Worksheet # 22 Solution

(From Lecture #22 given on 4/10/2019)

$$
\begin{aligned}
S &\rightarrow \text{if } C \text{ then } S \text{ else } S && (1)\\
&\mid \text{if } C \text{ then } S && (2)\\
&\mid X && (3)\\
C &\rightarrow c_1 && (4)\\
&\mid c_2 && (5)\\
X &\rightarrow x_1 && (6)\\
&\mid x_2 && (7)
\end{aligned}
$$

a. Give two distinct right-most derivations for the string **`if c1 then if c2 then x1 else x2'.**

b. Can you also find two distinct left-most derivations for the above string? If so, provide both derivations. If not, explain why not.

$$
\begin{aligned}
S &\to \text{if } C \text{ then } S \text{ else } S && (1)\\
&\mid \text{if } C \text{ then } S && (2)\\
&\mid X && (3)\\
C &\to c_1 && (4)\\
&\mid c_2 && (5)\\
X &\to x_1 && (6)\\
&\mid x_2 && (7)
\end{aligned}
$$

- Two distinct **right-most derivations** for the string **`if c1 then if c2 then x1 else x2'.**

$$
\begin{aligned}
\underline{S} &\overset{1}{\to} \text{if } C \text{ then } S \text{ else } \underline{S}\\
&\overset{3}{\to} \text{if } C \text{ then } S \text{ else } \underline{X}\\
&\overset{7}{\to} \text{if } C \text{ then } \underline{S} \text{ else } x_2\\
&\overset{2}{\to} \text{if } C \text{ then if } C \text{ then } \underline{S} \text{ else } x_2\\
&\overset{3}{\to} \text{if } C \text{ then if } C \text{ then } \underline{X} \text{ else } x_2\\
&\overset{6}{\to} \text{if } C \text{ then if } \underline{C} \text{ then } x_1 \text{ else } x_2\\
&\overset{5}{\to} \text{if } \underline{C} \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2\\
&\overset{4}{\to} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2
\end{aligned}
$$

$$
\begin{aligned}
\underline{S} &\overset{2}{\to} \text{if } C \text{ then } \underline{S}\\
&\overset{1}{\to} \text{if } C \text{ then if } C \text{ then } S \text{ else } \underline{S}\\
&\overset{3}{\to} \text{if } C \text{ then if } C \text{ then } S \text{ else } \underline{X}\\
&\overset{7}{\to} \text{if } C \text{ then if } C \text{ then } \underline{S} \text{ else } x_2\\
&\overset{3}{\to} \text{if } C \text{ then if } C \text{ then } \underline{X} \text{ else } x_2\\
&\overset{6}{\to} \text{if } C \text{ then if } \underline{C} \text{ then } x_1 \text{ else } x_2\\
&\overset{5}{\to} \text{if } \underline{C} \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2\\
&\overset{4}{\to} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2
\end{aligned}
$$

- Two distinct **left-most derivation**s for the same string.

$$
\begin{aligned}
S &\to \text{if } C \text{ then } S \text{ else } S && (1)\\
&\mid \text{if } C \text{ then } S && (2)\\
&\mid X && (3)\\
C &\to c_1 && (4)\\
&\mid c_2 && (5)\\
X &\to x_1 && (6)\\
&\mid x_2 && (7)
\end{aligned}
$$

$$
\begin{aligned}
\underline{S} &\xrightarrow{1} \text{if } \underline{C} \text{ then } S \text{ else } S\\
&\xrightarrow{4} \text{if } c_1 \text{ then } \underline{S} \text{ else } S\\
&\xrightarrow{2} \text{if } c_1 \text{ then if } \underline{C} \text{ then } S \text{ else } S\\
&\xrightarrow{5} \text{if } c_1 \text{ then if } c_2 \text{ then } \underline{S} \text{ else } S\\
&\xrightarrow{3} \text{if } c_1 \text{ then if } c_2 \text{ then } \underline{X} \text{ else } S\\
&\xrightarrow{6} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } \underline{S}\\
&\xrightarrow{3} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } \underline{X}\\
&\xrightarrow{7} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2
\end{aligned}
$$

$$
\begin{aligned}
\underline{S} &\xrightarrow{2} \text{if } \underline{C} \text{ then } S\\
&\xrightarrow{4} \text{if } c_1 \text{ then } \underline{S}\\
&\xrightarrow{1} \text{if } c_1 \text{ then if } \underline{C} \text{ then } S \text{ else } S\\
&\xrightarrow{5} \text{if } c_1 \text{ then if } c_2 \text{ then } \underline{S} \text{ else } S\\
&\xrightarrow{3} \text{if } c_1 \text{ then if } c_2 \text{ then } \underline{X} \text{ else } S\\
&\xrightarrow{6} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } \underline{S}\\
&\xrightarrow{3} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } \underline{X}\\
&\xrightarrow{7} \text{if } c_1 \text{ then if } c_2 \text{ then } x_1 \text{ else } x_2
\end{aligned}
$$