UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: AST4320 — Cosmology and Extragalactic Astronomy

Day of examination: Wednesday 20. December 2017

Examination hours: 9.00 – 13.00

This problem set consists of 13 pages.

Appendices: None

Permitted aids: Formula book by Rottmann.
Calculator that meets standard requirements by university.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Preface: some useful relations:

- $H(a) = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda + \Omega_{\rm rad}a^{-4}}$, where H_0 is the Hubble parameter today, Ω's denote the density parameters in matter, dark energy, and radiation.
- $a = \frac{1}{1+z}$.
- Einstein-de Sitter Universe: $\Omega_{\rm m} = 1.0$, $\Omega_\Lambda = 0.0$.
- $\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm crit}}$, $\Omega_{\rm b} = \frac{\rho_{\rm b}}{\rho_{\rm crit}}$, where $\rho_{\rm m}$ is the mean matter density of the Universe, $\rho_{\rm b}$ is the mean baryon density of the Universe, $\rho_{\rm crit} = \frac{3H^2}{8\pi G}$
- Planck cosmology parameters: $h = 0.678$, $\Omega_{\rm m} = 0.308$, $\Omega_\Lambda = 0.692$, $\Omega_{\rm b} = 0.048$.
- Hubble parameter at z = 0: $H_0 = 67.8$ km s^{-1} Mpc^{-1} = 2.20×10^{-18}s^{-1}, critical density at z = 0: $\rho_{\rm crit,0} = 8.64\times 10^{-30}$g cm^{-3}
- Units and constants: $1M_\odot = 2\times 10^{33}$ g, 1pc $= 3.086\times 10^{18}$ cm, 1 year $= 3.15\times 10^7$ s, gravitational constant $G = 6.67\times 10^{-8}$ cm^3 g^{-1}s^{-2}, mass of a hydrogen atom $m_H = 1.67\times 10^{-24}$ g, Boltzmann's constant $k_B = 1.38\times 10^{-16}$ cm^{-2}g s^{-2}K^{-1}, speed of light $c = 3.0\times 10^{10}$cm s^{-1}.
- The sound speed of non-relativistic ideal gas $c_{\rm s} = \sqrt{\frac{k_BT}{\mu m_H}}$.
- Indefinite integrals: $\int \frac{1}{x}dx = \ln(x) + c$, $\int x^2dx = \frac{x^3}{3} + c$, $\int x^4dx = \frac{x^5}{5} + c$ and $\int \frac{1}{x^2}dx = -\frac{1}{x} + c$

(Continued on page 2.)

Be brief in your answers. Lengthy derivation are never required

Problem 1 Concepts (weight 25%)

a (3 points) Describe briefly what a Gaussian random field is, and **qualitatively** what the two-point correlation function $\xi(r)$ describes (I have stressed "qualitatively" because you do not need to provide with the equation describing a multi-variate Gaussian).

Solution: A Gaussian Random Field (GRF) is a field of variables, $x_1, x_2, x_3, ...$ each of which obey Gaussian statistics (i.e. each x_i is drawn from a Gaussian distribution). The two point correlation function $\xi(r)$ fully specifies the statistical properties of the field, i.e. it describes the standard deviation of each field variable x_i.

b (3 points) Explain what the galaxy 2-point correlation function $\xi_{gal}(r)$ is, and why it can be different than the $\xi(r)$ describing the matter density field. The difference is called "bias". Describe how the "bias" behaves for galactic halos in different mass regimes.

Solution: $\xi_{gal}(r)$ denotes the excess probability over random of finding two galaxies separated by r, i.e. the probability of finding two galaxies in two identical volume elements dV separated by r equals to $P = dV^2n^2(1+\xi_{gal}(r))$, where ndV denotes the probability of finding a galaxy in a volume element dV. Galaxies forms in collapsed objects, so $\xi_{gal}(r)$ is related to the 2-point correlation of collapsed matter. Because not all matters in the GRF are part of collapsed objects, $\xi_{gal}(r)$ differs from the underlying matter 2-point correlation function $\xi_(r)$.

