HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

ĐẠI HỌC
BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY
OF SCIENCE AND TECHNOLOGY

PLANNING OPTIMIZATION

MODELLING

ONE LOVE. ONE FUTURE.

CONTENT

- Modelling
- Constraint linearization
- Examples

MODELLING

- Modelling consists of specifying
  - Decision variables
  - Constraints
  - Objective functions
- A problem can be modelled in different ways (how to define variables)
- Take into account the modelling languages of software tools
  - Constraint Programming solvers: constraints can be stated in flexible ways

CONSTRAINT LINEARIZATION

- Motivation
  - Linear Programming solvers are very efficient
- Examples
  - How to model $X = \min\{x_1, x_2\}$?

CONSTRAINT LINEARIZATION

- Motivation
  - Linear Programming solvers are very efficient
- Examples
  - How to model $X = \min\{x_1, x_2\}$?

Solution: define an auxiliary binary variable y, use big constant M

- $x_1 \geq X$
- $x_2 \geq X$
- $X \geq x_1 - M(1-y)$
- $X \geq x_2 - My$

CONSTRAINT LINEARIZATION

- Examples
  - How to model ($x = 1$) $\Rightarrow$ ($z \geq y$) where x is binary variable, y and z are real variables ?

CONSTRAINT LINEARIZATION

- Examples
  - How to model (x = 1) $\Rightarrow$ ($z \geq y$) where x is binary variable, y and z are real variables ?

Solution: use big constant M

- $M(x\text{-}1) + y \leq z$

CONSTRAINT LINEARIZATION

- Example
  - ($x > 0$) $\Rightarrow$ ($z \geq y$) in which x, y and z are real variables (and x >=0) ?

CONSTRAINT LINEARIZATION

- Example
  - $(x > 0) \Rightarrow (z \geq y)$ in which x, y and z are real variables (and $x \geq 0$) ?

Solution:

- Let M be a very big constant,
- Introduce a binary variable $t \in \{0,1\}$:
  - $t = 1$ indicates that $x > 0$, and $t = 0$ indicates that $x = 0$
- Equivalent linear constraints
  - $x \leq M.t$
  - $z + (1-t)M \geq y$

EXAMPLES

- **Balanced Course Assignment Problem**
  - At the beginning of the semester, the head of a computer science department *D* have to assign courses to teachers in a balanced way. The department *D* has *m* teachers T={0, 2, ..., *m-1*} and *n* courses C={0, 2,..., *n-1*}.
    - Each teacher $t \in T$ has a preference list which is a list of courses he/she can teach depending on his/her specialization. The preference information is represented by a 0-1 matrix $A_{m \times n}$ in which $A(t,c) = 1$ indicates that teacher t can teach the course c and $A(t,c) = 0$, otherwise
    - We know a set *B* of pairs of conflicting two courses that cannot be assigned to the same teacher as these courses have been already scheduled in the same slot of the timetable.
    - The load of a teacher is the number of courses assigned to her/him. How to assign *n* courses to *m* teacher such that each course assigned to a teacher is in his/her preference list, no two conflicting courses are assigned to the same teacher, and the maximal load among teachers is minimal.

EXAMPLES

- **Balanced Course Assignment Problem**

Course	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
credits	3	3	4	3	4	3	3	3	4	3	3	4	4

Teachers	**Preference Courses**
0	0, 2, 3, 4, 8, 10
1	0, 1, 3, 5, 6, 7, 8
2	1, 2, 3, 7, 9, 11, 12

Conflict courses

0	2
0	4
0	8
1	4
1	10
3	7
3	9
5	11
5	12
6	8
6	12

EXAMPLES

- **Balanced Course Assignment Problem**

Course	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
credits	3	3	4	3	4	3	3	3	4	3	3	4	4

Teachers	**Preference Courses**
0	0, 2, 3, 4, 8, 10
1	0, 1, 3, 5, 6, 7, 8
2	1, 2, 3, 7, 9, 11, 12

Conflict courses

0	2
0	4
0	8
1	4
1	10
3	7
3	9
5	11
5	12
6	8
6	12

Teacher	**Assigned courses**	**Load**
0	2, 4, 8, 10	15
1	0, 1, 3, 5, 6	15
2	7, 9, 11, 12	14

