Scientific Computing (MATH6183001)

Problem Set 4 - Regular Expressions and Non-Regular Languages

July 23, 2024

Solve totally 3 problems and at least one subproblem from each of the sections.

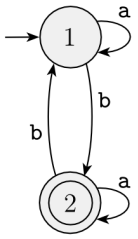
1 Regular Expressions

Problem 1. For each of the following languages, give two strings that are members and two strings that are not members — a total of four strings for each part. Assume the alphabet $\Sigma = \{a,b\}$ in all parts.
a. $a(ba)^*b$
b. $\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*$
c. $(a \cup ba \cup bb)\Sigma^*$

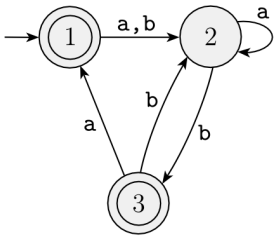
Problem 2. Convert the following regular expressions to nondeterministic finite automata.
a. $(0 \cup 1)^*000(0 \cup 1)^*$
b. $(((00)^*(11)) \cup 01)^*$
c. $(0 \cup \epsilon)1^*$

Problem 3. Give regular expressions generating the following languages.
a. {w | the length of w is at most 5}.
b. {w | w contains at least three 1s}.
c. {w | w contains at least two 0s and at most one 1}.

Problem 4. Convert the following finite automata to regular expressions.



(a)



(b)

2 Non-Regular Languages and Pumping Lemma

Problem 5. Use the pumping lemma to show that the following languages are not regular.
a. $A = \{\text{www} \mid \text{w} \in \{a, b\}^*\}$
b. $\{0^n 1^m 0^n \mid m, n \geq 0\}$
c. $\{wtw | w, t \in \{0, 1\}^+\}$

Problem 6. For any string $w = w_1 w_2 ... w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n ... w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.

Problem 7. Let $\Sigma = \{1, 0\}$ and let $Y = \{w | w = x_1 0 x_2 0 ... 0 x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$. Prove that Y is not regular.