





UNIT 6, LESSON 1

Relationships between Quantities



1.1 Warm-up

10 mins

Pricing Theater Popcorn

Standards Alignment

Building On Building Toward



7.RP.A.2



7.EE.B

Materials

None

Activity Narrative

Students generate two sets of values: one set could be values in a proportional relationship, and the other set could not. The purpose of this *Warm-up* is to remind students of some characteristics that make a relationship proportional or not proportional, so that later in the lesson, they are better equipped to recognize that a relationship is not proportional and explain why.

Look for students who have a reasonable way to explain why their set of numbers is not proportional, such as "The unit price is different for each size," or "Each size costs a different amount per ounce."

Launch

Invite students to share experiences going to the movies. What do they know about the popcorn for sale? What sizes does it come in? About how much does it cost?

Tell students that in this activity, they will come up with prices for different sizes of popcorn one set of prices in which the price is in proportion to the size, and another set of prices in which the price is not in proportion to the size, but is still reasonable. Ask students to be ready to explain the reasons they chose the numbers they did. If needed, review what it means for a relationship to be proportional: the values for one quantity are each multiplied by the same number to get the values for the other quantity.

Arrange students in groups of 2. Give 2 minutes of guiet work time and then invite students to share their response with their partner, followed by whole-class discussion.



Student Task Statement

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

Complete one column of the table with prices where popcorn is priced at a constant rate. That is, the amount of popcorn is proportional to the price of the bag. Then complete the other column with realistic prices where the amount of popcorn and price of the bag are not in proportion.

volume of popcorn (ounces)	price of bag, proportional (\$)	price of bag, not proportional (\$)
10	6	6
20		
35		
48		

Student Response



<u>Sign in</u> to access Student Response.

Activity Synthesis

The purpose of this discussion is to elicit different ways of viewing a proportional relationship. For example, for 20 ounces and 35 ounces, students might move from row to row and think in terms of scale factors. This approach is less straightforward for 48 ounces, and some students may shift to thinking in terms of unit rates.

Invite a student to share their prices for the proportional relationship and how they decided on those numbers. Ask if any students thought of it in a different way.

Then invite a student to share their prices for the relationship that is not proportional and record these for all to see. Ask students to explain ways you can tell that the relationship is not proportional.

1.2 Activity

10 mins

Entrance Fees

Standards Alignment

Building On Building Toward



7.RP.A.2.a



7.EE.B.4

7.EE.B.4.a

Instructional Routines

MLR6: Three Reads

Materials

None

Activity Narrative

In this activity, students use any strategy to extend a non-proportional pattern. This context was used in an earlier unit as an example of a relationship that is not proportional. However, a different rule for determining the entrance fee is used here.

Watch for students who organize the given information in a table or another visual representation, and for unique, correct approaches to the first two questions.

As students analyze several pairs of values in the relationship and then encapsulate the relationship with a rule, they look for and express regularity in repeated reasoning (MP8).

Launch

Tell students that unlike the previous activity where they could choose any numbers, this activity has a relationship where there is a pattern, and part of the work is to figure out what

that pattern is. This activity has to do with an entrance fee to a park, where the fee is based on the number of people in the vehicle.



Access for English Learners

MLR6 Three Reads. Keep books or devices closed. Display only the problem stem, without revealing the questions. Say, "We are going to read this problem 3 times."

- After the 1st read: "Tell your partner what this situation is about."
- After the 2nd read: "List the quantities. What can be counted or measured?"
- For the 3rd read: Reveal and read the first question. Ask, "What are some ways we might get started on this?"

Advances: Reading, Representing



Access for Students with Disabilities

Representation: Access for Perception. Provide visual representations of anything that is read aloud.

Supports accessibility for: Language, Memory



Student Task Statement

A state park charges an entrance fee based on the number of people in a vehicle. A car containing 2 people is charged 14, acarcontaining4peopleischarged20, and a van containing 8 people is charged \$32.

- 1. How much do you think a bus containing 30 people would be charged?
- 2. If a bus is charged \$122, how many people do you think it contains?
- 3. What rule do you think the state park uses to decide the entrance fee for a vehicle?

Student Response



Sign in to access Student Response.

Building on Student Thinking

Students may misunderstand that the first two questions require noticing and extending a pattern, and (because of the Warm-up) think that any reasonable number is acceptable.

Encourage them to organize the given information and think about what rule the park might use to determine the entrance fee based on the number of people in the vehicle.

Students may come up with "rules" that aren't supported by the context or the given information. For example, they may notice that each additional person costs **3**, butthenreasonthat30peoplemustcost90. Whatever their rule, ask students to check whether it works for all of the information given. For example, since 2 people cost **14**, wecantellthat\(\mathbf{3}\)3 per person" is not the rule.

Activity Synthesis

The purpose of this discussion is to elicit different ways of describing the rule for determining the entrance fee based on the number of people, and to notice that the relationship is not proportional.

Invite a student who organized the given information in a table to share. If no students did this, display this table for all to see:

number of people	entrance fee in dollars	
2	14	
4	20	
8	32	
30		
	122	

Ask: "What are some ways that you can tell that this relationship is not proportional?" Sample responses:

- 2 people to 4 people is double, but 14 to 20 is not double.
- $14 \div 2 = 7$, but $20 \div 4 = 5$. If the entrance fee was in proportion to the number of people, each quotient would be equal.
- You can't describe the situation with an equation like px = q.

