CHAPTER 1

Vectors

In physics, a **vector** is a quantity that has both a magnitude (size) and a direction. Familiar examples of vectors include velocity, force, and electric field. For any applications beyond one dimension, you need at least two numbers to describe a vector precisely: one for the magnitude of the quantity, and one for the direction. In three dimensions you need three numbers for a complete description. In this chapter you will learn how to describe a vector in the precise language of mathematics. You will also learn how to combine several vectors—that is, to perform the arithmetic of vectors.

1.1 Scalars

A **scalar** is a quantity that requires only one number to quantify it. For example, to quantify the mass of something, you need only specify the value of its mass: 3.5 kg or 1.67×10^{-27} kg. There is no direction associated with mass, just its magnitude—how big it is. Other examples of scalars are pressure, temperature, and speed. Each of these has a magnitude but no direction, so a single number (with units) specifies the scalar quantity completely.

1.2 Vector Notation

When you refer to a vector quantity there are two common ways to represent it. In either case, you first give the vector quantity a descriptive letter name, and then use either boldface type or a small arrow above the name to stand for vector. For example, you could represent a force vector as **F** or $\vec{F}$. In this book, the upper-arrow convention will be followed. In addition, vectors are frequently depicted graphically. By this convention, you use an arrow to represent the vector. The length of the arrow indicates the magnitude of the vector (according to some scale) and the direction of the arrow indicates the direction of the vector. Figure 1.1 shows examples of vectors.

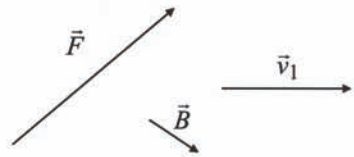

Figure 1.1

Sometimes you will want to specify only the magnitude of a vector. Although you could use the notation $|\vec{F}|$ for the magnitude of a force vector, it is easier to adopt the convention of just using the letter name without the arrow. Thus $F \equiv |\vec{F}|$. When a vector has a magnitude equal to one unit, it is called a **unit vector,** in which case it is represented with a carat over it instead of an arrow. Unit vectors for the three vectors above are given in Figure 1.2.

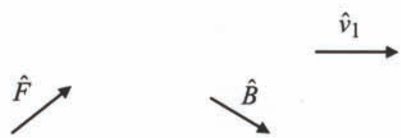

Figure 1.2

$\hat{F}$ is a vector of magnitude 1 unit pointing in the same direction as the vector $\vec{F}$. Of course, if $\vec{F}$ is a force vector, the unit is measured in Newtons (N), and if $\vec{v}_1$ is a velocity vector, the unit is measured in meters per second (m/s). Special names are given to the unit vectors that point along the positive coordinate axes: $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ represent unit vectors along the positive x-, y-, and z-axes, respectively, in a rectangular coordinate system, as indicated in Figure 1.3.

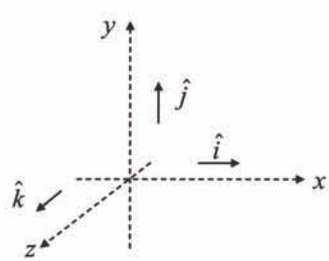

Figure 1.3

The direction of a vector is usually specified precisely by indicating the angle(s) made by the vector with respect to the coordinate axes. In two dimensions, you need to specify just one angle (as illustrated in Figure 1.4), whereas in three dimensions, two angles would be needed.

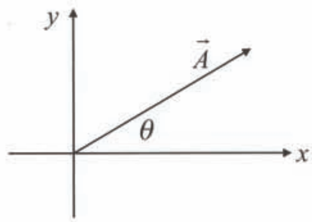

Figure 1.4 The direction of a vector is usually specified by an angle measured with respect to a coordinate axis.

1.3 Combining Vectors and Scalars

A vector can be "multiplied" by a scalar. As a result, the vector after multiplication lies in exactly the same direction, but its magnitude is now changed by a factor equal to the magnitude of the scalar. For example, the vector $2\vec{R}$ points in the same direction as $\vec{R}$, but it has twice the magnitude. Further examples are illustrated in Figure 1.5.

