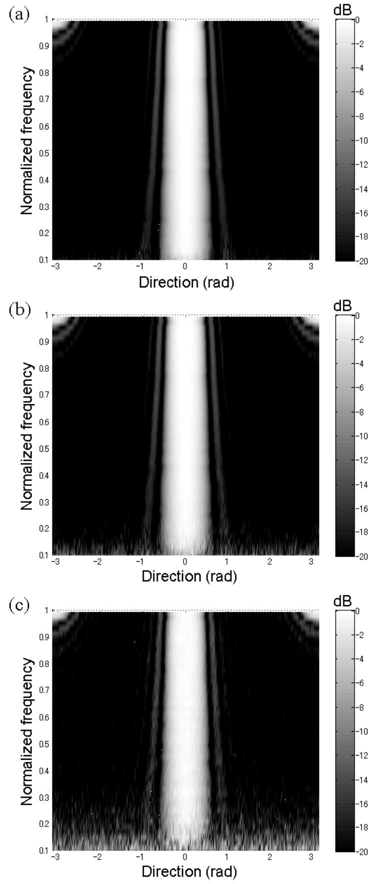


Figure 10 Directivity pattern of the designed constant beamwidth beamformer with a relative noise level of: (a) -30 dB, (b) -20 dB, (c) -10 dB.

The concept of harmonic-nesting based CDB activates sensor pairs with a wider spacing at lower frequencies. This can be done by applying low pass filters, that have different cut-off frequencies, to each pair of sensors located at symmetric positions with respect to the center of the array [8, 9]. If the array is continuous, the constant beamwidth can be achieved when the total activated aperture size is maintained by Bc/f, where B is a constant determined by the target directivity. For a general discrete array system, the filter gain should be interpolated to virtually have the target aperture size. Ward, et al. suggested a method for the proper positioning of transducers and the filter estimation [9]. With this approach, the total aperture size of an array should be increased almost exponentially to extend the lower bound of the frequency range.

To reduce the aperture size, the optimal beamformer can be adopted. Figure 11 depicts a sensor array set to implement a constant beamwidth array using the harmonic nesting. The sensor pair at each end of the array is only for the lowest frequency bin; however, the total array size becomes twofold by this pair. If the optimal beamformer, which can obtain the higher directivity with the same aperture size, is applied to the low frequency range, the size of the array can be drastically reduced.

In Fig. 12, the directivity pattern of the array is presented with the array set as shown in Fig. 11. At 200 Hz, the sensor pair at each end of the array was not activated. The optimal beamformer was applied to the same activated sensor set at 200 Hz and it was observed that almost the same directivity pattern can be achieved by proper selection of the stability factor. Therefore, the combination of different beamforming methods could be useful to optimize the array configuration.

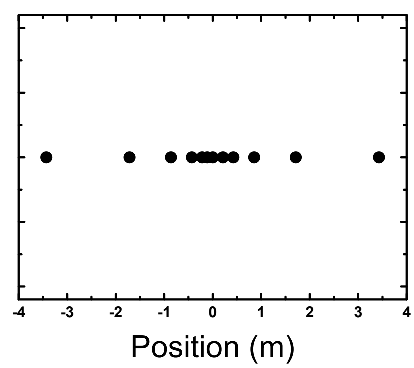


Figure 11 Selected positions of sensors to implement a constant beamwidth array using harmonic nesting.

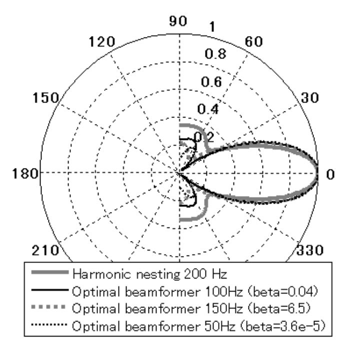


Figure 12 Comparison of directivity patterns estimated by the harmonic nesting and optimal beamformer.

4. APPLICATION TO A LOUDSPEAKER ARRAY

4.1. Beamforming for loudspeaker array systems

Reciprocally, the beamforming method can also be applied to a loudspeaker array system [2]. In this case the noise sensitivity is interpreted as the reciprocal of the white noise array gain (AG). Hence, AG = - NS (in dB). However, there remain several concerning points due to the size of the transducers. First, scattering effects induced by the loudspeaker body are larger than from the microphone. In particular, the phase shift due to scattering cannot be ignored for an endfire configuration. Another point is the directivity of the transducer. The size of a loudspeaker body as well as its diaphragm is larger than a microphone in general. Therefore, the directivity pattern of a single loudspeaker cannot be assumed as omni-directional. For these reasons, several assumptions enabling a simple estimation for the microphone array cannot be applied to the loudspeaker array.

