UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: AST4320 — Cosmology and Extragalactic Astronomy

Day of examination: Friday December 11th 2015

Examination hours: 14.30 – 18.30

This problem set consists of 6 pages.

Appendices: None

Permitted aids: Formula book by Rottmann.
Calculator that meets standard requirements by university.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Structure Formation in the Universe (weight 40 %)

General remark: when you are asked to provide a description of something, be brief. For example, when we ask what the 'Jeans' mass is, there is no need to present a derivation.

The Universe was not perfectly smooth or homogeneous even at the very beginning. The density of matter was varying from place-to-place throughout the Universe. We quantify these fluctuations with the density contrast $\delta(\mathbf{r}) \equiv \frac{\rho(\mathbf{r})-\bar{\rho}}{\bar{\rho}}$, where $\rho(\mathbf{r})$ denotes the mass density at position $\mathbf{r}$, and $\bar{\rho}$ denotes the mean density of the Universe at a given time.

a Briefly describe and sketch how δ evolves for with time t (or if you prefer, scale factor a) for pressureless dark matter in a region that is initially overdense (i.e $\delta > 0$). Include the concept of 'horizon entry' in this discussion/sketch.

$\delta \propto a$ during matter domination. The meszaros effect suppresses growth and makes it $\delta \sim$ constant, or $\delta \propto \log a$. However, this is only the case when the region is smaller than the horizon, i.e. prior to horizon entry. Prior to horizon entry a perturbation grows as $\delta \propto a^2$.

b How does δ evolve for ordinary baryons? Include the concept of 'Jeans Mass' in your discussion.
Baryonic overdensities grow the same, provided the perturbation is larger than the so-called Jeans mass. If the perturbation is smaller than the Jeans length, pressure forces prevent collapse.

c Describe briefly how dark matter helps the formation of structures that we currently observe in the Universe. Include the amplitude

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of temperature fluctuations that we observe in the Cosmic Micrwave Background in this discussion.

Prior to recombination the Jeans mass is very large. Baryonic structures cannot collapse. The CMB temperature fluctuations trace the baryonic density fluctuations at $z \sim 10^3$, or $a \sim 10^{-3}$. The observed amplitude of intensity fluctuations in the CMB is $\Delta T/T \sim 10^{-5} \sim \delta_{\rm baryon}$. Because $\delta \propto a$ there is not enough time to grow structures. Fortunately, the dark matter was able to collapse to higher overdensities. Once recombination finishes, the Jeans mass dropped and baryons were able to flow into the gravitational potential wells created by dark matter.

d The statistical properties of the density field $\delta(\mathbf{r})$ are quantified by the power spectrum $P(k)$. Sketch a plot of $P(k)$ vs k for (*1*) a density field that has the strongest fluctuations on smaller scales, and (*2*) a density field that has most power on large scales.

1. Small scales equals large k. $P(k)$ is larger for large k. The opposite is true for the reverse.

e In the lectures we introduced another statistical measurement of the density field $\delta(\mathbf{r})$, namely $\sigma(M)$. Sketch what this quantity is. *Hint:* the mass scale M corresponds to the total mass enclosed in sphere of radius R.

Random density field. Randomly lie down sphere of radius R, such that enclosed (average) mass with these spheres is M. If we plot a histogram of density inside all spheres, we would find is Gaussian with mean $\delta = 0$ and standard deviation $\sigma(M)$.

f The figure below shows observational constraints on the matter power spectrum.

Discuss the shape of $P(k)$. Discuss what sets the 'turn-over', and what sets the difference in $P(k)$ at scales smaller and larger than this turn-over scale. Also, mention qualitatively where and how we would expect a modification of the power spectrum if dark matter were 'warm' rather than 'cold'.

