Particle Motion Problem

Introduction

We are given that a particle is moving along the x-axis with an acceleration $a(t)$ described by the equation:

$$a(t) = 20t^3$$

The initial velocity of the particle at $t = 0$ is $2\,\text{m/s}$.

Objective

The goal is to find the displacement of the particle between $t = 2\,\text{s}$ and $t = 3\,\text{s}$.

Solution

Step 1: Find the Velocity Function

To find the velocity function $v(t)$, we integrate the acceleration function $a(t)$:

$$v(t) = \int a(t)\, dt = \int 20t^3\, dt$$

Integrating, we get:

$$v(t) = 5t^4 + C$$

Where C is the constant of integration. Using the initial condition $v(0) = 2\,\text{m/s}$, we find $C = 2$.

$$v(t) = 5t^4 + 2$$

Step 2: Find the Displacement

The displacement Δx between $t = 2\,\text{s}$ and $t = 3\,\text{s}$ is given by:

$$\Delta x = \int_2^3 v(t)\, dt$$

Using Equation 2, this becomes:

$$\Delta x = \int_2^3 (5t^4 + 2)\, dt$$

On integrating, we get the displacement Δx of 213   m.

$$\Delta x = 213\,\text{m}$$