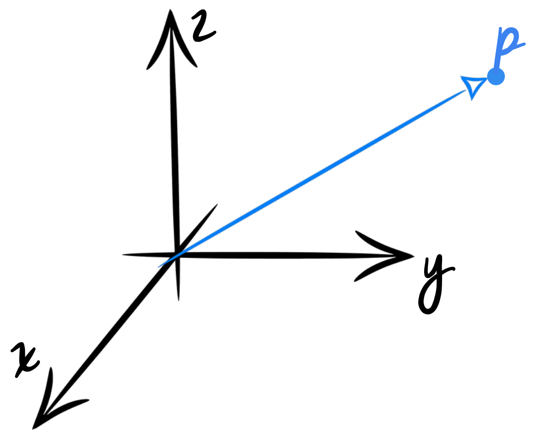
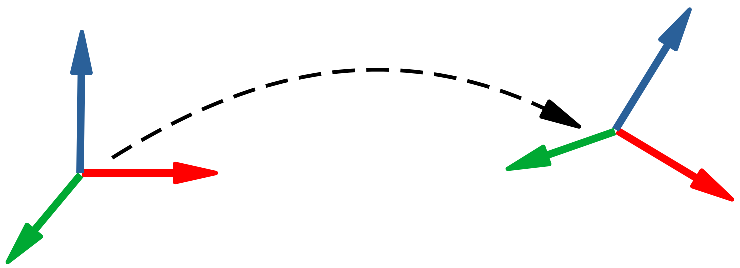
Computer Vision

Class 02 - Complement



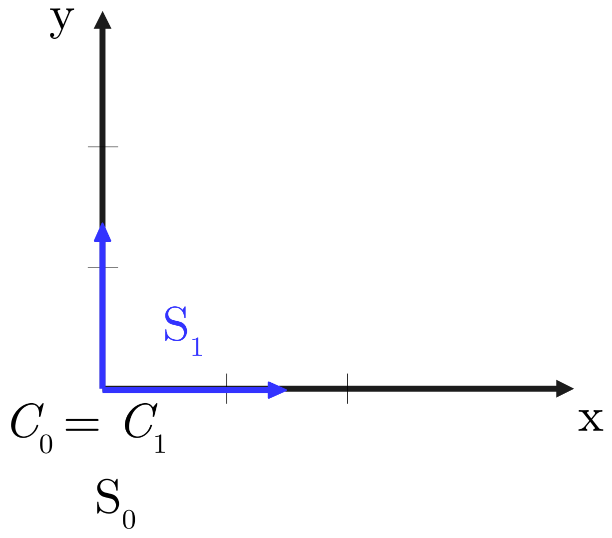
Raquel Frizera Vassallo

Object
Transformation
X
Changing Reference
Frame



Let's use an example to explain the difference

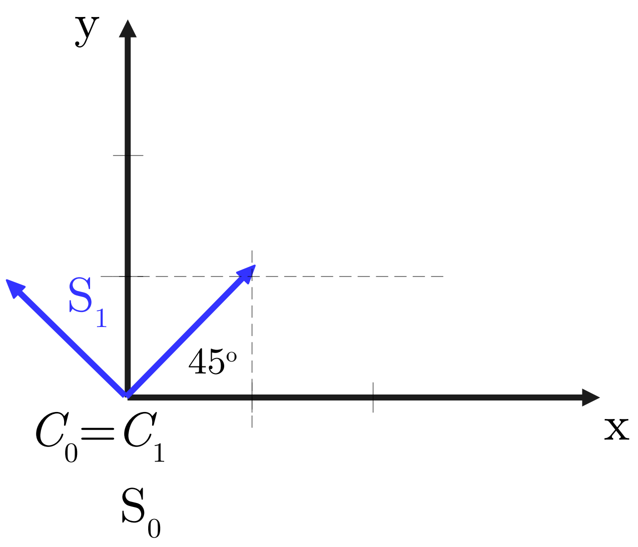
Consider the following reference frames :



Both reference frames are aligned and placed at the same position.

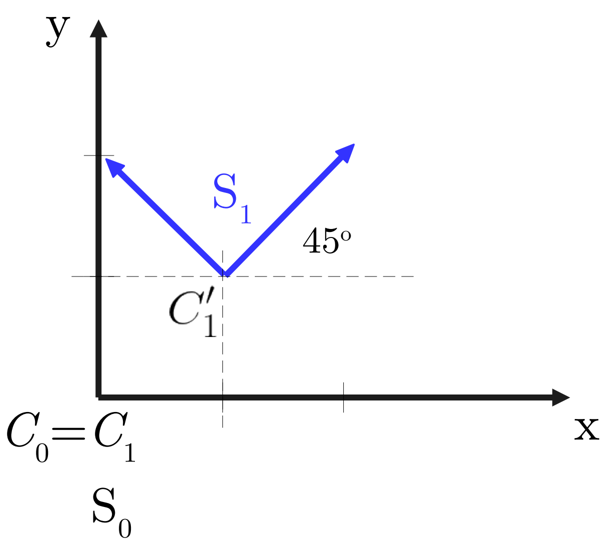
$$C_0 = C_1 = [0,0,1]^T$$

Now, transforming the reference frame S_1 as an object in frame S_0:



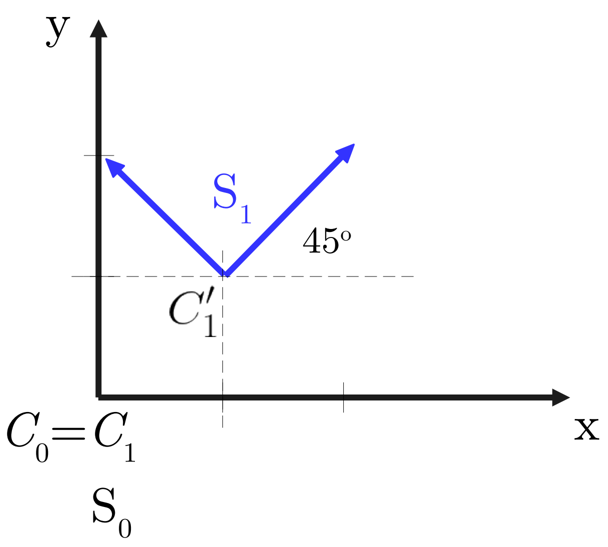
1. A rotation of 45°

Now, transforming the reference frame S_1 as an object in frame S_0:



1. A rotation of 45°
2. Translation of (1,1)

Now, transforming the reference frame S_1 as an object in frame S_0:

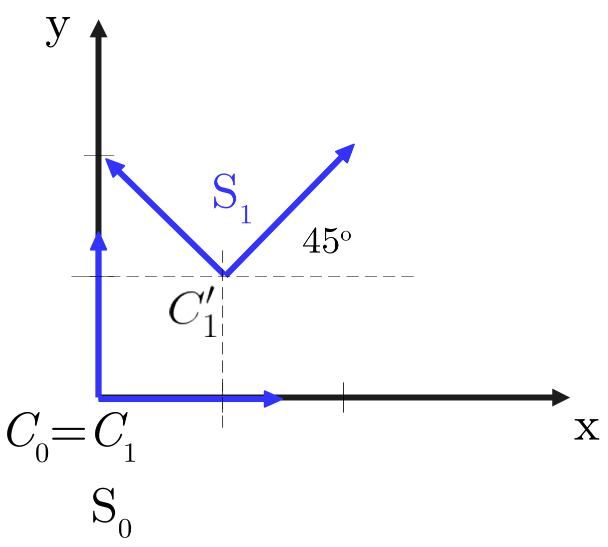


1. A rotation of 45^o
2. Translation of (1,1)

$$C'_1 = T(1,1).R(45^o).C_1$$

$$C_1 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$$
$$C'_1 = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$$

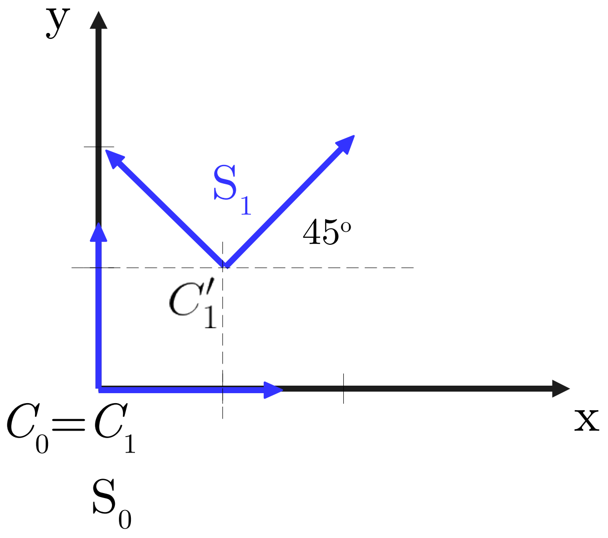
Now, transforming the reference frame S_1 as an object in frame S_0:



$$C_1' = T(1,1).R(45^o).C_1$$

$$C_1' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now, transforming the reference frame S_1 as an object in frame S_0:



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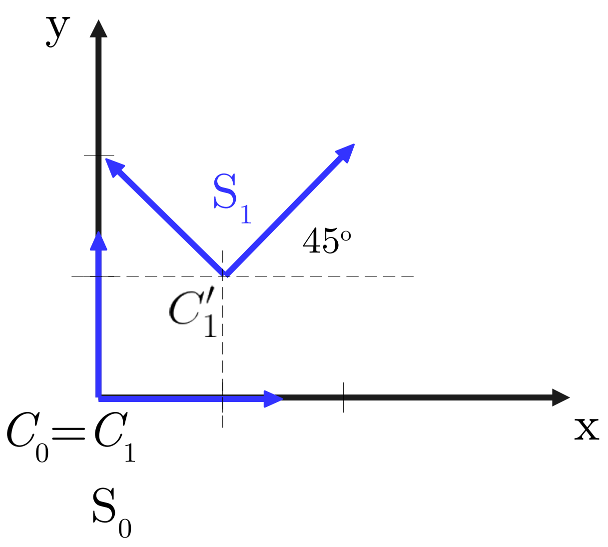

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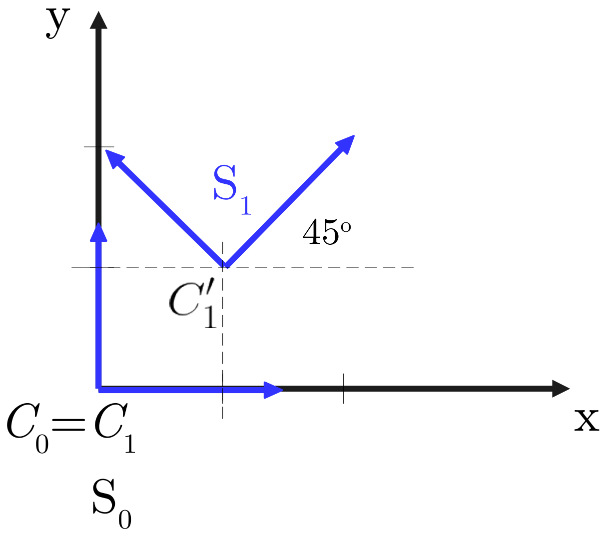
$$C_1' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

Now, transforming the reference frame S_1 as an object in frame S_0:



$$C_1' = T(1,1).R(45^o).C_1$$

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Rotation $R(45^o)$

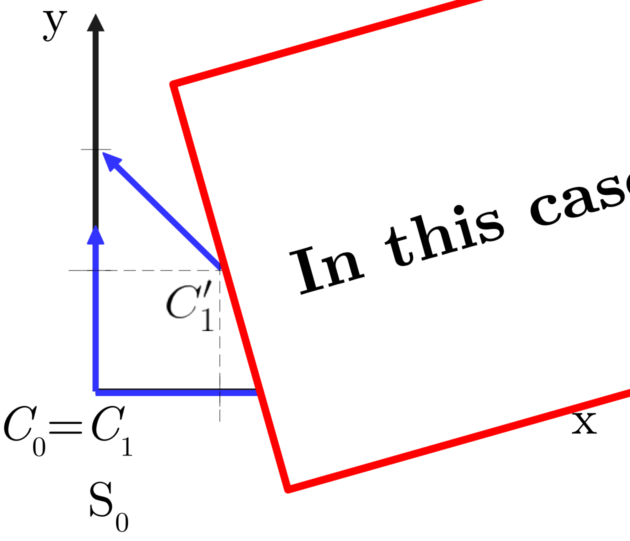
$$C_1' = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Translation $T(1,1)$

$$C_1' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

New point coordinates in S_0.

Now, transforming the reference frame S_1 as an objec



In this case, we transformed an object!

$$C_1' = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

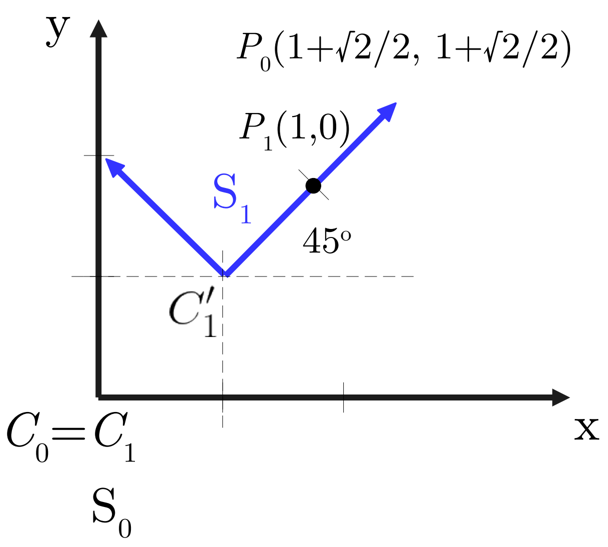
Translation $T(1,1)$

$$C_1' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

New point coordinates in S_0.

Now, let's define a point P in S_1 with coordinates $P_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T$

It is easy to infer that such point in S_0 has the coordinates $P_0 = \begin{bmatrix} 1 + \frac{\sqrt{2}}{2} & 1 + \frac{\sqrt{2}}{2} & 1 \end{bmatrix}^T$



Now, let's define a point P in S_1 with coordinates $P_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T$

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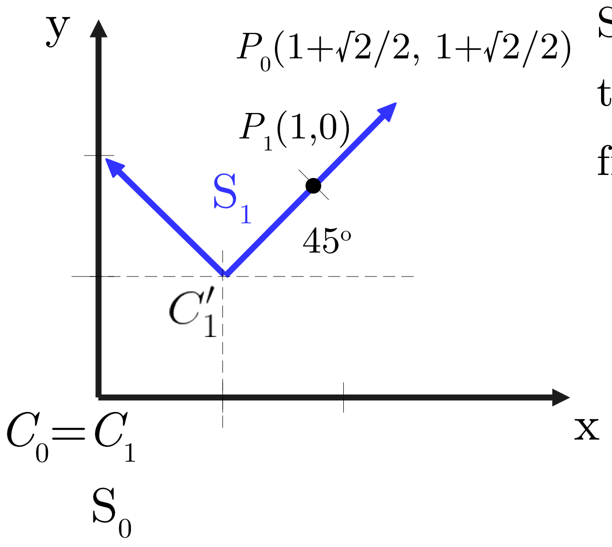

To change coordinates that are known in S_1 to the reference frame S_0, we can use the transformation defined previously when we transformed the "object" S_1. Thus to obtain the coordinates P_0 from the known coordinates P_1, we can do as:

$$P_0 = T(1,1).R(45^o).P_1$$

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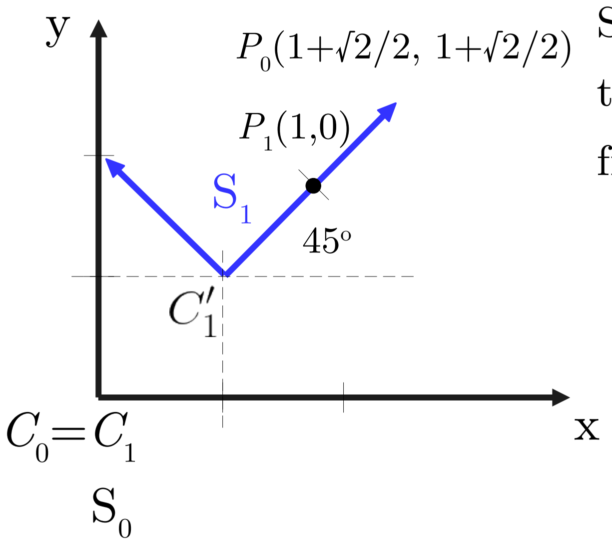

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$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

