AST3220, spring 2020: Project 1

Important moral speech

This project consists of a set of tasks, some analytical, some numerical. You should structure your answers as a report with an introduction, methods, results, discussion and conclusion. It is important that you explain how you think, just writing down a bunch of equations with no explanations will not give you a maximum score. I recommend that you write the report using LaTeX. Posting handwritten lecture notes and solutions to problems is a privilege that belongs to the lecturer alone. Your figures should have a clear layout with proper axis labels and units, and with a caption explaining what the figure shows. The figures should be referenced in the main text. You are also required to hand in your source code in a form that can be easily compiled.

The project: A universe with an extra spatial dimension

The idea that space may have more than 3 dimensions was first explored in attempts to unify the description of electromagnetism and gravity. None of these were entirely successful, but later, when string theory was developed, extra spatial dimensions turned out to be necessary to make the theory mathematically consistent.

Since we have seen no indications that space may have more than 3 dimensions, it was thought that the extra dimensions were curled up and extremely small, of the order of the Planck length. However, in the early 2000s it was realised that this need not be the case. One or several of the extra spatial dimensions could in fact be large, even infinite, without coming into conflict with experiments. This lead to several phenomenological models with large extra dimensions being proposed in order to solve a range of problems in fundamental physics. In particular, one can modify how gravity works on large scales and explain the accelerated expansion of the Universe without a cosmological constant or any form of dark energy. In this project, you will study one such model, see how well it fits a selected data set, and compare it to the standard ΛCDM model.

The first thing we want to do is to understand the effect of a large extra dimension on the behaviour of gravity. Since the gravitational field is a complicated object in general relativity, we will study the simpler problem of a scalar field $\phi(\mathbf{x}, t)$, and we will do it by following the paper "Exploring extra dimensions with scalar fields" by K. Brown, H. Mathur, and M. Verostek in American Journal of Physics 86 (2018), page 327. A link to the paper and the supplementary material is provided on the course webpage.

a) Read the introduction. It mentions that Gauss' law implies that in three dimensions the electric (and the gravitational field) must fall like $1/r^2$. Prove this.

b) Prove equation (3).

c) Fill in all the missing steps needed to obtain equation (4).

The model we are interested in is discussed in section V of the paper, the DGP model. The models described before that are mostly of interest for explaining the so-called hierarchy problem in the Standard Model of particle physics.

d) Explain briefly what the hierarchy problem is.

We are interested in the static point-source solution for the scalar field in the DGP model. Problem 4 in the supplementary material to the paper guides you through the analysis.

e) Solve problem 4.

In the DGP model, the first Friedmann equation on the 3 dimensional brane we live on becomes modified to

$$\frac{H^2(z)}{H_0^2} = \left(\sqrt{\Omega_{\mathrm{m0}}(1+z)^3 + \Omega_{\mathrm{rc}}} + \sqrt{\Omega_{\mathrm{rc}}}\right)^2 + \Omega_{\mathrm{k0}}(1+z)^2,$$

where $\Omega_{\mathrm{rc}} = \frac{1}{4H_0^2 r_{\mathrm{c}}^2}$ is a new parameter associated with the length scale r_{c} where the extra dimension manifests itself.

f) What is the sum rule for the density parameters in this model?

We want to compare this model to some measured luminosity distances and redshifts and try to find the parameters Ω_{m0} and Ω_{rc} which best fit the data. In order to do so, it is a useful exercise to first carry out the same analysis for the ΛCDM model.

In all of the following you may assume the value $h = 0.7$ for the dimensionless Hubble constant.

Forgetting about the DGP model for a minute, consider a universe with non-relativistic matter, a cosmological constant, and spatial curvature.

g) Write a code that takes Ω_{m0}, $\Omega_{\Lambda 0}$, and the redshift z as inputs and calculates the luminosity distance at z. You should include in your code a check that the right-hand side of the first Friedmann equation is always positive on the interval $0 \leq a/a_0 \leq 1$ for the given combination of parameters, otherwise you will have numerical problems. Plot the luminosity distance in units of c/H_0 as a function of z for z between 0 and 2 for a couple of parameter sets of your choice.

h) Find two limiting cases where you can calculate the luminosity distance analytically. Plot them, and show that your code agrees with them.

