CS7.404: Digital Image Processing

Monsoon 2023: Image Restoration

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A Quick Recap

Image Enhancement

- “Improve” the appearance of an image; a subjective process.

Original Image

Blurred Image

Image Restoration

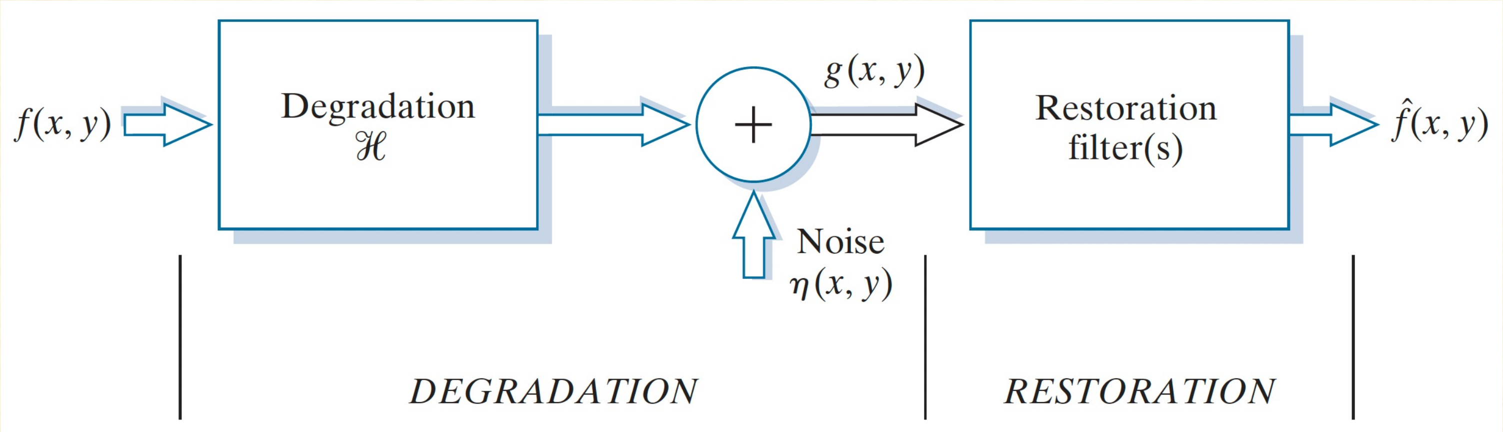
- Remove distortions from an image to go back to the “original” image; an objective process.

Original Image

Degraded Image

A Model for Image Degradation and Restoration

- Image Restoration
  - Use a priori knowledge of the degradation
  - Modeling the degradation and apply the inverse process
  - Formulate and evaluate objective criteria of goodness

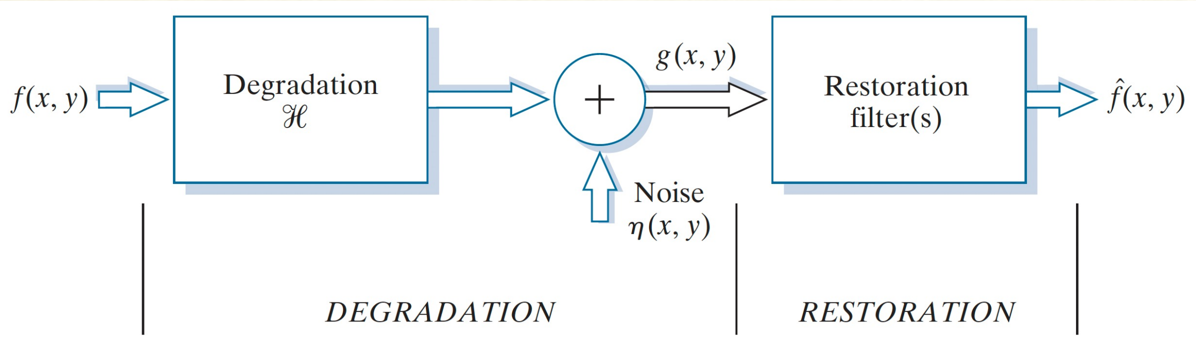


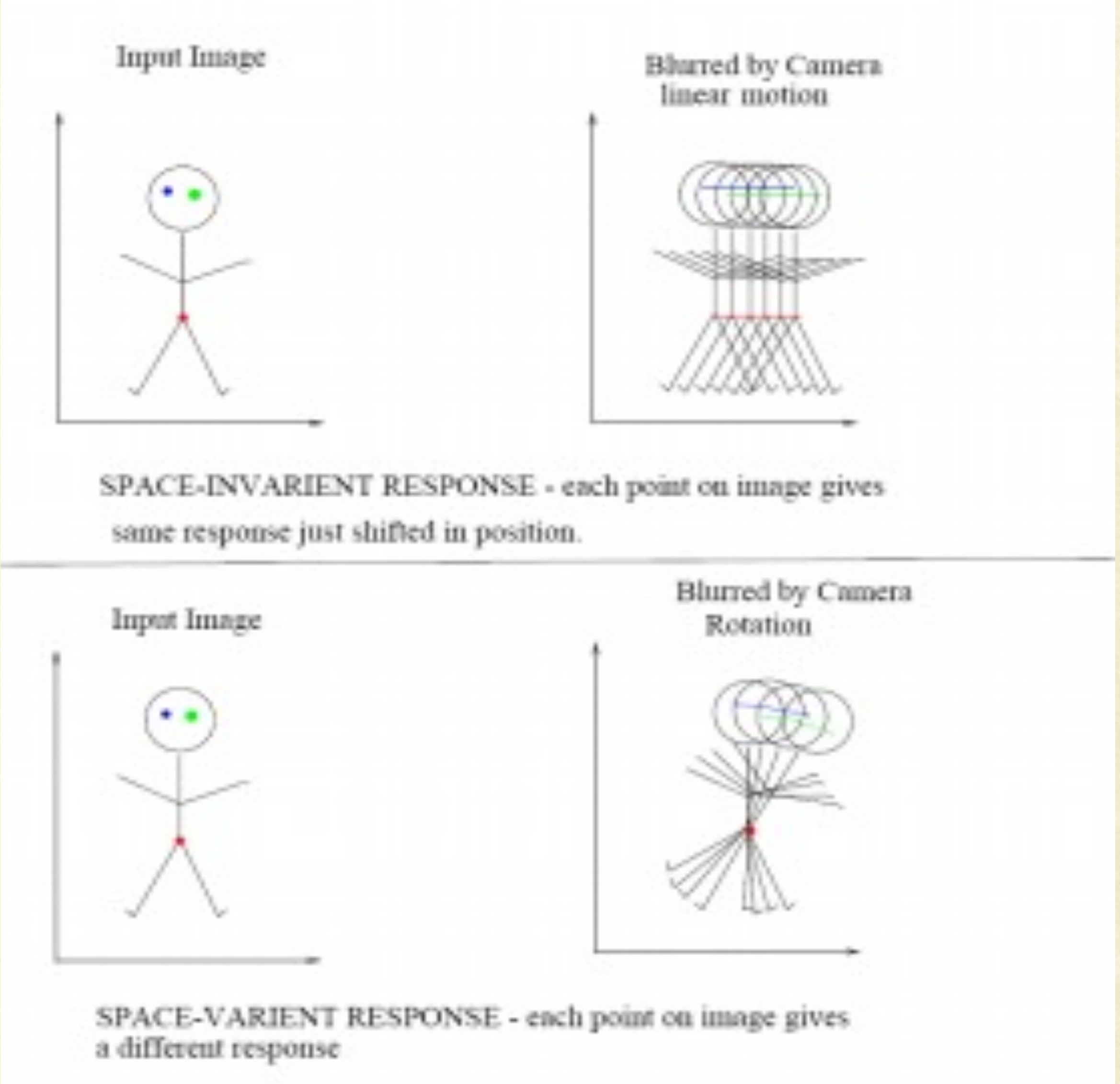
$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

  - Design the restoration filters such that $\hat{f}(x, y)$ is as close to $f(x, y)$ as possible.

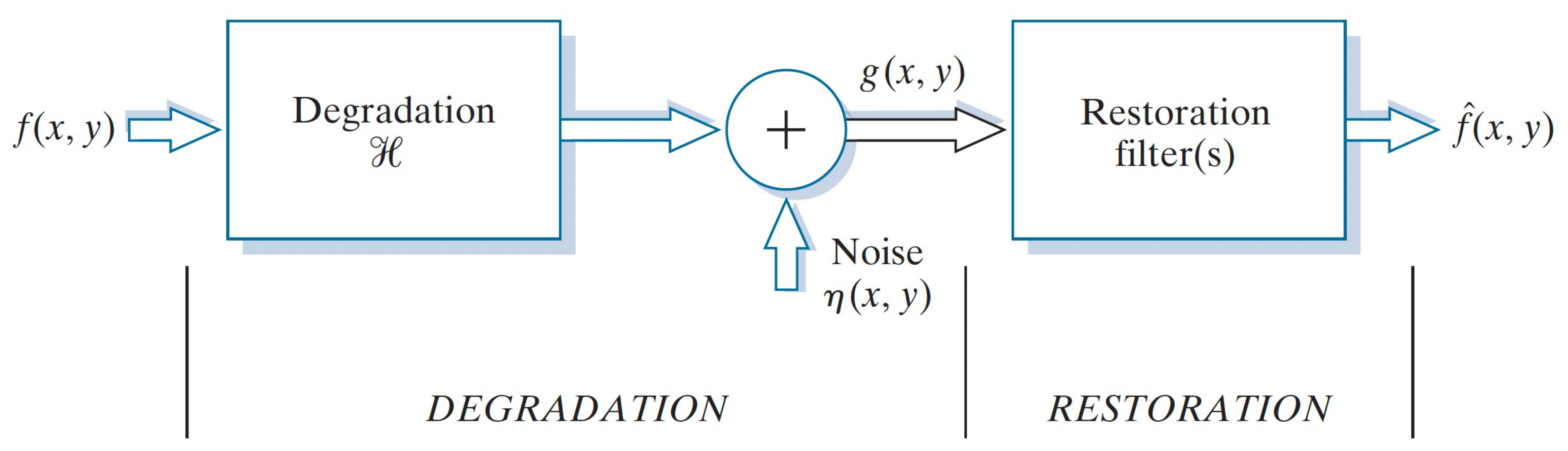
Assumptions for the Distortion Model

- Noise
  - Independent of spatial location
    - except for periodic noise
  - Uncorrelated with the image
- Degradation function, H
  - Linear
  - Position-invariant





Mathematical model of Image Degradation/Restoration



$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

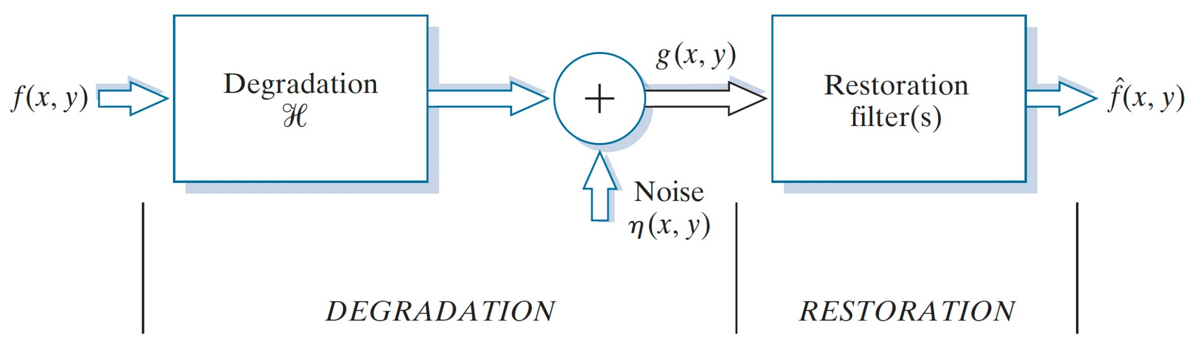
If H is a linear, position-invariant process,

