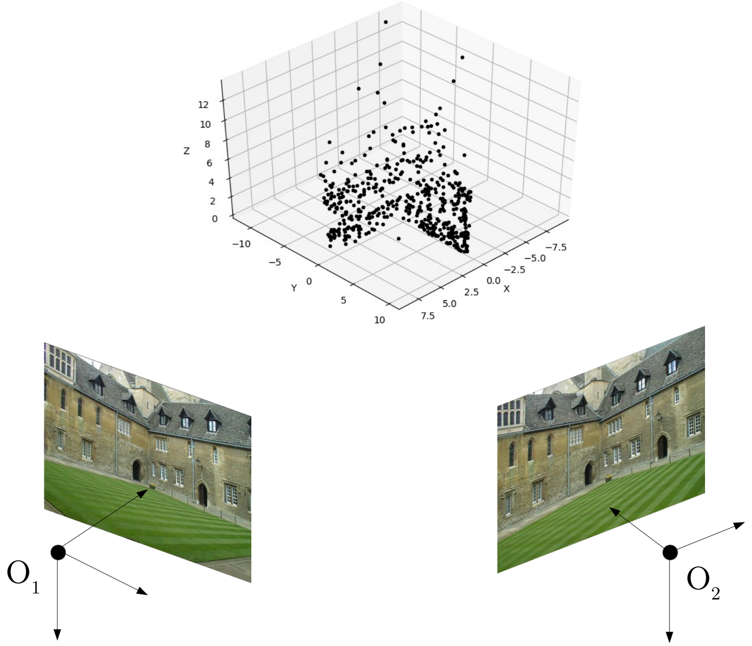
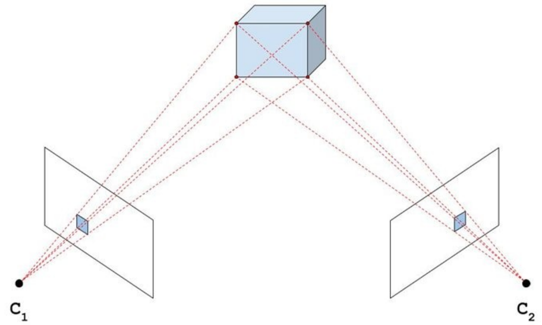
Computer Vision

Class 09



Raquel Frizera Vassallo

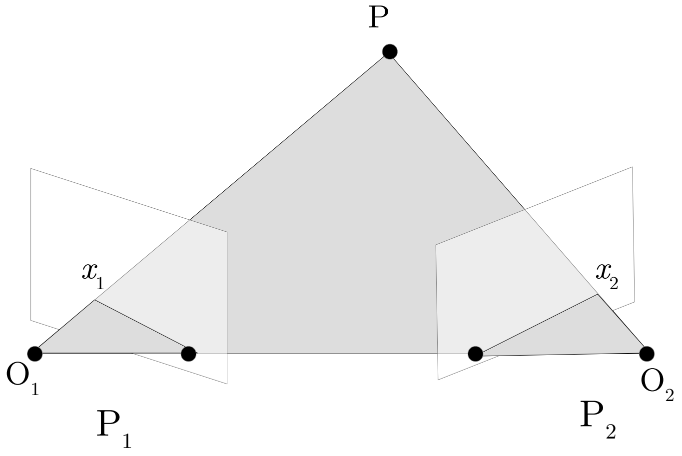
Reconstruction for Calibrated and Partially Calibrated Systems



Summary

- Triangulation for Stereo Calibrated Systems
- Triangulation for Stereo Partially Calibrated Systems

