UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: AST4320 — Cosmology and Extragalactic Astronomy

Day of examination: Friday 18. December 2020

Examination hours: 15.00 – 19.30

This problem set consists of 6 pages.

Appendices: None

Permitted aids: Formula book by Rottmann. Course notes, lecture slides and reference books.
Calculator that meets standard requirements by university.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Preface: some useful relations:

- $H(a) = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda + \Omega_{\rm rad}a^{-4}}$, where H_0 is the Hubble parameter today, Ω's denote the density parameters in matter, dark energy, and radiation.
- $a = \frac{1}{1+z}$.
- Einstein-de Sitter Universe: $\Omega_{\rm m} = 1.0$, $\Omega_\Lambda = 0.0$.
- $\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm crit}}$, $\Omega_{\rm b} = \frac{\rho_{\rm b}}{\rho_{\rm crit}}$, where $\rho_{\rm m}$ is the mean matter density of the Universe, $\rho_{\rm b}$ is the mean baryon density of the Universe, $\rho_{\rm crit} = \frac{3H^2}{8\pi G}$
- Planck cosmology parameters: $h = 0.678$, $\Omega_{\rm m} = 0.308$, $\Omega_\Lambda = 0.692$, $\Omega_{\rm b} = 0.048$.
- Hubble parameter at z = 0: $H_0 = 67.8$ km s^{-1} Mpc^{-1} = 2.20×10^{-18}s^{-1}, critical density at z = 0: $\rho_{\rm crit,0} = 8.64 \times 10^{-30}$g cm^{-3}
- Units and constants: $1M_\odot = 2 \times 10^{33}$ g, 1pc $= 3.086 \times 10^{18}$ cm, 1 year $= 3.15 \times 10^7$ s, gravitational constant $G = 6.67 \times 10^{-8}$ cm^3 g^{-1}s^{-2}, mass of a hydrogen atom $m_H = 1.67 \times 10^{-24}$ g, Boltzmann's constant $k_B = 1.38 \times 10^{-16}$ cm^{2}g s^{-2}K^{-1}, speed of light $c = 3.0 \times 10^{10}$cm s^{-1}.
- The sound speed of non-relativistic ideal gas $c_{\rm s} = \sqrt{\frac{k_B T}{\mu m_H}}$.
- Indefinite integrals: $\int \frac{1}{x}dx = \ln(x)+c$, $\int x^2 dx = \frac{x^3}{3}+c$, $\int x^4 dx = \frac{x^5}{5}+c$ and $\int \frac{1}{x^2}dx = -\frac{1}{x} + c$

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Be brief in your answers. Lengthy derivation are never required

Problem 1 Lyα Forest and Reionization (weight 25%)

Figure 1 shows two spectra from distant quasars. Both spectra are in the observed wavelength frame.

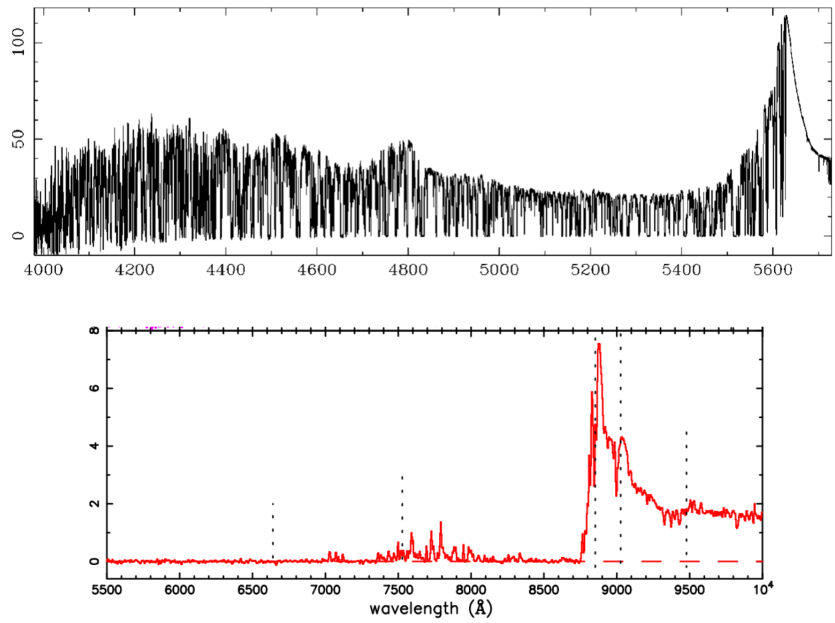

Figure 1:

a Determine the redshift of the quasar in the upper and lower panel of Figure 1. Note that the rest-frame wavelength of Lyα emission/absorption is 1216 Å. (4 points)

b The absorption features to the left of the emission line in the upper panel is called the Lyα forest. Briefly explain what the origin of Lyα forest is, what it tells us about the physical properties of the intergalactic medium, what over-density range it probes, and why it disappears in the lower panel. (5 points)

c Explain **qualitatively** why the temperature of the intergalactic medium probed by the Lyα forest has temperature of few times 10^4 K? You may find it helpful to provide a sketch to support your description. (4 points)

d Derive the comoving wavenumber beyond which we expect Jeans smoothing to suppress power in Lyα forest at $z = 3$. Assuming the gas has temperature $T = 2 \times 10^4$ K, mean molecular weight $\mu = 0.59$, cosmological parameter $\Omega_m = 0.308$ and $\Omega_\Lambda = 0.692$ (Planck Cosmology). (6 points)

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e How does the Jeans smoothing scale compare to the smoothing that is introduced by the finite width of the Lyα absorption cross-section due to thermal broadening? Assume the same gas temperature and cosmological parameters as in the question above. (6 points)

Problem 2 Cooling and Gas Accretion (weight 20%)

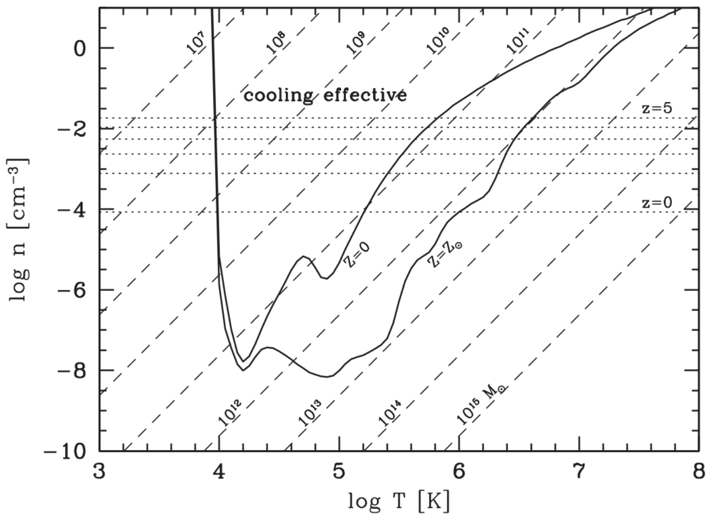

Figure 2:

a Consider a virialized spherical halo with total mass M and gas mass M_{gas}, so $f_{gas} \equiv M_{gas}/M$ is the gas fraction, and assuming the gas and dark matter are distributed uniformly and the gas is monoatomic, use the Virial Theorem *for the gas* to derive an expression of M_{gas} as a function of temperature T, number density $n \equiv \rho/(\mu m_p)$ and gas fraction f_{gas}. Here μ is the mean molecular weight and m_p is the proton mass. The dashed lines with mass labels in Figure 2 indicate lines with constant gas mass, using the expression you derive, explain why the slope of these lines are what you see in the figure. (*Hint:* the kinetic energy of the gas is the thermal energy, i.e. $E_k = \frac{3}{2}NkT$). (10 points)

b Assuming $\mu = 0.59$ and $f_{gas} = 0.15$, calculate M_{gas} for $T = 1 \times 10^6$ K and $n = 10^{-3}$ cm^{-3}, is this mass similar to what you read from Figure 2? (5 points)

c What do the solid lines in Figure 2 indicate? Comparing the solid and dashed lines, one can see that cooling is effective for certain mass but ineffective for others. Early theories use this result to explain why galaxy formation is not efficient for massive galaxies and clusters, discuss possible flaws in this picture. (5 points)