EXAMPLES

- **Balanced Course Assignment Problem: A Constraint Programming model**
  - Decision variables
    - $X(i)$: teacher assigned to course i, $\forall i \in C$, domain $D(X(i)) = \{t \in T \mid A(t,i) = 1\}$
    - $Y(i)$: load of teacher i, domain $D(Y(i)) = \{0,1,\ldots,n-1\}$
    - Z: maximum load among teachers
  - Constraints
    - $X(i) \neq X(j)$, $\forall (i,j) \in B$
    - $Y(i) = \sum_{j\in C}(X(j) = i)$, $\forall i \in T$
    - $Z \geq Y(i)$, $\forall i \in T$
  - Objective function to be minimized: Z

EXAMPLES

- **Balanced Course Assignment Problem: An Integer Linear Programming model**
  - Decision variables
    - $X(i,j) = 1$: teacher i is assigned to course j, and $X(i,j) = 0$, otherwise, $\forall\ i \in T, j \in C$, domain $D(X(i,j)) = \{0,1\}$
    - $Y(i)$: load of teacher i, domain $D(Y(i)) = \{0,1,\ldots,n\}$
    - Z: maximum load among teachers
  - Constraints
    - $\sum_{i \in T} X(i,j) = 1, , \forall j \in C$
    - $X(t,i) + X(t,j) \leq 1, \forall (i,j) \in B, t \in T$
    - $Y(i) = \sum_{j \in C} X(i,j), \forall i \in T$
    - $Z \geq Y(i), \forall i \in T$
  - Objective function to be minimized: Z

EXAMPLES

- **Travelling Salesman Problem (TSP)**
  - A salesman departs from point 1, visiting N-1 points 2, ..., N and comes back to the point 1. The travelling distance from point i and point j is d(i,j), i,j = 1,..., N. Compute the route of minimal total travelling distance

EXAMPLES

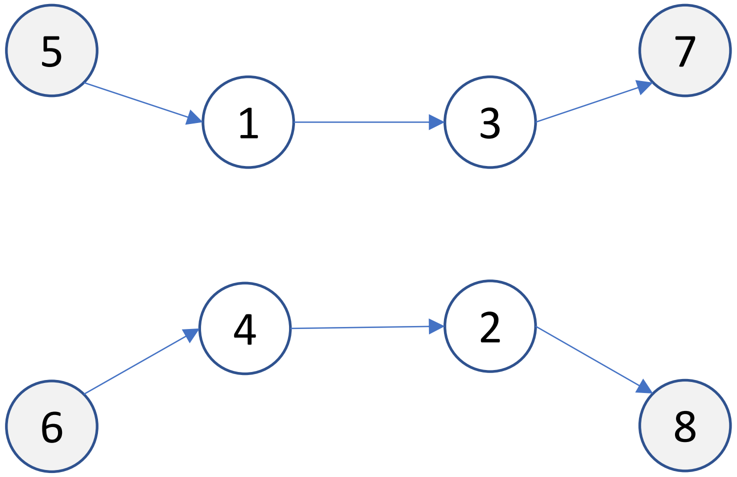
- **Travelling Salesman Problem (TSP): An Integer Linear Programming model**
  - Decision variables
    - Binary variable $X(i,j)$ = 1 if the route traverses from point i to point j, and $X(i,j)$ = 0, otherwise.
  - Constraints
    - $\sum_{j=1}^{N} X(i,j) = \sum_{j=1}^{N} X(j,i) = 1, \forall i \in \{1,2,\dots,N\}$
    - $\sum_{(i,j)\in S} X(i,j) \leq |S| - 1, \forall S \subseteq \{1,2,\dots, N\}$ and $|S| < N$
  - Objective function to be minimized

$$f(X) = \sum_{j=1}^{N} \sum_{i=1}^{N} d(i,j) X(i,j)$$

EXAMPLES

- **Capacitated Vehicle Routing Problem**
  - A fleet of K trucks 1, 2, …, K must be scheduled to visit N customers 1, 2, …, N for collecting items
    - Customer i located at point i and requests to be collected $r(i)$ items, i = 1, 2, …, N
    - Truck k (k = 1,…, K)
      - Departs from point $N+k$ and terminates at point $N + K + k$ ($N+k$ and $N+K+k$ might refer to the central depot)
      - has capacity $c(k)$ which is the maximum number of items it can carry at a time
    - Travel distance from point i to point j is $d(i,j)$, i,j = 1,…, $N + 2K$
  - Compute the delivery solution such that the total travelling distance is minimal
    - Satisfy capacity constraints
    - Each customer is visited exactly once by exactly one truck

