UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: AST4320 — Cosmology and Extragalactic Astronomy

Day of examination: Friday 13. October 2017

Examination hours: 14:30 – 17:30

This problem set consists of 12 pages.

Appendices: None

Permitted aids: Formula book by Rottmann.
Calculator that meets standard requirements by university.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Preface: some useful relations and constants

- $H(a) = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda + \Omega_{\rm rad}a^{-4}}$, where H_0 is the Hubble parameter today, Ω's denote the density parameters in matter, dark energy, and radiation.
- $a = \frac{1}{1+z}$.
- $\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm crit}}$, $\Omega_{\rm b} = \frac{\rho_{\rm b}}{\rho_{\rm crit}}$, where $\rho_{\rm m}$ is the mean matter density of the Universe, $\rho_{\rm b}$ is the mean baryon density of the Universe, $\rho_{\rm crit} = \frac{3H^2}{8\pi G}$
- Einstein-de Sitter Universe: $\Omega_{\rm m} = 1.0$, $\Omega_\Lambda = 0.0$.
- Planck cosmology parameters: $h = 0.678$, $\Omega_{\rm m} = 0.308$, $\Omega_\Lambda = 0.692$, $\Omega_{\rm b} = 0.048$.
- Hubble parameter at z = 0: $H_0 = 67.8$ km s^{-1} Mpc^{-1} $= 2.20 \times 10^{-18}$s^{-1}, critical density at z = 0: $\rho_{\rm crit,0} = 8.64 \times 10^{-30}$g cm^{-3}
- Units and constants: $1M_\odot = 2 \times 10^{33}$ g, 1pc $= 3.086 \times 10^{18}$ cm, 1 year $= 3.15 \times 10^7$ s, gravitational constant $G = 6.67 \times 10^{-8}$ cm^3 g^{-1}s^{-2}, mass of a hydrogen atom $m_H = 1.67 \times 10^{-24}$ g, Boltzmann's constant $k_B = 1.38 \times 10^{-16}$ cm^{-2}g s^{-2}K^{-1}, speed of light $c = 3.0 \times 10^{10}$cm s^{-1}.

Please show important intermediate steps in your answers, but try to be concise. Lengthy derivations are not required.

Problem 1 Jeans Mass and Growth of Structure (weight 35%)

a In lectures we derived the Jeans length $\lambda_J = c_s\sqrt{\frac{\pi}{G\rho}}$ from Newtonian linear perturbation theory. Write down an expression for the Jeans

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mass (M_J). Explain briefly what the Jeans length and Jeans mass are, and how they are relevant for the formation of structure in the Universe.

Solution: The Jeans mass is the mass that is within a sphere of radius $(\lambda_J/2)$ with density ρ:

$$M_J = \frac{4\pi}{3}\rho(\lambda_J/2)^3 = \frac{\pi}{6}\rho\lambda_J^3$$

Physically, we can understand the existence of this critical length scale by considering the balance between gravity and thermal pressure. If we take a small part of the pre-galactic gas and perturb it adiabatically, its density and temperature will increase. It will therefore be over-pressured relative to the surrounding gas, and the pressure gradients that we have created will try to smooth out the perturbation. Our perturbation will survive and grow only if its self-gravity i.e. the gravitational force acting on the perturbation due to the perturbation's own mass, is larger than the pressure forces acting to smooth out the perturbation. It should be plain that for very small perturbations, with very low masses, pressure will overcome gravity. Similarly, it should be clear that on very large scales, gravity will win. There must therefore be some intermediate scale in which we go from being pressure-dominated to being gravity-dominated. This critical scale is the Jeans length.

b The recombination process occurs at $z \sim 1100$. The Universe is matter-dominated at this epoch, and the photons and gas are still coupled prior to recombination. The sound speed of the gas is

$$c_s = \frac{c}{\sqrt{3}}(\frac{3\rho_m}{4\rho_\gamma} + 1)^{-1/2}, \quad (1)$$

where c is the speed of light, ρ_m and ρ_γ are the average density of matter and radiation, respectively. Express the Jeans length and mass as a function of the scale factor a for $\frac{\rho_m}{\rho_\gamma} \gg 1$.