Using the Press-Schechter Formalism, $\xi_{gal}(r) = b_E^2\xi(r)$, where $b_E \equiv 1 + \frac{\nu 2-1}{\sigma(M)\nu}$ is the Eulerian bias. $\nu \equiv \delta_{crit}/\sigma(M)$ and $\sigma(M)^2$ is the variance of the density filed. There are four regimes:

(a) $\nu = 1 \Rightarrow b_\mathrm{E} = 1$. Collapsed objects with the characteristic mass of the PS mass-function have the same 2-pt function $\xi(r)$ as all matter.

(b) $\nu < 1 \Rightarrow b_\mathrm{E} < 1$. Collapsed objects with $\nu < 1$ (i.e. objects in the power-law low mass end of the PS function) have a 2-pt function $\xi(r)$ that is suppressed to that of all matter. These objects 'cluster less', or said to be 'anti-biased' tracers.

(c) $\nu > 1 \Rightarrow b_\mathrm{E} > 1$. Collapsed objects with $\nu > 1$ (i.e., objects in the exponential tail of the halo mass function) have a 2-pt function $\xi(r)$ that is enhanced to that of all matter. These objects 'cluster stronger', or said to be 'biased' tracers.

(d) $\nu \gg 1 \Rightarrow b_\mathrm{E} \sim \frac{\nu}{\sigma}$. Note that $\nu \equiv \delta_\mathrm{crit}/\sigma(M) \gg 1$. i.e. $\sigma(M) \ll 1$. Therefore $b_\mathrm{E} \gg \nu \gg 1$. The rarest objects are very strongly clustered.

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c (3 points) Assume that the Universe is presently dominated by cold dark matter and that the primordial density fluctuations were generated by quantum fluctuations during inflation. Sketch a plot of the present-day linear power spectrum $P(k)$ as a function of wavenumber k. Indicate the approximate scale where the "turn-over" occurs, and the power law slope at scales smaller and larger than this turn-over.

Solution: The power spectrum looks like Figure 1. Students should indicate the turn-over scale (or wavenumber) k_0, and $P(k) \propto k$ for $k \ll k_0$ and $P(k) \propto k^{-3}$ for $k \gg k_0$.

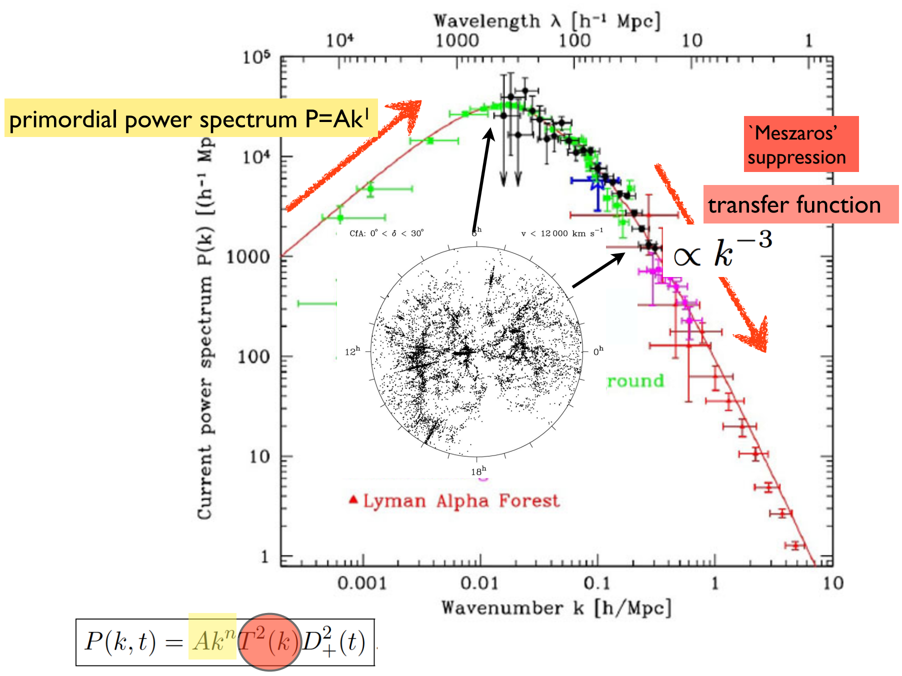

Figure 1: .

d (4 points) Explain the shape of $P(k)$. Discuss what sets the "turn-over", and discuss qualitatively the difference in $P(k)$ at scales smaller and larger than this turn-over radius.