- **Capacitated Vehicle Routing Problem**



1	0	2	3	4	3	3	3	3
2	4	0	2	6	1	1	1	1
3	2	4	0	2	1	1	1	1
4	5	7	7	0	4	4	4	4
5	3	1	5	7	0	0	0	0
6	3	1	5	7	0	0	0	0
7	3	1	5	7	0	0	0	0
8	3	1	5	7	0	0	0	0

	1	2
c	10	10

	1	2	3	4
r	4	3	6	5

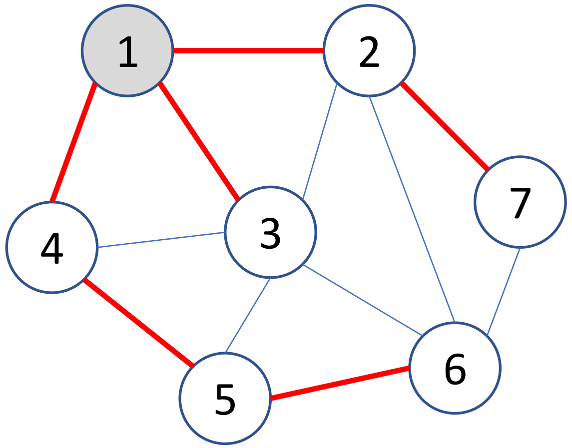
EXAMPLES

- **Capacitated Vehicle Routing Problem**
  - Notations
    - $B = \{1,\ldots, N+2K\}$
    - $F_1 = \{(i, k+N) \mid i \in B, k \in \{1,\ldots,K\}\}$
    - $F_2 = \{(k+K+N, i) \mid i \in B, k \in \{1,\ldots,K\}\}$
    - $F_3 = \{(i, i) \mid i \in B\}$
    - $A = B^2 \setminus F_1 \setminus F_2 \setminus F_3$
    - $A^+(i) = \{ j \mid (i, j) \in \mathrm{A}\}$, $A^-(i) = \{ j \mid (j, i) \in \mathrm{A}\}$
  - Decision variables
    - $X(k,i,j) = 1$ if truck k travel from point i to point j, $\forall k = 1,\ldots,K$, $(i,j) \in A$
    - $Y(k,i)$: number of items on truck k after leaving point i, $\forall k = 1,\ldots,K$, $\forall i = 1,\ldots,N+2K$
    - $Z(i)$: index of truck visiting point i, $\forall\, i = 1,2,\ldots, N+2K$

EXAMPLES

- **Capacitated Vehicle Routing Problem**
  - Constraints
    - $\sum_{k=1}^{K} \sum_{j \in A^{+}(i)} X(k,i,j) = \sum_{k=1}^{K} \sum_{j \in A^{-}(i)} X(k,j,i,) = 1, \forall\ i = 1,\ldots, N$
    - $\sum_{j \in A^{+}(i)} X(k,i,j) = \sum_{j \in A^{-}(i)} X(k,j,i), \forall i = 1,\ldots,N, k = 1,\ldots, K$
    - $\sum_{j=1}^{N} X(k,k+N,j) = \sum_{j=1}^{N} X(k,j,k+K+N) = 1, \forall k = 1,\ldots, K$
    - $M(1\text{-}X(k,i,j)) + Z(i) \geq Z(j), \forall\ (i,j) \in A, \forall k = 1,\ldots,K$
    - $M(1\text{-}X(k,i,j)) + Z(j) \geq Z(i), \forall\ (i,j) \in A, \forall k = 1,\ldots,K$
    - $M(1\text{-}X(k,i,j)) + Y(k,j) \geq Y(k,i) + r(j), \forall\ (i,j) \in A, \forall k = 1,\ldots, K$
    - $M(1\text{-}X(k,i,j)) + Y(k,i) + r(j) \geq Y(k,j), \forall\ (i,j) \in A, \forall k = 1,\ldots, K$
    - $Y(k,k+K+N) \leq c(k), \forall k = 1,\ldots,K$
    - $Y(k,k+N) = 0, \forall k = 1,\ldots,K$
    - $Z(k+N) = Z(k+K+N) = k, \forall k = 1,\ldots, K$
  - Objective function
    - $f(X,Y,Z) = \sum_{k=1}^{K} \sum_{(i,j) \in A} X(k,i,j)d(i,j) \rightarrow \min$