The turn-over is due to the meszaros effect, which states that perturbations cannot grow during radiation domination, unless they were larger than the horizon size. Smaller perturbations, i.e. larger k entered the horizon earlier and were therefore more subject to the Meszaros effect. Warm dark matter suppresses fluctuations on small scales, i.e. larger k. This means that this suppresses $P(k)$ for large k.

g What is the mass scale M that corresponds to $k = 10^{-1}$/Mpc, $k = 10^0$/Mpc, and $k = 10^1$/Mpc (you can ignore the factor h on the horizontal axis of the plot), assuming that $\Omega_{\rm m} = 0.3$ and a Hubble parameter of $H_0 = 70$ km/s/Mpc (1 Mpc$=3.08 \times 10^{24}$ cm). *Hint: the critical density of the Universe today is* $\rho_{\rm crit} = \frac{3H_0^2}{8\pi G}$, *where* $G = 6.67 \times 10^{-8}$ $cm^3\ g^{-1}\ s^{-2}$.

$M = \frac{4\pi}{3}R^3\bar{\rho}$, where $R = 2\pi/k$ and $\bar{\rho} = \Omega_{\rm m}\rho_{\rm crit}$. Substituting numbers we get $M = 4.2 \times 10^{16} M_\odot$ for $k = 10^{-1}$/Mpc, $M = 4.2 \times 10^{13} M_\odot$ for $k = 10^0$/Mpc, and $M = 4.2 \times 10^{10} M_\odot$ for $k = 10^1$/Mpc.

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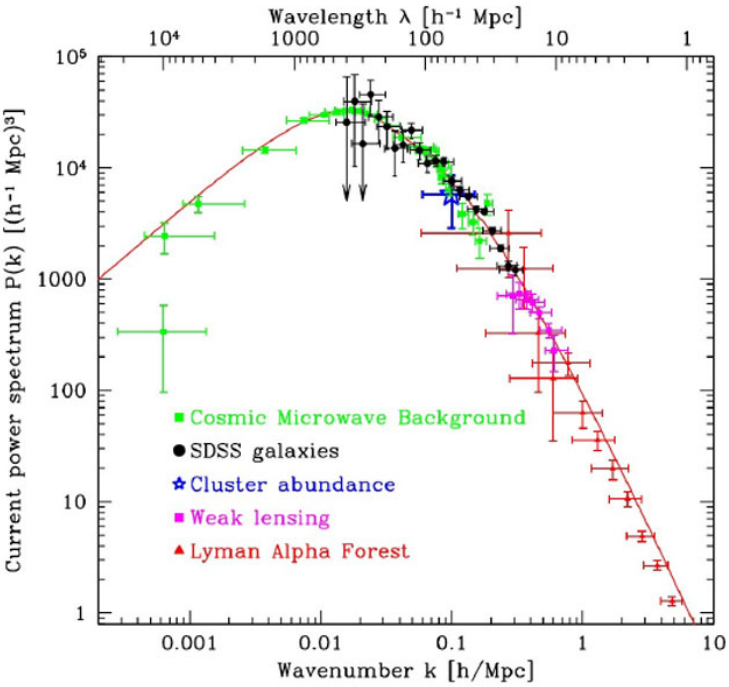

h In the lecture, we derived that

$$\sigma^2(M) \propto k^{3+n} \propto R^{(-n-3)} \propto M^{(-n-3)/3}. \tag{1}$$

, for $P(k) \propto k^n$. By reading off the slope in the range of k that is relevant for galaxies, estimate the the mass dependence of $\sigma^2(M)$, and argue why we expect low-mass dark matter halos to form first (answers to previous questions may be helpful). Lets pick a mass-scale for $10^{12} M_\odot$ like our Milky Way as representative for galaxies. This corresponds to $k \sim 3.4/\text{Mpc}$. From the plot we see that the powerspectrum has a slope of roughly $P(k) \propto k^{-2}$ (2 orders of mag drop in P over 1 order of mag increase in k), in which case $\sigma(M) \propto M^{-2}$. In other words, $\sigma(M)$ is larger for lower mass galaxies. We therefore have a larger variance in the fluctuations on these mass-scales. It is therefore more likely that randomly chosen spheres with lower masses M has a 'critical' overdensity that is associated with collapse, $\delta_{\text{crit}} = 1.69$ (this likelyhood also grows with time, because $\sigma(M) \propto a^2$ during matter domination).

