UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: AST4320 — Cosmology and Extragalactic Astronomy

Day of examination: Monday 12. December 2016

Examination hours: 14.30 – 18.30

This problem set consists of 10 pages.

Appendices: None

Permitted aids: Formula book by Rottmann.
Calculator that meets standard requirements by university.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Preface: some useful relations:

- $H(a) = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda + \Omega_{\rm rad}a^{-4}}$, where $H_0 = 2.27 \times 10^{-18}$ $\rm s^{-1}$ is the Hubble parameter today, Ω's denote the density parameters in matter, dark energy, and radiation in units of the critical density today (see next bullet point).
- $\Omega_{\rm m} = \frac{\bar{\rho}_{\rm m}}{\rho_{\rm crit}}$, where $\rho_{\rm crit} = \frac{3H_0^2}{8\pi G}$, $G = 6.67 \times 10^{-8}$ $\rm cm^3\ g^{-1}s^{-2}$, and $\bar{\rho}_{\rm m}$ denotes the average mass density of the Universe today.
- Einstein-de Sitter Universe: $\Omega_{\rm m} = 1.0$, $\Omega_\Lambda = 0.0$.
- Dynamical time $t_{\rm dyn} = \frac{1}{\sqrt{4\pi G\rho}}$. Cooling time is $t_{\rm cool} = \frac{3n_{\rm H}k_{\rm B}T}{2n_{\rm H}^2\Lambda(T)}$.
- The sound speed $c_{\rm s} \approx 10(T/10^4\ {\rm K})^{1/2}$ km $\rm s^{-1}$.
- Collapse criteria: $\delta_{\rm lin} = 1.69$, $\delta_{\rm vir} = 178$.
- $1M_\odot = 2 \times 10^{33}$ g, 1 Mpc$=3.08 \times 10^{24}$ cm, $k_{\rm B} = 1.38 \times 10^{-16}$ erg $\rm K^{-1}$.

Be brief in your answers. Lengthy derivation are never required.

Problem 1 Concepts (weight 20%)

Please answer in **max. 4 sentences**. If you wish you can sketch a picture.

a What is the Eddington luminosity?

The Eddington luminosity is the maximum accretion luminosity associated with accretion onto a black hole. At the Eddington luminosity, radiation pressure by photons balances the gravitational pull by the black hole.

(Continued on page 2.)

b What is the cusp-core problem? Simulated dark matter halos have 'cusps', i.e. the dark matter density evolves as $\rho \propto r^{-1}$ for small r. Observations prefer the density of the innermost regions of dark matter halos to contain 'cores', in which the dark matter density is constant.

c Why do we expect gas to be 'stable' at certain temperatures, but not all? When cooling efficiency increases with T (i.e. $d\Lambda/dT > 0$), only then do we expect gas to return to its original temperature after a small perturbation of its temperature. For example, we can locally boost T. If $d\Lambda/dT > 0$, the more efficient cooling will bring the gas temperature down again.

d In an adiabatically expanding Universe, the temperature of (non-relativistic) baryonic gas and of radiation evolves with scale factor a as $T \propto a^{-p}$. Briefly discuss what p is for both fluids.
See lecture notes + assignment 1.

Problem 2 Cold vs Hot Mode Accretion (weight 25%)

Consider a dark matter halo with a total mass $M = 3 \times 10^{13} M_{\odot}$ that 'lives' in an Einstein-de Sitter Universe $[(\Omega_{\rm m}, \Omega_{\lambda}) = (1.0, 0.0)]$.

a Show that the virial radius (in proper, i.e. physical, units) of the dark matter halo equals

$$R_{\rm vir} = 668(1+z_{\rm vir})^{-1}\ {\rm kpc} \tag{1}$$

, where $z_{\rm vir}$ denotes the redshift of virialization.