Solution: For large scales - i.e. small k - we have the 'primordial' power spectrum $P(k) = Ak^n$. The slope has been inferred to be $n = 1$ (from the CMB). For small scales (i.e. large k) we have $P(k) = Ak^n T^2(k)$ where $T(k) \propto k^{-2}$ denotes the transfer function. The transfer function quantifies how the 'Meszaros' effect suppresses the growth of small scale perturbations: these perturbations entered the horizon prior to matter radiation equality. The Meszaros effect suppressed the growth of these perturbations during radiation domination. The turn-over in the power spectrum occurs on a scale which corresponds to the size of a perturbation that entered the horizon at matter-radiation equality.

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e (5 points) Which processes (broadly) set the range of masses of galaxies? Use concepts like collapse, virialization, gas cooling.

Solution: When a gas cloud collapses, it virializes and reaches the virial temperature. Mass range is set broadly by comparing the cooling time of this virialized gas vs dynamical time of the overall collapsed gas cloud. The cooling time is the time it takes for gas to radiate away its thermal energy ($t_{cool} \propto E/C(T)$). The dynamical time corresponds to the time-scale over which structure changes in the gas become apparent ($t_{dyn} \propto \rho^{-1/2}$). The lower limit is set by inefficient cooling. Below $T = 10^4 K$ cooling is extremely inefficient (gas is too cold for collisional processes to excite any transitions of atomic H). For high T cooling is efficient, by the energy content is so large that the cooling time remains very large.

f (4 points) What is Toomre instability and how it differs from the Jeans instability? Explain why Toomre instability is relevant for the formation of Giant Molecular Clouds in galactic disks.

Solution: Toomre instability is a gravitational instability occurring in rotating thin disks, where the centrifugal and Coriolis force can act like pressure and prevent gravitational collapse. It differs from Jeans instability because it takes rotation into account, whereas Jeans instability only consider thermal pressure. For Jeans Instability there always exists a scale beyond which collapse will occur (i.e. the Jeans length or Jeans Mass), but this is not the case for Toomre instability. The Toomre parameter $Q \sim \frac{c_s \Omega}{G\Sigma}$, where c_s is the sound speed, Ω characterises the rotation of the disk and Σ is the disk surface density. When $Q > 1$, the disk is stable against perturbations of *all wavelength*. When $Q < 1$, the disk is subject to gravitational instability for certain perturbations. Because galactic disks rotates, Toomre instability is more relevant for GMC formation in galactic disks.

g (3 points) What is the Eddington luminosity?

Solution: The Eddington luminosity is the maximum accretion luminosity a black hole or a star can reach from the gravitational energy of the accreting material. At the Eddington luminosity, radiation pressure by photons balances the gravitational force of the black hole or the star.

Note: There is no need to give or derive the expression for the Eddington luminosity.

Problem 2 Lyα Forest and Reionization (weight 25%)

Figure 2 shows two spectra from distant quasars. Both spectra are in the observed wavelength frame.