To overcome these problems, instead of the simplified expression in Eq. (3), Eq. (2) should be used. Also, the description of the propagation delay vector should be changed to

$$\mathbf{W}_g(\theta,\phi,\omega) = \boldsymbol{\Gamma}(\theta,\phi,\omega)\cdot\mathbf{W}(\omega), \tag{11}$$

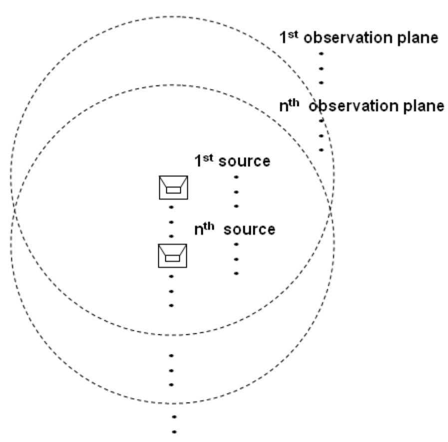


Figure 13 Observation plane to obtain the directional pattern of the total sound field for each source in the array system [2].

where $\boldsymbol{\Gamma}(\theta, \phi, \omega)$ means the vector describing the directivity patterns of the single loudspeakers in an array setup. To obtain an exact value of $\boldsymbol{\Gamma}(\theta, \phi, \omega)$, the measurement or calculation should be carried out on the observation plane as shown in Fig.13 with the actual array system.

4.2. Example

4.2.1. System configuration and modeling

As an example, an endfire array system consisted of 8 loudspeakers was controlled to obtain the directivity patterns specified in terms of the directivity index. Figure 14 shows the configuration of the array system. The spacing between the sources was 0.1m. As a single source, a spherical loudspeaker enclosure of 0.08 m in diameter was used with a loudspeaker having a diaphragm diameter of 0.05 m. To consider the scattering effect induced by each loudspeaker body, the boundary element model of the array system was constructed as shown in Fig. 14(a). The linear triangular elements were used and the high frequency limit of the model was 2 kHz based on the 6 over wavelength criterion. With this boundary element model, the directivity pattern of each elementary source in the array system was estimated.

4.2.2. Results

First of all, the controllable range was estimated with the given loudspeaker array system. For the endfire array with 8 elementary sources, the controllable range is depicted in Fig. 15. As target radiation patterns, 3 types of constant beamwidth filters, having 8, 12, and 16 dB in DI, respectively, were chosen to be manipulated. Figure 16 shows the selected stability factor. One can observe that the target DI cannot be achieved at some frequencies.

The radiated sound field was simulated by the boundary element models as shown in Sec. 4.2.1. Figure 17 shows the estimated radiation pattern and the actual directivity index was obtained as shown in Fig. 18. The performance of the array system can be changed within a specific frequency range; however, its performance, viz., directivity, was rather degraded. This is because the selection process of the stability factor was done under the assumption of omni-directional radiation and ignoring the scattering effect. Also notice that the output of the loudspeaker array will be small in the low frequency region because of the low AG.

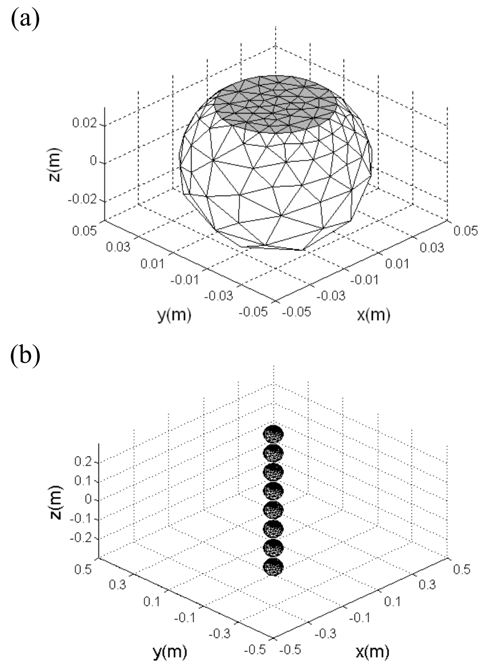


Figure 14 Boundary element model of the system: (a) single loudspeaker, (b) array system.