A virialized object has a density that is 178 times denser than the average. The average density inside the collapsed virialized halo is the $178 \times \bar{\rho}_{\rm m} = 178 \times \rho_{\rm crit} = 1.6 \times 10^{-27}(1+z)^3$ gr cm^{-3}. The radius of the dark matter halo is given by

$$R_{\rm vir} = \left(\frac{3M}{4\pi G \times 178 \times \rho_{\rm crit}}\right)^{1/3} = 668(1+z_{\rm vir})^{-1}\ {\rm kpc}. \tag{2}$$

b Show that the circular velocity, and virial temperature equal

$$v_{\rm circ} = 441(1+z)^{1/2}\ {\rm km\ s^{-1}} \tag{3}$$

$$T_{\rm vir} = 1.1 \times 10^{7}(1+z)\ {\rm K} \tag{4}$$

We can get the circular velocity from $v_{\rm circ} = \sqrt{\frac{GM}{R_{\rm vir}}}$. The fact that $R_{\rm vir} \propto (1+z)^{-1}$ causes $v_{\rm circ} \propto (1+z)^{1/2}$. The virial temperature is defined as $k_{\rm B}T_{\rm vir} = \frac{m_{\rm p}v_{\rm circ}^2}{2}$.

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c Evaluate the cooling time $t_{\rm cool}$ as a function of $z_{\rm vir}$. You can approximate the cooling function $\Lambda(T) = \Lambda_0(T/T_0)^{1/2}$, with $\Lambda_0 = 10^{-23}$ erg s^{-1} cm^3, $T_0 = 10^6$ K. When converting density (in g cm^{-3}) into a number density (in cm^{-3}) assume that Helium accounts for $Y_{\rm He} = 0.25\%$ of the total baryonic mass, and that the gas is fully ionized.

The cooling time is defined as

$$t_{\rm cool} = \frac{3n_{\rm H}k_{\rm B}T_{\rm vir}}{2n_{\rm H}^2\Lambda(T)} = \frac{k_{\rm B}T_{\rm vir}}{n_{\rm H}\Lambda(T_{\rm vir})}. \tag{5}$$

Using now that $n_{\rm H} = 178 \times \rho_{\rm crit}/[\mu m_{\rm p}]$ (where μ denotes the mean particle mass in units of $m_{\rm p}$), and substituting expressions for $T_{\rm vir}$ and $\Lambda(T)$ we get

$$t_{\rm cool} = \frac{k_{\rm B}T_0^{1/2}}{\Lambda_0}\frac{T_{\rm vir}^{1/2}}{\mu n_{\rm H}} = 3.63(1+z)^{-5/2}\ {\rm Gyr}, \tag{6}$$

where we used that $\mu = 0.59$ for a fully ionized gas. We can get μ from

$$\bar{n} \equiv \frac{\rho_{\rm tot}}{\mu m_{\rm p}} = 2\frac{(1-Y_{\rm He})\rho_{\rm tot}}{m_{\rm p}} + 3\frac{Y_{\rm He}\rho_{\rm tot}}{4m_{\rm p}}, \tag{7}$$

where the first term each H-nucleus (mass $m_{\rm p}$) contributes 2 particles, where each He-nucleus (mass $4m_{\rm p}$) contributes 3 particles. We ignored the mass of the electron.

d Compare the cooling time to the dynamical time of the dark matter halo, as a function of $z_{\rm vir}$. At what redshifts does the dark matter halo accrete via hot and cold mode accretion?

The dynamical time is

$$t_{\rm dyn} = \frac{1}{\sqrt{4\pi G\rho}} = \frac{1}{\sqrt{4\pi G178 \times \bar{\rho}_{\rm m}}} = 0.85(1+z)^{-3/2}\ {\rm Gyr} \tag{8}$$

When $t_{\rm cool} > t_{\rm dyn}$, we have hot mode accretion. The two time-scales are equal when $(1+z) \sim 3.63/0.85 \sim 4.3$. Cold mode accretion dominates for $z > 3.3$.