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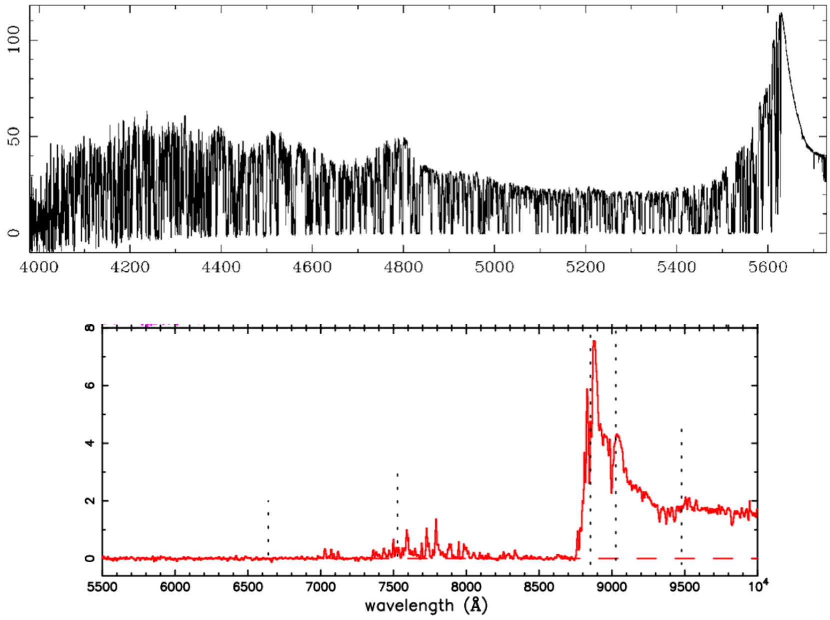

Figure 2:

a (2 points) Determine the redshift of the quasar in the upper and lower panel of Figure 2. *Hint:* The rest-frame wavelength of Lyα emission/absorption is 1216 Å.

Solution: use $\lambda_{obs} = \lambda_{rest}(1+z)$. We get $z \approx 3.6$ in the upper panel and $z \approx 6.3$ in the lower panel.

b (4 points) Describe the absorption features to the left of the emission line in the upper panel. Briefly explain what is the origin of this feature, what it tells us about the physical properties of the intergalactic medium, what over-density range it probes, and why it disappears in the lower panel.

Solution: The absorption features to the shorter wavelength of the emission line is the Lyα Forest, it is caused by absorption of photons (when redshifted to the Lyα rest-frame wavelength) by neutral hydrogen (H I) between the quasar (or galaxy) and us. The photon causes the $1s \rightarrow 2p$ transition for the electron in a hydrogen atom. The fluctuations in Lyα opacity in the Ly forest correspond to density fluctuations along the line-of-sight. So Lyα Forest can be used to probe underlying density distribution of dark matter. Most of these absorption lines correspond to mildly overdense regions $\delta \sim 1$, which is very different than the densities probed by virialized structures.

If the gas in the Universe were neutral, the optical depth along the line of sight is of order 10^5 and therefore we should see no flux at frequencies higher than the redshifted Lyα emission. The fact that we see absorption lines and a continuum in the upper panel implies that the intergalactic medium, must have been highly ionized. In the lower panel, there is little flux at shorter wavelength of the emission,

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implying the IGM is mostly neutral (Gunn-Peterson trough). The transition of the IGM from mostly neutral to mostly ionized is called the "reionization".

c (5 points) Explain qualitatively why the intergalactic medium at the density range probed by the Lyα forest has temperature of few times 10^4 K? You may find it helpful to provide a sketch to support your description. **Note:** this is a question on the general properties of the IGM, you do **not** need to discuss how the temperature is determined using the Lyα spectra.

Solution: The Lyα forest mostly probe the IGM with overdensity $\delta \sim 1$. At this density, the temperature is set by the equilibrium between photoionization heating and radiative cooling, at $T \sim 10^4$ K, the heating rate is roughly the same as cooling rate (see Figure 3).