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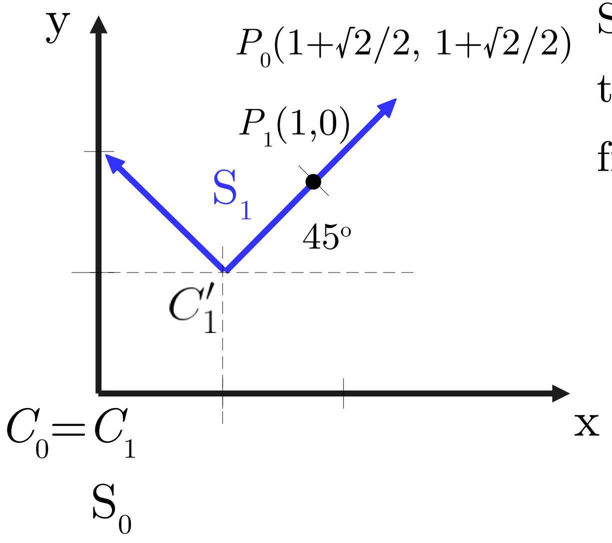

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To ... ference frame

... hen we

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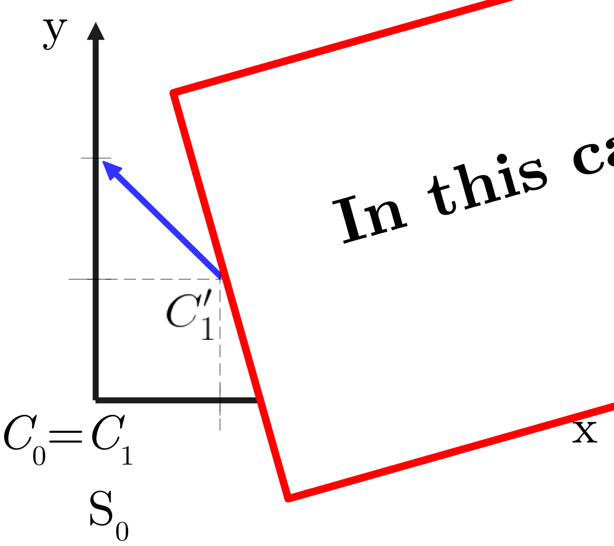


In this case, we converted from one frame to another!

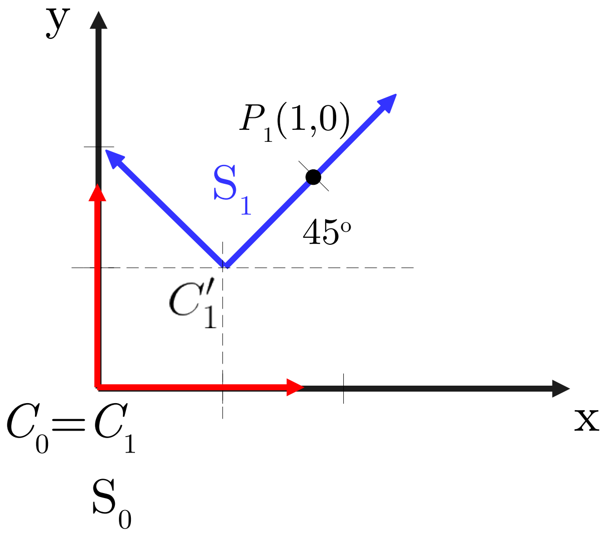
$$... (45^\circ).P_1$$

$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{P_0 = \left[1+\frac{\sqrt{2}}{2}\ \ 1+\frac{\sqrt{2}}{2}\ \ 1\right]^T}$$

Note that even if we apply the idea of successive transformations using intermediate frames, we get the same result.

Using such method, to change from S_1 to S_0, we transform S_0 until it overlaps S_1.



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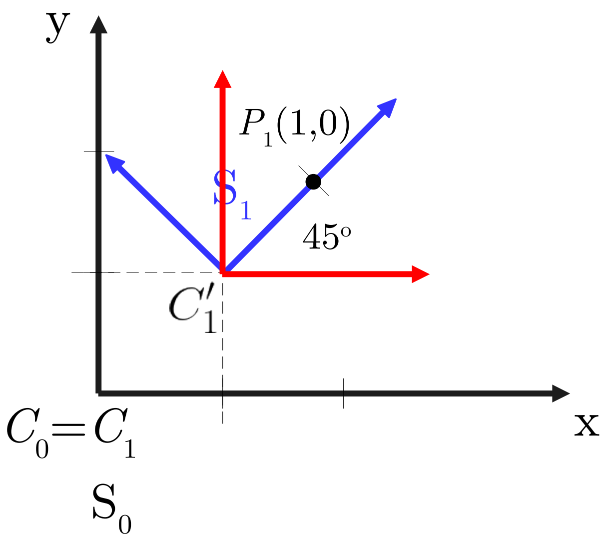

1. Translate (1,1)

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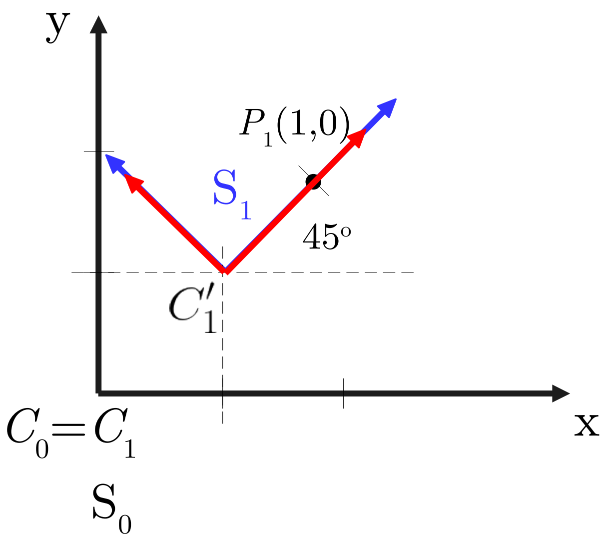

1. Translate (1,1)
2. Rotate 45^o

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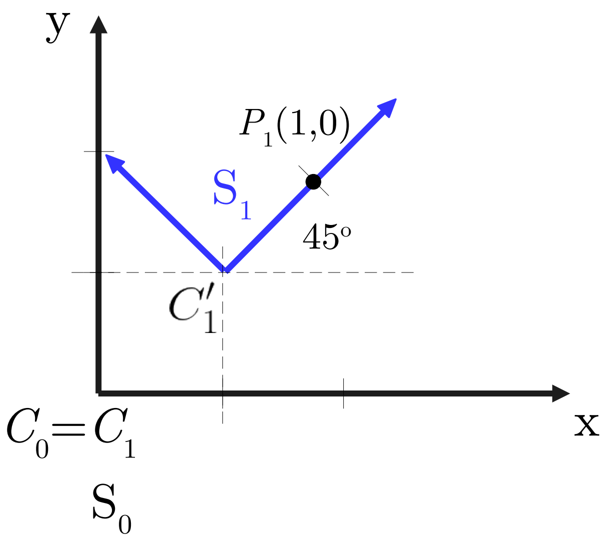


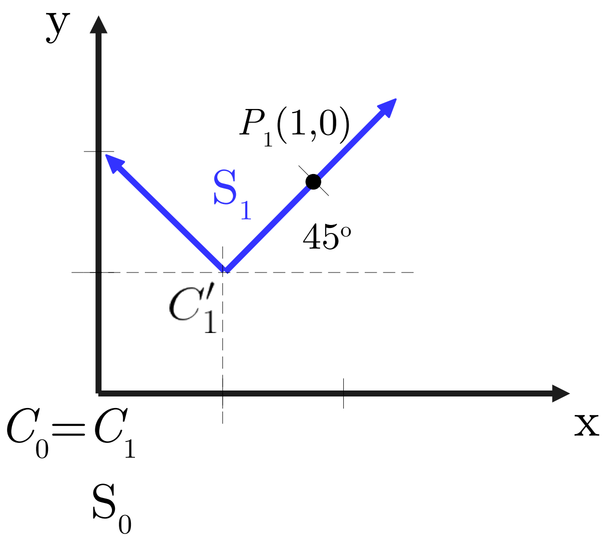
1. Translate (1,1)
2. Rotate 45^o

$$\boxed{P_0 = T(1,1).R(45^o).P_1}$$

Convert from S_1 to S_0.