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Problem 2 Galaxies, Stars & Black holes (weight 45 %)

a Which processes (broadly) set the range of masses of galaxies? Use concepts like collapse, virialization, gas cooling.
When a gas cloud collapses, it virializes and reaches the virial temperature. Mass range is set broadly by comparing the cooling time of this virialized gas vs dynamical time of the overall collapsed gas cloud. The cooling time is the time it takes for gas to radiate away its thermal energy ($t_{\rm cool} \propto E/C(T)$). The dynamical time corresponds to the time-scale over which structure changes in the gas become apparent ($t_{\rm dyn} \propto \rho^{-1/2}$). The lower limit is set by inefficient cooling. Below $T = 10^4$ K cooling is extremely inefficient (gas is to cold for collisionals to excite any transitions of atomic H). For high T cooling is efficient, by the energy content is so large that the cooling time remains very large.

b Which process set the the minimum ($m_{\rm min}$) and maximum mass ($m_{\rm max}$) of stars?
The minimum mass is set by the central density of the stars which must be large enough to trigger nuclear fusion. At masses below $m_{\rm min} \sim 0.07M_{\odot}$ the central temperature is too low to ignite any fusion processes. The maximum mass is set by radiation pressure. the luminosity of stars increases so rapidly with stellar mass that the pressure exerted by the stellar radiation blows off mass from the star. In class I showed some images of a Wolf-Rayet star which is a massive star which shed its outer mass.

c Describe what the initial mass function (IMF) $\frac{dn}{dm_*}$ is.
The IMF describes the number of stars that form during star formation as a function of their mass.

d A galaxy undergoes a burst of star formation at time $t = 0$. Assume that the IMF is a power law of the form $\frac{dn}{dm_*} \propto m_*^{-x}$, and that the mass-luminosity relation of stars equals $L \propto m_*^y$. Write down an expression for the luminosity of the galaxy. Throughout we will assume that $x = 2.35$ and $y = 3$.

$$L = \int_{m_{\rm min}}^{m_{\rm max}} dm_* \, m_* \frac{dn}{dm_*} L(m_*) \propto \int_{m_{\rm min}}^{m_{\rm max}} dm_* m_*^{y-x} \propto \tag{2}$$

$$\int_{m_{\rm min}}^{m_{\rm max}} dm_* m_*^{0.65} \propto [m_{\rm max}^{1.65} - m_{\rm min}^{1.65}] \tag{3}$$

e The life-time of stars scales as $t_* = m_*/L$. If $m_{\rm max} \gg m_{\rm min}$, how does the luminosity of the galaxy evolve with time? Comment briefly on the time evolution of the spectrum of the radiation (focus on UV, optical and IR bands. Use that the spectrum of our sun, a 1 solar mass star, peaks in the optical). The life time of our sun $t_{\odot}$ is approximately 10 Gyr (the mass of our sun is denoted with $M_{\odot} = 2 \times 10^{33}$ g)

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$$L \propto [m_{\rm max}^{1.65} - m_{\rm min}^{1.65}] \approx m_{\rm max}^{1.65} \quad (4)$$

Since $t_* \propto m_*/L \propto m_*^{1-y} = m^{-2}$, we have $m_{\rm max} \propto t^{1/2}$. Therefore

$$L(t) \propto m_{\rm max}^{1.65} \propto t^{-1.65/2} \approx t^{-0.8} \quad (5)$$

The luminosity decreases with time. The most massive stars are bright in the UV. The loss in luminosity starts with the loss of UV flux and then moves down to the optical and subsequently into the IR.

f Which processes set the range of masses of black holes? In class we discussed the minimum mass required for black hole formation via stellar evolution (you do not have to worry about alternative, more exotic mechanisms that may form lower mass black holes).
Minimum mass is the Chandrasekhar mass, which is $\sim 1.4M_\odot$. Anything above that will collapse into a BH. The maximum mass is set by the maximum mass accretion rate.