Solution: We know that the universe is matter dominated at this point, such that $\rho_m >> \rho_\gamma$ (before matter-radiation equilibrium $\frac{\rho_m}{\rho_\gamma} << 1$, this term is negligible). We also know how the matter and radiation densities evolve as a function of the scale factor $\rho_m = \rho_{m,0}a^{-3}$ and $\rho_\gamma = \rho_{\gamma,0}a^{-4}$. Inserting this and we get:

$$c_s = \frac{c}{\sqrt{3}}\left(\frac{3\rho_{m,0}}{4\rho_{\gamma,0}}a^{-1} + 1\right)^{1/2} \propto a^{-1/2}$$

However instead of relating it to the current time values, it is nicer to relate the ratio $\frac{\rho_m}{\rho_\gamma}$ to the matter-radiation value (which is equal to 1):

$$\frac{\rho_m}{\rho_\gamma} = \frac{\rho_{m,eq}}{\rho_{\gamma,eq}}\frac{a_{eq}}{a} = \frac{a_{eq}}{a}$$

(Continued on page 3.)

$$c_s = \frac{c}{\sqrt{3}}\left(\frac{3a_{eq}}{4}a^{-1}+1\right)^{1/2}$$

Which we can then simply insert into the jeans length and mass, the proportionality to the scale factor goes as (remember matter dominated so $\rho \propto a^{-3}$):

$$\lambda_J \propto a$$
$$M_J \propto a^0 = Constant$$

c Explain briefly what Particle Horizon is. The general formula to calculate the particle horizon at time t is given by:

$$r_{\rm H}(t) = a(t)\int_0^t c\ d\tau/a(\tau). \tag{2}$$

Derive an expression of the particle horizon as a function of time t and the scale factor a, assuming an Einstein-de Sitter Universe.

Solution: The particle horizon is the maximum distance in which particles could have traveled to the observer at time t from the beginning of the universe. Or in other words, the region which have been in casual contact with the observer (however the region which the observer can casually effect in the future is set by the cosmic horizon). In the Einstein-de Sitter Universe we have from the friedmann equation:

$$\frac{\dot{a}}{a} = H_0 a^{-3/2} \rightarrow a = \left(\frac{3H_0}{2}\right)^{2/3} t^{2/3}$$

The constant does not matter as we have a^{-1} inside and a outside the integral. Inserting and solving we get:

$$r_H = 3ct = \frac{2c}{H_0}a^{3/2}$$

d Assuming all matters in the Universe are baryons, and the matter-radiation equality occurs at $z_{eq} = 3390$, compute the Jeans mass and the total mass within the particle horizon ($M_H = \frac{4\pi}{3}\rho_m(\frac{r_H}{2})^3$) at redshift $z = 1100$ **prior** to recombination, how do the two masses compare? And how does the Jeans mass compare to the typical mass of galaxies (M $= 10^{10} - 10^{13} M_\odot$)? Discuss the implication of your results in the context of galaxy formation. *Hint:* $\rho_m/\rho_\gamma = 1$ at $z_{eq} = 3390$, you can compute $\frac{\rho_m}{\rho_\gamma}$ at z $= 1100$ from how this ratio evolves as a function of a and hence compute c_s.

Solution: The total mass within the particle horizon as a function of scale factor becomes:

$$M_H = \frac{4\pi}{3}\rho_{m,0}a^{-3}(\frac{c}{H_0}a^{3/2})^3 = \frac{4\pi}{3}\rho_{m,0}a^{-3}(\frac{c}{H_0}a^{3/2})^3 = \frac{4\pi c^3}{3H_0^3}\rho_{m,0}a^{3/2}$$

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We know $a = (1+z)^{-1}$ and $\rho_{m,0} = \Omega_m \rho_{c,0}$, inserting this and $z = 1100$ we get our answer. Similarly using the speed of sound expression in b with the jeans length and mass of a, we can calculate the jeans mass at z=1100.

$$M_J \approx 5 \cdot 10^{17}$$

$$M_H \approx 10^{18}$$

The two masses are comparable to eachother and as $M_J \gg M_{galaxy}$, galaxy formation cannot occur at this time.