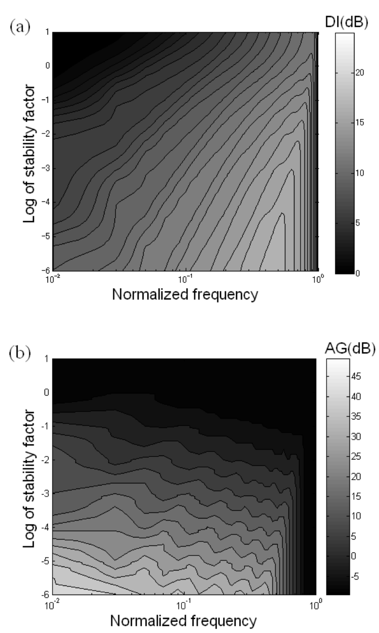


Figure 15 Controllable range of endfire array with 8 elementary sources: (a) directivity index, (b) array gain.

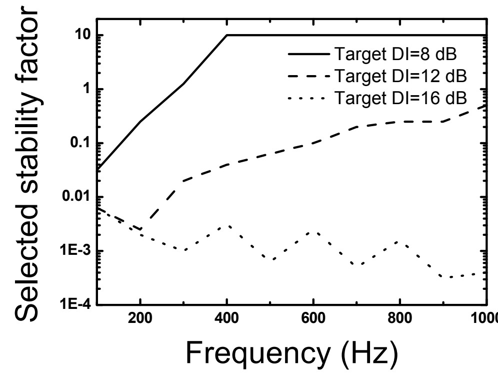


Figure 16 Selected stability factors.

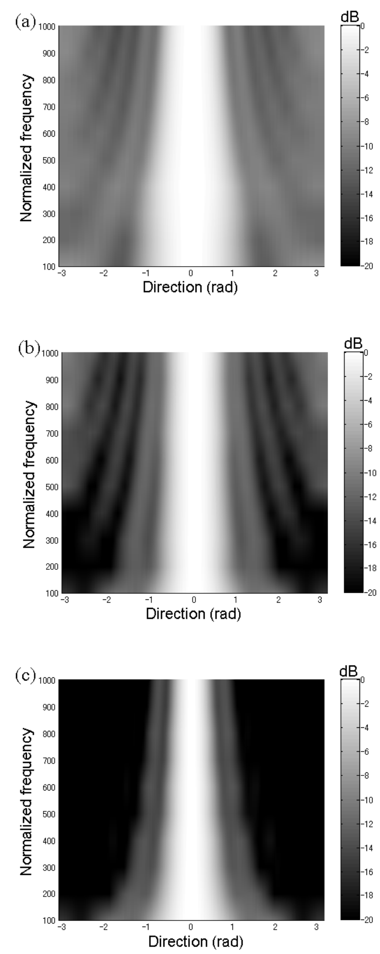


Figure 17 Generated directivity pattern with CBA filter estimated by the optimal beamforming with the target field of: (a) DI=8 dB, (b) DI=12 dB, (c) DI=16 dB.

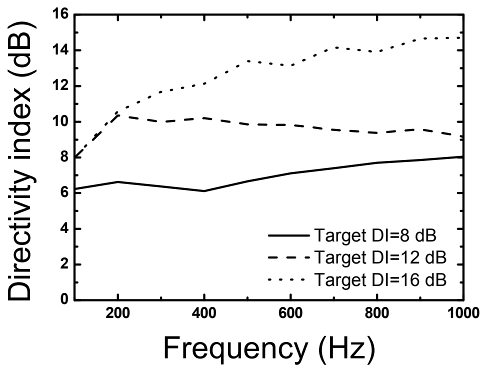


Figure 18 Actual directivity index of designed beamformer.

5. CONCLUSIONS

In this paper, the directivity control for the fixed configuration of an array system was conducted by using the stability factor as normally used in the optimal beamformer. A method using a contour plot of DI and NS (or AG) was suggested to control the directional characteristics of the optimal beamformer without changing the array configuration. It was observed that the controllable range also depended on the initial configuration of the system and the endfire configuration could be controlled over a wide frequency range.

The constant beamwidth beamformer was implemented by using the optimal beamforming and it was compared with the method using harmonic nesting. The optimal beamforming could achieve a constant beamwidth with a small aperture size of the array, but the noise sensitivity should be considered too. It is also found that the combination of these two beamforming methods would be useful to optimize the array configuration.

It was found that the suggested methods in this paper can be applied to the loudspeaker array system under the theory of reciprocity. However, the prediction and design would be more complicated then for microphone arrays due to the size of the elementary sources involved in the array. Another concern is the low array gain at low frequencies that will restrict the acoustical power here.

6. ACKNOWLEDGEMENTS

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