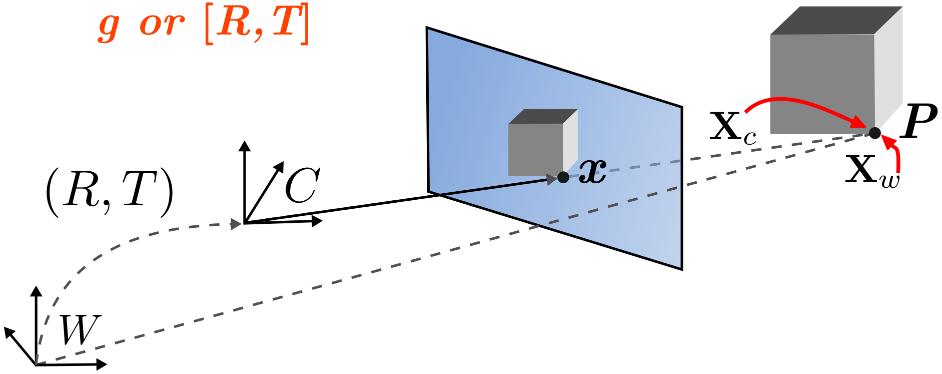
Calibrated System



Quick Review - General Projection Matrix

$$\lambda \underbrace{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}}_{\text{2D Image Point}} = \underbrace{\begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{\text{Intrinsic Parameter Matrix} \\ \boldsymbol{K}}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\substack{\text{Projection Matrix} \\ \boldsymbol{\Pi_0}}} \underbrace{\begin{bmatrix} R_{cw} & T_{cw} \\ 0 & 1 \end{bmatrix}}_{\substack{\text{Extrinsic Parameter Matrix} \\ \boldsymbol{g} \text{ or } [\boldsymbol{R}, \boldsymbol{T}]}} \underbrace{\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}}_{\text{3D World Point}}$$

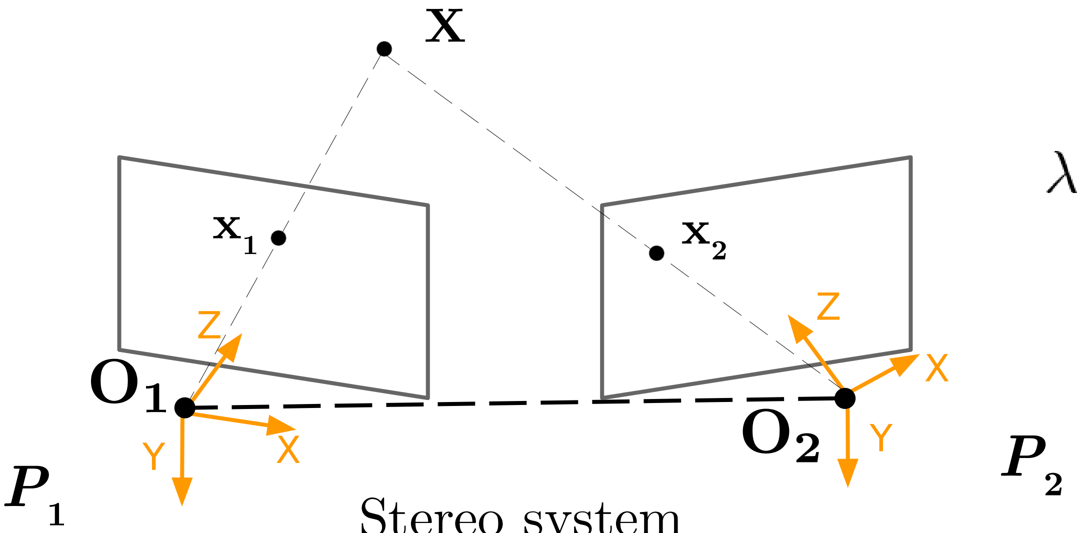
$$\boxed{\lambda x' = K\,\Pi_0\, g\, X_w = \mathrm{P} X_w}$$



Calibrated Cameras

- Given known camera matrices, a set of point correspondences can be triangulated to recover the 3D positions of these points in the world reference frame.

