Scientific Computing (MATH6183001)

Problem Set 9 - Decidability

August 6, 2024

Problem 1. Show that the following languages are decidable.
a. $\{a^n b^n c^n \mid n \geq 0\}$
b. $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$
c. $\{ww \mid w \in \{0,1\}^*\}$
d. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Problem 2. Let $S = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$. Show that S is decidable.

Problem 3. Let $ALL_{DFA} = \{\langle A \rangle \mid \text{A is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

Problem 4. Prove that the language $A_\epsilon = \{\langle G \rangle \mid \text{G is a CFG that generates } \epsilon\}$.

Problem 5. Let L_1 and L_2 be decidable languages. Prove that $L_1 \cup L_2$ is also decidable.

Problem 6. Show that the class of Turing-decidable languages is closed under: (a) union, (b) intersection, (c) concatenation, and (d) star.

Problem 7. Let $A = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.

Problem 8. Let $A = \{\langle R \rangle \mid R \text{ is a regular expression describing a language containing at least one string } w \text{ that has 111 as a substring (i.e., } w = x111y \text{ for some } x \text{ and } y)\}$. Show that A is decidable.