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Problem 3 Star Formation (weight 25%)

a The Larson's law (Larson 1981) describes an observed relation between turbulent velocity dispersion (σ_{turb}) and the size of a star forming region (L):

$$\sigma_{turb} = 1.1\ L^{0.38} \tag{1}$$

where σ_{turb} is in units of km s^{-1}, and L is in units of pc. Consider a GMC with a uniform temperature $T = 10$ K, number density $n = 10^3$ cm^{-3} and mean molecular weight $\mu = 1.2$, compute 1) the Jeans length of the GMC ($\lambda_J = c_s\sqrt{\frac{\pi}{G\rho}}$); 2) the velocity dispersion for a cloud with size of the Jeans length $L = \lambda_J$. How does the turbulent velocity dispersion at this length scale compare to the sound speed? Discuss briefly the implication of your results on star formation in GMCs. (10 points)

b There is observational evidence for magnetic fields in the ISM. Magnetic fields play an important role in star formation because it provides additional support against gravitational collapse. The magnetic pressure is given by $P_m = \frac{B^2}{8\pi}$, where B is the strength of the magnetic field. By analogy with thermodynamics, the magnetic energy E_m is given by $E_m = P_m V = \frac{B^2 V}{8\pi}$, where V is the volume of the cloud. The Virial theorem in the presence of a magnetic filed is

$$U + 2T + E_m = 0 \tag{2}$$

Here U and T are the potential and kinetic energy as before. Consider a uniform density, spherically symmetric cloud of mass M, initial radius R_0 and volume V_0, threaded by a magnetic filed of strength B_0. If the magnetic pressure is the dominant support mechanism (i.e. $E_m \gg T$), then the virial theorem reduces to $U + E_m = 0$. A slight compression decreases R_0 to radius R, forcing the field lines closer and increasing the magnetic field strength because the conservation of magnetic flux:

$$B_0 R_0^2 = B_R R^2, \tag{3}$$

where B_R is the new magnetic field strength after the compression. Using the virial theorem, show that the minimum mass against collapse in this case, the so-called magnetic critical mass, M_c, is given by

$$M_c = \frac{1}{3}\sqrt{\frac{5}{2G}} B_0 R_0^2 \tag{4}$$

Hint: the gravitational potential energy for a uniform spherical cloud with radius R is $U = -\frac{3GM^2}{5R}$. (10 points)

c Assuming $R_0 = \frac{1}{2}\lambda_J$, where λ_J is the Jeans length calculated in **Problem 3a**, and the magnetic field strength is about 2×10^{-5} Gauss, compute the magnetic critical mass in solar masses. How does it compare to the Jeans mass? (5 points)

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Problem 4 The Cusp-Core Problem (weight 30%)

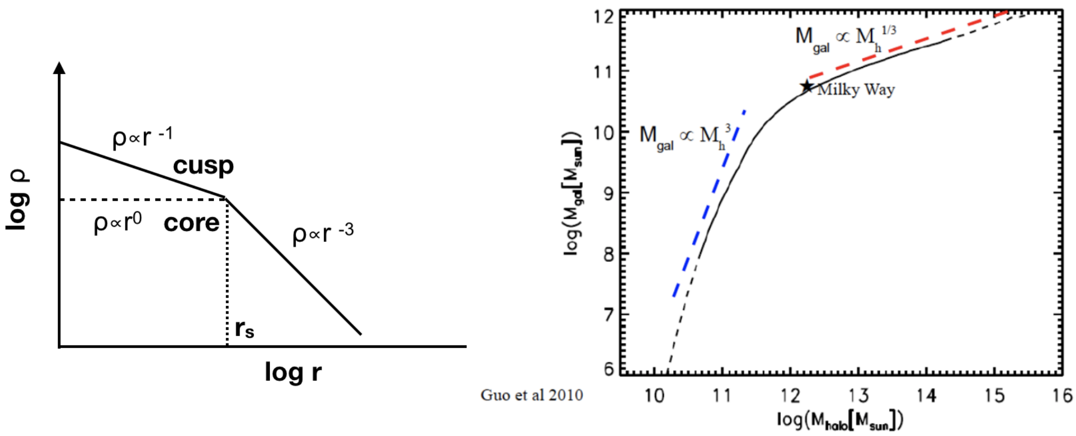

Figure 3:

In lectures we discussed the "cusp-core" problem and how stellar feedback (such as supernova explosions) can convert an initially cusp profile to a "cored" one. Assume a dark matter halo has initially a cuspy density profile which is described by:

$$\rho^{\mathrm{cusp}}(r) = \begin{cases} \rho_0 \left(\frac{r}{r_s}\right)^{-1}, & \text{if } r < r_s \\ \rho_0 \left(\frac{r}{r_s}\right)^{-3}, & \text{if } r \geq r_s \end{cases}$$

Here, ρ_0 is a normalization parameter and r_s is the scale length. Later, star formation and subsequent supernova feedback create a "cored" profile, which is described by:

$$\rho^{\mathrm{core}}(r) = \begin{cases} \rho_0 & \text{if } r < r_c \\ \rho_0 \left(\frac{r}{r_c}\right)^{-3}, & \text{if } r \geq r_c \end{cases}$$

Here, r_c is the size of the core. For simplicity, we adopt $r_c = r_s$ for this exercise, and assume ρ_0 is the same in both profiles. The profiles are illustrated in the left panel of Figure 3. Simple integration gives the total mass enclosed within radius r:

$$M^{\mathrm{cusp}}(<r) = \begin{cases} 2\pi\rho_0 r_s r^2, & \text{if } r < r_s \\ 2\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\mathrm{lnr} - \mathrm{lnr_s}), & \text{if } r \geq r_s \end{cases}$$

for the cuspy profile, and

$$M^{\mathrm{core}}(<r) = \begin{cases} \frac{4}{3}\pi\rho_0 r^3, & \text{if } r < r_s \\ \frac{4}{3}\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\mathrm{lnr} - \mathrm{lnr_s}), & \text{if } r \geq r_s \end{cases}$$

for the cored profile.

a Assuming the system is in equilibrium before and after the profile is changed, use the Virial theorem, derive that the minimum total energy needed to create a cored profile from a cuspy profile is given by

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$\Delta E = (W^{core} - W^{cusp})/2$, where W is the total gravitational potential energy. (4 points)

b Show that if the density distribution is spherical symmetric, the gravitational potential energy at the virial radius r_{vir} is

$$W = -4\pi G \int_0^{r_{vir}} \rho(r) M(< r) r dr \tag{5}$$

(4 points)

c Using the density and mass profile given above, show that for $r_s \ll r_{vir}$, the minimum energy needed to create a cored profile is

$$\Delta E = \frac{32}{15}\pi^2 G \rho_0^2 r_s^5 \tag{6}$$

Hint: You may want to use the integration formula given in the beginning of the test sheet. **Please expand all the terms before you do the actual integrations because you do not need to do all of them!** (8 points)

d Consider a dwarf galaxy at z = 0 with $M_{vir} = 3 \times 10^{10}$ M$_\odot$, $R_{vir} = 45$ kpc and scale length $r_s = 1$kpc, compute the energy needed to create a dark matter core with $r_c = r_s = 1$ kpc. (4 points)

e The right panel of Figure 3 shows a relationship between the total stellar mass of a galaxy and the virial mass (M$_{gal}$ in this figure is the same as total stellar mass). Assuming the number of supernova explosions per solar mass formed is $\xi = 0.004$ and energy injection per supernova is $E_{SN} = 10^{51}$ ergs, use the information in the figure to compute the total energy available from supernova feedback for a dwarf galaxy with the virial mass specified in Problem 4d. How does it compare to the energy required to generate 1 kpc core in the dark matter profile? (5 points)

f Consider a galaxy with 10 times smaller virial mass (i.e. $M_{vir} = 3 \times 10^9$M$_\odot$), but **the same virial radius and scale length**, use the information given in the right panel of Figure 3 (especially the scaling relations), comment on whether supernova energy is sufficient to generate a 1 kpc core in this halo. *Hint:* you do not need to compute any numbers here, simply observe the dependencies on M_{vir} for both the energy required and the energy provided by supernova. (5 points)