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y), \text{ or}$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Divide and Conquer: Step #1

- Image degraded only by noise



- Assuming ‘H’ is identity, the model reduces to:

$$g(x,y) = f(x,y) + \eta(x,y)$$

or

$$G(u,v) = F(u,v) + N(u,v)$$

Noise Models

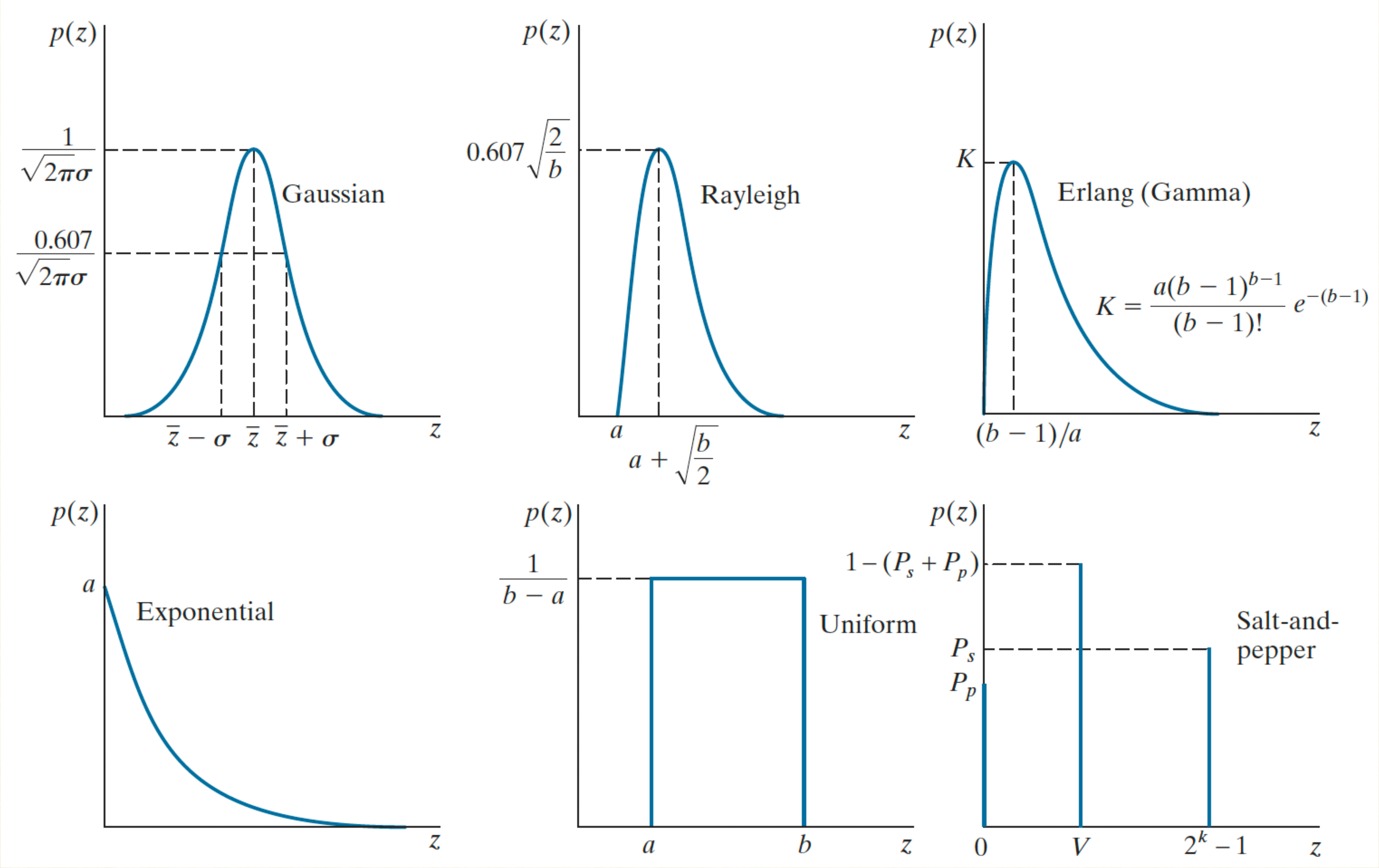
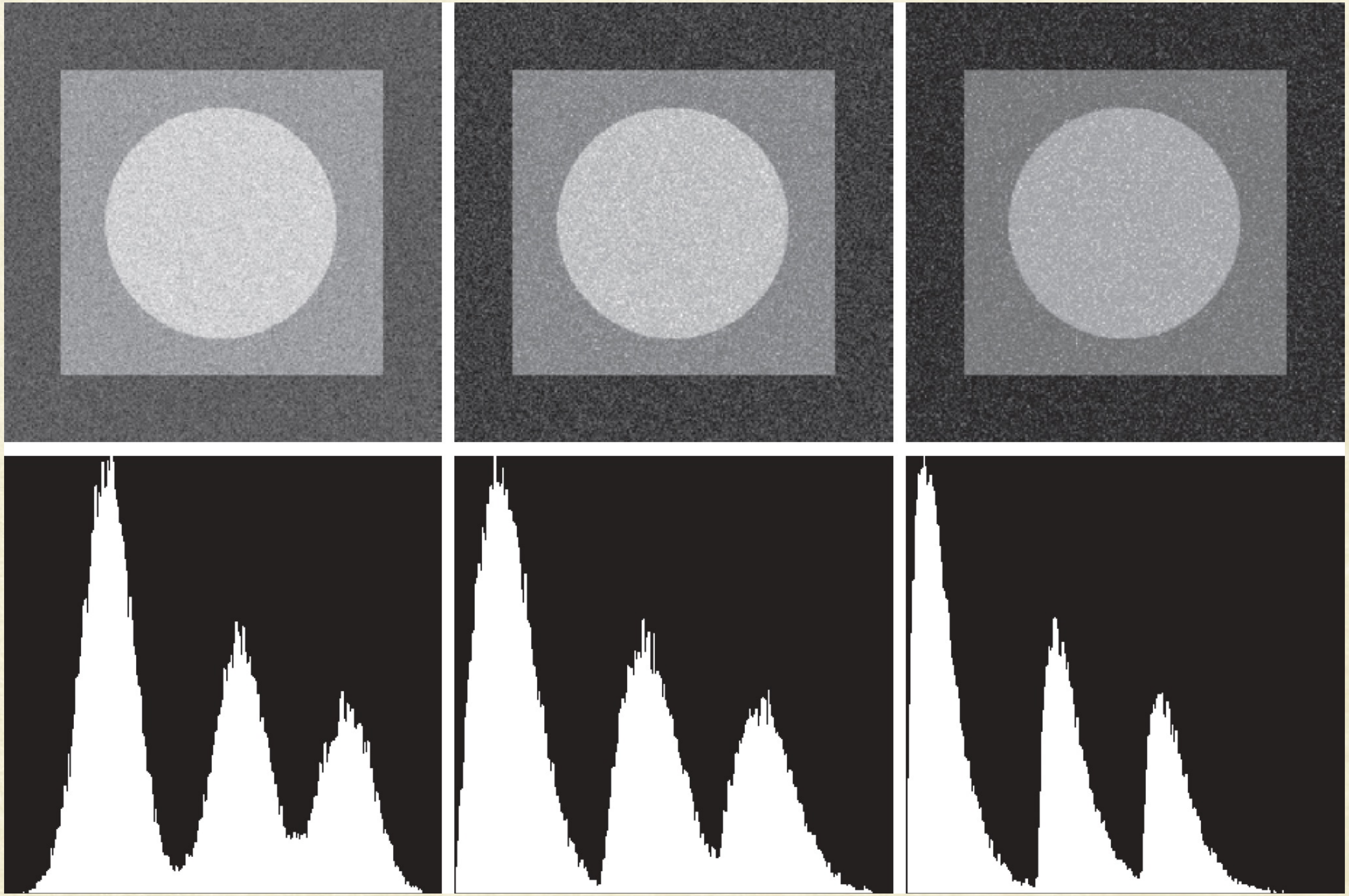


Illustration of Noise Models

- Visually similar.
- Not easy to determine noise model from appearance



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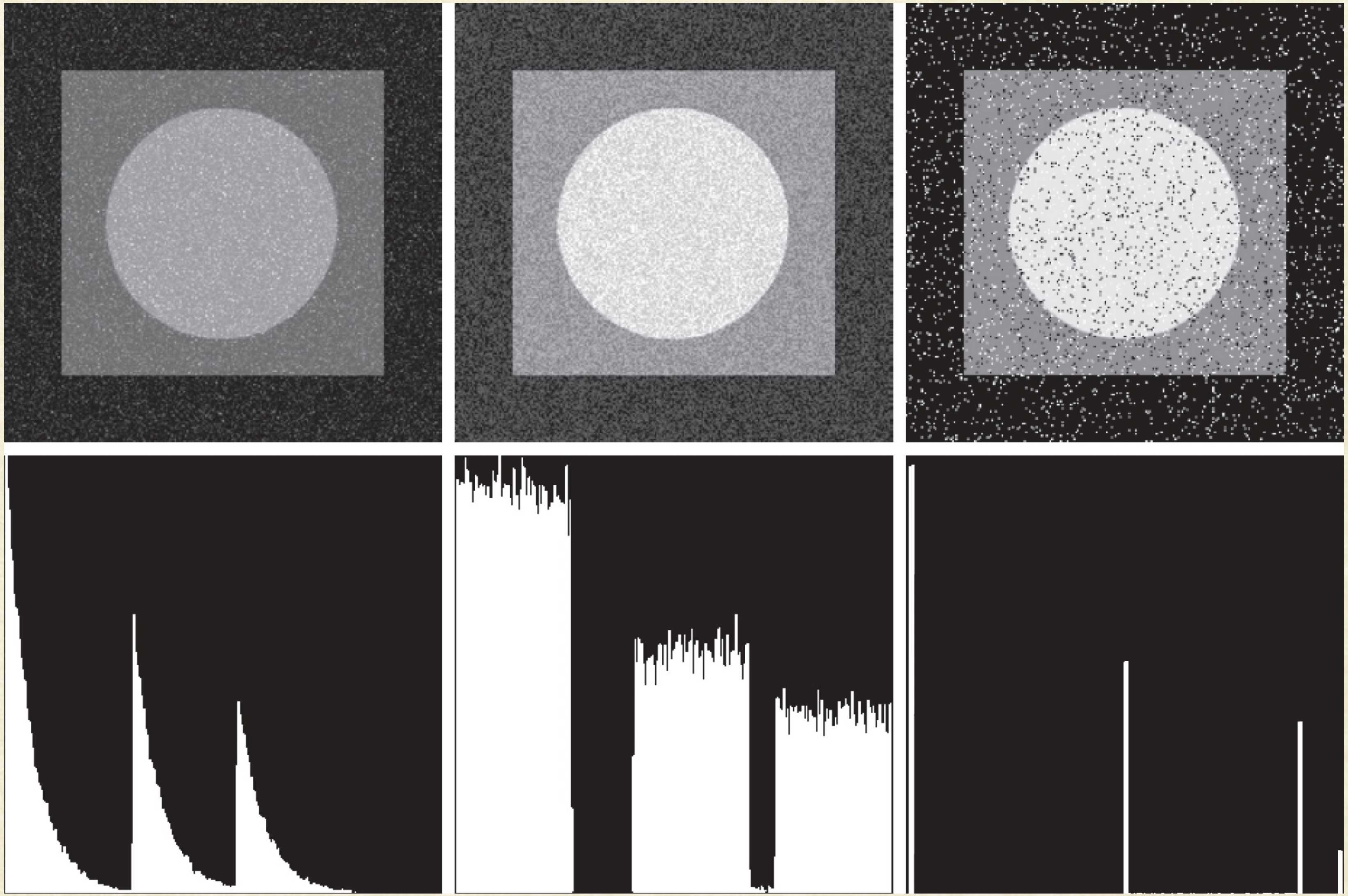