Problem 3 Lyα Forest (weight 20%)

a Describe what the Lyα forest is, what its origin is, and and what it tells us about the physical properties of the intergalactic medium. You may find it helpful to provide a sketch that supports your description.

The Lyα forest is a collection of Lyα absorption lines that have been observed in quasar (but also galaxy) spectra. In the lecture we derived that if the gas in the Universe were neutral, that then we should absolutely see no flux at frequencies higher than the redshifted Lyα frequency. The fact that we see absorption lines and a continuum implies that gas in the Universe, the intergalactic medium must have been highly ionized. The fluctuations in Lyα opacity in the Lyα forest correspond to density fluctuations along the line-of-sight. Most of these

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absorption lines correspond to mildly overdense regions $\delta \sim 1$, which is very different than the densities probed by virialized structures. We also learned in the lecture that the temperature of photoionized intergalactic medium is $T = 10^4$ K. For lower temperatures, cooling is too inefficient and the gas efficiently heats up. For higher temperatures, the gas is too highly ionized for the radiation to insert more energy.

b Derive the following expression for the Jeans length by evaluating the total energy of a spherical, self-gravitating ball of gas with radius R, mass M, and temperature T (i.e. sound speed $c_{\rm s}$).

$$\lambda_{\rm J} = c_{\rm s}\sqrt{\frac{3}{8\pi G\rho}} \tag{9}$$

This follows the lecture notes on gravitational instabilities in self-gravitating disks. The total energy, $U_{\rm tot}$, of this cloud is

$$U_{\rm tot} = \frac{1}{2}Mc_{\rm s}^2 - \frac{GM^2}{R}, \tag{10}$$

where we used that the 3D velocity dispersion of atoms/nuclei inside the cloud equals the sounds speed (to within a factor of order unity), i.e. $\sigma^2 = c_{\rm s}^2$. The gas cloud is unstable and will collapse if $U_{\rm tot} < 0$. The onset of instability thus occurs when $U_{\rm tot} = 0$, which translates to

$$R_{\rm J} = c_{\rm s}\sqrt{\frac{3}{8\pi G\rho}} \tag{11}$$

c Derive the comoving wavenumber beyond which we expect Jeans smoothing to suppress power in Lyα forest at $z = 3$. Assume Einstein-de Sitter.

Compute the Jeans length of the intergalactic medium. The average density of the IGM is that of the Universe as a whole. Substituting numbers, and using that $T \sim 10^4$ K, we get

$$R_{\rm J} = 145 \text{ kpc} \tag{12}$$

This uses physical units. The comoving Jeans mass is $(1+z) = 4$ times larger, i.e. $R_{\rm J,c} \sim 580$ ckpc. To convert this to a wavenumber, we use $k_{\rm J} = 2\pi/R_{\rm J} \sim 10$ cMpc^{-1}. Using $k_{\rm J} = 1/R_{\rm J}$ would have been fine too.

d How does the Jeans smoothing scale compare to the smoothing that is introduced by the finite width of the Lyα absorption cross-section. Assume that the gas temperature is $T = 10^4$ K. **Note:** if you do not know the width of the Lyα absorption cross-section (in km s^{-1}), discuss how you would solve the problem if you had known. Assume that the FWHM$\sim 2\sigma \sim 20$ km s^{-1}. This corresponds to a proper distance $L = 2\sigma/H(z = 3) \sim 35$ kpc in redshift space. This is comparable to the Jeans smoothing scale.

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Problem 4 Top-Hat Model & Redshift Space Distortions (weight 25%)

Consider the spherical perturbation shown in Figure 1. We have indicated 3 points, and 2 concentric spherical shells within the perturbation (one of which contains point '1'. The other one contains both points '2' and '3').