On the webpage for this course you can find a table of measured luminosity distances with associated errors. The table is in the format (redshift, luminosity distance, error). The distances and the errors are given in units of Gpc (1 Gpc $= 10^9$ pc). You will now use this measurements to find empirical constraints on the parameters Ω_{m0} and $\Omega_{\Lambda 0}$. Let us call the expression for the luminosity distance based on these two parameters for our model. Given values for Ω_{m0} and $\Omega_{\Lambda 0}$, we want to know the probability of the model, given the data, $P(\text{model}|\text{data})$. There is no ready recipe for calculating this probability, but a result known as Bayes' theorem says that

$$P(\text{model}|\text{data}) = \frac{P(\text{data}|\text{model})P(\text{model})}{P(\text{data})} \tag{1}$$

The second factor in the numerator is the probability we would assign to the model before obtaining the data, and it is called the *prior*. The factor in the denominator is known as the *evidence*. We will, as is quite common, consider both of these factors to be constants, and we then have the result

$$P(\text{model}|\text{data}) \propto P(\text{data}|\text{model}). \tag{2}$$

The probability on the right-hand side is known as the *likelihood*, and the point is that it is possible to work out how to calculate it. For example, we will assume that the observations are drawn from a Gaussian distribution.

This means that we assume that if we measure the luminosity distance to the ith redshift z_i to be d_{L}^i with measurement error σ_i, then the probability distribution for the true luminosity distance $d_{\mathrm{L}}(z_i)$ is

$$P(d_{\mathrm{L}}(z_i)) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{(d_{\mathrm{L}}(z_i) - d_L^i)^2}{2\sigma_i^2}\right].$$

If we also assume that the measurements are uncorrelated, it can be shown that the likelihood is given by

$$P(\text{data}|\text{model}) = \frac{1}{(2\pi\prod_{i=1}^N \sigma_i^2)^{1/2}} \exp\left[-\frac{1}{2}\sum_{i=1}^N \frac{(d_L(z_i;\vec{p}) - d_L^i)^2}{\sigma_i^2}\right],$$

where $d_L(z_i, \vec{p})$ is the model prediction for the luminosity distance to redshift z_i for given parameter vector $\vec{p}$, and N is the number of observations. In our case, $\vec{p} = (\Omega_{\mathrm{m}0}, \Omega_{\Lambda 0})$. To find the most probable values of $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$, we want to maximize the likelihood as a function of these two parameters, and this is equivalent to minimizing the quantity

$$\chi^2(\vec{p}) = \sum_{i=1}^N \frac{(d_L(z_i;\vec{p}) - d_L^i)^2}{\sigma_i^2}.$$

i) Write a code that finds the most probable value for $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$ given the data in the file you can download from the course webpage.

j) We cannot be sure that the most probable value is the *true* value. All the data allow us to find is the most probable value, and the range in which the true value probably lies. In the case you consider here, it can be shown that there is a 95 % probability that the true values of $\Omega_{\mathrm{m}0}$ and $\Omega_{\lambda 0}$ if found in the region which satisfies

$$\chi^2(\vec{p}) - \chi^2_{\mathrm{min}} < 6.17,$$

where χ^2_{min} is the minimum you found in i). Find this region and plot it in the $\Omega_{\mathrm{m}0}$-$\Omega_{\Lambda 0}$ plane.

k) Plot the data with associated errors along with a few theoretical $d_{\mathrm{L}}(z)$ curves with parameters chosen from the region in ii). In the same figure, also plot the luminosity distances for the EdS and the dS models.

l) Repeat the analysis for the DGP model. Start by writing a code that takes the redshift z, Ω_{m0}, and Ω_{rc} as input and calculates the luminosity distance (also in this case you should include a test for non-negativity of the right-hand side of the first Friedmann equation). Once this is in place, you should be able to reuse most of the code you wrote for the ΛCDM analysis. Find the best-fit values of Ω_{m0} and Ω_{rc} and make a plot similar to the one in j).

m) Which model fits the data best?