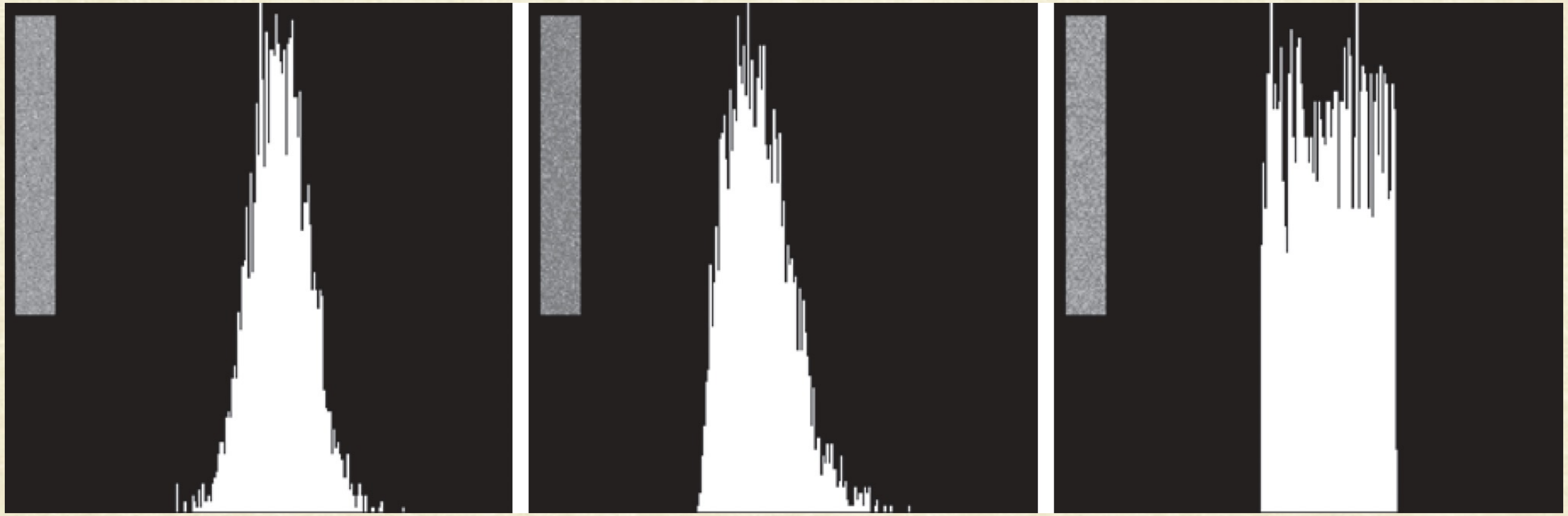
Exponential Uniform Salt & Pepper

Understanding System Noise

- Case 1: Imaging system available
  - Noise Calibration: Capture a set of ‘flat environments’ (e.g. solid gray board, object at fixed location)
  - Select the model with better statistical test scores (Akaike Information Criteria (AIC) or Likelihood Ratio Test (LRT))
  - Compute model parameters (mean, variance, etc.) from the statistics of pixel values.

Understanding System Noise

- Case 2: Only images available
  - Estimate from patches of constant intensity
  - For impulse noise, use a mid-gray patch/area



Gaussian Rayleigh Uniform

Restoration (in presence of noise only)

- Mean filters
  - Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} g(r,c)$$

  - Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(r,c)\in S_{xy}} g(r,c) \right]^{\frac{1}{mn}}$$

Restoration (in presence of noise only)

- Mean filters
  - Harmonic mean filter (Gaussian, Salt)

$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(r,c)\in S_{xy}} \frac{1}{g(r,c)}}$$

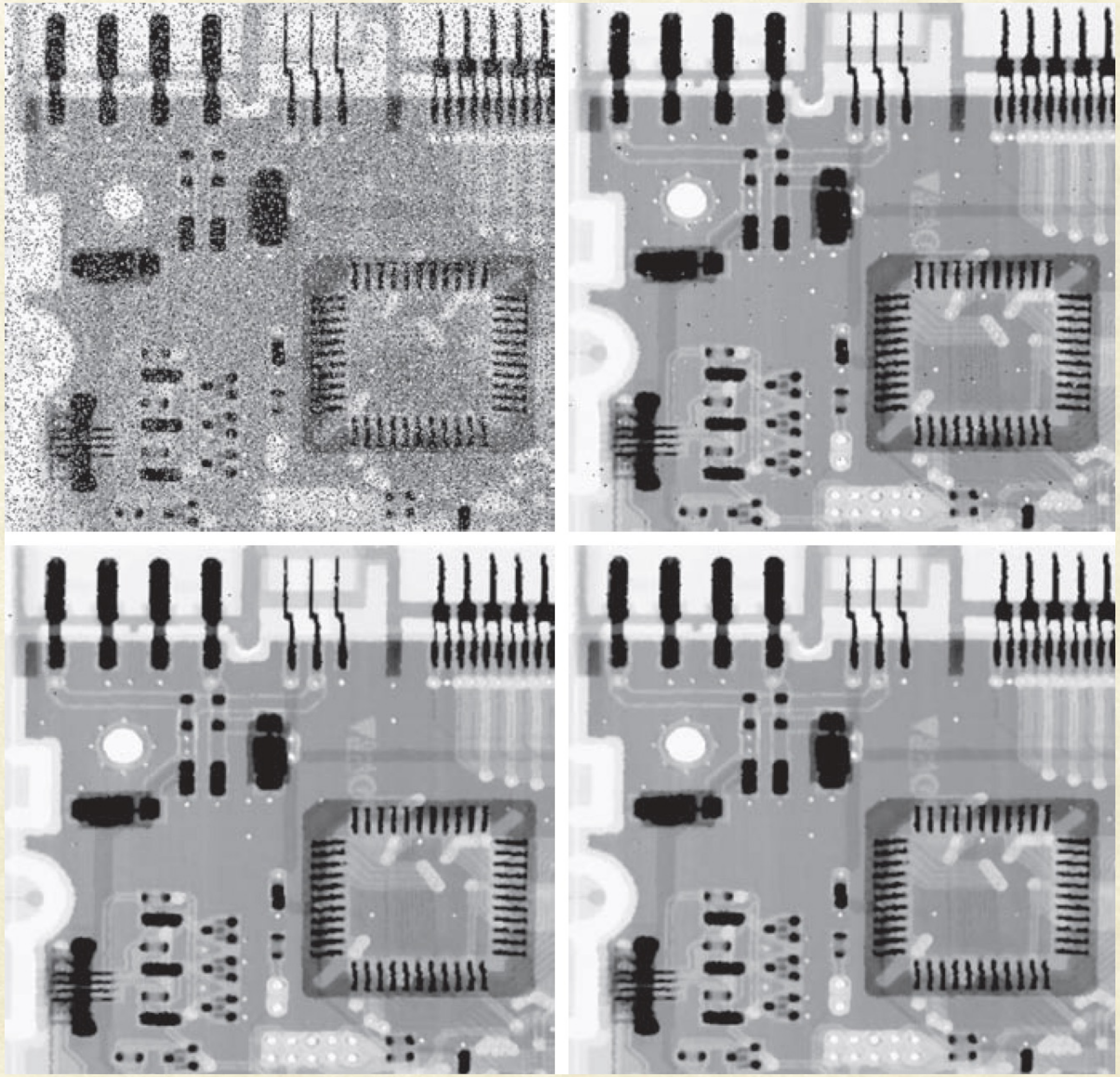
  - Contraharmonic mean filter (Salt & Pepper)

$$\hat{f}(x,y) = \frac{\displaystyle\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\displaystyle\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$$

Q = *order* of the filter
Good for salt-and-pepper noise.
Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$
NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

Restoration (in presence of noise only)

- Median filter

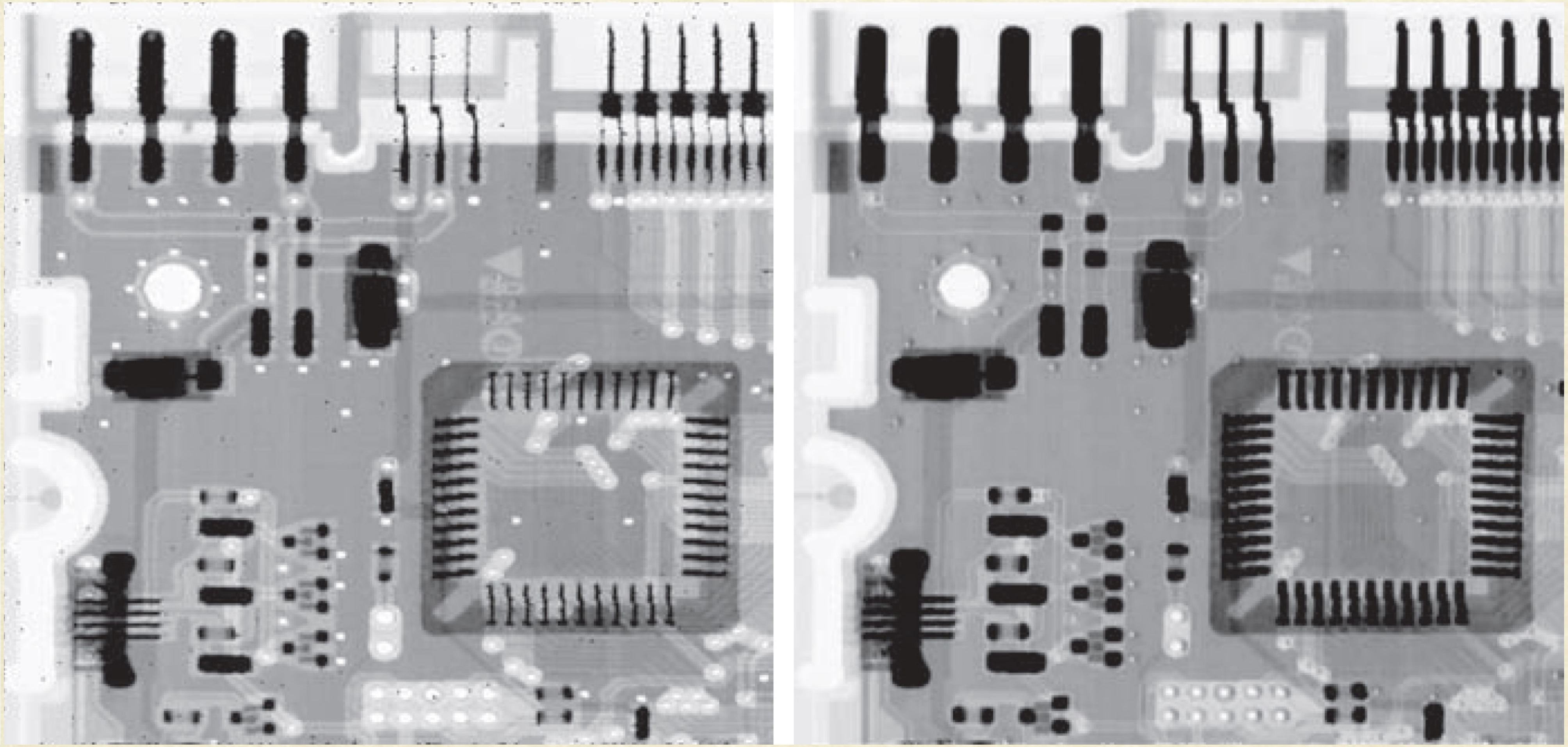


a	b
c	d

FIGURE 5.10 (a) Image corrupted by salt-and- pepper noise with probabilities $P_s = P_p = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Restoration (in presence of noise only)