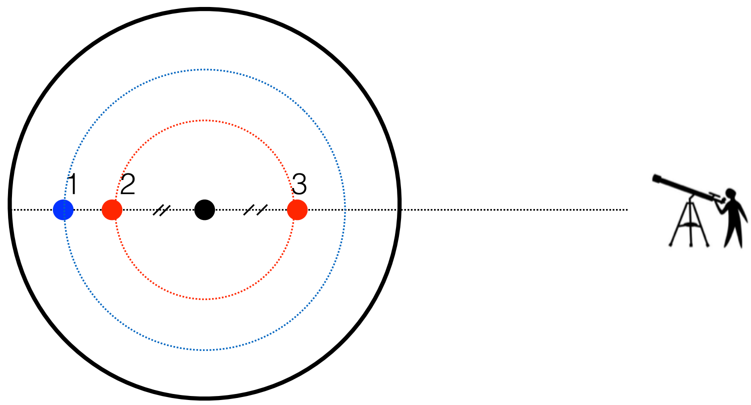

Figure 1:

a Assume the perturbation has a top-hat density profile (i.e. constant overdensity within it). Show how the three points move in redshift space as the perturbation expands, turns-around and virializes. Figure 2 shows the type of diagram you should use. Here, we have already shown the location of the center of the top-hat in redshift space, Indicate where each point at some arbitrary early time, at turn-around, and just before and after virialization. Provide a 1-2 sentence motivation for the location of your choices.

Fig 4 shows the solution. Initially, the perturbation expands closely following the expansion of Universe. Order of points in z-space is same as in redshift space. As expansion slows the points move closer in z-space. At turn-around, all points are stationary and occupy the same location in z-space. For top-hat perturbation turn-around happens at the same time for all points. After turn-around the order of points is reversed. Everything virializes at the same time, and the outermost points fall in with the largest velocity ($v_{\rm cric}$) just before virialization. Post virialization, everything is static again and in the same location in z-space.

b Repeat the same exercise as above, but now assume that the density profile is more realistic, with a maximum overdensity in its center, and

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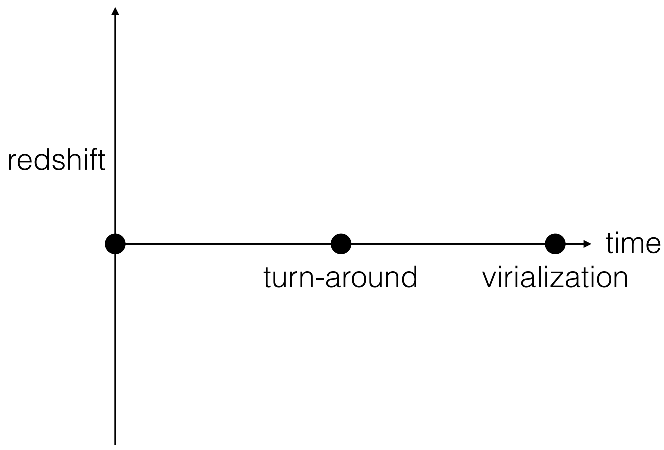

Figure 2:

overdensity decreasing outward. Only explain the differences from the previous plot.

The inner-most turn-around, and virialize earlier than the outer points.

c Material generally reaches the virial radius at the corresponding circular velocity. Show that the circular velocity can be written as (assume Einstein-de Sitter, as usual)

$$v_{\rm circ} = R_{\rm vir} H \times \sqrt{89} \sim 9 H R_{\rm vir} \tag{13}$$

$$v_{\rm circ}^2 = \frac{GM}{R_{\rm vir}} = \frac{G\frac{4\pi}{3} \times 178 \rho_{\rm crit} R_{\rm vir}^3}{R_{\rm vir}} = R_{\rm vir}^2 \times \frac{178}{2} \times H^2, \tag{14}$$

where in the last step I used the expression for $\rho_{\rm crit}$.

d Consider the spherical mass shell that encloses point '1' (indicated as the *blue dotted circle*). Show the time evolution of this spherical shell in *real and redshift* space at some arbitrary early time, at turn-around, and just before virialization (again assume Einstein-de Sitter). Use a diagram as shown in Figure 5. Comment on the shape and relative size of the shell where possible.