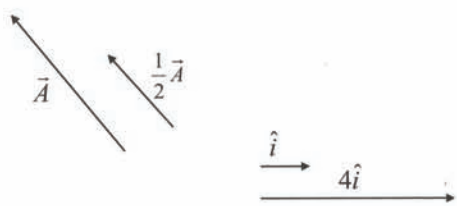

Figure 1.5

Note that when you multiply a unit vector by a scalar, the magnitude of the new vector is the magnitude of the scalar. You can produce a unit vector out of any vector by multiplying the vector by the inverse of the magnitude of the vector.

$$\hat{F} = \frac{1}{F}\vec{F} = \frac{\vec{F}}{F}$$

1.4 Vector Addition

Two vectors can be added to produce a new vector called the sum, or **resultant,** of the two original vectors. As you might suspect, vector addition is more complicated than the addition of simple numbers. Intuitively you know that equal forces applied to the same object will produce many different results, depending on the direction in which the forces are applied. In two dimensions, each vector is described by two numbers—one for the magnitude and one for the direction. But how do these different quantities combine? The rule for vector addition is easy to explain graphically. Take one of the vector arrows to be added and place its tail at the tip of the other vector to be added. The resultant vector, the sum of the two, is the vector you draw from the tail of the first to the head of the second. Figure 1.6 illustrates the process.

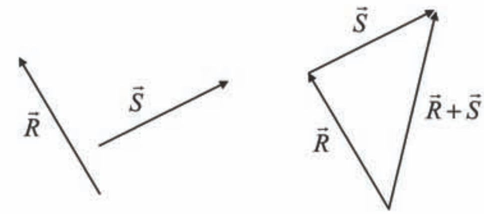

Figure 1.6 Adding vectors.

Also note that the order in which you add the vectors does not matter: the resultant is the same, as shown in Figure 1.7.

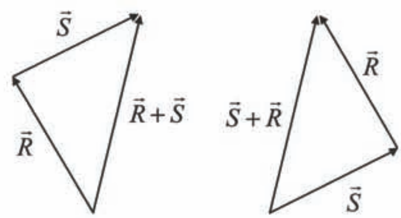

Figure 1.7 Vector addition is commutative.

To add more than two vectors, just extend this process. Starting with any of the vectors, keep putting them head to tail, making a train of all the vectors in the sum. The resultant will be the vector you draw from the tail of the first to the head of the last.

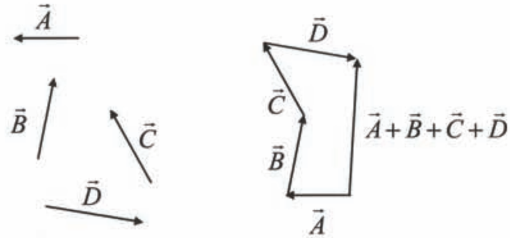

Figure 1.8 Adding several vectors.

Subtraction of two vectors may be thought of as the addition of one vector to the negative of the other vector, obtained by multiplying the other vector by the scalar -1. This step keeps the magnitude the same but reverses the direction. Symbolically you write

$$\vec{V}_1 - \vec{V}_2 = \vec{V}_2 + (-\vec{V}_2)$$

Figure 1.9 illustrates this process.

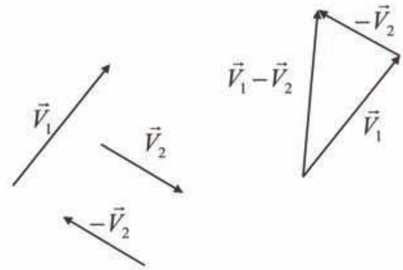

Figure 1.9 Subtraction of vectors.

1.5 Vector Components

Representing vectors with diagrams is a good way to gain insight into their basic relationships and properties, but clearly it is not a precise method of analysis. Although you could use scale drawings and a protractor to gain precision, there will always be limitations to such a graphic approach. It is better to have an analytic method that uses the numbers describing the vectors directly and does not rely on any diagrams. Such an approach is possible and is facilitated by the concept of vector components.

Any vector $\vec{V}$ in two dimensions can be written as the sum of two vectors. Provided that the tail of the first vector begins at the tail of $\vec{V}$ and the head of the second vector ends at the head of $\vec{V}$, the two vectors will add to $\vec{V}$. There are an infinite number of ways to make this sum. (See Figure 1.10.)

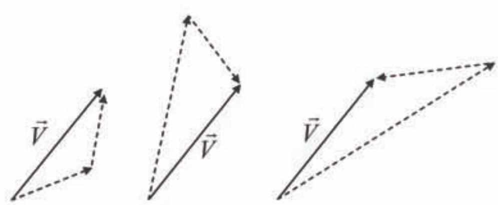

Figure 1.10 A vector can be represented as the sum of two vectors in an infinite number of ways.

