Scientific Computing (MATH6183001)

Problem Set 2 - Deterministic Finite Automata

July 11, 2024

1 Formal definition of a finite automaton

Problem 1. The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, where δ is given by the Table 1. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

Table 1: DFA

Problem 2. Give the formal description of the machines M_1 and M_2 pictured in Figure 1 and 2.
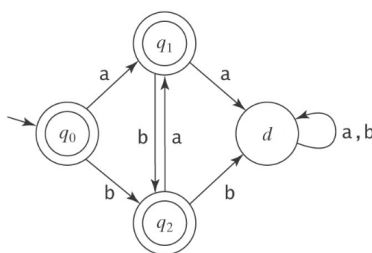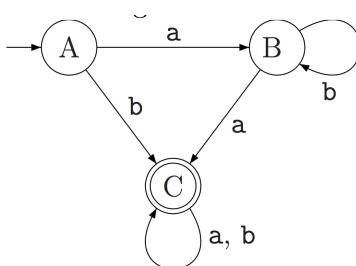

Figure 1: DFA M_1

Figure 2: DFA M_2

Problem 3. Give the 3 examples of acceptable and 3 examples of not acceptable strings for the following DFAs:
1) DFA in the Problem 1.
2) DFA M_1 in the Problem 2.
3) DFA M_2 in the Problem 2.

2 Designing finite automata

Problem 4. Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.
a. {w | w has at least three a's and at least two b's}
b. {w | w has an even number of a's and one or two b's}
c. {w | w starts with an a and has at most one b}
d. {w | w has an odd number of a's and ends with a b}
e. {w | w has even length and an odd number of a's}

Problem 5. Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.
a. {w | w contains neither the substrings ab nor ba}
b. {w | w is any string not in a^*b^*}
c. {w | w is any string not in $(ab+)^*$}
d. {w | w is any string not in $a^* \cup b^*$}
e. {w | w is any string that doesn't contain exactly two a's}
f. {w | w is any string except a and b}

Problem 6. Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0,1\}$.
a. {w | w begins with a 1 and ends with a 0}
b. {w | w contains at least three 1s}
c. {w | w contains the substring 0101 (i.e., w = x0101y for some x and y)}
d. {w | w has length at least 3 and its third symbol is a 0}
e. {w | w starts with 0 and has odd length, or starts with 1 and has even length}
f. {w | w doesn't contain the substring 110}
g. {w | the length of w is at most 5}
h. {w | w is any string except 11 and 111}
i. {w | every odd position of w is a 1}
j. {w | w contains at least two 0s and at most one 1}
k. $\{\epsilon, 0\}$
l. {w | w contains an even number of 0s, or contains exactly two 1s}
m. The empty set
n. All strings except the empty string