Real space: perturbation is always spherical. At turn-around it is twice as large as at virialization. At early times it is smaller than at virialization. *Redshift space:* at early times the perturbation is practically expanding along with the Universe. The shape (and size) in redshift space is then the same as in real space. At virialization, all points within the shell are stationary, and so the shell is compressed

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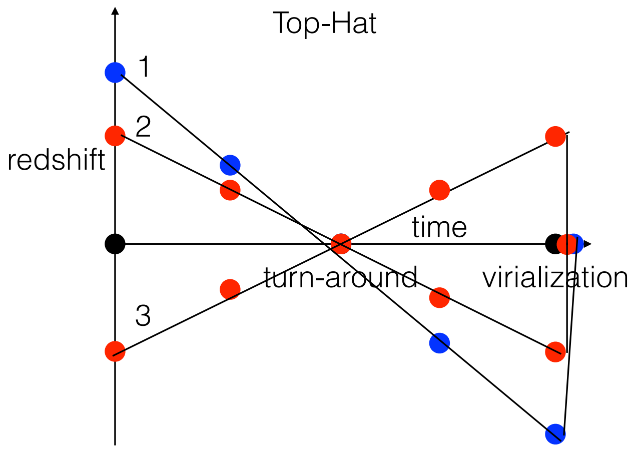

Figure 3:

into a line. The angular size of the perturbation is not affected. Just before virialization, the perturbation is mass shell is reaching $R_{\rm vir}$ at $v_{\rm circ} \approx 9 R_{\rm vir} H$. The perturbation is therefore stretched out in redshift space as if it were 9 times longer.

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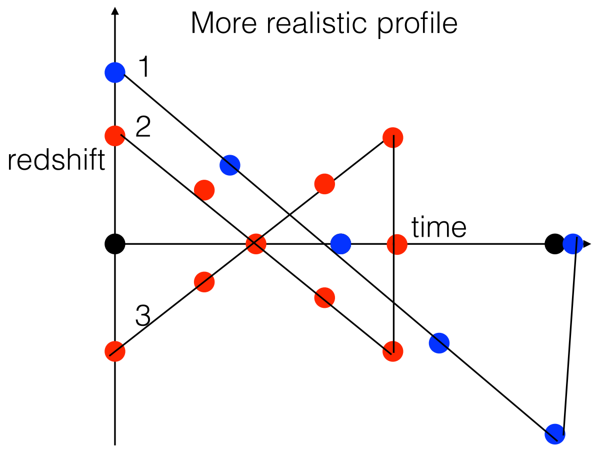

Figure 4:

Problem 5 Bonus Questions (weight 10%)

Please note *all the correct answers* for each question. It is sufficient to write down the number and letters of the correct answers. For example, "42. a, b, c" if in question number "42" the answers "a", "b", and "c" are correct. Not also that there is 10/11∼0.9% point per correct answer to be gained. *This means that you should not spend too much time on this exercise!*

1. What properties characterizes a 'Population III' star compared to a 'Population I' star?

   (a) Less metals
   (b) More metals
   (c) (On average) heavier
   (d) (On average) lighter

2. What is a consequence of inflation?

   (a) A slowly rolling scalar field
   (b) Flatness ($\Omega_k \sim 0$)
   (c) Homogeneity
   (d) Structure formation