On the other hand, to change from S_0 to S_1, we can invert the previous transformation.

$$P_1 = [T(1,1).R(45^o)]^{-1}.P_0$$



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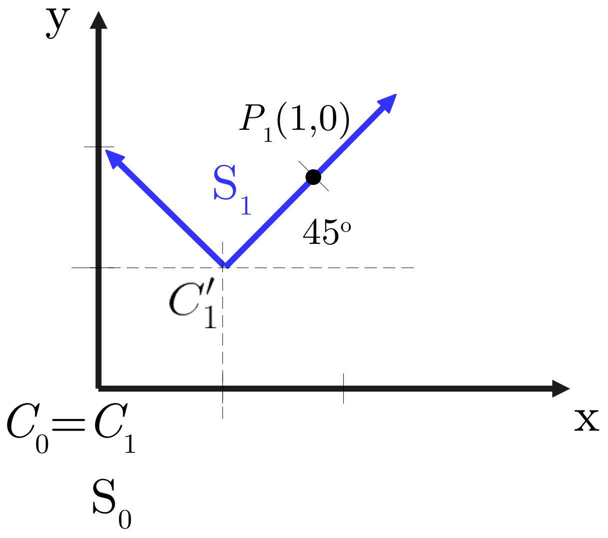
$$P_1 = R(45^o)^{-1}.T(1,1)^{-1}.P_0$$



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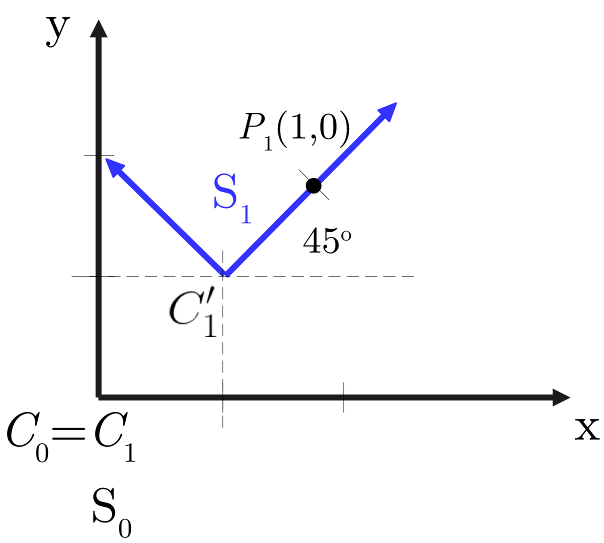
$$\boxed{P_1 = R(-45^o).T(-1,-1).P_0}$$

Convert from S_0 to S_1.



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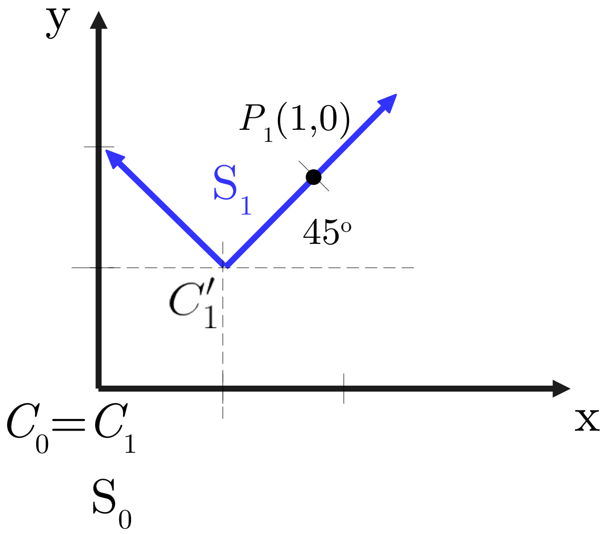

$$P_1 = [T(1,1).R(45^o)]^{-1}.P_0$$

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$$\boxed{P_1 = R(-45^o).T(-1,-1).P_0}$$

Convert from S_0 to S_1.

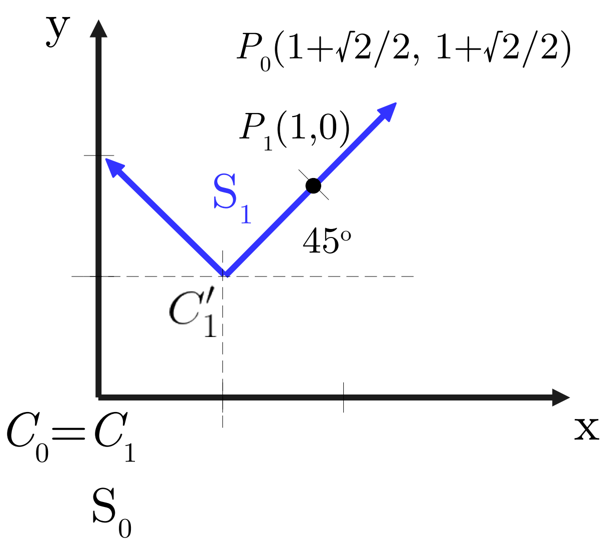
When using intermediate frames, that corresponds to:

1. Rotate -45°
2. Translate (-1,-1)

Thus, if we consider that we know P_0 and want to obtain P_1:

$$P_0 = \begin{bmatrix} 1+\frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} & 1 \end{bmatrix}^T$$

$$P_1 = R(-45^o).T(-1,-1).P_0$$



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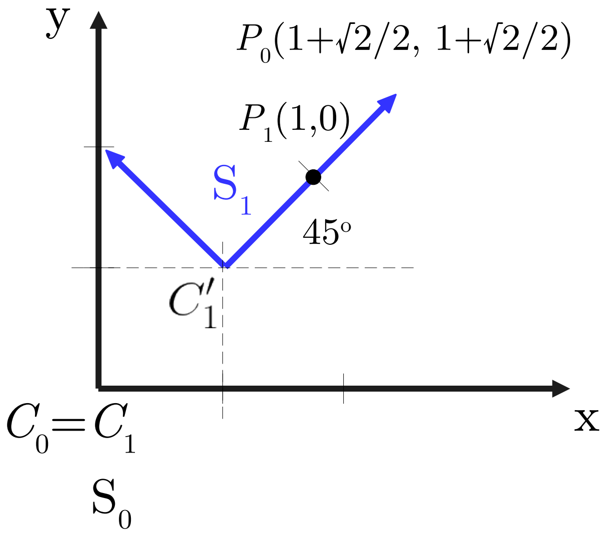

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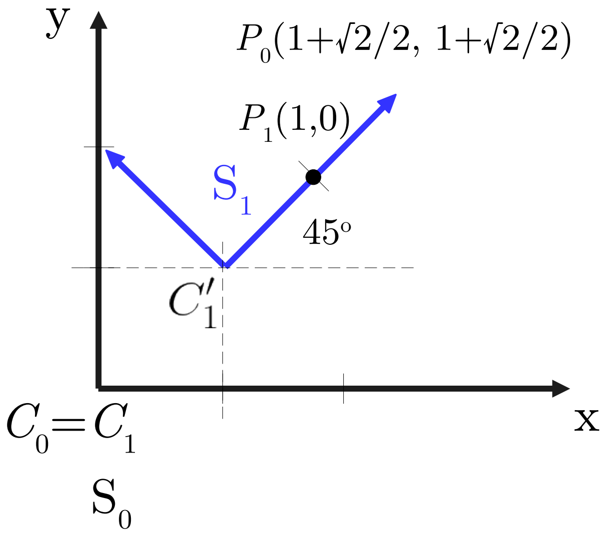

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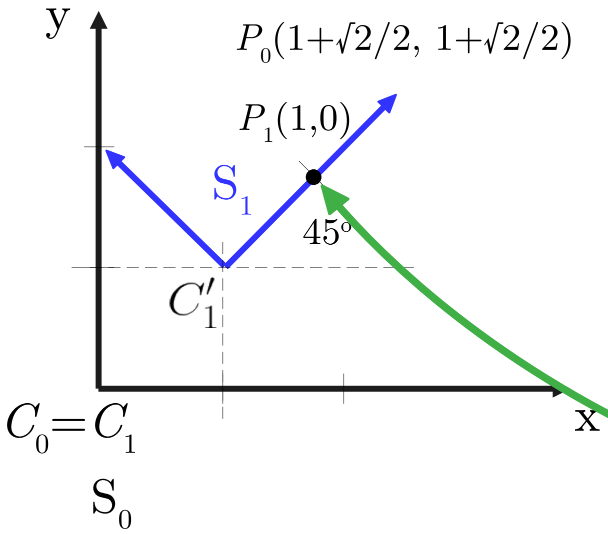

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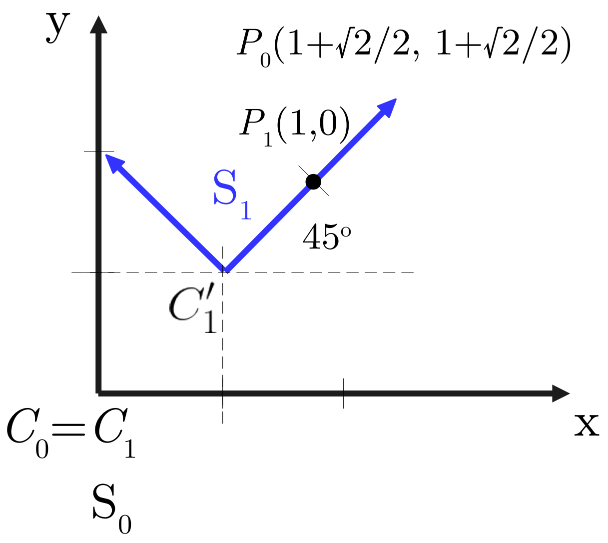
$$\boxed{P_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T}$$

IMPORTANT:

When considering intermediate reference frames, translation and rotation must be defined according to the current frame.