e After recombination is completed, gas decouples from photons and becomes non-relativistic. For ideal gas, the sound speed becomes

$$c_s = \sqrt{\frac{k_B T}{\mu m_H}}. \tag{3}$$

Here k_B is the Boltzmann's constant, T is the temperature of the gas, μ is the mean molecular weight and m_H is the mass of the hydrogen atom. Express the Jeans length and mass as a function of the scale factor a, and compute the Jeans mass at z = 1100 **after** the recombination (assuming $\mu = 1$). How does it compare to the Jeans mass before recombination? Discuss the implication of your results. (*Hint*: consider the evolution of gas temperature $T_{\text{gas}} \propto a^{-2}$ and photon temperature $T_\gamma \propto a^{-1}$ and that the CMB temperature today $T_{\text{CMB}}(z = 0) = 2.73$ K).

Solution: Given the speed of sound, we can get an expression for the Jeans length and mass. The only thing we need is the full expression for the temperature of the gas. At recombination we know that $T_{gas} = T_\gamma$. The radiation in the CMB have evolved according to $T_\gamma \propto a^{-1}$. With these two considerations together with the current CMB temperature we get:

$$T_{gas} = 2.73(1 + z_{req})$$

We can then simply just insert equation 3 in our Jeans length and mass expressions given before (and remember that we are in matter dominated epoch).

$$M_J \approx 10^5$$

Which is much less than the Jeans mass before recombination, which will allow galaxies to start forming.

f For non-relativistic ideal gas, the temperature and density follows the relationship $T \propto \rho^{\gamma-1}$. Here γ is the adiabatic index. Show that the Jeans length and mass follow the relationships:

$$\begin{aligned} \lambda_J &\propto \rho^{\frac{1}{2}(\gamma-2)} \\ M_J &\propto \rho^{\frac{3}{2}(\gamma-\frac{4}{3})} \end{aligned} \tag{4}$$

(Continued on page 5.)

Solution: We know that the jeans length and mass are proportional to:

$$\lambda \propto c_s \rho^{-1/2}$$

$$M_J \propto \lambda^3 \rho$$

The speed of sound of an non-relativistic ideal gas is given in previous exercise:

$$c_s \propto T^{1/2} \propto \rho^{\frac{1}{2}(\gamma-1)}$$

Which by inserting into the jeans length and mass gives the requested relations.

g When gas in galaxies (in the form of giant molecular clouds) exceeds the Jeans mass and collapse, it usually does not collapse into a single star (or a black hole). What instead happens is that sub regions within the cloud will start to **locally** collapse and fragment. This fragmentation is what eventually leads to formation of star clusters with individual stars much less massive than the cloud. This fragmentation is directly linked to the Jeans mass, explain why and for what values of γ fragmentation can occur in a collapsing cloud.

Solution: During a collapse the density increases, how the Jeans mass will change depends on the adiabatic index γ. For a $\gamma < 4/3$ the Jeans mass will decrease during collapse, which will allow smaller regions within the cloud to surpass their Jeans mass and collapse on their own, leading to fragmentation. One could also add that sub regions will have a shorter t_{ff} which means that these sub-regions collapse faster. For $\gamma > 4/3$ the Jeans mass will increase and thus we will have no fragmentation.

Problem 2 Spherical Top-Hat Model (weight 15%)

The non-linear evolution of a spherical density perturbation of radius R (with uniform density, enclosing a total mass M) was given by a parameterised solution:

$$\begin{aligned} R &= A(1-\cos\theta) \\ t &= B(\theta - \sin\theta), \end{aligned} \tag{5}$$

where $A^3 = GMB^2$, in which M denotes the total mass of the perturbation.

a Sketch the time-evolution of a mass-shell within the top-hat model (i.e. radius R vs time t). Indicate 'turn-around', 'virialization', 'collapse', and indicate how the evolution at small R and t is compatible with the result obtained from linear theory (do **not** derive the linear theory result:that would take too long!).

Solution: Sketched in the lecture slides and notes.

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b Consider a stationary, spherical object of mass M. Use the parametrised solution to show that it takes a time $t_{\rm coll}$ for this object to collapse into a point, where the collapse time $t_{\rm coll}$

$$t_{\rm coll} = \sqrt{\frac{3\pi}{32G\rho_i}}, \tag{6}$$

where ρ_i denotes the initial density of the object (i.e. the density before it starts collapsing).