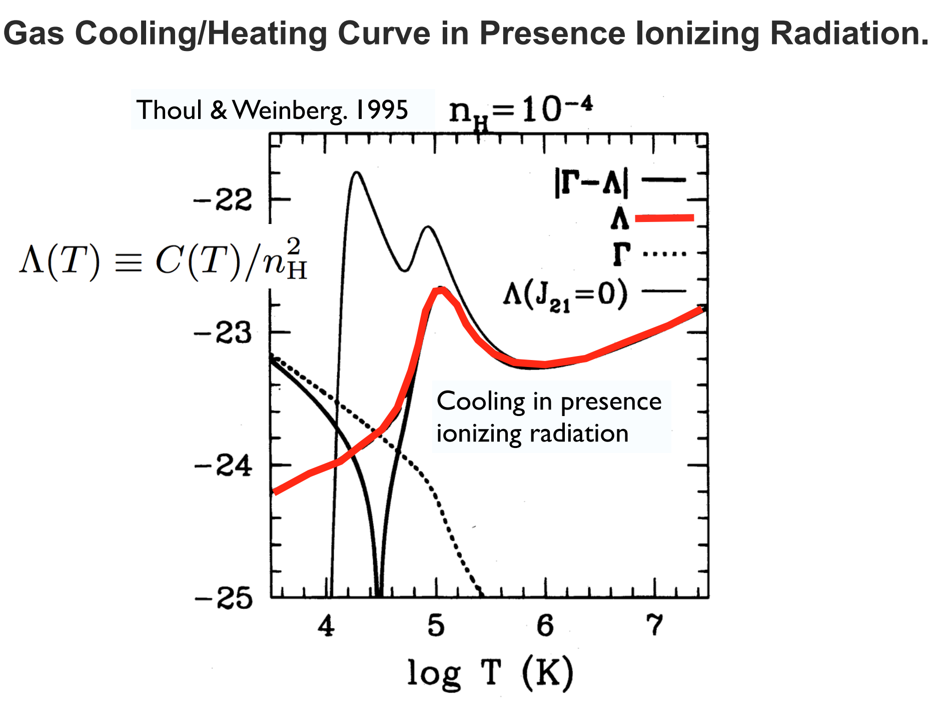

Figure 3:

d (5 points) Derive the comoving wavenumber beyond which we expect Jeans smoothing to suppress power in Lyα forest at $z = 3$. Use the Jeans length $\lambda_J = 2R_J$ where R_J is given by Equation (1) in Problem 3 (next page), assuming $T = 10^4$ K, $\mu = 0.59$, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$.

Solution: see Assignment 3 solutions.

e (4 points) How does the Jeans smoothing scale compare to the smoothing that is introduced by the finite width of the Lyα absorption cross-section? Assume that the same gas temperature cosmological parameters as in the question above. Note: if you do not know the width of

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the Lyα absorption cross-section (in km s^{-1}), discuss how you would solve the problem if you had known.

Solution: see Assignment 3 solutions. The Jeans smooth scale should be a bit larger than the broadening due to the Lyα absorption cross-section.

f (5 points) Describe **qualitatively** how the smallest scale probed by the Lyα forest derived above can provide a lower limit on dark matter mass if dark matter is "warm".

Solution: If dark matter is warm, DM particles would be relativistic in the early Universe and free-streaming, and density fluctuations would not be able to grow under gravity. As the Universe expands and cools, the DM particles become non-relativistic and structure can grow. This occurs at $k_bT \sim mc^2$. We denote the scale factor at which DM particles become non-relativistic as a_{nr}, so perturbations with scales smaller than the comoving particle horizon scale at a_{nr}, $r^c_{\mathrm{H}}(a_{nr})$, would have been washed out completely by the free-streaming of particles. If $a_{nr} \ll a_{eq}$, where a_{eq} denotes the scale factor of the Universe at matter-radiation equality, then $r^c_{\mathrm{H}} \propto a$, and T $\propto a^{-1}$, so $r^c_{\mathrm{H}}(a_{nr})/r^c_{\mathrm{H}}(a_{eq}) = (k_bT_{eq})/(k_bT_{nr}) \sim 10^{-3}$keV/mc^2 (since k_bT_{eq} ~1 eV). We know that $r^c_{\mathrm{H}}(a_{eq}$ corresponds to the wave number (k_0) at which the power spectrum turns over due to Meszaros effect. From the above derivations we expect structure to be washed out for $k > k_{fs} = 10^3(mc^2/\mathrm{keV})$ and this is the regime probed by the Lyα forest. The smallest scale (largest k) that can be probed is set by the broadening due to the Lyα absorption cross-section, which gives a lower limit of DM particle mass.

