Scientific Computing (MATH6183001)

Problem Set 5 - Context-Free Grammars and Context-Free Languages

July 23, 2024

Solve totally 3 problems and at least one subproblem from each of the sections.

1 Understanding CFG

Problem 1. Show the derivation and the parse tree for the string 011100 generated by the following CFG:
$S \to 0S1S|1S0S|\epsilon$

Problem 2. Show the derivation and the parse tree for the string 01011010 generated by the following CFG:
$S \to 0X|1Y|\epsilon$
$X \to 0S|1Z$
$Y \to 1S|0Z$
$Z \to 0Y|1X$

Problem 3. Describe the language that is generated by the following CFG. Give 5 examples from this language.
$S \to aSa$
$S \to bSb$
$S \to c$

2 Designing CFG

Problem 4. Convert the following DFA to a corresponding CFG.

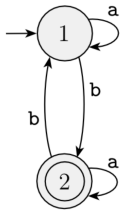

Figure 1: (a)

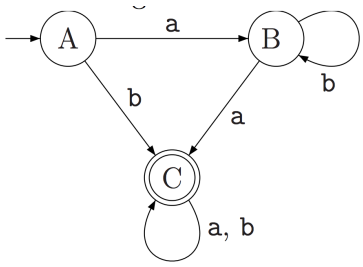

Figure 2: (b)

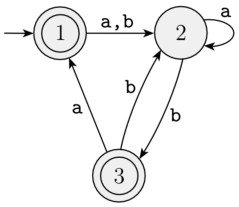

Figure 3: (c)

Problem 5. Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is {0,1}.
a. $\{w \mid w$ starts and ends with the same symbol$\}$
b. $\{w \mid$ the length of w is odd $\}$
c. $\{w \mid w = w^R$, that is, w is a palindrome $\}$
d. The empty set
e. The complement of the language $\{\ 0^n1^n \mid \text{n} \geq 0\}$

Problem 6. Give context-free grammars that generates the language:
A = $\{a^ib^jc^k \mid i = j$ or $j = k$ where $i, j, k \geq 0\}$.

3 Chomsky normal form

Problem 7. Convert the following CFG into an equivalent CFG in Chomsky normal form.
$A \to BAB|B|\epsilon$
$B \to 00|\epsilon$

Problem 8. Convert the following CFG into an equivalent CFG in Chomsky normal form.
$S \to ASB$
$A \to aAS|a|\epsilon$
$B \to SbS|A|bb$

Problem 9. Convert the following CFG into an equivalent CFG in Chomsky normal form.
$S \to a|aA|B$
$A \to aBB|\epsilon$
$B \to Aa|b$