Solution: Stationary sphere:

$$R = R_{max} = 2A,\ t = \pi B$$

Collapse to a point happens at

$$R = 0,\ t = 2\pi B$$

So we have $\delta t = \pi B$. B is given by:

$$B = \sqrt{\frac{A^3}{GM}}$$

A and M is given by:

$$A = R_{max}/2$$
$$M = \frac{4\pi}{3}\rho R_{max}^3$$

Combining all these together and we get:

$$\Delta t = \sqrt{\frac{3\pi}{32G\rho}} = t_{coll}$$

c In the spherical top-hat model, we assume that δ =constant up to some radius $R_{\rm th}$. Now instead assume that $\delta = \delta_0/r$. If a mass-shell initially at R_1 collapses at t_1, compute at what time a mass shell at $R_2 = 3R_1$ collapses. Assume an Einstein-de-Sitter Universe in which $\delta \propto a$ in the linear regime. *Hint:* use linear theory results to identify when a mass-shell collapses.

Solution: Total enclosed overdensity within radius R:

$$\delta = \frac{\int_0^R 4\pi r^2 \frac{\delta_0}{r} dr}{\frac{4}{3}\pi R^3} = \frac{3\delta_0}{2R} \propto 1/R$$

So at $R_2 = 3R_1$ we have:

$$\frac{\delta_2}{\delta_1} = \frac{R_1}{R_2} = \frac{1}{3} \rightarrow \delta_2 = \frac{\delta_1}{3}$$

We see that the perturbation needs to grow by a factor of three to collapse. In EdS universe we know that:

$$\delta \propto a \propto t^{2/3}$$

The time of collapse of the outer shell will then be:

$$t_2 = 3^{3/2} t_1$$

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Problem 3 Gas Cooling (weight 23%)

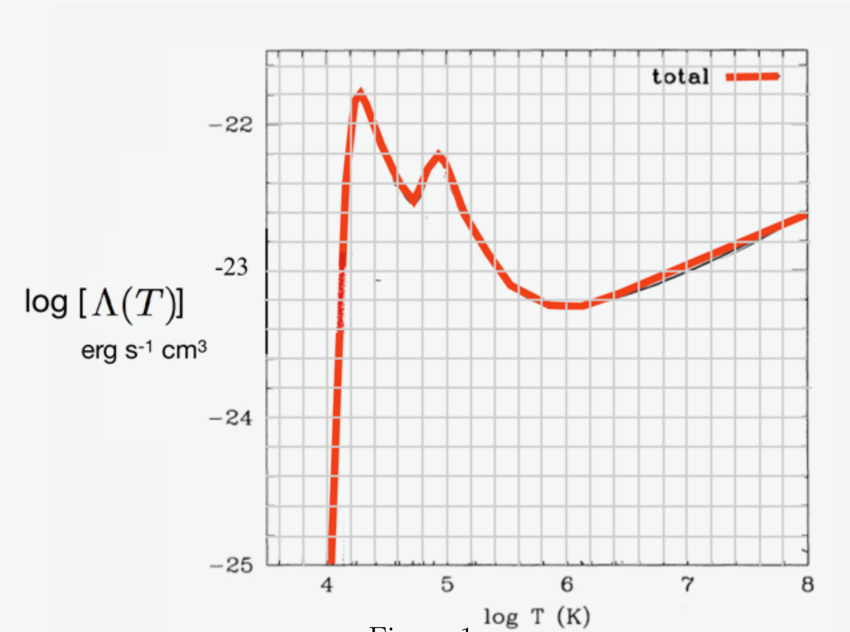

Figure 1:

a Describe three main cooling processes of an astrophysical gas and explain **briefly** why these processes results in the cooling of gas.(*Hint*: we referred to one of the three as bound-bound).

Solution: Brehmstrahlung/Free-Free: Occurs when a free electron is accelerated by a nearby ion. The accelerated electron radiates, which causes the electron to loose energy.

Recombination/Free-Bound: When a free electron is captured by an ion, its kinetic + binding energy is radiated away. Only the loss of kinetic energy counts towards cooling of the gas, as the binding energy was counted already when the electron was detached from the atom.