$$\lambda_1 \mathbf{x_1} = P_1 \mathbf{X}$$



$$\lambda_2 \mathbf{x_2} = P_2 \mathbf{X}$$

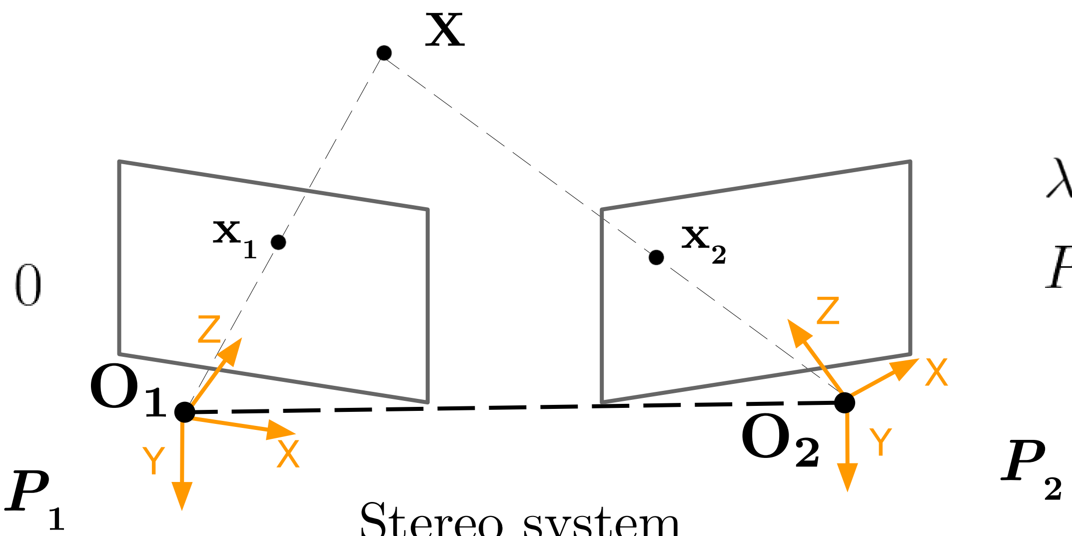
Stereo system

Calibrated Cameras

- Given known camera matrices, a set of point correspondences can be triangulated to recover the 3D positions of these points in the world reference frame.

$$\lambda_1 \mathbf{x_1} = P_1 \mathbf{X}$$

$$P_1 \mathbf{X} - \lambda_1 \mathbf{x_1} = 0$$



Stereo system

$$\lambda_2 \mathbf{x_2} = P_2 \mathbf{X}$$

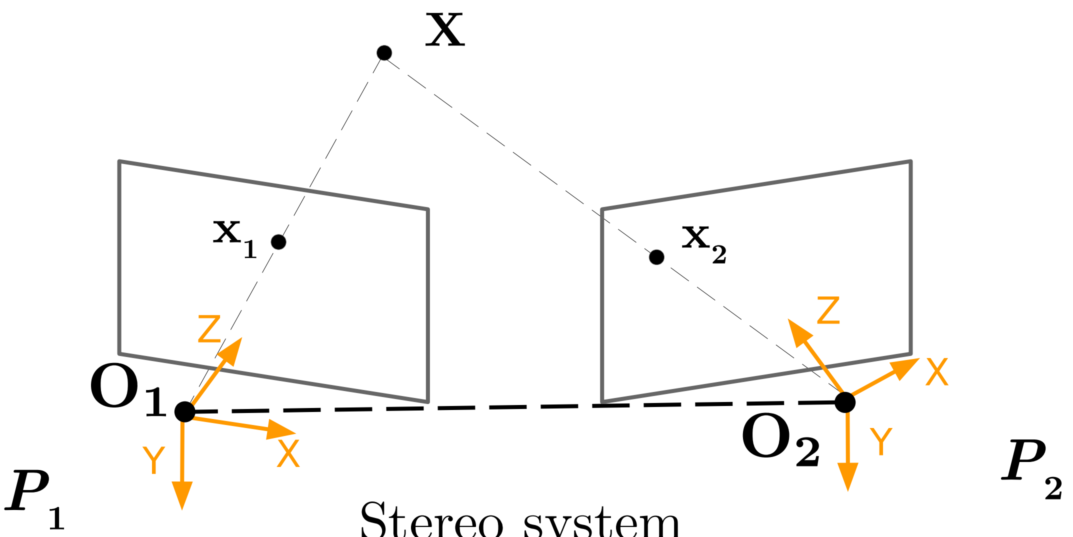
$$P_2 \mathbf{X} - \lambda_2 \mathbf{x_2} = 0$$

Calibrated Cameras

- Given known camera matrices, a set of point correspondences can be triangulated to recover the 3D positions of these points in the world reference frame.

$$P_1\mathbf{X} - \lambda_1\mathbf{x_1} = 0$$
$$P_2\mathbf{X} - \lambda_2\mathbf{x_2} = 0$$

$$\begin{bmatrix} P_1 & -\mathbf{x_1} & 0 \\ P_2 & 0 & -\mathbf{x_2} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = 0$$



Stereo system

Calibrated Cameras

- Given known camera matrices, a set of point correspondences can be triangulated to recover the 3D positions of these points in the world reference frame.

$$P_1\mathbf{X} - \lambda_1\mathbf{x_1} = 0$$
$$P_2\mathbf{X} - \lambda_2\mathbf{x_2} = 0$$

$$\begin{bmatrix} P_1 & -\mathbf{x_1} & 0 \\ P_2 & 0 & -\mathbf{x_2} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} & -x_1 & 0 \\ [P_1]_{3\times4} & -y_1 & 0 \\ & -1 & 0 \\ & 0 & -x_2 \\ [P_2]_{3\times4} & 0 & -y_2 \\ & 0 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Calibrated Cameras

- There might not be an exact solution to these equations due to image noise, errors in the camera matrices, or other sources of errors. Using SVD, we can get a least squares estimate of the 3D point.