- Max, Min filters



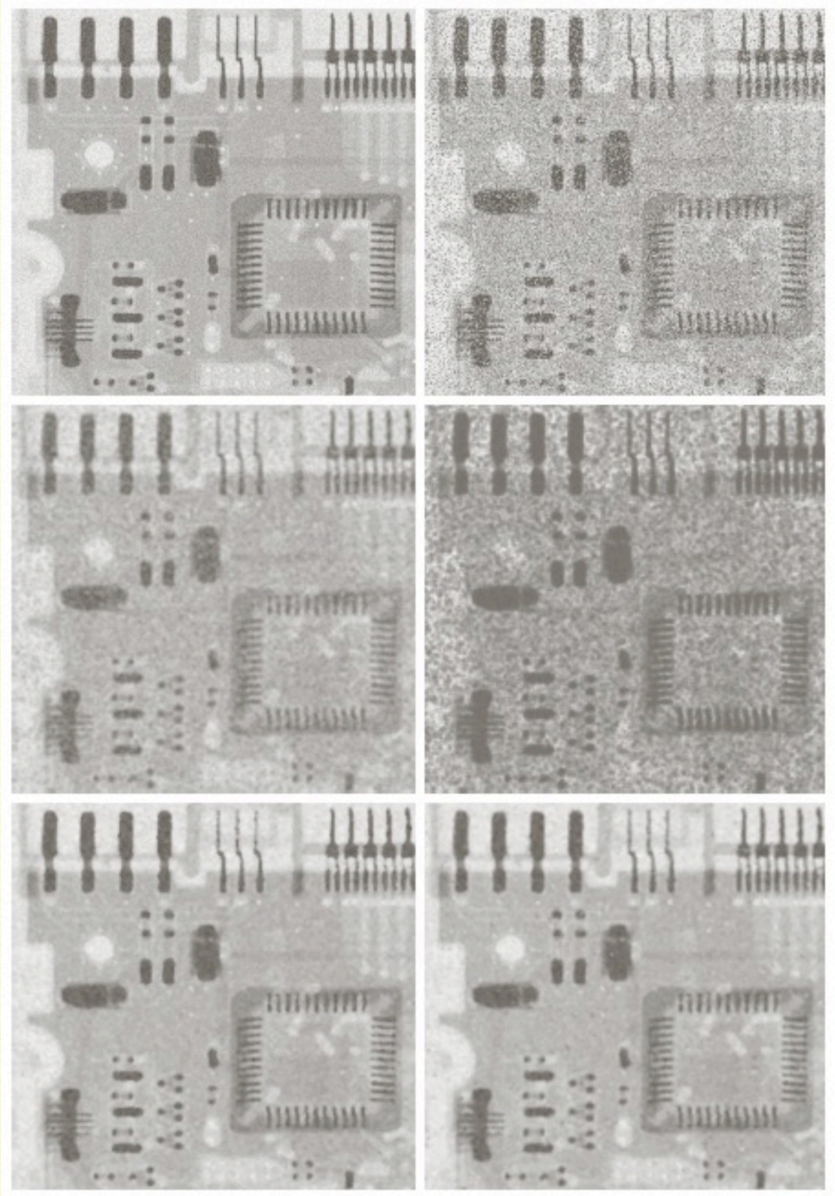
a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3.
(b) Result of filtering Fig. 5.8(b) with a min filter of the same size.

Restoration (in presence of noise only)

original

Original + salt and pepper noise



Arithmetic mean filter

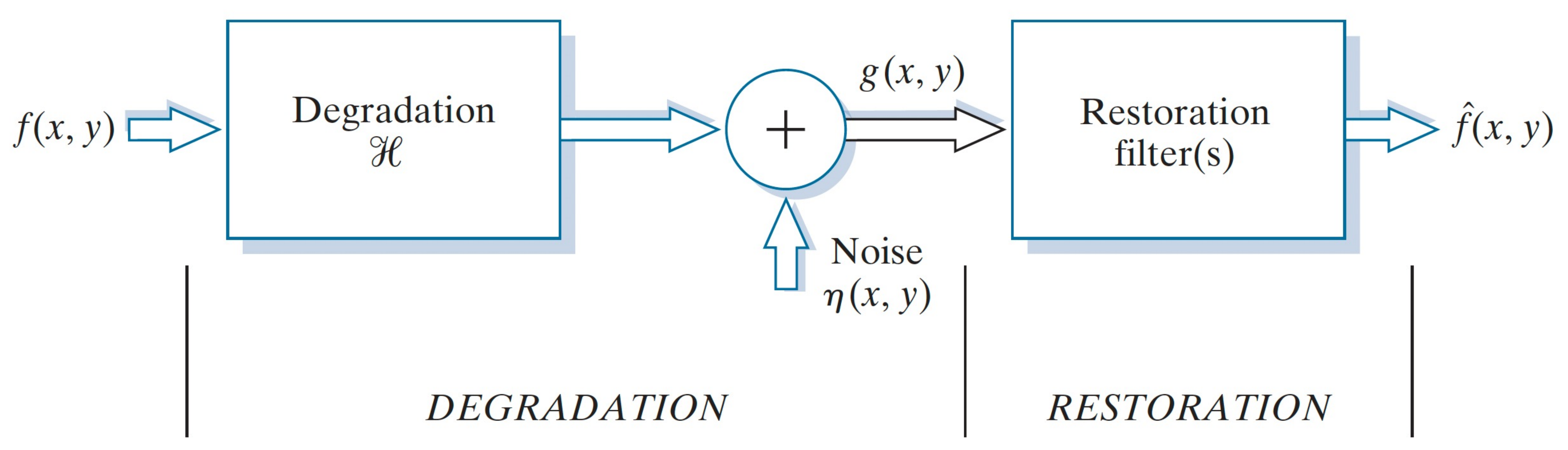
Geometric mean filter

Median filter

Alpha Trimmed filter

Questions?

Image Restoration: Periodic Noise



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y), \text{ or}$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Assuming only additive noise: $g(x, y) = f(x, y) + \eta(x, y)$

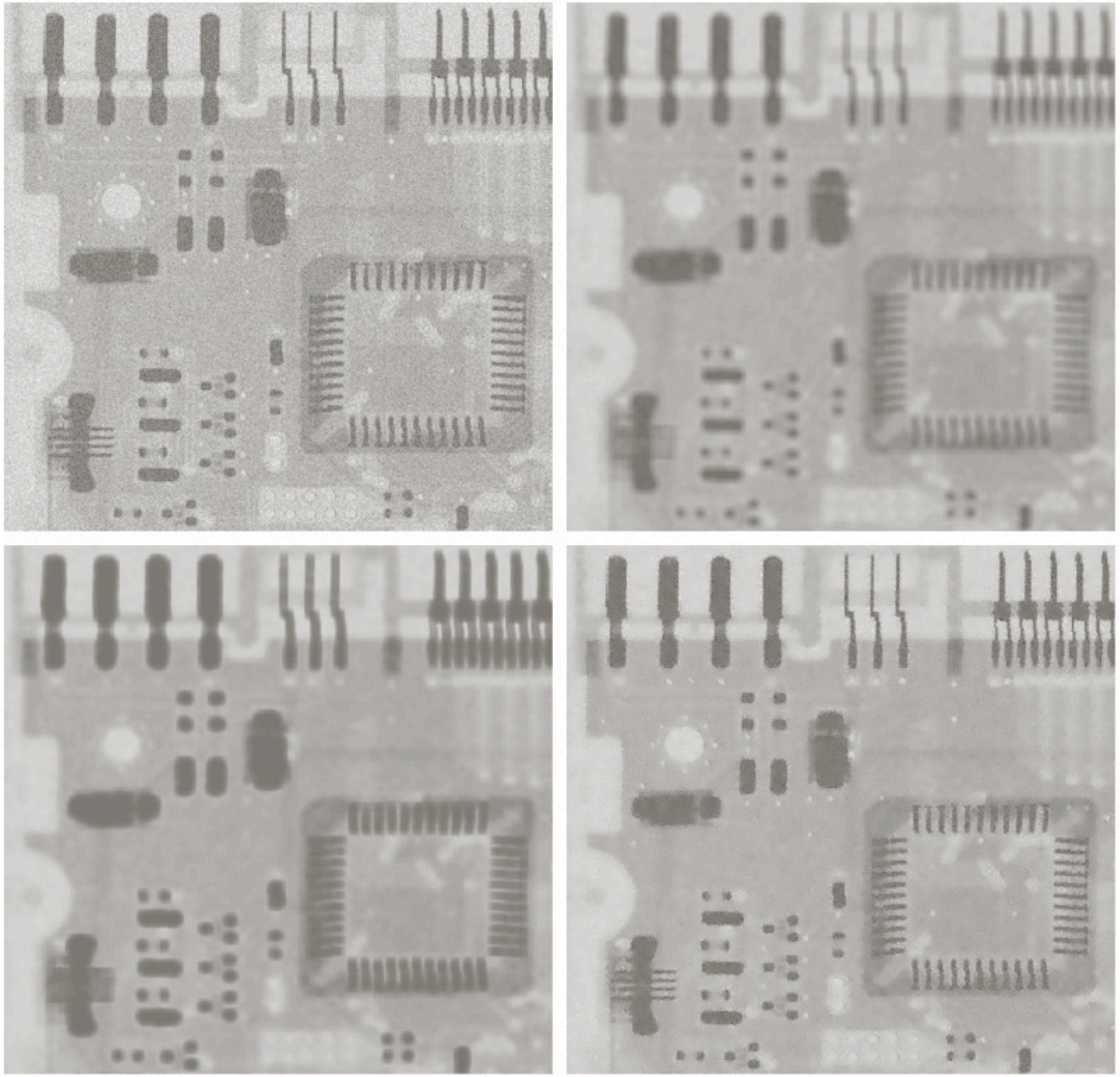
Adaptive Local Noise Reduction Filter

- Mean filter blurs edges while removing additive noise
- Adaptive filter: Reduce averaging around edges

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

  - $\sigma_\eta^2 = 0$: Image is noise free
  - $\sigma_L^2 \gg \sigma_\eta^2$: Local variation is very high (presence of edges)
  - $\sigma_L^2 = \sigma_\eta^2$: Local variation similar to overall noise (mean filter)

Adaptive mean filtering

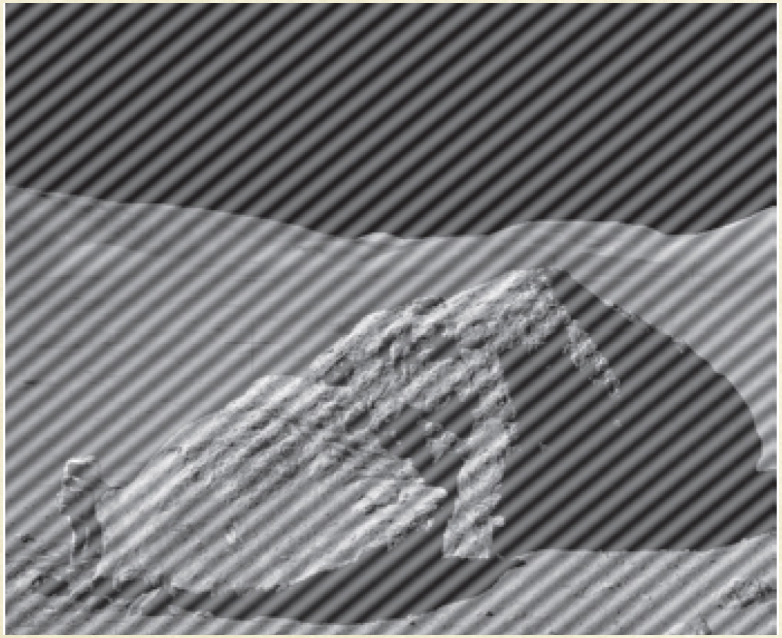


a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Restoration (in presence of noise only)