Ionization/Bound-Free: A free electron collides with an atom, which ionizes the atom, at the expense of the collisional kinetic energy.

Excitation/Bound-Bound: A free electron collides with an atom and excites it to a higher energy state. This comes at the expense of the collisional kinetic energy. The excited atom radiatively (i.e. by emitting radiation) transitions back to its ground state.

b **Briefly** explain the shape of the cooling curve shown in Figure 1.

Solution: When $T < 10^4 K$ the gas is mostly neutral and cooling very inefficient. As we increase the temperature the gas becomes energetic enough to start excite and ionize the gas. Cooling from excitation will be dominant and will peak at around $2 \times 10^4 K$, when most of the hydrogen have been ionized. Past this the cooling will decrease rapidly from both excitation and ionization, until around $3-4 \times 10^4 K$ in which

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this decrease is counteracted by the excitation and ionization of ionized helium. This eventually also peaks(around $10^5 K$), when most of the helium is doubly ionized. The decrease in cooling is eventually stopped by the increased cooling from the free-free process, as the gas becomes fully ionized.

c Describe the conditions that allow gas inside a dark matter halo to collapse into its center.

Solution:

To collapse, the gas needs to cool so fast that the gas cannot re-establish hydrostatic equilibrium. In other words, we require that the cooling time for the gas is less than the dynamical time-scale:

$$t_{cool} < t_{dyn}$$

d A perturbation of mass M virializes at $z = 3$. Use the expression from **Problem 2b** to compute the collapse time of the virialized halo. Assume **Planck cosmology** and use the parameters and the values of H_0 and $\rho_{crit,0}$ given at the beginning of the test sheet. Use overdensity $\delta = 178$ for virialized halos.

Solution: We begin by calculating the mass density of the universe at z=3 in Planck cosmology:

$$\rho_m = \Omega_m \rho_{c,0}(1+z)^3$$

We are given that virilization occurs for an overdenisty of 178:

$$\rho_i = 178\Omega_m \rho_{c,0}(1+z)^3$$

Which gives the collapse time:

$$t_{coll} = \sqrt{\frac{3\pi}{32 \cdot 178 G\Omega_m \rho_{c,0}(1+3)^3}}$$

e The cooling time is given by $t_{\rm cool} = \frac{3k_{\rm b}T}{2n\Lambda(T)}$, n denotes the number density of **gas**. Gas in galaxies has the virial temperature given by:

$$T_{\rm vir} \equiv \frac{m_{\rm p}v_{\rm circ}^2}{2k_{\rm B}} \approx 2\times 10^4\ {\rm K}\left(\frac{M}{10^8\ M_\odot}\right)^{2/3}\left(\frac{1+z}{10}\right). \tag{7}$$

Using the answer to **Problem 3c**, estimate minimum gas number density $n_{\rm gas}$ needed for the gas to collapse into the centre of the dark matter halo, for halo mass $M = 10^{10}$ M$_\odot$, 10^{11} M$_\odot$ and 10^{12} M$_\odot$. If we assume a spherical top-hat density distribution and that gas follows the distribution of dark matter, how does $n_{\rm gas}$ in each case compare to the average gas (i.e. baryon) density of the halo? Use **Planck cosmology**

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and note that $\Omega_\mathrm{b} = \rho_\mathrm{b}/\rho_\mathrm{crit}$, and $n_\mathrm{gas} = \rho_\mathrm{gas}/m_H$. Again, use overdensity $\delta = 178$ for virialized halos. Discuss your results and comment on which halo(s) in the three cases can form galaxies.

Solution: Basically, use equation 7 to figure out the temperature for all the masses. Then extract Λ from the cooling curve figure. The number density is given by rearranging the cooling time expression:

$$n_{gas} = \frac{3kT}{2\Lambda t_{coll}}$$

We use our t_{coll} from the previous question and then calculate the number density for all the three cases. The mean density of the gas for all the halos is given by the baryon content times the overdenisty required for virilization:

$$< n_{gas} >=< \rho_{gas} > /m_H = \delta\rho_b/m_H = 178\Omega_b\rho_{c,0}(1+z)^3/m_H$$

For the cases which require a smaller number density than the average we can have collapse into galaxies.