From S_1 to $S_0 \rightarrow P_0 = T(1,1).R(45^o).P_1$

From S_0 to $S_1 \rightarrow P_1 = R(-45^o).T(-1,-1).P_0$

IMPORTANT:

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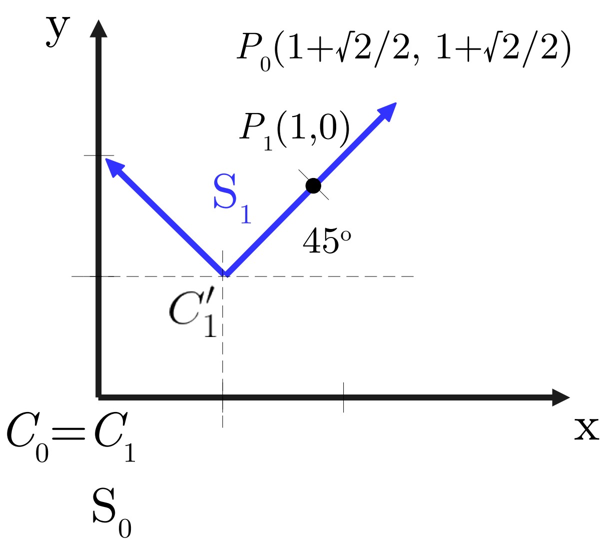


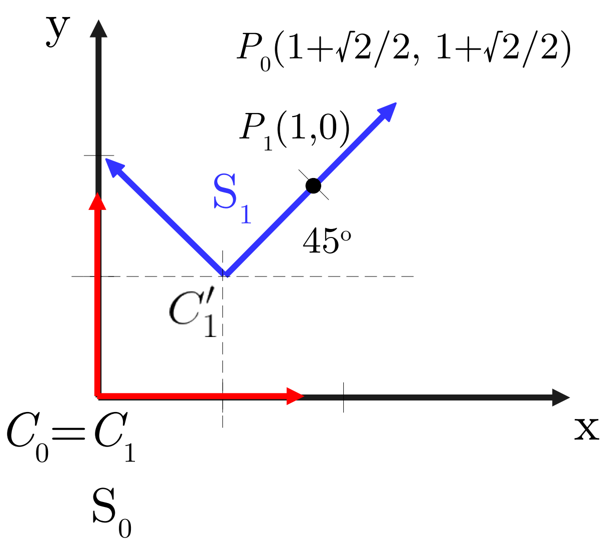
From S_1 to $S_0 \rightarrow$ $P_0 = T(1,1).R(45^o).P_1$

From S_0 to $S_1 \rightarrow$ $P_1 = R(-45^o).T(-1,-1).P_0$

Note that in both cases above, the translation is defined according to the orientation of S_0. Thus before performing such translation we must align the intermediate frame with S_0, so it can be applied correctly.

Now, let's try a different order of intermediate transformations.

From S_1 to S_0 $\rightarrow$ Transform S_0 until it overlaps S_1:



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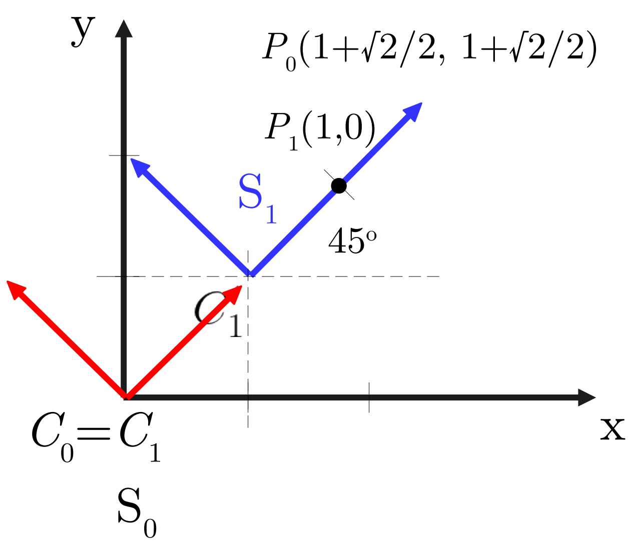
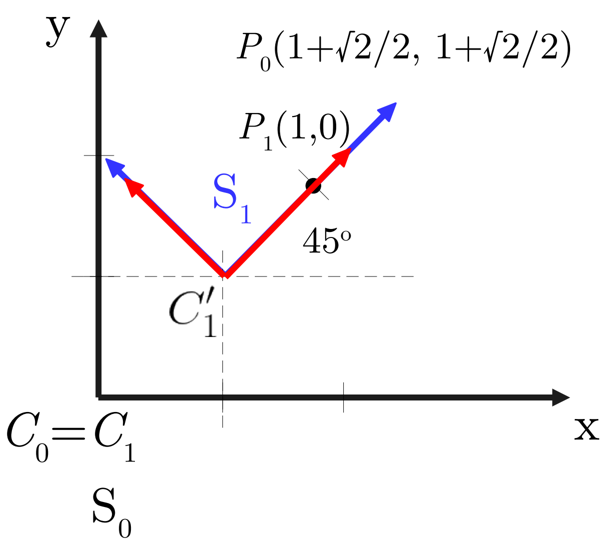


From S_1 to S_0 $\rightarrow$ Transform S_0 until it overlaps S_1:

1. Rotate 45°

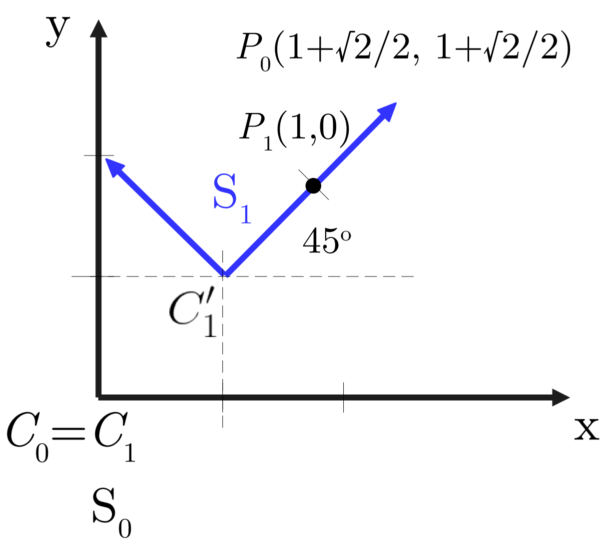
Now, let's try a different order of intermediate transformations.



From S_1 to S_0 $\rightarrow$ Transform S_0 until it overlaps S_1:

1. Rotate 45^o
2. Translate $(\sqrt{2},0)$

Now, let's try a different order of intermediate transformations.

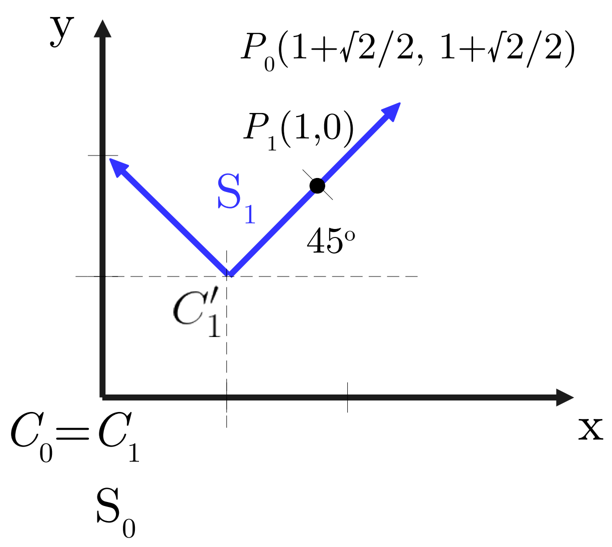


From S_1 to S_0 $\rightarrow$ Transform S_0 until it overlaps S_1:

1. Rotate 45°
2. Translate $(\sqrt{2},0)$

Defined according to the new orientation of the intermediate frame.

Now, let's try a different order of intermediate transformations.