Problem 3 Star Formation (weight 25%)

a (6 points) Consider a uniform spherical giant molecular cloud (GMC) has temperature T, density ρ, radius R and mass M, and all pressure support is from thermal pressure. Using the Virial theorem, show that if $R > R_J$, the cloud will collapse under gravitational force, where

$$R_J = \sqrt{\frac{15k_BT}{4\mu m_H \pi G\rho}} = c_s\sqrt{\frac{15}{4\pi G\rho}}, \tag{1}$$

is half of the Jeans length. Here, c_s is the sound speed, μ is the mean molecular weight and m_H is the mass of an hydrogen atom. *Hint*: the gravitational potential energy for a uniform spherical cloud is $U = -\frac{3GM^2}{5R}$.

b (6 points) The Larson's law (Larson 1981) describes an observed relation between turbulent velocity dispersion (σ_{turb}) and the size of a star forming region (L):

$$\sigma_{turb} = 1.1\ L^{0.38} \tag{2}$$

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where σ_{turb} is in units of km s^{-1}, and L is in units of pc. Compute the velocity dispersion for a cloud with size $L = 2R_J$, assuming the GMC has uniform temperature $T = 10$ K, number density $n = 10^3$ cm^{-3} and mean molecular weight $\mu = 1.2$. How does the turbulent velocity dispersion at this length scale compare to the sound speed? Discuss briefly the implication of your results on star formation in GMCs. Specifically mention how turbulent velocity dispersion affects the Jeans mass.

c (7 points) For ideal gas, the temperature and density follows the relationship $T \propto \rho^{\gamma-1}$, where γ is the adiabatic index. In the midterm, we derived:

$$\lambda_J \propto \rho^{\frac{1}{2}(\gamma-2)} \tag{3}$$

$$M_J \propto \rho^{\frac{3}{2}(\gamma-\frac{4}{3})} \tag{4}$$

For a $\gamma < 4/3$ the Jeans mass will decrease during collapse which leads to fragmentation. For a $\gamma > 4/3$ the Jeans mass will increase and we have no fragmentation. Initially GMCs contract isothermally ($\gamma = 1$), which allows for fragmentation to occur within the cloud. This will be true as long as the heat generated from the gravitational collapse is efficiently radiated away (the cloud is optically thin). Given that the cloud is in free fall with velocity $v_{ff} = \sqrt{\frac{GM}{R}}$, show that the heating rate (i.e. luminosity) that is gravitationally generated by the collapse is:

$$L_{grav} \approx G^{3/2} \left(\frac{M_J}{R_J}\right)^{5/2} \tag{5}$$

Hint: Use the Virial theorem to derive total energy release during the collapse, and use the free-fall velocity to derive a time scale. You can approximate the free-fall velocity by a constant $v_{ff} \approx \sqrt{\frac{GM_J}{R_J}}$ to avoid doing integrations.

d (6 points) As density increases, the cloud becomes optically thick and energy is radiated away through blackbody radiation. The luminosity of a blackbody is given by $L_{rad} = 4\pi R^2 \sigma T^4 f$, where f is an efficiency factor with $0 \leq f \leq 1$, and σ is the Stefan-Boltzmann Constant. At a certain critical point when radiative cooling equals to or slower than heating, the cloud will heat up and contract adiabatically ($\gamma > 4/3$) and the fragmentation will stop. Using the information from above, derive an expression for the mass of the smallest fragment in terms of temperature (T), the efficiency factor (f) and the mean molecular weight (μ). What parameter will this mass strongly depend on?

Problem 4 The Cusp-Core Problem (weight 25%)

a (3 points) Describe briefly what the cusp-core problem is, and why it represents a "crisis" for the standard cold dark matter paradigm.

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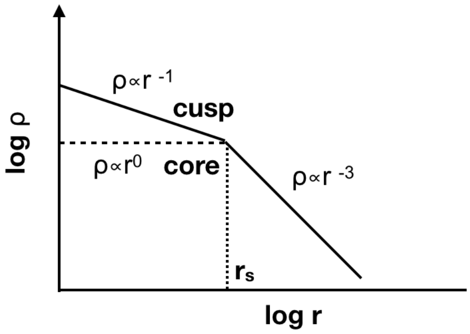