f In reality, the dark matter and gas density distributions are not uniform, density is higher in the centre than the outskirt. If mass in a shell at $R = R_\mathrm{th}$ where $t_\mathrm{coll} = t_\mathrm{cool}$, discuss the behaviour of the gas in shells at $R < R_\mathrm{th}$ and $R > R_\mathrm{th}$. How does this impact your result in **Problem 3e**?
Solution: From the previous exercises we can see that the collision time and cooling time follows:

$$t_{coll} \propto \rho^{-1/2} \propto n^{-1/2}$$

$$t_{cool} \propto n^{-1}$$

As we go deeper into the centre the number density will increase, thus the cooling time will decrease faster for $R < R_{th}$, which should then be able to collapse.

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Problem 4 Press-Schechter Formalism and Dark Matter Halo Mass Function (weight 22%)

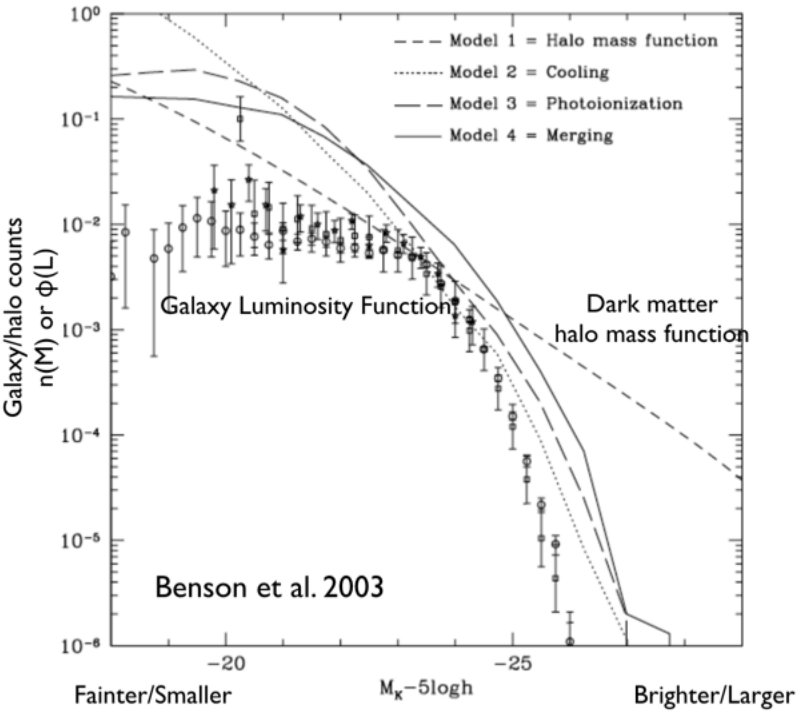

Figure 2:

The Press-Schechter (PS) theory can be thought of as a first attempt to connect statistical aspects of a Gaussian random density field to observed statistical descriptions of galaxies. The PS formalism is flawed in many ways, but it predicts a halo mass function with similar features as the observed galaxy luminosity function.

a The Press-Schechter formalism is based on several assumptions. **Briefly** describe these assumptions. In particular, describe how to determine the probability that mass at a position x is part a collapsed object with mass $> M$, $P(> M)$. (Note: just stating the assumptions is sufficient, you do not need to derive the halo mass function.)

   Solution: Three assumptions are:

   (a) density evolves linearly at all times $\delta \propto a$
   (b) An object collapses once $\delta > \delta_{crit} = 1.69$
   (c) The probability that mass at point x is part of a collapsed object with mass $>$ M equals $P(> M) = P(\delta > \delta_{crit}|M)$

b **Briefly** explain the main problems with the assumptions in the Press-Schechter formalism.

   Solution: The two main problems are

   (a) Density does not always grow linearly.

