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PARTIAL DERIVATIVES: SIGNIFICANCE, MIXED 2ND ORDER DERIVATIVES, CHAIN RULE

- Definition
- Partial Derivative Rules
- Example with Calculation
- Chain Rule
- Application in our engineering field
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❖ HISTORY OF PARTIAL DERIVATIVE

- In mathematics, a partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant (as opposed to the total derivative, in which all variables are allowed to vary). Partial derivatives are used in vector calculus and differential geometry.

- The symbol used to denote partial derivatives is ∂. One of the first known uses of this symbol in mathematics is by Marquis de Condorcet from 1770, who used it for partial differences. The modern partial derivative notation was created by Adrien-Marie Legendre (1786), although he later abandoned it; Carl Gustav Jacob Jacobi reintroduced the symbol in 1841.

❖Definition :

Suppose, we have a function f(x, y), which depends on two variables x and y, where x and y are independent of each other. Then we say that the function f partially depends on x and y. Now, if we calculate the derivative of f, then that derivative is known as the partial derivative of f. If we differentiate the function f with respect to x, then take y as a constant and if we differentiate f with respect to y, then take x as a constant.

Partial Derivative Formula :

If f(x,y) is a function, where f partially depends on x and y and if we differentiate f with respect to x and y then the derivatives are called the partial derivative of f. The formula for partial derivative of f with respect to x taking y as a constant is given by;

$$f_x = \frac{\partial f}{\partial x} = lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

And partial derivative of function f with respect y keeping x as constant, we get;

$$f_y = \frac{\partial f}{\partial y} = lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

❖Partial Derivative Rules :

Same as ordinary derivatives, partial derivatives follow some rule like product rule, quotient rule, chain rule etc.

❖ Product Rule :

If u = f(x,y).g(x,y), then,

$$u_x = \frac{\partial u}{\partial x} = g(x,y)\frac{\partial f}{\partial x} + f(x,y)\frac{\partial g}{\partial x}$$

$$And, u_y = \frac{\partial u}{\partial y} = g(x,y)\frac{\partial f}{\partial y} + f(x,y)\frac{\partial g}{\partial y}$$

❖ Quotient Rule :

If u = f(x,y)/g(x,y), where g(x,y) ≠ 0, then;

$$u_x = \frac{g(x,y)\frac{\partial f}{\partial x} - f(x,y)\frac{\partial g}{\partial x}}{[g(x,y)]^2}$$

And $u_y = \frac{g(x,y)\frac{\partial f}{\partial y} - f(x,y)\frac{\partial g}{\partial y}}{[g(x,y)]^2}$

❖Example with Calculation :

Example 1 : Find the partial derivative of $f(x,y) = x2y + \sin x + \cos y$.

Solution:

Now, find out fx first keeping y as constant

$fx = \partial f/\partial x = (2x)\, y + \cos x + 0$

$= 2xy + \cos x$

When we keep y as constant cos y becomes a constant so its derivative becomes zero.

Similarly, finding fy

$fy = \partial f/\partial y = x2 + 0 + (-\sin y)$

$= x2 – \sin y$

❖Example with Calculation :

Example 2 :Let f(x,y)=x2y+2x+y3 . Find fx(x,y) using the limit definition.

Solution:

$$F_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 y + 2(x+h) + y^3 - (x^2 y + 2x + y^3)}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 y + 2xhy + h^2 y) + 2x + 2h + y^3 - (x^2 y + 2x + y^3)}{h}$$

$$= \lim_{h \to 0} \frac{2xhy + h^2 y + 2h}{h}$$

$$= \lim_{h \to 0} 2xy + 2 = \underline{2xy + 2}$$

We have found fx(x,y)=2xy+2.

❖ Chain Rule :

o Chain Rule for One Independent variable :

Consider that, if x = g(t) and y=h(t) are the differentiable functions of t, and z = f(x, y) which is a differentiable function of x and y. Thus z can be written as z = f(g(t), h(t)), is a differentiable function of t, then the partial derivative of the function with respect to the variable "t" is given as:

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

Here, the ordinary derivatives are determined at "t", whereas the partial derivatives are evaluated at (x, y)

o Chain Rule for Two Independent variables :

Assume that x = g (u, v) and y = h (u, v) are the differentiable functions of the two variables u and v, and also z = f (x, y) is a differentiable function of x and y, then z can be defined as z = f (g (u, v), h (u, v)), which is a differentiable function of u and v. Thus, the partial derivative of the function with respect to the variables are given as:

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

and

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

❖Application in our engineering field :

Partial differential equations are used to mathematically formulate, and thus aid the solution of, physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electrostatics, electrodynamics, etc.

❖ References :

1. R. Wrede; M.R. Spiegel (2010). Advanced Calculus (3rd ed.).
2. Miller, Jeff (2009-06-14). "Earliest Uses of Symbols of Calculus". Earliest Uses of Various Mathematical Symbols.
3. Partial derivative - Wikipedia

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