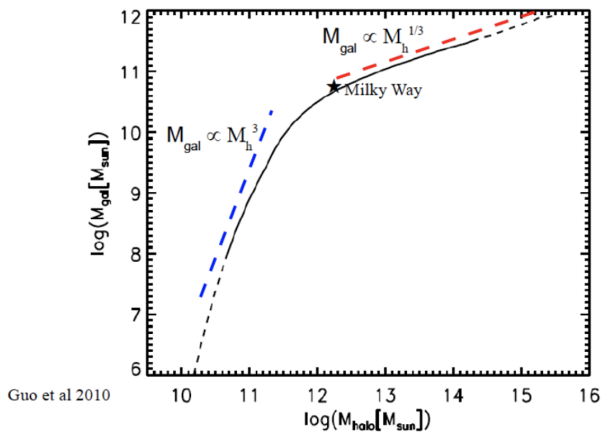

Figure 4:

Solution: In CDM simulations, simulated dark matter halos have 'cusps', i.e. the dark matter density evolves as $\rho \propto r^{-1}$ for small r. Observations prefer the density of the innermost regions of dark matter halos to contain 'cores', in which the dark matter density is constant. This is the 'cusp-core' problem. Since cuspy profile is a robust prediction of the cold dark matter scenario, the fact that DM profiles is cored represents a crisis of the CDM paradigm.

b (4 points) Assume a dark matter halo has a cuspy density profile which is described by:

$$\rho^{\rm cusp}(r) = \begin{cases} \rho_0 \left(\frac{r}{r_s}\right)^{-1}, & \text{if } r < r_s \\ \rho_0 \left(\frac{r}{r_s}\right)^{-3}, & \text{if } r \geq r_s \end{cases}$$

Here, ρ_0 is a normalization parameter and r_s is the scale length. In lectures we discussed how baryonic processes such as supernova feedback can result in the formation of a dark matter core. If we assume the resultant "cored" profile is described by:

$$\rho^{\rm core}(r) = \begin{cases} \rho_0 & \text{if } r < r_c \\ \rho_0 \left(\frac{r}{r_c}\right)^{-3}, & \text{if } r \geq r_c \end{cases}$$

Here, r_c is the size of the core. For simplicity, we adopt $r_c = r_s$ for this exercise and assume ρ_0 is the same in both profiles. The profiles are illustrated in the left panel of Figure 4. Show that the total mass enclosed within radius r is

$$M^{\rm cusp}(<r) = \begin{cases} 2\pi\rho_0 r_s r^2, & \text{if } r < r_s \\ 2\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\rm{ln r} - \rm{ln r_s}), & \text{if } r \geq r_s \end{cases}$$

for the cuspy profile, and

$$M^{\rm core}(<r) = \begin{cases} \frac{4}{3}\pi\rho_0 r^3, & \text{if } r < r_s \\ \frac{4}{3}\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\rm{ln r} - \rm{ln r_s}), & \text{if } r \geq r_s \end{cases}$$

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for the cored profile.

c (3 points) From Virial theorem we can derive that the minimum total energy needed to create a cored profile from a cuspy profile is given by $\Delta E = (W^{core} - W^{cusp})/2$, where W is the total gravitational potential energy. Show that if the density distribution is spherical symmetric, the gravitational potential energy at the virial radius r_{vir} is

$$W = -4\pi G \int_0^{r_{vir}} \rho(r) M(<r) r dr \tag{6}$$

d (5 points) Using the density and mass profile given in the previous questions, show that for $r_s \ll r_{vir}$, the minimum energy needed to create a cored profile is

$$\Delta E = \frac{32}{15}\pi^2 G \rho_0^2 r_s^5 \tag{7}$$

Hint: You may want to use the integration formula given in the beginning of the test sheet. **Please expand all the terms before you do the integrations because you do not need to do all of them!**

e (3 points) Consider a dwarf galaxy at z = 0 with $M_{vir} = 3 \times 10^{10}$ M$_\odot$, $R_{vir} = 45$ kpc and scale length $r_s = 1$kpc, compute the energy needed to create a dark matter core with $r_c = r_s = 1$ kpc.