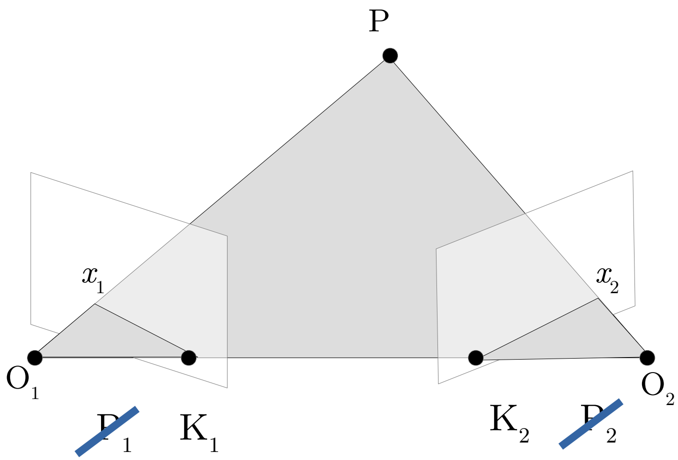
$$\begin{bmatrix} P_1 & -\mathbf{x_1} & 0 \\ P_2 & 0 & -\mathbf{x_2} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = 0$$

$$M \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = 0$$

$$\text{SVD}\ (M) = \text{USV}^{\text{T}}$$

The first four values in the last column of matrix V are the 3D coordinates in homogeneous coordinates.

Partially Calibrated System



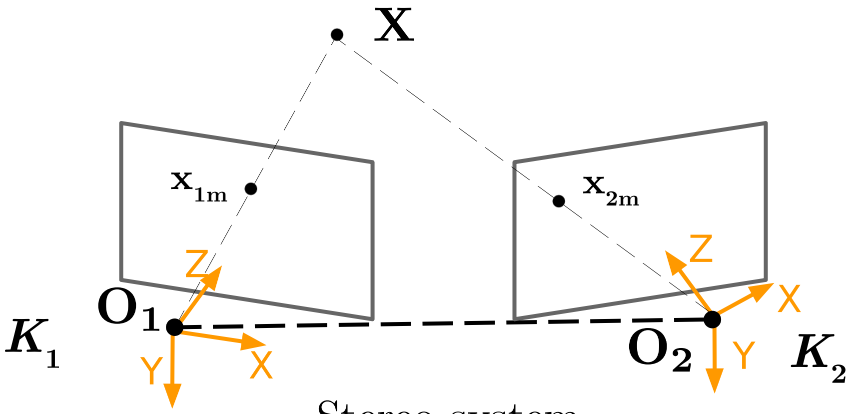
Partially Calibrated Cameras

For the first camera we can:

- Convert pixels to metric points using the intrinsic parameter matrix K_1.
- Consider the camera frame as the world frame → extrinsic parameters matrix has R = I and T = 0.

$$\lambda_1 \mathbf{x_1} = P_1 \mathbf{X} \quad \text{and} \quad \begin{aligned} \overset{\text{pixel}}{\mathbf{x_1}} &= K_1 \overset{\text{metric}}{\mathbf{x_{1m}}} \\ \mathbf{x_{1m}} &= {K_1}^{-1} \mathbf{x_1} \end{aligned}$$

$$P_1' = \Pi_0 \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Stereo system

Partially Calibrated Cameras

For the first camera we can:

- Convert pixels to metric points using the intrinsic parameter matrix K_1.
- Consider the camera frame as the world frame → extrinsic parameters matrix has R = I and T = 0.