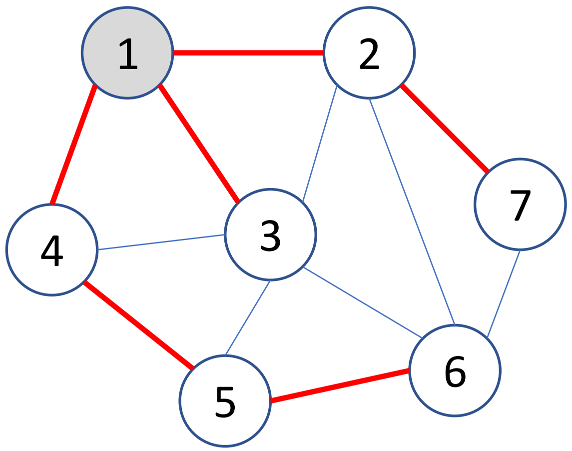
- **MultiCast Routing Problem**
  - Given a network $V = \{1,\ldots, N\}$ is the set of nodes, $E \subseteq V^2$ is the set of links between nodes. A node $s \in V$ is the source node which will transmit a package to others nodes. A node receiving the package can continue transmit this package to adjacent nodes.
    - $t(i,j)$ and $c(i,j)$ are transmission time and transmission cost when transmitting the package from node i to node j
  - Compute the set of links used for broadcasting the package from the source node to all other nodes such that
    - Total transmission time from s to any node cannot exceed a given value L
    - Total transmission cost is minimal



EXAMPLES

- **MultiCast Routing Problem**
  - Denote $A(i) = \{j \in V \mid (i,j) \in E\}$, M is a big constant
  - Decision variables
    - Binary variable $X(i,j) = 1$ if the package is transmitted from node i to node j, and $X(i,j) = 0$, otherwise, $\forall (i,j) \in E$
    - $Y(i)$: time-point when the package arrives at node i, $\forall i \in V$
  - Constraints
    - $\sum_{i \in A(j)} X(i,j) = 1, \forall j \in V \setminus \{s\}$
    - $Y(i) + t(i,j) + M(1-X(i,j)) \geq Y(j), \forall (i,j) \in E$
    - $Y(i) + t(i,j) + M(X(i,j)-1) \leq Y(j), \forall (i,j) \in E$
    - $Y(i) \leq L, \forall j \in V \setminus \{s\}$
    - $Y(s) = 0$
  - Objective function to be minimized

$$f(X) = \sum_{(i,j) \in E} c(i,j) X(i,j)$$



EXAMPLES

- **Facility Location Problem**
  - There are M sites 1, 2, ..., M that can be used to open facility for servicing N customers 1, 2, ..., N.
    - $f(i)$ is the cost for opening the site i
    - $Q(i)$ is the capacity of site i (maximum amount of good it can serve customers)
    - $c(i,j)$ is the cost for transporting unit of good from site i to customer j
    - $d(j)$ is the total demand (amount of goods) of customer j
  - Compute a planning (which site to be opened and amount of good each opened site serves a customer) such that
    - Capacity constraint is satisfied
    - Total cost is minimal

EXAMPLES

- **Facility Location Problem**
  - Decision variables
    - *Y*(*i*) – binary variable, *Y*(*i*) = 1 means that the site *i* is opened, and *Y*(*i*) = 0, otherwise
    - *X*(*i*,*j*) – amount of good site *i* serves customer *j*
  - Constraints
    - $\sum_{i=1}^{M} X(i,j) = d(j), \forall\, j = 1,\ldots, N$
    - $\sum_{j=1}^{N} X(i,j) \le Q(i)Y(i), \forall\, i = 1,\ldots, M$
    - $0 \le X(i,j) \le d(j)Y(i), \forall\, i = 1,\ldots, M, \forall\, j = 1,\ldots, N$
  - Objective functions

$$\sum_{i=1}^{M} f(i)Y(i) + \sum_{i=1}^{M} \sum_{j=1}^{N} c(i,j)X(i,j) \rightarrow \min$$

HUST

hust.edu.vn fb.com/dhbkhn

THANK YOU !