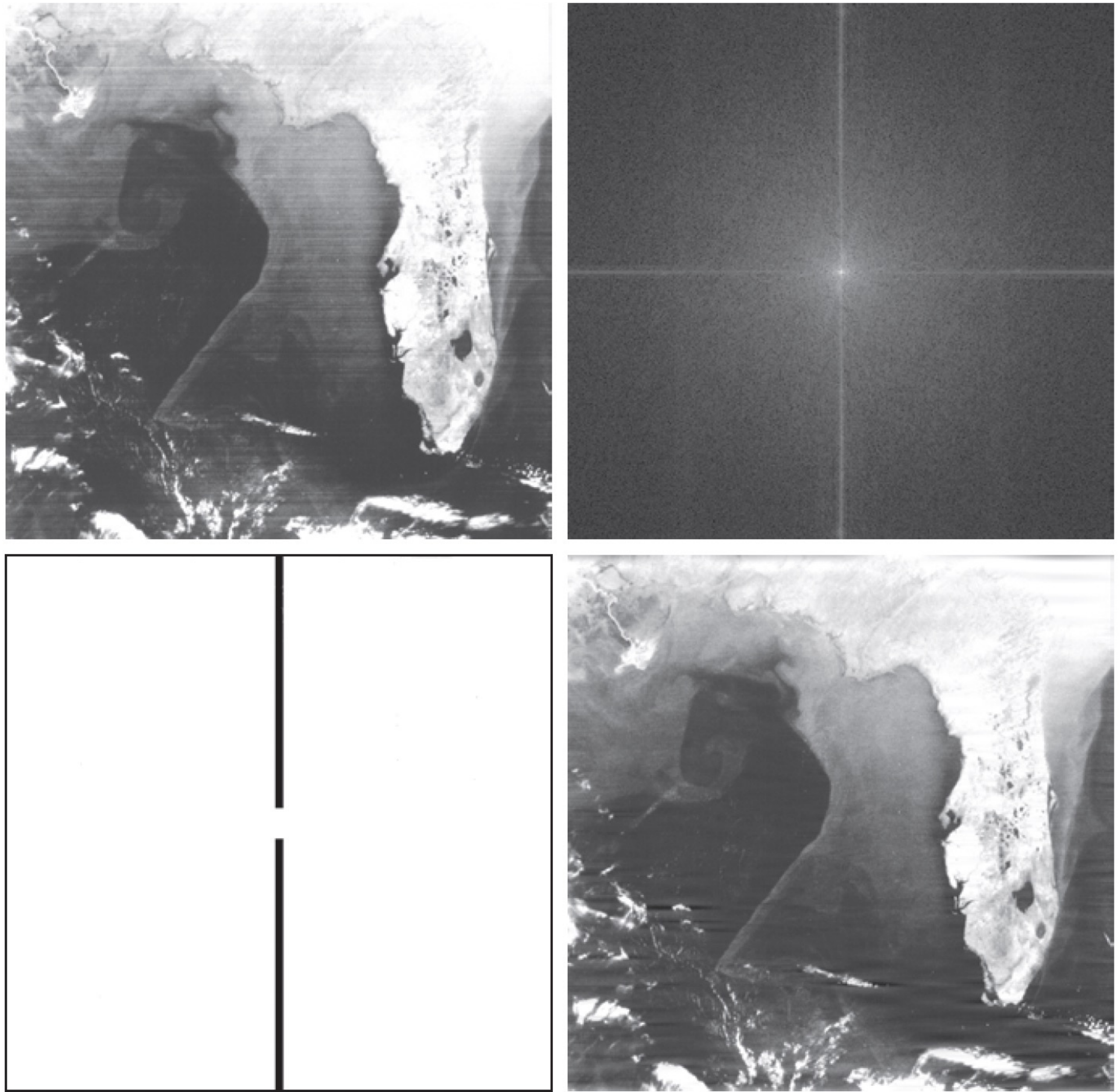
- Band pass/reject



Very difficult to get result of this quality via spatial domain filtering

Restoration (in presence of noise only)

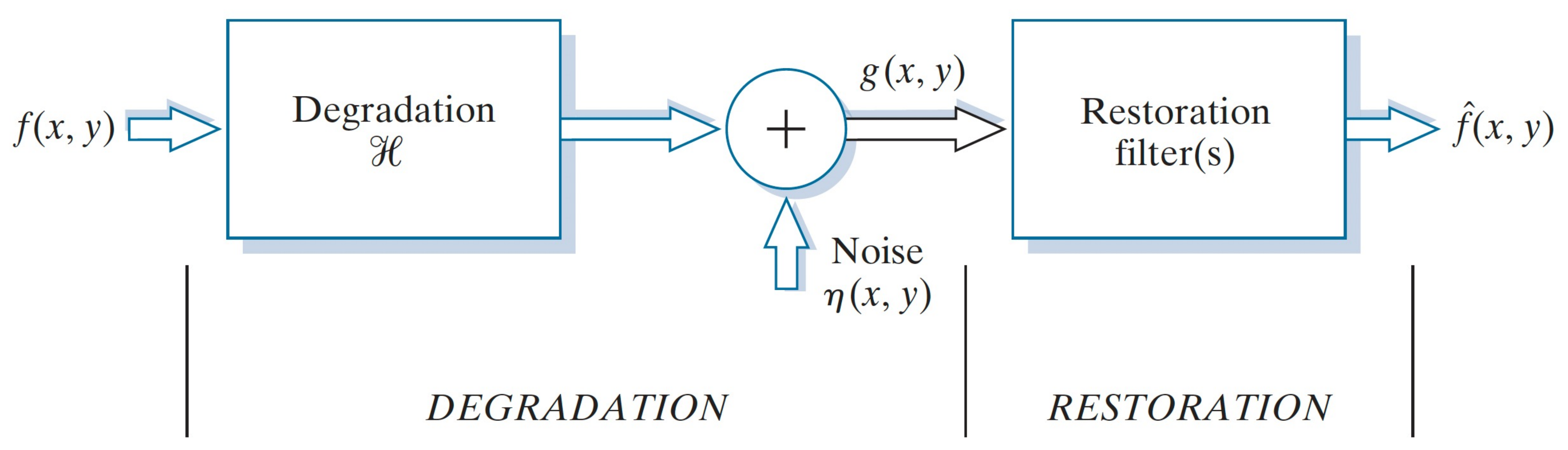
- Notch pass/reject



a b
c d

FIGURE 5.18 (a) Satellite image of Florida and the Gulf of Mexico. (Note horizontal sensor scan lines.) (b) Spectrum of (a). (c) Notch reject filter transfer function. (The thin black border is not part of the data.) (d) Filtered image. (Original image courtesy of NOAA.)

Image Restoration: Modeling Degradation, $H(u, v)$



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y), \text{ or}$$

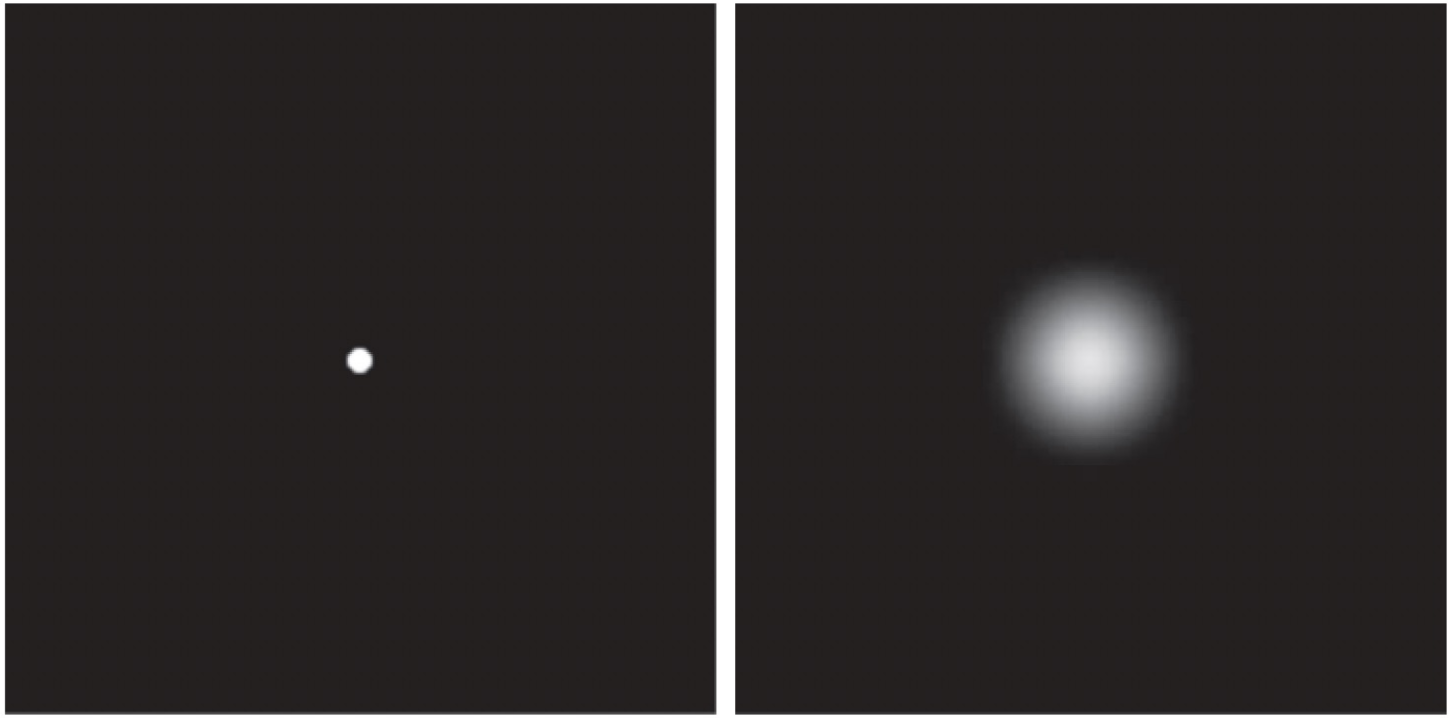
$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Assuming No Noise, $G(u, v) = H(u, v)\, F(u, v)$

Estimation of degradation function

- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration
  - Mathematical modelling

$$H(u, v) = \frac{G(u, v)}{A}$$



a b

FIGURE 5.24 Estimating a degradation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Motion Blur

- Exposure
- If amount of light hitting the sensor changes significantly over exposure period → Motion Blur
- Causes (one or more of)
  - Camera motion
  - Subject motion

Motion blur effect

```
#define filterWidth 9
#define filterHeight 9

double filter[filterHeight][filterWidth] =
{
  1, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 1, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 1, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 1, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 1, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 1, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 1, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 1,
};

double factor = 1.0 / 9.0;
double bias = 0.0;
```

https://lodev.org/cgtutor/filtering.html

Estimation by Modeling (uniform motion blurring)