Invite students who had different strategies for answering the first two questions to share their responses. Include as many unique strategies as time allows. For each strategy, ask students to state their rule that the park uses to decide the entrance fee. Record all unique, correct rules, so all students can see different ways of expressing the same idea. For example, the rule might be expressed in the following ways:

- 8 dollars for the vehicle plus 3 dollars per person
- 3 dollars for every person and an additional \$8

- 3 times the number of people plus 8
- 8 + 3 · people

Note: We have the entire rest of the unit to systematically develop relationships like these. There is no need to formalize or generalize anything yet!

1.3

Activity
Optional

10 mins

Making Toast

Standards Alignment

Building Toward



7.EE.B

Instructional Routines

MLR7: Compare and Connect

Materials

None

Activity Narrative

In this activity, students are presented with a different relationship that is not proportional and also doesn't fit a pattern that can be characterized by an equation in the form y = px + q (like the previous activity could be). This optional activity is a good opportunity for students to interpret another context and describe a relationship, but it can be safely skipped if the previous activity takes too much time.

Students must make sense of the problem and persevere in problem solving (MP1) in this activity because there are many viable ways to represent the relationship and solve the problems, none of which are demonstrated first.

Launch

Keep students in the same groups. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.



Access for Students with Disabilities

Representation: Internalize Comprehension. Provide a blank two-column table for students to process and organize information. Invite students to share their column labels (for example, number of slices and number of seconds) and how they organized the given information.

Supports accessibility for: Organization, Attention



Student Task Statement

A toaster has 4 slots for bread. Once the toaster is warmed up, it takes 35 seconds to make 4 slices of toast, 70 seconds to make 8 slices, and 105 seconds to make 12 slices.

- 1. How long do you think it will take to make 20 slices?
- 2. If someone makes as many slices of toast as possible in 4 minutes and 40 seconds, how many slices do you think they can make?

Student Response



Sign in to access Student Response.



Are You Ready for More?



Activity Synthesis

Invite students to share their responses and their reasoning. Select as many unique approaches as time allows.



Access for English Learners

MLR7 Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

- "How are the tables or diagrams the same? How are they different?"
- "How does the quantity 20 slices show up in each representation?"
- "Are there any benefits or drawbacks to one representation compared to another?"

Advances: Representing, Conversing

Lesson Synthesis

The goal of this lesson is to recognize that there are situations in the world that are more complicated than what we have studied until this point, and to let students know this unit is about developing tools to solve some more sophisticated problems. Questions for discussion:

- "Describe some rules we encountered in this lesson for how one quantity was related to another quantity."
- "What made these situations more complicated than relationships we have seen in the past?"
- "What were some tools or strategies we used that were particularly helpful?"



Sign in to access Cool-down.

Student Lesson Summary

In much of our previous work that involved relationships between two quantities, we were often able to describe amounts as being so much more than another, or so many times as much as another. We wrote equations like x + 3 = 8 and 4x = 20 and solved for unknown amounts.

In this unit, we will see situations where relationships between amounts involve more operations. For example, a pizza store might charge the amounts shown in the table for delivering pizzas.

number of pizzas	total cost in dollars	
1	13	
2	23	

number of pizzas	total cost in dollars	
3	33	
5	53	

We can see that each additional pizza adds 10tothetotalcost, and that each totalincludesa 3 additional cost, maybe representing a delivery fee. In this situation, 8 pizzas will cost $8 \cdot 10 + 3$ and a total cost of \$63 means 6 pies were ordered.

In this unit, we will see many situations like this one, and will learn how to use diagrams and equations to answer questions about unknown amounts in the situation.

Lesson 1



- 1.1 Warm-up
- 1.2 Activity
- 1.3 Activity (Optional)

Lesson Synthesis

Cool-down

Student Lesson Summary



🛂 Student Lesson 🔀



🚱 Student Lesson in Spanish 🔼



Sign in to access protected content and invite other educators

> Sign in using your existing Kendall Hunt account. If you don't have one, create an educator account.

IM K-5 Math	Professional Learning	IM Blog
IM 6-8 Math	Resources	Facebook
IM 9-12 Math		X

Illustrative Mathematics® has operated as an independent 501(c)3 non-profit organization since 2013.

IM 6–8 Math was originally developed by Open Up Resources and authored by Illustrative Mathematics®, and is copyright 2017-2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum.

Adaptations and updates to IM 6–8 Math are copyright 2024 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution-Noncommercial 4.0 International License (CC BY 4.0).

Adaptations to add additional English language learner supports are copyright 2019 by Open Up Resources, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

The second set of English assessments (marked as set "B") are copyright 2019 by **Open Up Resources**, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Spanish translation of the "B" assessments are copyright 2020 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

The Illustrative Mathematics name and logo are not subject to the Creative Commons license and may not be used without the prior and express written consent of Illustrative Mathematics.

This site includes public domain images or openly licensed images that are copyrighted by their respective owners. Openly licensed images remain under the terms of their respective licenses. See the image attribution section for more information.