f (4 points) The right panel of Figure 4 shows a relationship between the total stellar mass of a galaxy and the virial mass (M$_{gal}$ in this figure is the same as total stellar mass). Assuming the number of supernova explosions per solar mass formed is $\xi = 0.004$ and energy injection per supernova is $E_{SN} = 10^{51}$ ergs, use the information in the figure to compute the total energy available from supernova feedback. How does it compare to the energy required to generate 1 kpc core in the dark matter profile?

g (3 points) Consider a galaxy with 10 times smaller virial mass (i.e. $M_{vir} = 3 \times 10^9$M$_\odot$), but **the same virial radius and scale length**, use the information given in the right panel of Figure 4 (especially the scaling relations), comment on whether supernova energy is sufficient to generate a 1 kpc core in this halo. *Hint:* you do not need to compute any numbers here, simply observe the dependencies on M_{vir} for both the energy required and the energy provided by supernova.

Problem 5 Bonus Questions (weight 10%)

Please note *all the correct answers* for each question. It is sufficient to write down the number and letters of the correct answers. For example, “1. a, b,

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c” if in question number “1” the answers “a”, “b”, and “c” are correct. There is a maximum of 10 points (1 point per question) to be gained for this exercise (i.e. the maximum score you can get is 110%). *This means that you should not spend too much time on this exercise!*

1. What is the dominant effect that brought about the transition from Population III star formation to Population II star formation? (c)
   (a) UV Radiation generated by Population III stars
   (b) Supernova explosions of Population III stars
   (c) Early metal enrichment by Population III stars
   (d) Dynamical interactions of Population III stars

2. What describes the ‘missing satellite’ problem? (b)
   (a) Too few satellites around the Earth.
   (b) Simulations with cold dark matter (CDM) show many more substructures than observed dwarf galaxies around the Milky Way.
   (c) Simulations with cold dark matter (CDM) show many more substructures than observed galaxies in galaxy clusters.
   (d) We observe many more galaxies in galaxy clusters compared to the number of substructures expected from simulations with cold dark matter (CDM).

3. What is the most (computationally) expensive algorithm for N-body simulations? (a)
   (a) Direct summation
   (b) Barnes-Hut
   (c) Binary Tree
   (d) Particle-Particle-Particle-Mesh

4. Smooth Particle Hydrodynamics has the advantages that... (a, c, d)
   (a) it is Galilean invariant
   (b) it resolves shocks and discontinuities better
   (c) it allows flexible geometry
   (d) it conserve mass and angular momentum exactly

5. Where can/could elements heavier than Iron (Fe) form? (d, e)
   (a) During inflation
   (b) During the ‘Big Bang nucleosynthesis’ (BBN).
   (c) During nucleosynthesis in stars
   (d) In supernova explosions
   (e) In compact binary mergers

6. What can/could power the Lyα emission in galaxies and their halo gas? (a, b, c)

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(a) Cooling radiation from gravitational collapse
(b) Recombination of H^+ and e^-
(c) Scattering of radiation from Active Galactic Nuclei (AGN) and stars
(d) Particles accelerated through magnetic fields

7. How do you call the phenomenon when we observe the light's trajectory being bent around *all sides* of an object? (b)

   (a) Einstein cross
   (b) Einstein ring
   (c) Weak lensing
   (d) Lensing arc

8. Which one of the following happens first in the future Universe? (d)

   (a) Gravitational radiation and decay of stellar orbits
   (b) Proton decay
   (c) Star formation through brown dwarf collisions
   (d) Dynamical relaxation of the Galaxy

9. Which one(s) of the following can contribute to the reionization of the Universe? (a, b, c, d)

   (a) Population III stars
   (b) Mini-quasars powered by intermediate mass black holes
   (c) Population II stars
   (d) Self-annihilating dark matter particles

10. Which one(s) of the following are the *direct* observational evidence of AGN feedback (a, b)?

   (a) High velocity (e.g. > 1000 km s^{-1}) galactic outflows
   (b) Bubbles (or cavities) in deep X-ray images of cool-core clusters
   (c) Relation between mass of central black holes and the velocity dispersions of their host galaxy (i.e., the $M - \sigma$ relation)
   (d) The reionization of the Universe

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