Scientific Computing (MATH6183001)

Problem Set 7 - Pumping Lemma for Non-CFL

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Problem 1. Use the pumping lemma to show that the following languages are not context free.
a. $\{a^n b^n c^n \mid n \geq 0\}$
b. $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$
c. $\{ww \mid w \in \{0,1\}^*\}$
d. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Problem 2. Let B be the language of all palindromes over {0,1} containing equal numbers of 0s and 1s. Show that B is not context free.

Problem 3. Let $\Sigma = \{1,2,3,4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s$\}$. Show that C is not context free.

Problem 4. Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:
$S \to TT|U$
$T \to 0T|T0|\#$
$U \to 0U00|\#$

Consider the language $B = L(G)$. The pumping lemma for context-free languages states the existence of a pumping length p for B. What is the minimum value of p that works in the pumping lemma? Justify your answer.

Problem 5*. Show that $F = \{a^i b^j \mid i = kj$ for some positive integer $k\}$ is not context free.