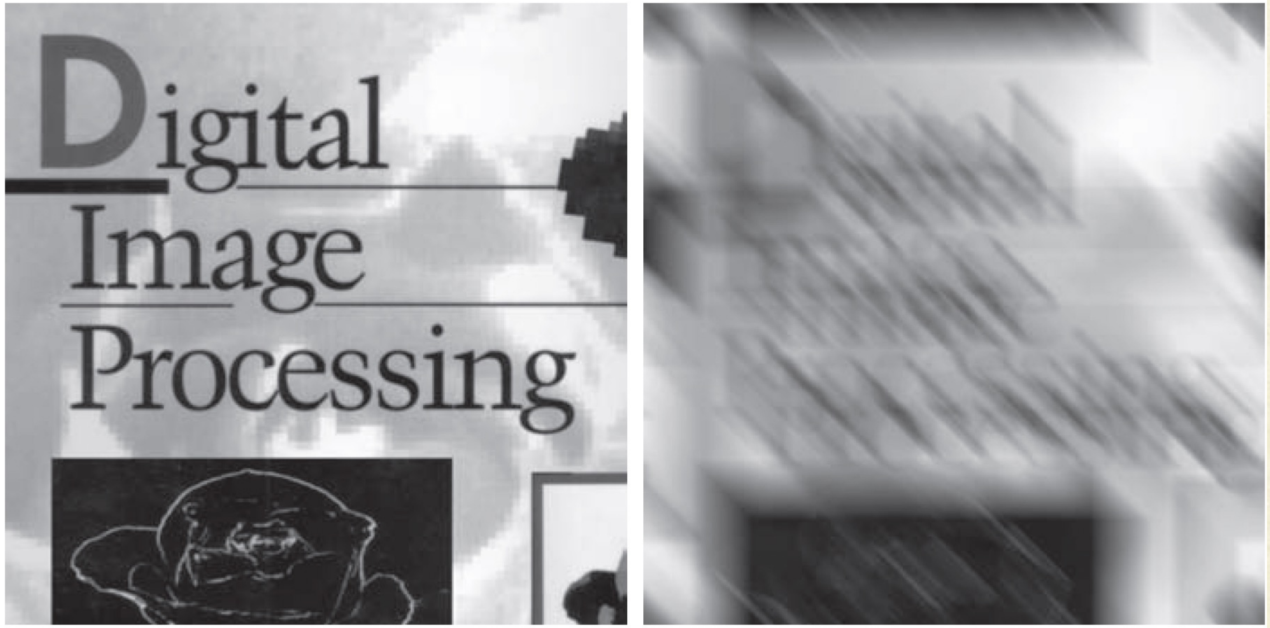


FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5-77) with $a = b = 0.1$ and $T = 1$.

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

$$G(u,v) = F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

putting, $x_0(t) = at/T$ and $y_0(t) = bt/T$

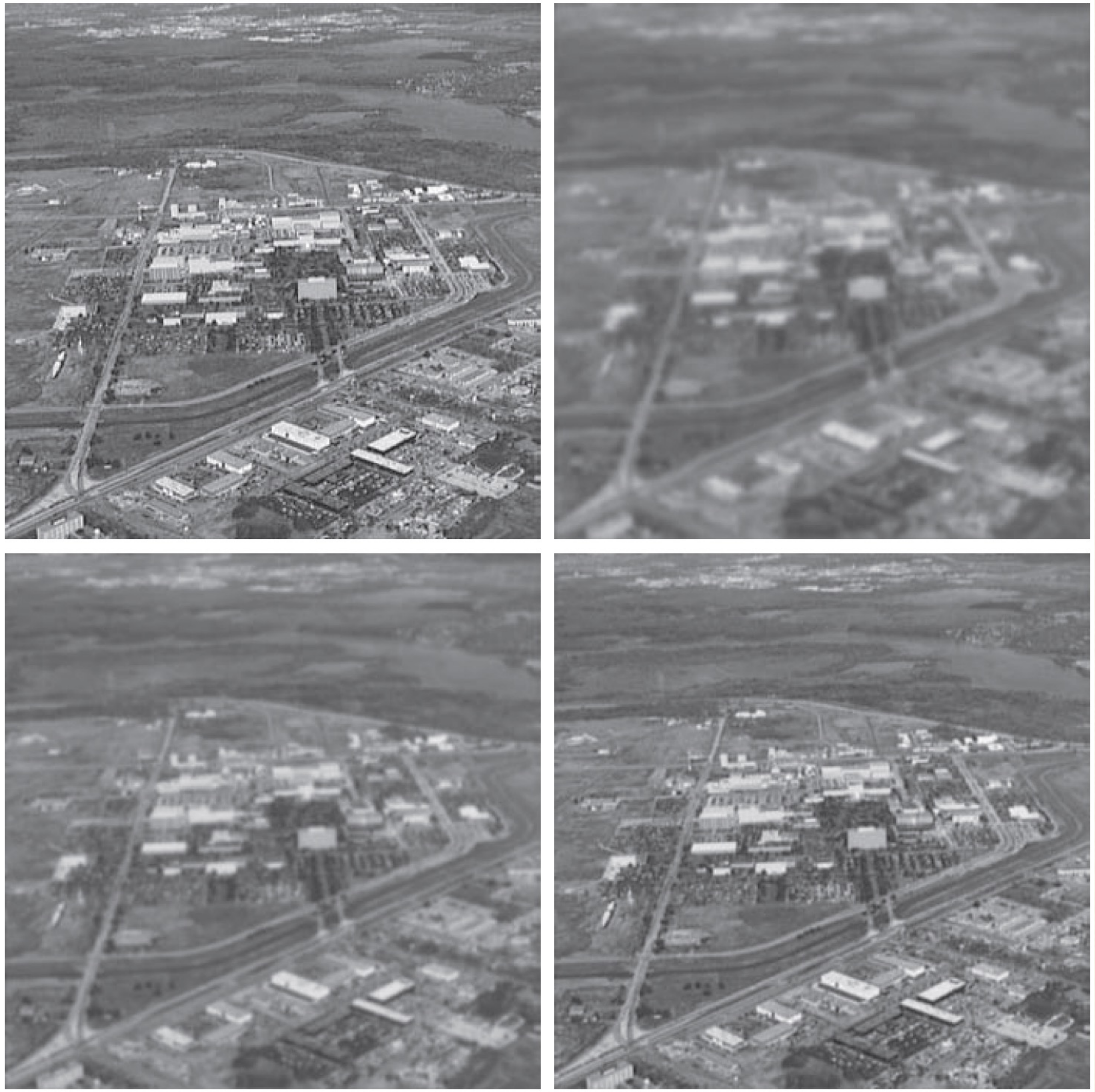
$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right] e^{-j\pi(ua+vb)}$$

Estimation by Modeling (atmospheric turbulence)

a b
c d

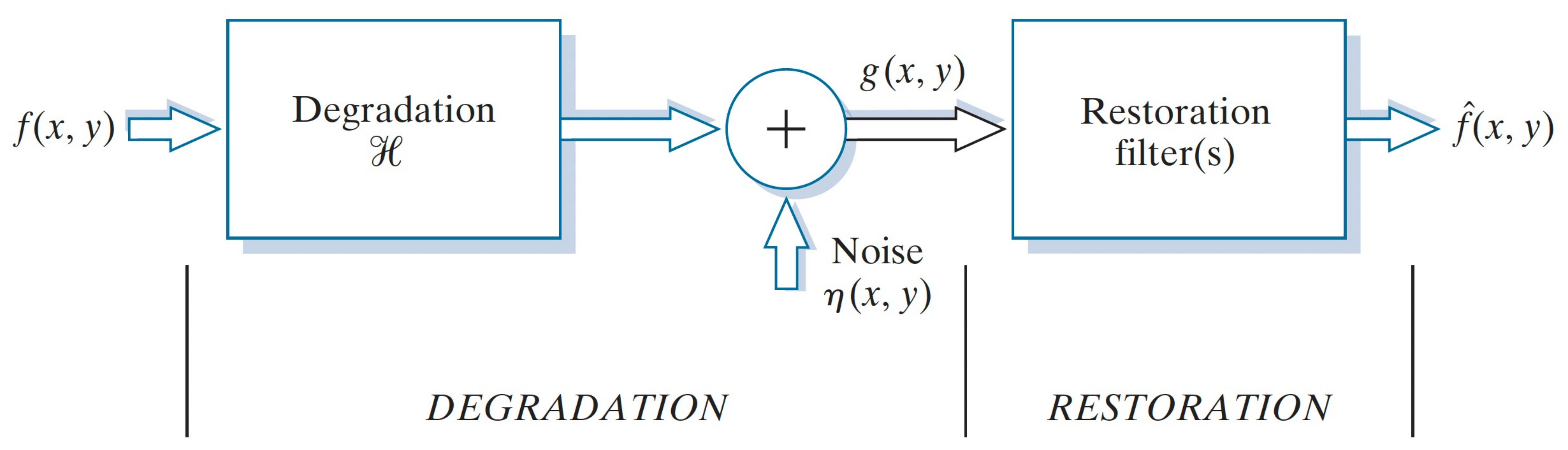
FIGURE 5.25 Modeling turbulence. (a) No visible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. All images are of size 480×480 pixels. (Original image courtesy of NASA.)



Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Image Restoration: Handling $H(u,v)$ and $N(u,v)$



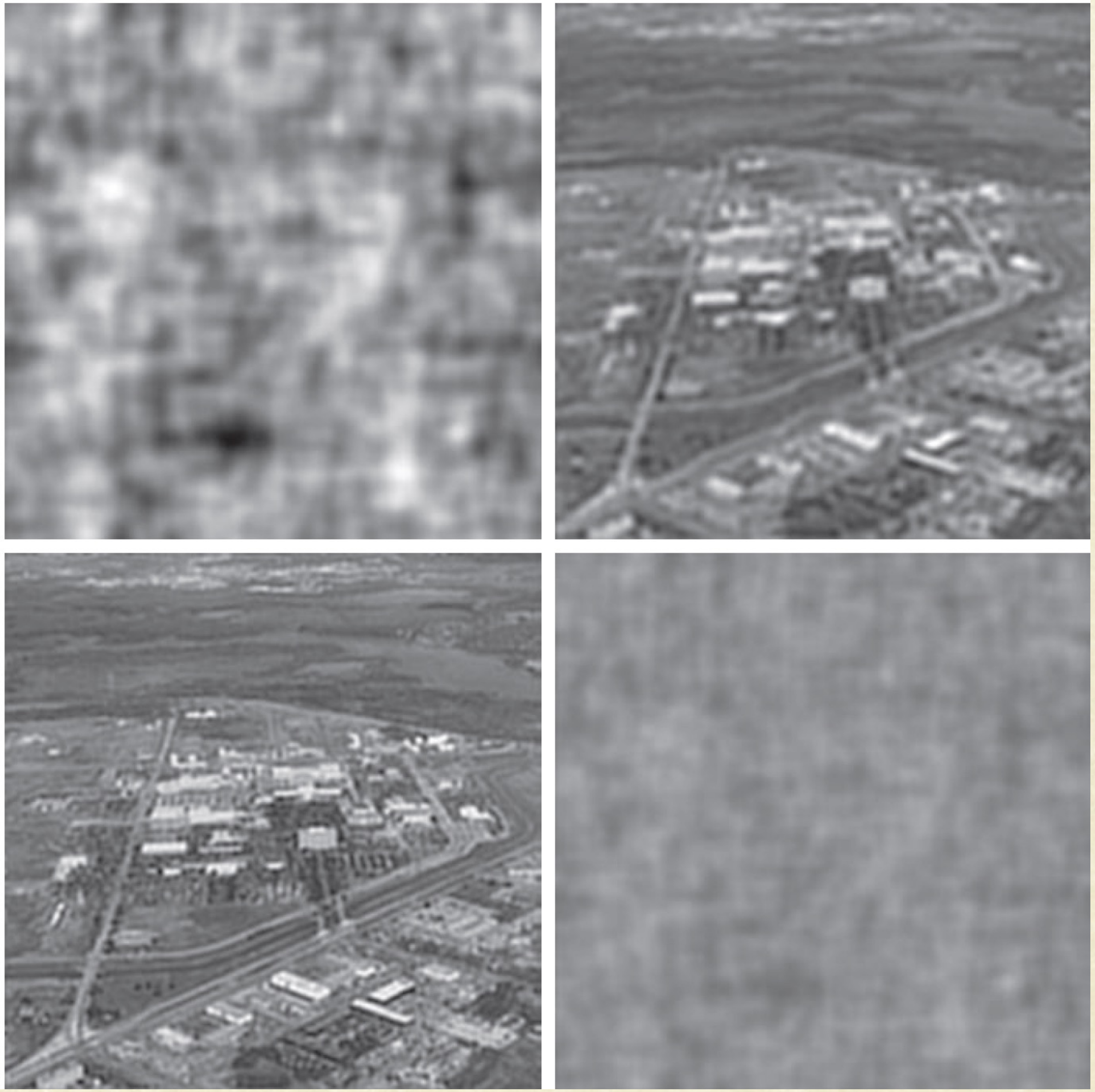
$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y), \text{ or}$$
$$G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

Recovering Image (both Noise and degradation)

- Even if we know the H(u,v), we cannot recover the original image!!
  - $\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$
  - $G(u,v) = H(u,v)F(u,v) + N(u,v)$
  - $\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$
- Two problems:
  1. N(u,v) is a random function whose Fourier Transform is not known
  2. If degradation has zero or small values → N(u,v)/H(u,v) will dominate

Recovering Image (both Noise and degradation)

Degraded Image (with known model)



a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) using Eq. (5-78). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40. (c) Result with *H* cut off outside a radius of 70. (d) Result with *H* cut off outside a radius of 85.