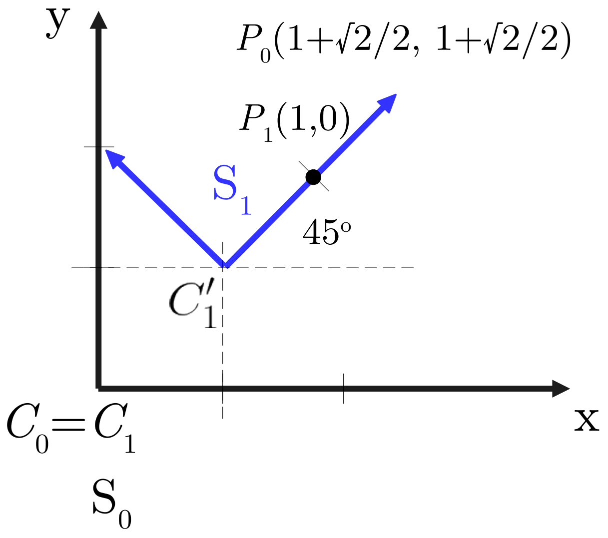


From S_1 to S_0 $\rightarrow$ Transform S_0 until it overlaps S_1:

1. Rotate 45°
2. Translate $(\sqrt{2},0)$

$$P_0 = R(45^o).T(\sqrt{2},0).P_1$$

Now, let's try a different order of intermediate transformations.



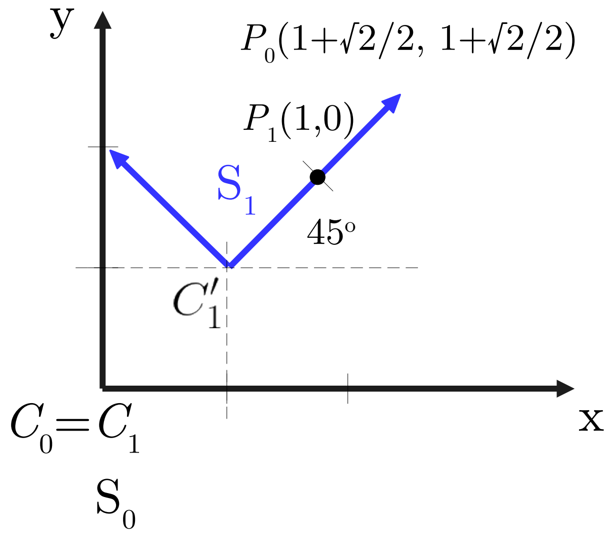
From S_1 to S_0 → Transform S_0 until it overlaps S_1:

1. Rotate 45°
2. Translate ($\sqrt{2}$,0)

$$P_0 = R(45^o).T(\sqrt{2},0).P_1$$

$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Now, let's try a different order of intermediate transformations.



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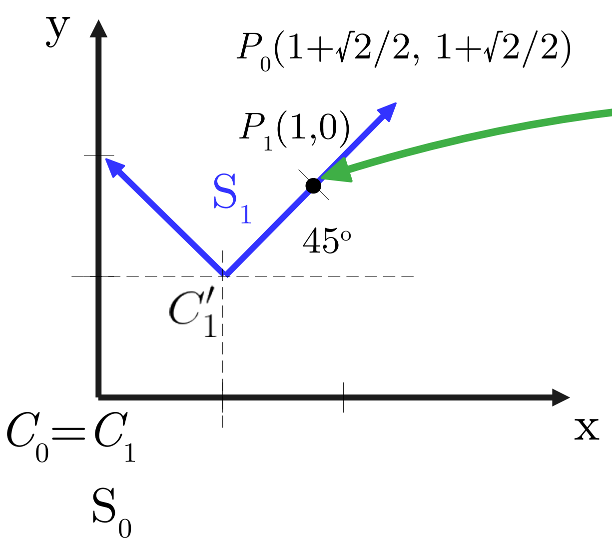
$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Now, let's try a different order of intermediate transformations.

From S_1 to S_0 → Transform S_0 until it overlaps S_1:

1. Rotate 45°
2. Translate (√2,0)



$$P_0 = R(45^o).T(\sqrt{2},0).P_1$$

$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{P_0 = \begin{bmatrix} 1+\frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} & 1 \end{bmatrix}^T}$$

Now, let's try a different order of intermediate transformations.

From S_1 to S_0 → Transform S_0 until it overlaps S_1:

1. Rotate 45°
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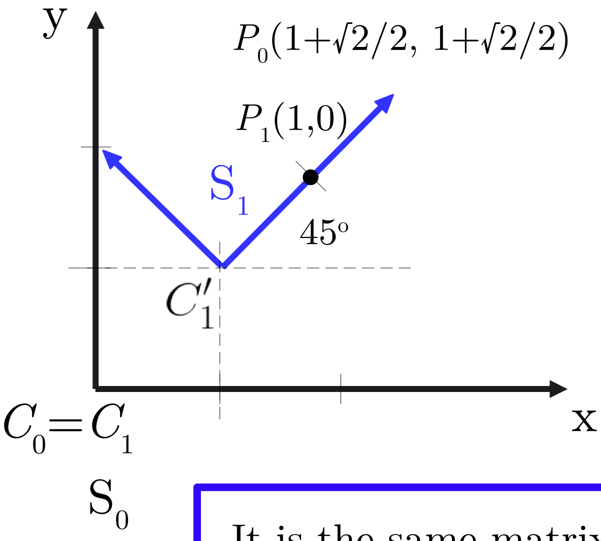


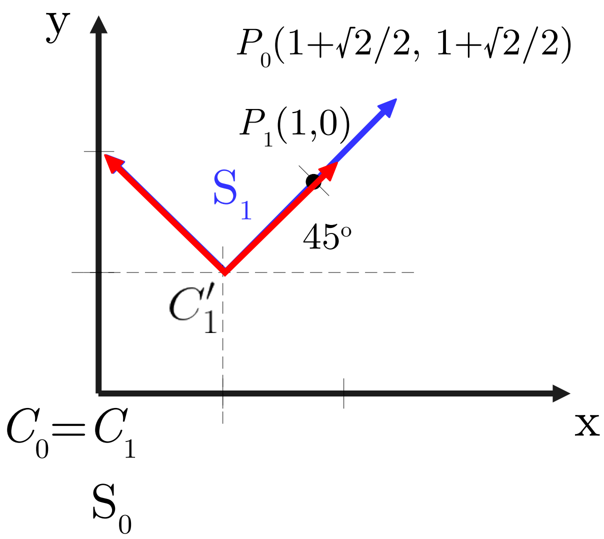
$$P_0 = R(45^o).T(\sqrt{2},0).P_1$$

$$P_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$P_0 = \boxed{\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad P_0 = \begin{bmatrix} 1+\frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} & 1 \end{bmatrix}^T$$

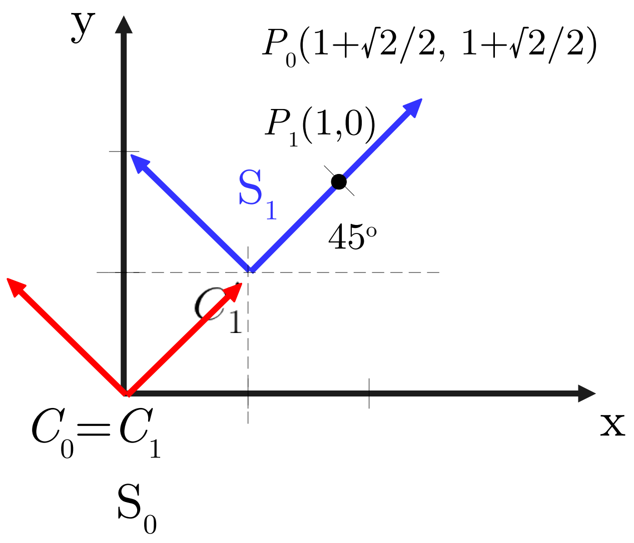
It is the same matrix obtained from T(1,1).R(45°)!!!!