pixel

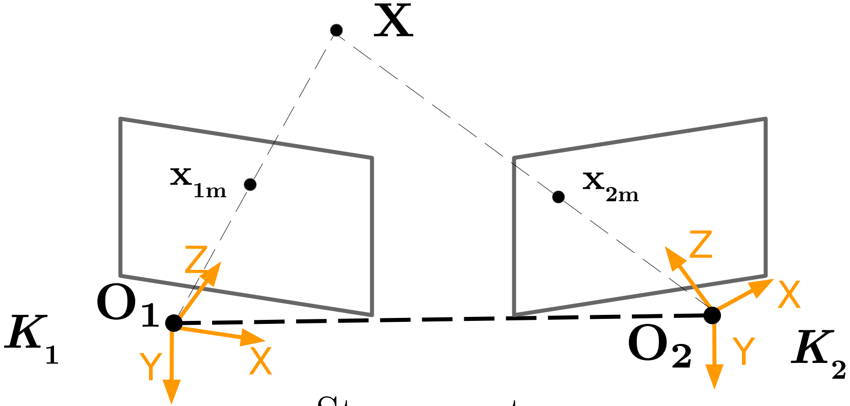
metric

$$\lambda_1 \mathbf{x_1} = P_1 \mathbf{X} \quad \text{and} \quad \begin{aligned} \mathbf{x_1} &= K_1 \mathbf{x_{1m}} \\ \mathbf{x_{1m}} &= {K_1}^{-1} \mathbf{x_1} \end{aligned}$$

$$\lambda_1 \mathbf{x_1} = K_1 \Pi_0 \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \mathbf{X}$$

$$\lambda_1 {K_1}^{-1} \mathbf{x_1} = {K_1}^{-1} K_1 {P_1}' \mathbf{X}$$

$$\lambda_1 \mathbf{x_{1m}} = {P_1}' \mathbf{X} \qquad \text{with} \quad {P_1}' = \Pi_0 \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

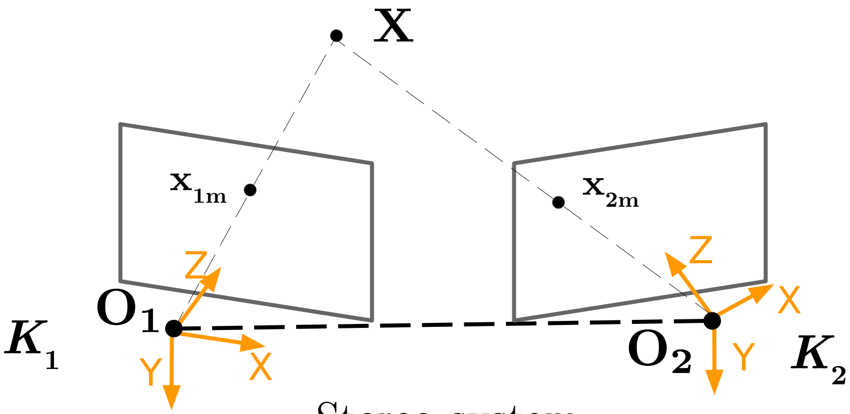


Stereo system

Partially Calibrated Cameras

For the second camera we can:

- Convert pixels to metric points using the intrinsic parameter matrix K_2.
- Estimate the Essential Matrix and obtain R,T.



Stereo system

$$\mathbf{x_{2m}} = {K_2}^{-1}\mathbf{x_2}$$

$$\mathbf{x_{2m}}^T E\ \mathbf{x_{1m}} = 0$$

$R \qquad T$

$$P_2' = \Pi_0 \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \qquad P_2' = \begin{bmatrix} R & T \end{bmatrix}_{3\times 4}$$

Partially Calibrated Cameras

- Recover the 3D positions of the points up to a scale factor in the reference of the first camera.

$$P_1'\mathbf{X} - \lambda_1 \mathbf{x_{1m}} = 0$$
$$P_2'\mathbf{X} - \lambda_2 \mathbf{x_{2m}} = 0$$

$$\begin{bmatrix} P_1' & -\mathbf{x_{1m}} & 0 \\ P_2' & 0 & -\mathbf{x_{2m}} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = 0$$

with

$$\mathbf{x_{1m}} = K_1^{-1}\mathbf{x_1}$$
$$\mathbf{x_{2m}} = K_2^{-1}\mathbf{x_2}$$

and

$$P_1' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P_2' = \begin{bmatrix} R & T \end{bmatrix}_{3\times 4}$$

Credits

- Jan Erik Solem.
  Programming Computer Vision with Python.
  Published by O’Reilly Media, ISBN: 978-1-449-31654-9