No explicit provision for handling noise!

Weiner filter

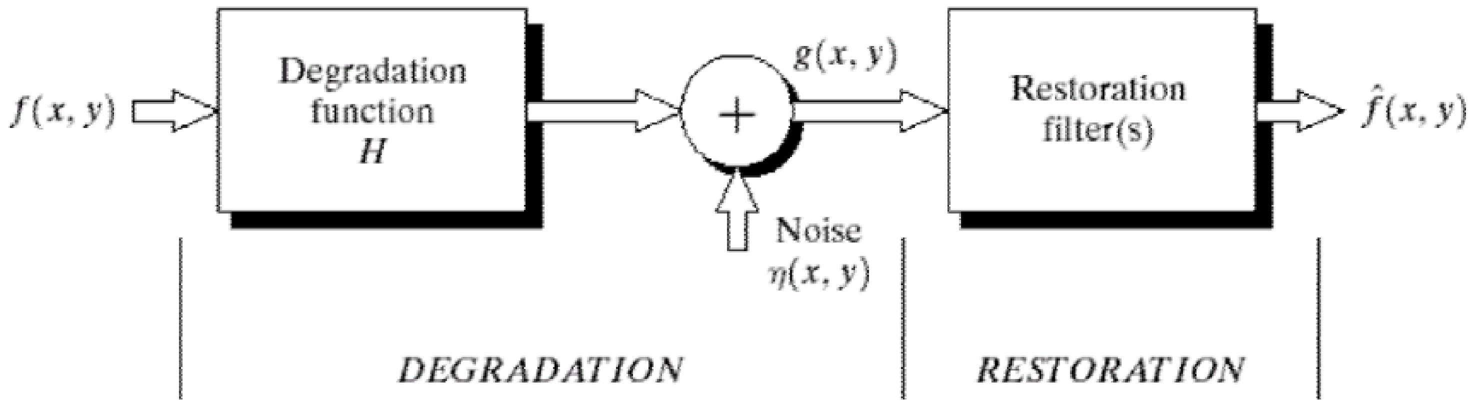


FIGURE 5.1 A model of the image degradation/restoration process.

Consider image and noise as random variables

$$e^2 = E\{(f - \hat{f})^2\}$$

Assumption:

- Noise and image are uncorrelated

Weiner filter

$$e^2 = E\{(f - \hat{f})^2\}$$

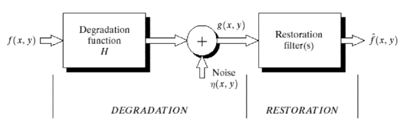


FIGURE 5.1 A model of the image degradation/restoration process.

$$\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\,\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)
\end{aligned}$$

Weiner filter

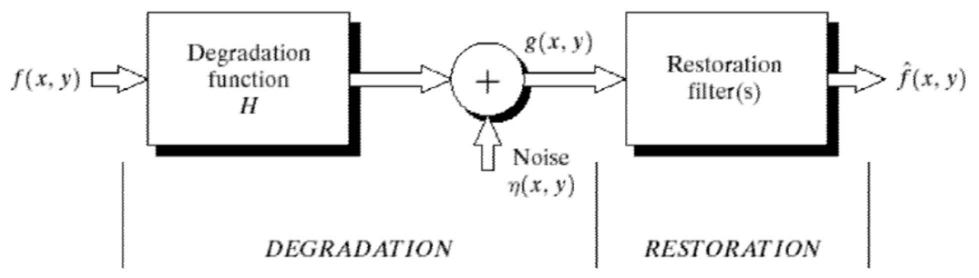


FIGURE 5.1 A model of the image degradation/restoration process.

$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function e is given by:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

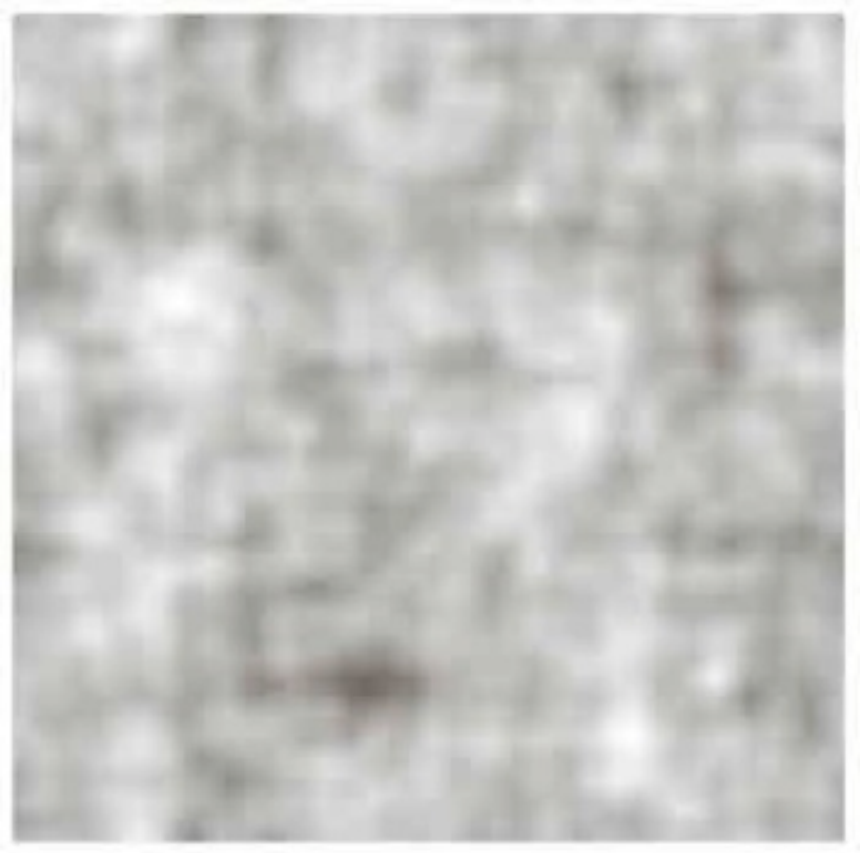
$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image

Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

Weiner filter



Weiner filter

Image Restoration

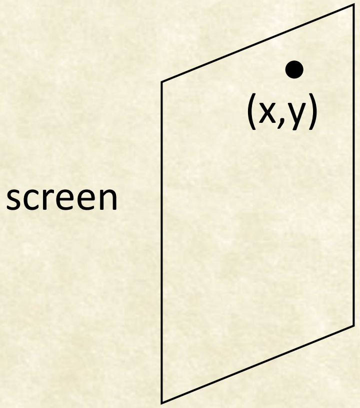
$$f \longrightarrow \boxed{H} \longrightarrow g$$

Inverse problems

Known	Problem type
H,g	Recovery
g	Blind recovery
g, H partially	Semi blind recovery
f,g	System identification

Geometric Distortion

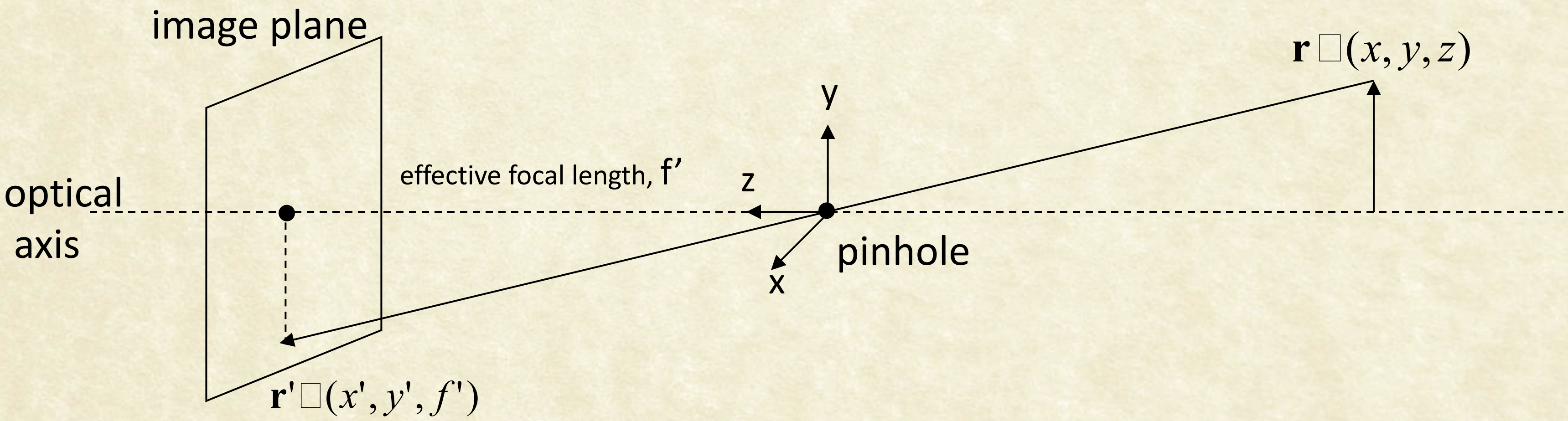
Pinhole and Perspective Projection



scene

Is an image being formed on the screen?

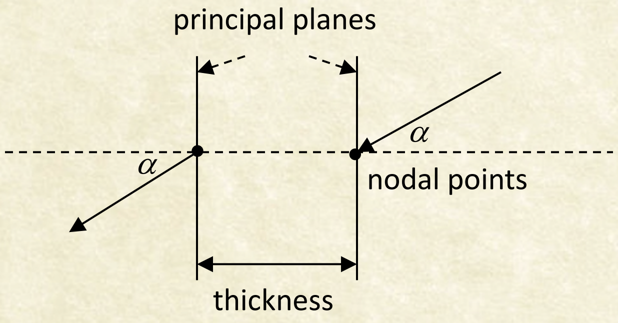
YES! But, not a “clear” one.



