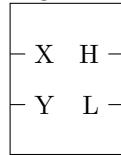


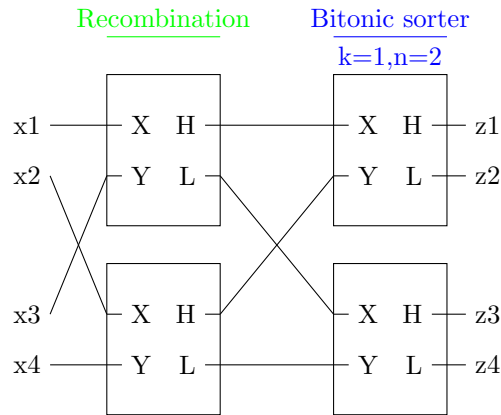
# 1 Bitonic Sorter

With a bitonic sorter, only bitonic sequences can be used as inputs. To sort arbitrary input sequences, a bitonic sorting network can be created from bitonic sorters. For bitonic sorting networks, see the next chapter.

A bitonic sorter for  $k=1$  ( $n=2$ ) is trivially just a comparator. Which we illustrate by using the given box layout with X and Y being the inputs and H the higher output and L the lower output, respectively.

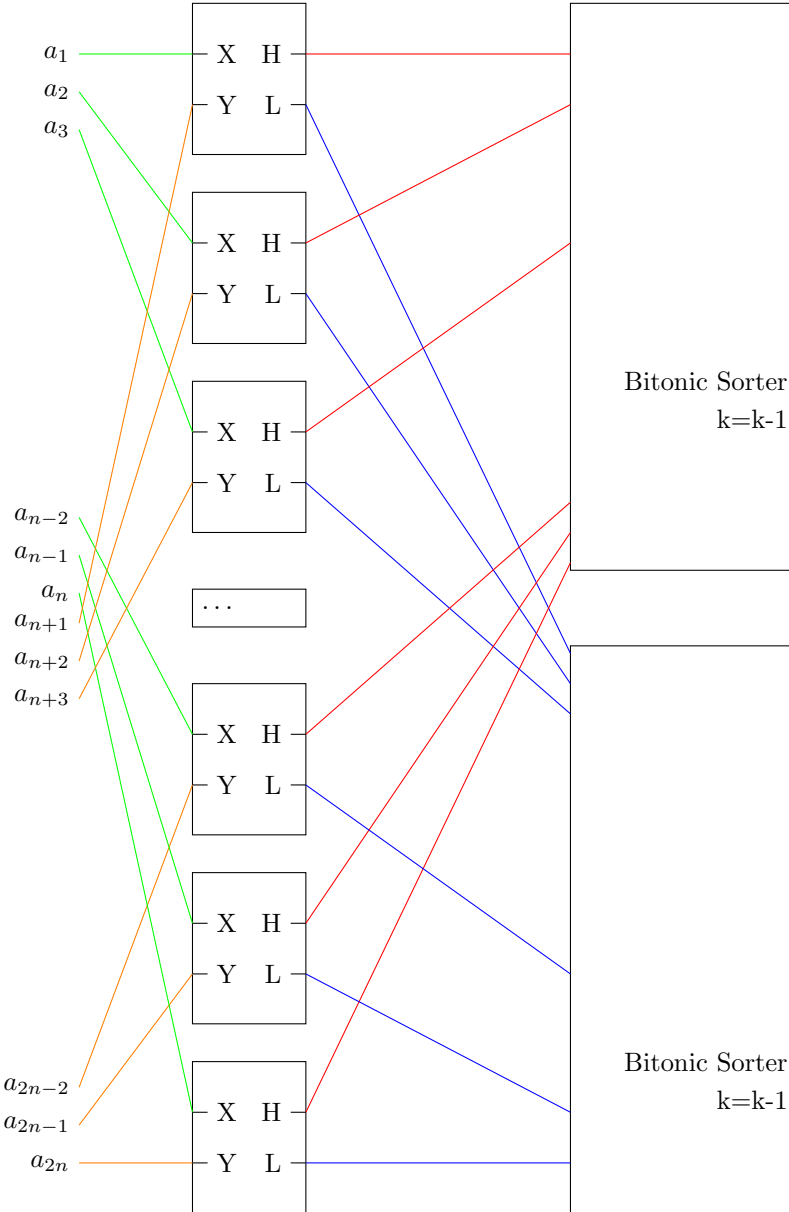


With the sorter for  $k=1$ , we can recursively create a sorter of higher order. For example, consider the following  $k=2$  ( $n=4$ ) bitonic sorter.



The bitonic sorter consists of two layers: a recombination layer and bitonic sorters of a lower order. The recombination layer recombines the bitonic inputs into two new bitonic sequences that are half as big as the original sequence. In the bitonic sort layer, there are two bitonic sorters of order  $k-1$  that take the bitonic input of the recombination and sort them again.

This can be done recursively for each  $k$  using an input of each monotonic sequence per comparator. The following illustration shows the connection of the recombination in order for this to work.



As you can see, the first half of each sequence is compared against the last half of each sequence. Both subsequences are already monotonically decreasing or increasing, respectively. Comparing each element of the (green) subsequence with the element of the other (orange) subsequence at the respective index produces two bitonic subsequences. These two bitonic series (blue and red, respectively) can then be fed into the next lower-order bitonic sorter. This can be done because all elements in the red sequence are guaranteed to be higher

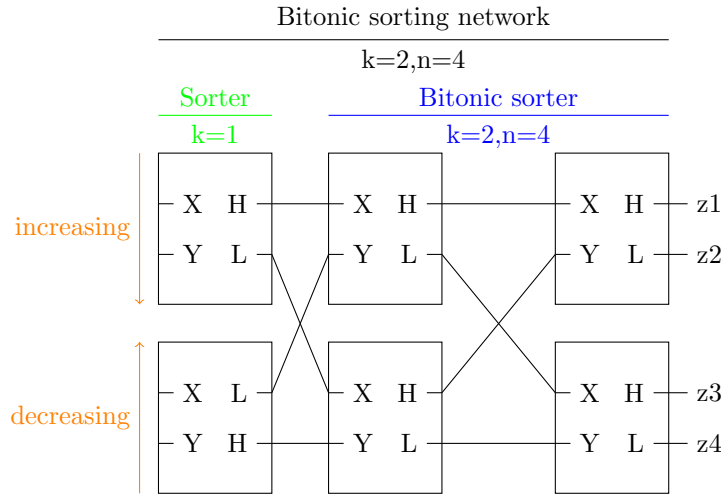
than all elements in the blue series.

### 1.1 Proof of the bitonic sorter

Ken Batcher provided some mathematical proof sketch in his paper.

## 2 Sorting network

A bitonic sorting network is created by using several bitonic sorters. These bitonic sorters are recursively used to create two monotonic sequences, one decreasing and one increasing, which are then put into the next stage. This creates a bitonic series for the next stage, which can then use this bitonic series as a monotonic series for the next stage. Consider the following example for an  $n=4$  bitonic sort network.



The bitonic sorting network for  $k=2$  can be created by using a  $k=2$  bitonic sorter and two  $k=k-1$  sorters. The two sorters create a decreasingly or increasingly sorted sequence in order to create a bitonic input for the bitonic sorter. Bitonic sorting networks of a lower order are mostly used for the two pre-sorters; therefore, a recursive definition of a bitonic sorting network from bitonic sorters can be described. In the above example, the two bitonic sorting networks are  $k=1$  networks; hence, they are just a comparator. The following figure shows the overall scheme.