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(b) In the classical PS-theory er state that $P(>M) = P(\delta > \delta_{crit}|M)$, however a point below the critical density at a small smoothed scale can at a larger smoothing be above its critical density. To account for these objects the fudge factor of 2 was introduced $P(>M) = 2P(\delta > \delta_{crit}|M)$.

c The dark matter halo mass function is given by:

$$n(M) = C\frac{\nu}{M^2}\exp(-\nu^2/2), \tag{8}$$

where M is the mass of the halo, C is a constant, and $\nu \equiv \delta_{crit}/\sigma(M)$. Here $\delta_{crit} = 1.69$, and for a matter power spectrum $P(k) \propto k^n$, we have $\sigma^2(M) \propto M^{-\frac{n+3}{3}}$. Describe what δ_{crit} and $\sigma^2(M)$ stand for, and the behaviour of the halo mass function $n(M)$ for small and large M. Note that for small scale perturbations, $n \sim -3$.

Solution: δ_{crit} is the linear extrapolation of the overdensity at the time of collapse. $\sigma^2(M)$ is the variance in the GDF, smoothed over a mass scale of M. Looking at the mass dependences in equation 8 we have:

$$n(M) \propto M^{\frac{n-9}{6}}\exp(-\nu^2/2) \quad \nu^2 \propto M^{\frac{n+3}{3}}$$

From this we can see that we get an exponential decrease in the number density for high mass halos ($\nu >> 1$) and that for low mass halos we instead get a number density that follows a power law (for $n = -3 \rightarrow n(M) \propto M^{-2}$)

d The characteristic mass scale, M_{crit}, is defined as the mass at which $\nu = 1$. M_{crit} evolves with cosmic time. Show an expression of M_{crit} as a function of the scale factor a under the assumption of the Einstein-de Sitter Universe.

Solution: At the critical mass we have that:

$$\frac{\delta_{crit}}{\sigma(M_{crit})} = 1 \rightarrow \delta_{crit} = \sigma(M_{crit})$$

So at the critical mass $\sigma(M_{crit})$ is a constant. What is important to note here is that $\sigma(M)$ is a function of both mass and time/scale factor:

$$\sigma(M) = A(M)B(a)$$

From PS-theory we know that:

$$\sigma(M) \propto M^{-\frac{n+3}{6}}$$

And from linear theory we know that it evolves with time as:

$$\sigma(M) \propto \delta \propto a \propto t^{2/3}$$

So we know that $A(M) = C_1 M^{-\frac{n+3}{6}}$ and $B(t) = C_2 a$, where C_1 and C_2 are constants. Inserting this into the previous equation for $\sigma(M)$ and using it with $\sigma(M_{crit}) = constant$, we get:

$$\sigma(M_{crit}) = C_1 C_2 a M^{-\frac{n+3}{6}} = constant \rightarrow M_{crit} \propto a^{\frac{6}{n+3}} \propto (1 + z^{-\frac{6}{n+3}})$$

(Continued on page 12.)

e If the critical mass $M_{crit} = 10^{14}\mathrm{M}_{\odot}$ at z = 0 and assuming $n = -1.5$, compute M_{crit} at z = 3, 5, 10, 20 and 30. Discuss how your result fits into the "hierarchical structure formation" picture.

Solution: At $z = 0$ we have $M_{crit} = 10^{14}$, so if $n = -1.5$ we have $M_{crit} = 10^{14}(1 + z)^{-4}$. We can then enter all the z values from above into the equation. What we see from this is that the critical mass decreases rapidly with increasing redshift. The hierachical structure of this can clearly be seen as small halos will be able to form first and larger ones later.

f Figure 2 shows a comparison between the halo mass function from N-body simulations (short dashed line in the figure) and the observed galaxy luminosity function (data points with error bars). It is evident that dark matter halos are more abundant than galaxies at the low and high mass (luminosity) regimes. Using the tools developed in **Problem 3**, explain why galaxies form inefficiently in dark matter halos less massive than $\sim 10^8 M_{\odot}$ and more massive than $\sim 10^{14} M_{\odot}$? What other mechanisms do you think can contribute to the mismatch? A qualitative explanation is sufficient.

Solution: Cooling is inefficient for halos $M < 10^8$, because the virial temperature is $T_{vir} < 10^4 K$ which means if we look at the cooling plot from exercise 3 that Λ is very low and t_{cool} is very long. For high mass halos $M > 10^{14}$, T_{vir} and Λ is higher, however cooling is still insufficent in regards to the high temperature, which means that the cooling time will still be very long. Another contribution to this discrepancy is additional mechanism such as feedback from supernovae and AGN.