From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:



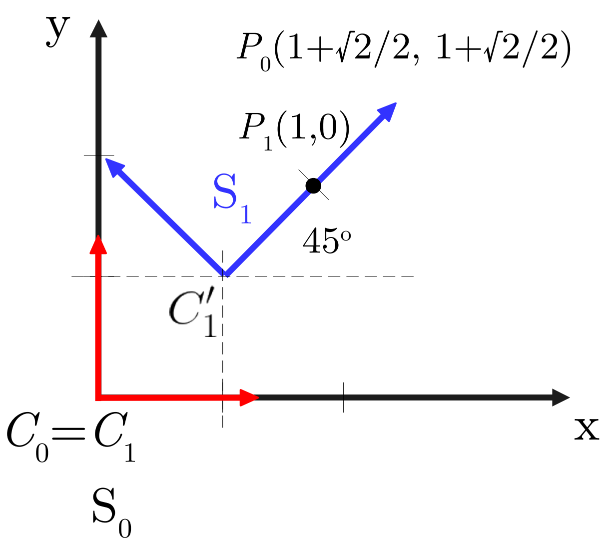
From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

1. Translate $(-\sqrt{2},0)$



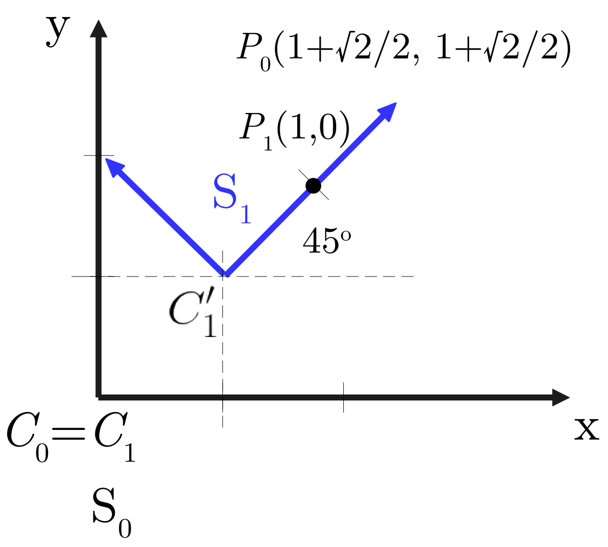
From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

1. Translate $(-\sqrt{2},0)$
2. Rotate -45^o



From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

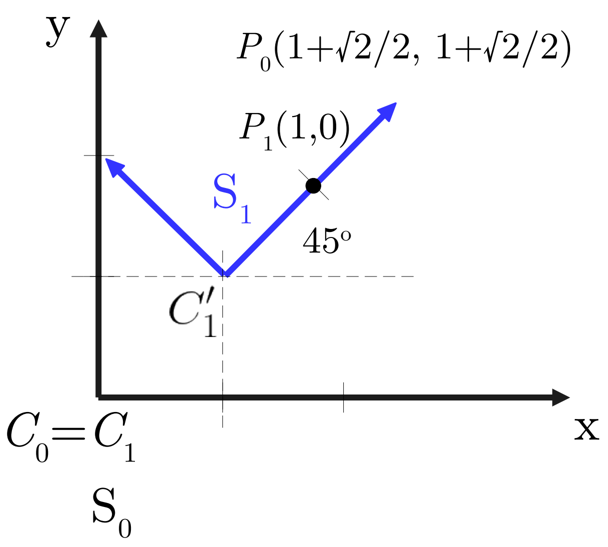
1. Translate $(-\sqrt{2},0)$
2. Rotate -45^o



$$P_1 = T(-\sqrt{2},0).R(-45^o).P_0$$

From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

1. Translate $(-\sqrt{2},0)$
2. Rotate -45^o



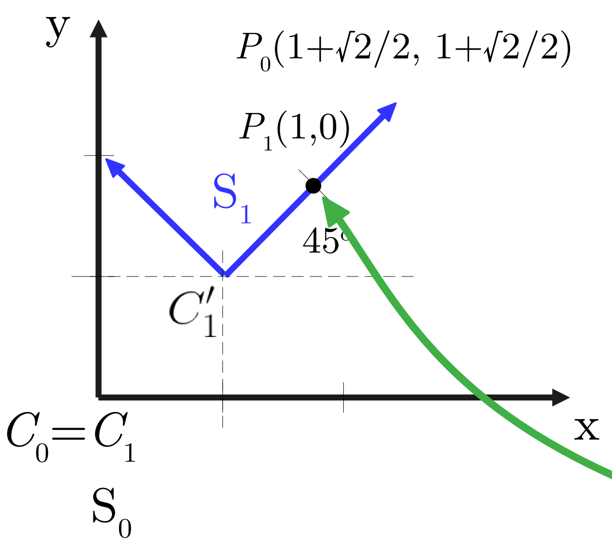
$$P_1 = T(-\sqrt{2},0).R(-45^o).P_0$$

$$P_1 = \begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

1. Translate $(-\sqrt{2},0)$
2. Rotate -45^o



$$P_1 = T(-\sqrt{2},0).R(-45^o).P_0$$

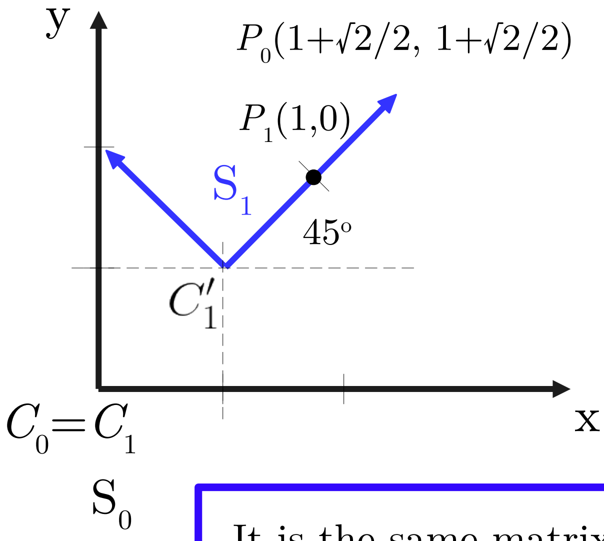
$$P_1 = \begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$\boxed{P_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T}$$

From S_0 to S_1 $\rightarrow$ Transform S_1 until it overlaps S_0:

1. Translate $(-\sqrt{2},0)$
2. Rotate -45^o



$$P_1 = T(-\sqrt{2},0).R(-45^o).P_0$$

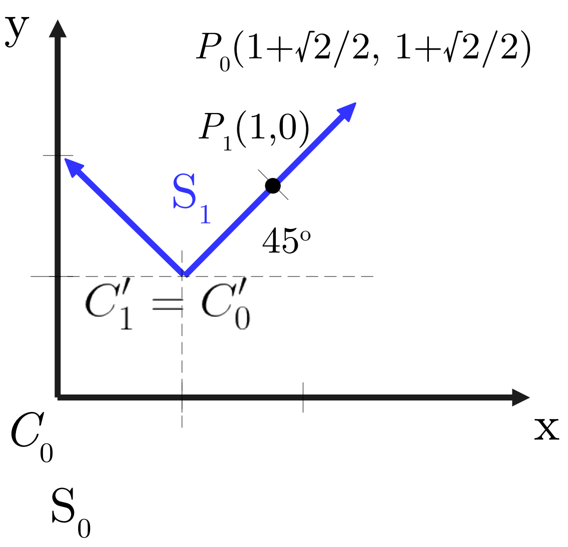
$$P_1 = \begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$P_1 = \boxed{\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1+\frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

It is the same matrix obtained from $R(-45^o).T(-1,-1)$!!!!

$$P_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T$$

Considering as object transformation, note that $P_1 = T(-\sqrt{2}, 0).R(-45^o).P_0$ can also be understood as a rotation of -45° followed by a translation of (-√2,0) in the reference frame S_1 when transforming an object S_0 that corresponds to a frame initially aligned with S_1.



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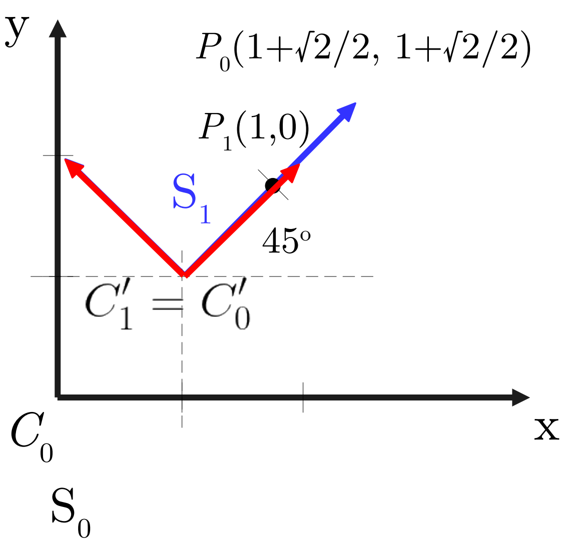
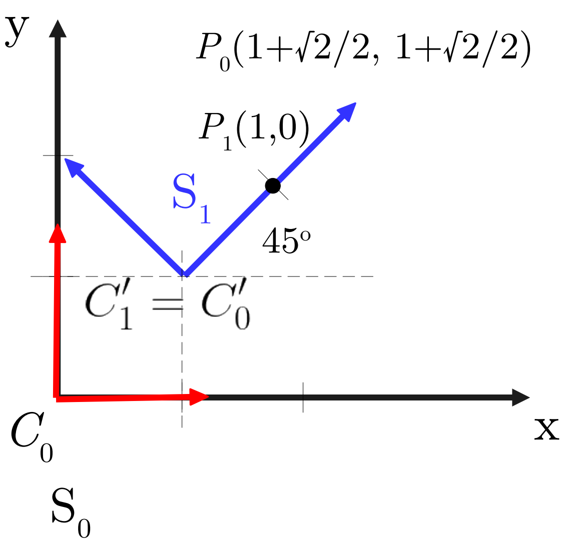


