1 Exercise 2: Time Integration

The purpose of this exercise is to gain further knowledge of various time integration schemes, how they work and also how to implement a specific method for computer simulations. Your task is to calculate the displacement as a function of time for a simple harmonic oscillator, using explicit and implicit time integration.

1.1 *Simple harmonic oscillator*

The one degree of freedom linear spring-mass system is shown in Figure 1, and a free-body diagram in Figure 2,

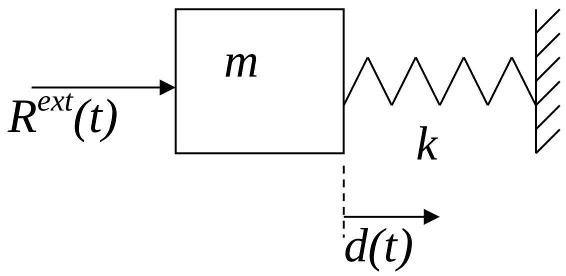

Figure 1. Moving 1-dof system, no damping.

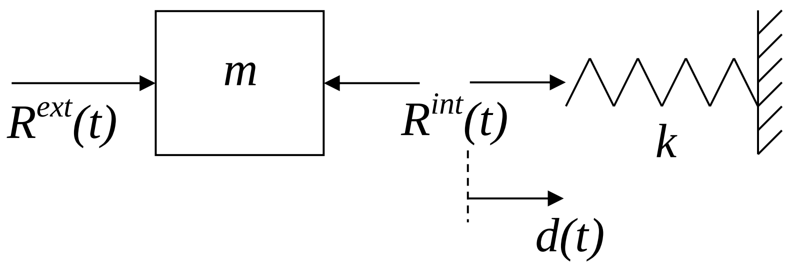

Figure 2. 1-dof system. Free-body diagram with internal force included.

Where R^{ext} is the external force, R^{int} the internal force, d the displacement relative to the equilibrium position, m the mass and k the spring stiffness coefficient. The equation of motion is given by:

$$ma = R^{ext} - R^{int}$$

where $R^{int} = kd$ and a is the acceleration. We will consider a case where the external forces are zero. The initial conditions are:

$$a_0 = 0$$
$$v_0 = A\omega$$
$$d_0 = 0$$

where $v_0$is the initial velocity, ω is the natural frequency of vibration (angular),

$$\omega = \sqrt{k/m},$$

and A is the displacement amplitude of vibration.

The analytical solution for this case is:

$$d(t) = Asin(\omega t).$$

1.2 Exercise assignments

Use Matlab to solve for the displacement as a function of time, using the explicit central difference method and implicit time integration by trapezoidal rule. Then,

1) Compare both numerical solutions with the analytic solution given above, in terms of displacement versus time plots. Choose a suitable value for the amplitude A.
2) Discuss, in general terms, which additions to your code that would be necessary if this was a non-linear problem, both in explicit and in implicit time integration setting.

Appendix 1

Critical time step length for explicit integration using central difference method:

$$t_{cr} = \frac{2}{\omega_{max}} = 2\sqrt{\frac{m}{k}}$$