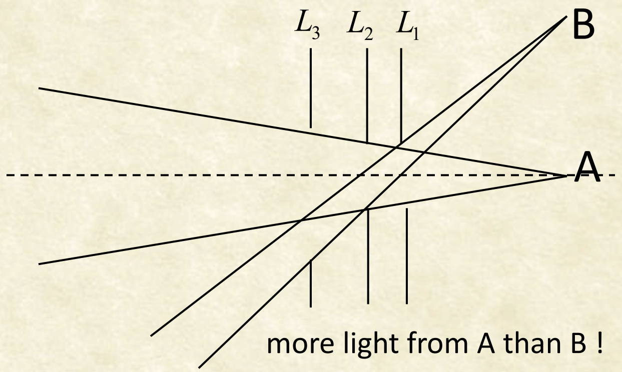
$$\frac{\mathbf{r}'}{f'} = \frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'} = \frac{x}{z} \qquad \frac{y'}{f'} = \frac{y}{z}$$

Common Lens Related Issues

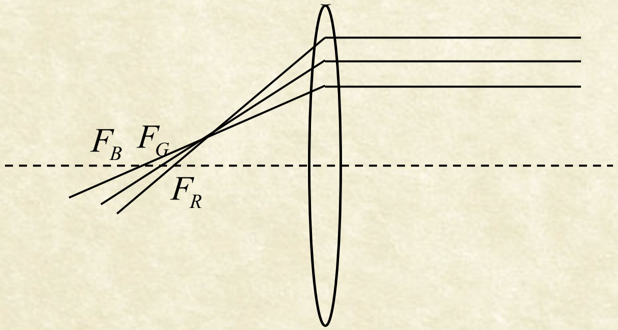
Compound (Thick) Lens



Vignetting



Chromatic Abberation



Lens has different refractive indices
for different wavelengths.

Radial and Tangential Distortion

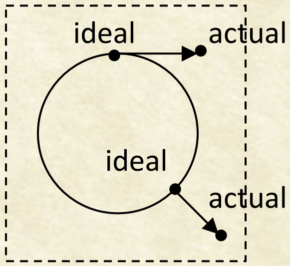


image plane

Lens Glare

- Stray interreflections of light within the optical lens system.
- Happens when very bright sources are present in the scene.

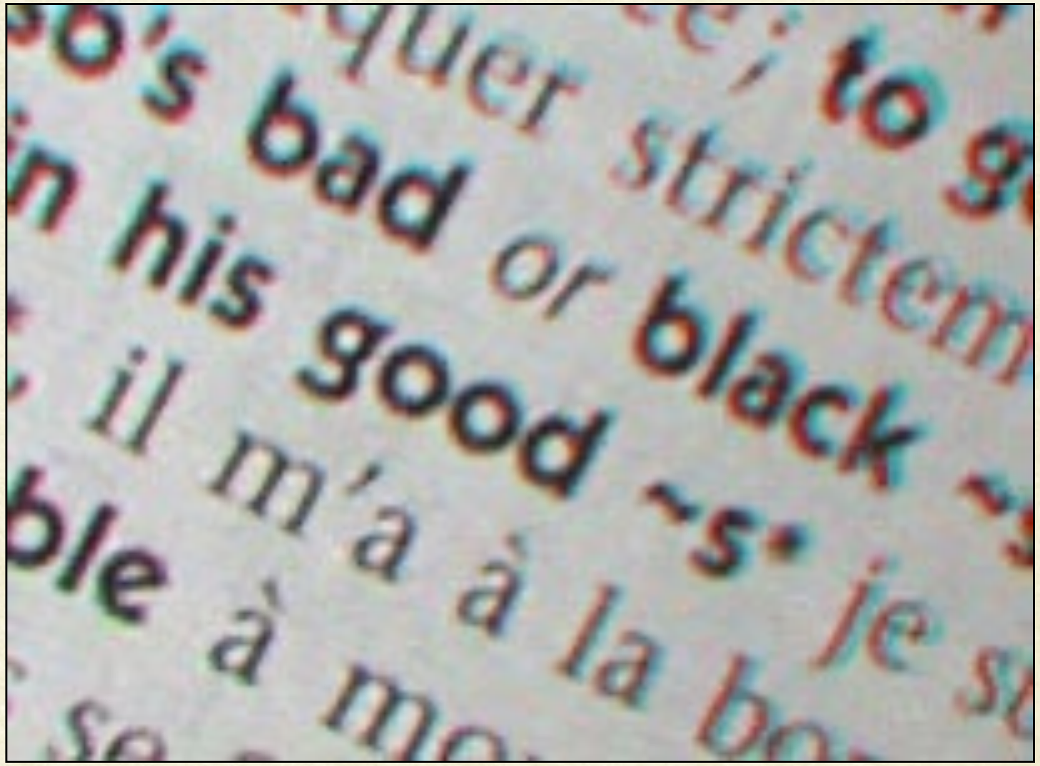
Reading: http://www.dpreview.com

Vignetting

photo by Robert Johnes

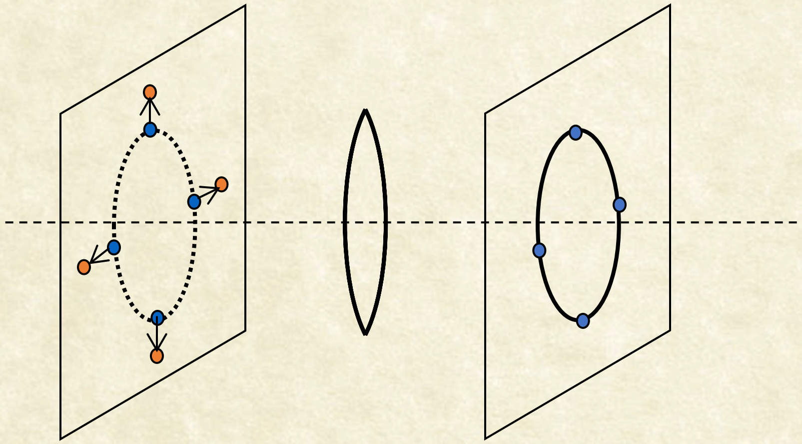
Chromatic Aberrations

longitudinal chromatic aberration
(axial)

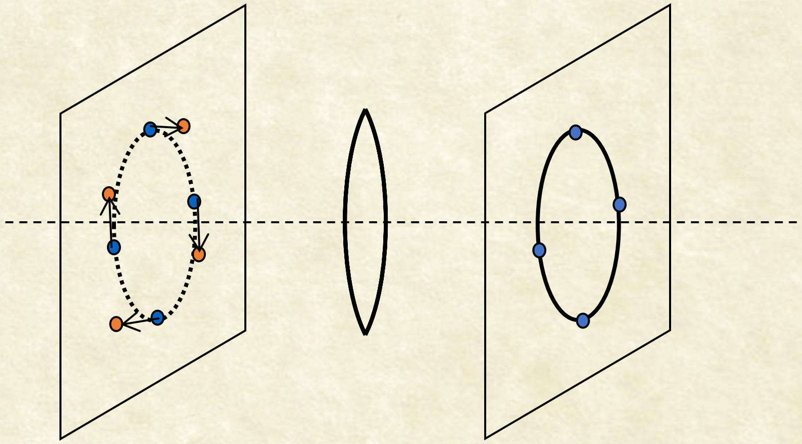


transverse chromatic aberration
(lateral)

Geometric Lens Distortions



Radial distortion



Tangential distortion

Photo by Helmut Dersch

Both due to lens imperfection

Rectify with geometric camera calibration

Is Distortion so bad?

Noise, Vignetting

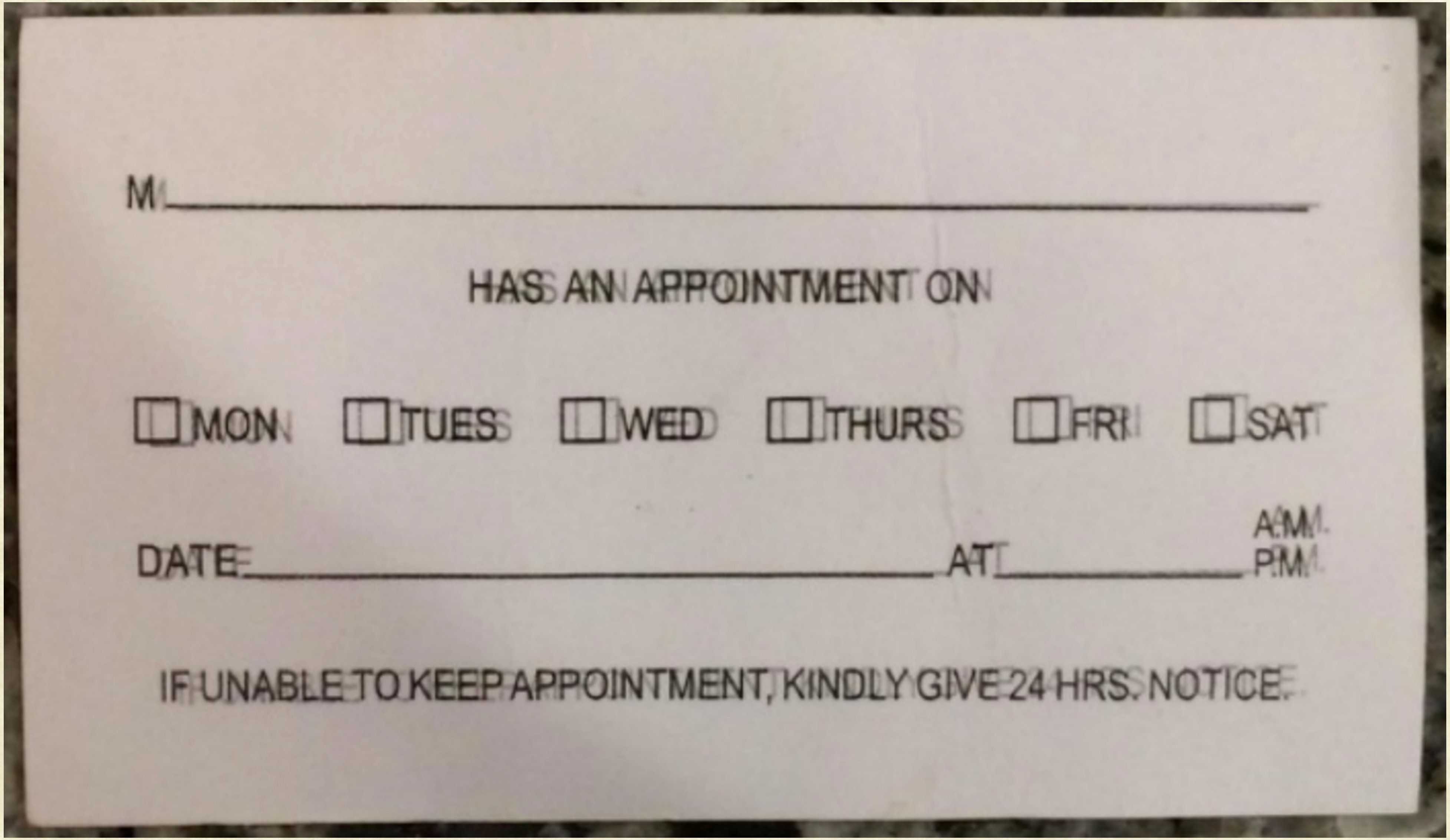
Motion Blur

Barrel Distortion

Other Degradations

Motion Blur

An eye doctor's business card


M______________________________

HAS AN APPOINTMENT ON

☐ MON ☐ TUES ☐ WED ☐ THURS ☐ FRI ☐ SAT

DATE______________________ AT______________ A.M. P.M.

IF UNABLE TO KEEP APPOINTMENT, KINDLY GIVE 24 HRS. NOTICE.

References

- http://www.robots.ox.ac.uk/~az/lectures/ia/lect3.pdf
- https://www.ece.iastate.edu/~namrata/EE528_Spring07/ImageRestoration1.pdf
- http://www.ee.columbia.edu/~xlx/ee4830/notes/lec7.pdf
- DIP Ch. 5 (G&W)