3. How do you call the phenomenon when we observe the light's trajectory being bent around *all sides* of an object?

   (a) Einstein cross
   (b) Einstein ring
   (c) Weak lensing
   (d) Lensing arc

4. What is *not* considered a possible seed for a 'supermassive black hole'?

   (a) Population III star supernovae
   (b) Primordial black holes

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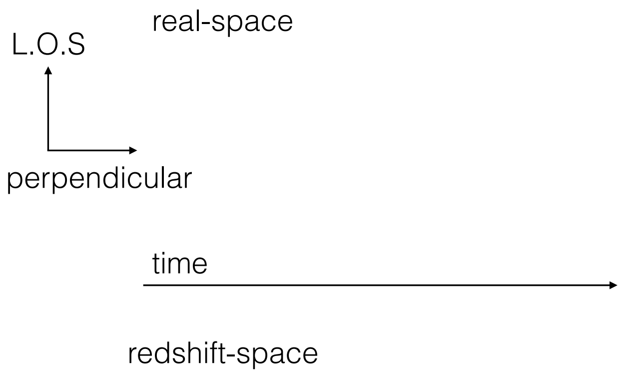

Figure 5:

(c) Population I supernovae
(d) A directly collapsing gas cloud

5. How long are typically ‘short’ Gamma Ray Bursts (GRBs)?

   (a) < 2 seconds
   (b) Several minutes
   (c) Hours
   (d) $\sim$ days

6. Why could it be that we not have detected a deviation from ‘general relativity’ (GR) yet?

   Because...

   (a) ... scientists have not been looking for deviations.
   (b) ... GR is the correct theory to describe gravity.
   (c) ... deviations are ‘screened’ in the regions which could so far probe GR to high precision.

7. What is the most (computationally) expensive algorithm for N-body simulations?

   (a) Direct summation
   (b) Barnes-Hut
   (c) Particle-Mesh

8. Where can / could elements heavier than Helium form?

   (a) During inflation.
   (b) During the ‘Big Bang nucleosynthesis’ (BBN).
   (c) Through the ‘r-process’
   (d) Through the ‘s-process’
   (e) Through the ‘t-process’

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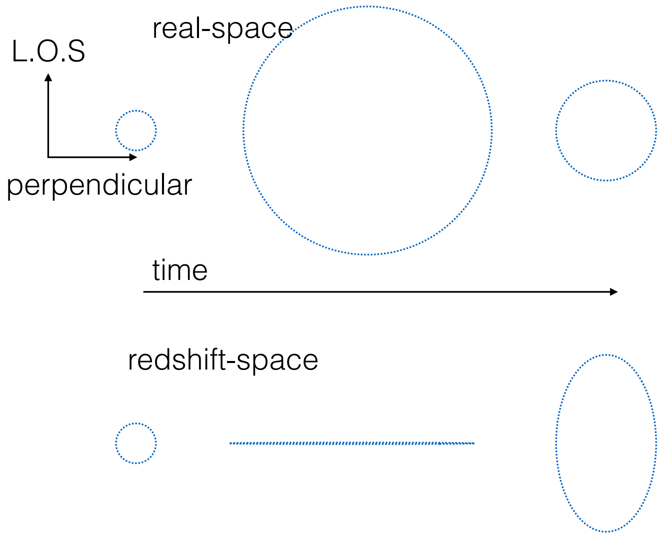

Figure 6:

9. Where does the 21cm-line originate?
   (a) De-excitation from the first excited state of hydrogen.
   (b) Deviation of an electron's trajectory due to a proton (Bremsstrahlung)
   (c) Electron-positron annihilation
   (d) Spin-flip from anti-aligned to aligned of the neutral hydrogen atom.

10. What describes the 'missing satellite' problem?
    (a) Too few satellites around the Earth.
    (b) Simulations with cold dark matter (CDM) show many more substructures than observed dwarf galaxies around the Milky Way.
    (c) We observe many more dwarf galaxies around the Milky Way compared to the number of substructures expected from simulations with cold dark matter (CDM).

11. Which of the statements about the galaxy luminosity function (LF) and the dark matter halo mass function (HMF) is correct?
    (a) The LF is proportional to the HMF.
    (b) The LF possesses a flatter low-mass slope than the HMF.
    (c) The LF has a cutoff at the high-mass end. The HMF does not.
    (d) The LF becomes negative when the HMF becomes positive, and the other way round.