g For a particle of mass m in a circular orbit around a black hole of mass M, show that the total energy of the particle $E_{\rm tot} = \frac{-GMm}{2r}$.
$E = U + K = -\frac{GMm}{r} + \frac{1}{2}mv^2$. Circular orbits we have $\frac{mv^2}{r} = \frac{GMm}{r^2}$. Combining the two gives us the desired answer.

h The *Inner Most Stable Circular Orbit* [ISCO] occurs at $r = 3R_{\rm s}$, where $R_{\rm s} = 2GM/c^2$ is the Schwarzschild Radius. Show that (1) each particle of mass m must radiate away $\Delta E = \eta mc^2$ before reaching the ISCO. This analysis will give you a numerical value for η; (2) the mass growth rate of the black hole at a fixed accretion luminosity L is given by $\dot{M}_{\rm BH} = \frac{1-\eta}{\eta c^2}L$.
(1) Substituting into above gives $E = \frac{-GMm}{12GM/c^2} = mc^2/12$, i.e. $\eta = 1/12$. (2) An accretion luminosity L is powered my mass accretion rate $\dot{M}_{\rm acc} = L/[\eta c^2]$. A fraction $1-\eta$ of this accreted mass is added to the black hole (as a fraction η is radiated away). We therefore have $\dot{M}_{\rm BH} = (1-\eta)\dot{M}_{\rm acc}$, which equals $\dot{M}_{\rm BH} = \frac{1-\eta}{\eta c^2}L$.

i A black hole with mass $1.3\times10^{10}M_\odot$ was found at a redshift of $z \sim 6.3$, when the Universe was ~ 1 billion years old. Why may such an object pose a problem for our current theory of structure formation. You can use the expression for the accretion rate given above, and use that the "Eddington luminosity" is $L = 1.3\times10^{38}(M/M_\odot)$ erg/s. You can also assume that $\eta = 0.1$. From above we know that the maximum black hole growth rate occurs when the BH emits at Eddington luminosity, in which case $\dot{M}_{\rm BH} = kM$, which has a solution $M = M_{\rm BH}(t=0)\exp\left(\frac{t}{t_{\rm s}}\right)$. Here, $t_{\rm s} = \eta c\sigma_{\rm T}/[(1-\eta)4\pi G m_{\rm p}] \sim 50$ Myr. The existence of super massive black holes of mass $M_{\rm BH} \sim 10^{10}M_\odot$ at $z \sim 6.3$ [when the Universe was only 1 Gyr old] is difficult to explain by having stellar mass black holes grow by $\sim 8-9$ orders of magnitude via the maximum allowed accretion rate over its entire life time.

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Problem 3 Lyα Forest (weight 15%)

a Describe what the Lyα forest is, what its origin is, and and what it tells us about the physical properties of the intergalactic medium. You may find it helpful to provide a sketch that supports your description. (40%) Lyα forest is a collection of Lyα absorption lines that have been observed in quasar (but also galaxy) spectra. In the lecture we derived that if the gas in te Universe were neutral, that then we should absolutely see no flux at frequencies higher than the redshifted Lyα frequency. The fact that we see absorption lines and a continuum implies that gas in the Universe, the intergalactic medium must have been highly ionized. The fluctuations in Lyα opacity in the Lyα forest correspond to density fluctuations along the line-of-sight. Most of these absorption lines correspond to mildly overdense regions $\delta \sim 1$, which is very different than the densities probed by virialized structures. We also learned in the lecture that the temperature of photoionized intergalactic medium is $T = 10^4$ K. For lower temperatures, cooling is too inefficient and the gas efficiently heats up. For higher temperatures, the gas is too highly ionized for the radiation to insert more energy.

b Describe what 'epoch of reionization' is, and how we know it must have happened. The EoR refers to the transformation of the gas in our Universe from neutral and cold, which it was following recombination, to ionized and hotter (which is what we infer from the Lyα forest). This transformation is likely linked to the formation of the first stars, galaxies and black holes throughout our Universe.

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