Thus, instead of converting coordinates, let's consider that we are transforming the frame S_0 in red so it ends up on the final position of the S_0 in black. The origin of the frame will be transformed as:

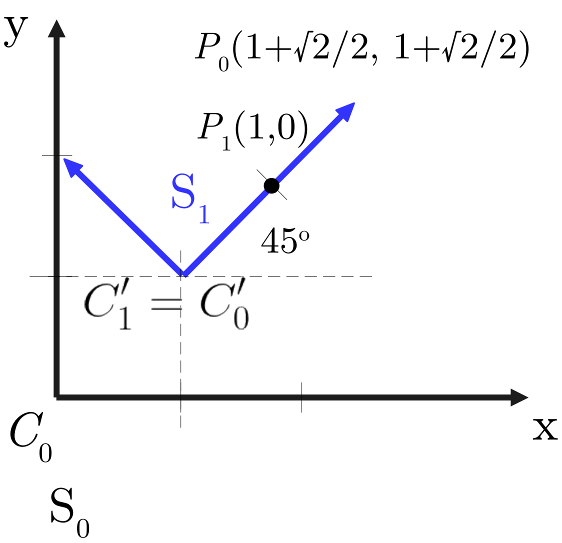
Considering as object transformation, note that $P_1 = T(-\sqrt{2}, 0).R(-45^o).P_0$ can also be understood as a rotation of -45° followed by a translation of (-√2,0) in the reference frame S_1 when transforming an object S_0 that corresponds to a frame initially aligned with S_1.



Thus, instead of converting coordinates, let's consider that we are transforming the frame S_0 in red so it ends up on the final position of the S_0 in black. The origin of the frame will be transformed as:

$$C_0 = T(-\sqrt{2}, 0).R(-45^o)C_0'$$

Considering as object transformation, note that $P_1 = T(-\sqrt{2}, 0).R(-45^o).P_0$ can also be understood as a rotation of -45° followed by a translation of (-√2,0) in the reference frame S_1 when transforming an object S_0 that corresponds to a frame initially aligned with S_1.

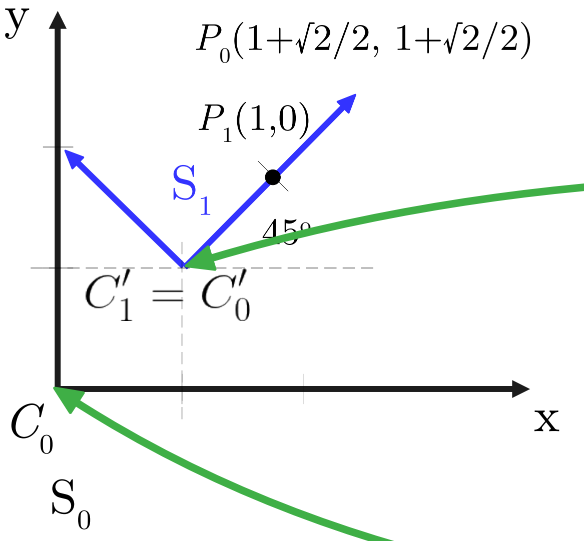


Thus, instead of converting coordinates, let's consider that we are transforming the frame S_0 in red so it ends up on the final position of the S_0 in black. The origin of the frame will be transformed as:

$$C_0 = T(-\sqrt{2}, 0).R(-45^o)C_0'$$

$$C_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} C_0'$$

Considering as object transformation, note that $P_1 = T(-\sqrt{2},0).R(-45^o).P_0$ can also be understood as a rotation of -45° followed by a translation of $(-\sqrt{2},0)$ in the reference frame S_1 when transforming an object S_0 that corresponds to a frame initially aligned with S_1.



Thus, instead of converting coordinates, let's consider that we are transforming the frame S_0 in red so it ends up on the final position of the S_0 in black. The origin of the frame will be transformed as:

$$C_0 = T(-\sqrt{2},0).R(-45^o)C_0'$$

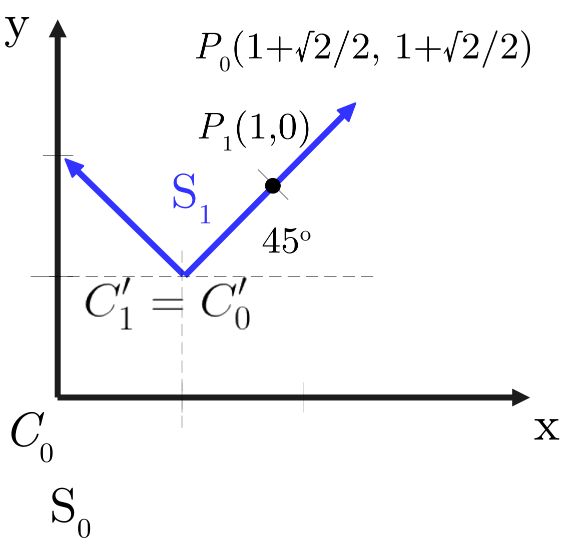
$$C_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} C_0'$$

$$\boxed{C_0' = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T}$$

$$\boxed{C_0 = \begin{bmatrix} -\sqrt{2} & 0 & 1 \end{bmatrix}^T}$$

In frame S_1!!!

Considering as object transformation, note that $P_1 = T(-\sqrt{2},0).R(-45^o).P_0$ can also be understood as a rotation of -45° followed by a translation of (-√2,0) in the reference frame S_1 when transforming an object S_0 that corresponds to a frame initially aligned with S_1.



Thus, instead of converting coordinates, let's consider that we are transforming the frame S_0 in red so it ends up on the final position of the S_0 in black. The origin of the frame will be transformed as:

$$C_0 = T(-\sqrt{2},0).R(-45^o)C_0'$$

Rotation

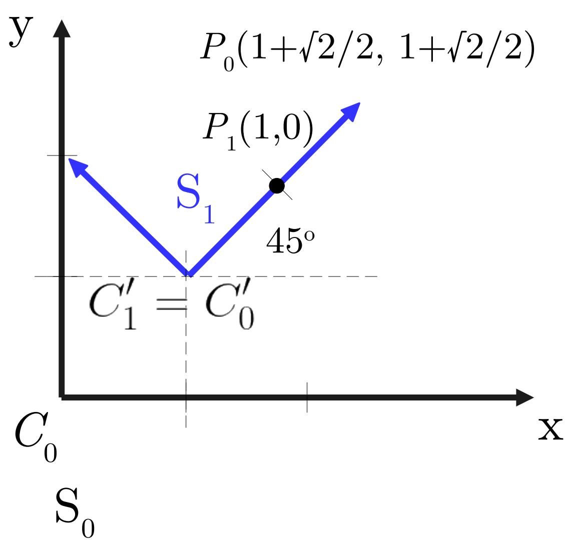
R(-45°)

Translation

T(-√2,0)

$$C_0 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} C_0'$$

Finally, note that:



From S_1 to S_0 $\xleftrightarrow{-1}$ **From S_0 to S_1**

Defined from S_0

$$M_{01} = T(1,1).R(45^o)$$

$$M_{01} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{10} = R(-45^o).T(-1,-1)$$

$$M_{10} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Defined from S_1

$$M_{01} = R(45^o).T(\sqrt{2},0)$$

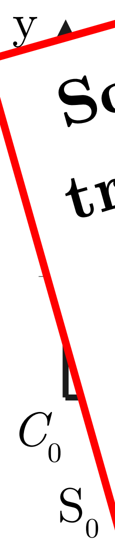
$$M_{01} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{10} = T(-\sqrt{2},0).R(-45^o)$$

$$M_{10} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Finally, note that:

From S_1 to S_0 ... **S_1**



... $(1, -1)$

So, if we use intermediate frames to define the transformation between two references, or if we define such conversion by transformations defined in a fixed frame...

We end up with the same result!

$$M_{01} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{10} = T(-\sqrt{2}, 0).R(-45^o)$$

$$M_{10} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$