However, if the two vectors that add to $\vec{V}$ are perpendicular to each other and are chosen to lie along the coordinate axes of your choosing, then this decomposition is unique. The two vectors that add to $\vec{V}$ in such a case are called the **vector components** of $\vec{V}$, and they are written $\vec{V}_x$ and $\vec{V}_y$.

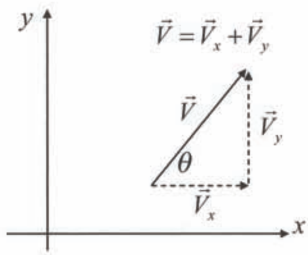

Figure 1.11

You can also use unit vector notation to represent this decomposition of the vector.

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

The quantities V_x and V_y are called, simply, the components of $\vec{V}$. Note that the components may be positive or negative. If θ is a second quadrant angle (between 90° and 180°), then the x component will be negative. In the third quadrant both components are negative, and in the fourth quadrant only the y component is negative.

Because the vector and its two components form a right triangle, you can use simple right triangle trigonometry to relate magnitude and direction information to the component information. If you know the magnitude and direction (V, θ), then you can find the components (V_x, V_y), and vice versa. First you draw the triangle, ignoring the arrows, and represent the magnitude of the legs by the magnitude of the components. The vector being decomposed is *always* the hypotenuse of the triangle.

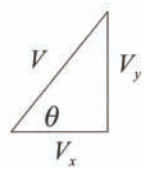

Figure 1.12

Now recall the basic definitions of trigonometry in terms of the sides of the triangle:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Since V_x is the leg adjacent to the given angle and V_y is the leg opposite, you can write

$$V_x = V\cos\theta \qquad V_y = V\sin\theta$$

Example 1 A force of 60 N is applied at an angle of 20° above the + x-axis. Find the x and y components of the force.

Solution Figure 1.13 depicts the triangle involved.

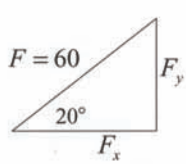

Figure 1.13

Note that F_x is the adjacent side to the angle given, and F_y is the opposite side. Thus you can write

$$F_x = 60\cos 20 = 56.38 \qquad F_y = 60\sin 20 = 20.52$$

Example 2 Find the magnitude and direction of the acceleration vector

$$\vec{a} = 4\hat{i} - 5\hat{j}$$

Solution As seen in Figure 1.14, since the y component is negative, the vector points below the x-axis.

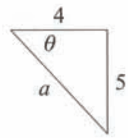

Figure 1.14

You can use the Pythagorean theorem to get the magnitude, and the tangent to get the angle directly.

$$a = \sqrt{4^2 + (-5)^2} = 6.40 \qquad \tan\theta = \frac{5}{4} \Rightarrow \theta = 51.34°$$

Note that while the depicted angle may be reported as 51.34°, the actual angle with the $+x$-axis is $-51.34°$.

1.6 Position and Displacement

A special vector in physics is the **position vector** $\vec{r}$ associated with a particle. It is defined to be the vector that points from the origin to the current position of the particle. Of course, the position of a particle in a coordinate system is determined by the value of its coordinates, and as you can see from Figure 1.15, the components of the position vector are the coordinates of the particle.

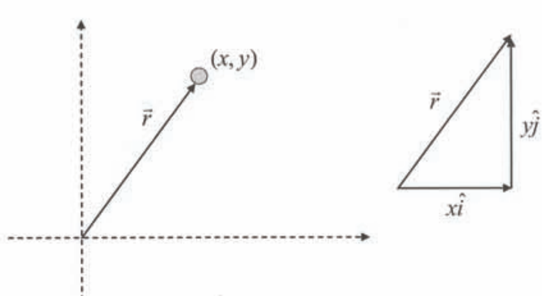

Figure 1.15

It follows from the rules for vector addition that you can write

$$\vec{r} = x\hat{i} + y\hat{j}$$

When an object changes position, you say it has undergone a displacement. The **displacement vector** is the change in the position vector and is written $\Delta\vec{r}$.

$$\Delta\vec{r} = \vec{r}_{final} - \vec{r}_{initial}$$

where *final* and *initial* refer to the values after and before the displacement occurred, respectively. (See Figure 1.16.)

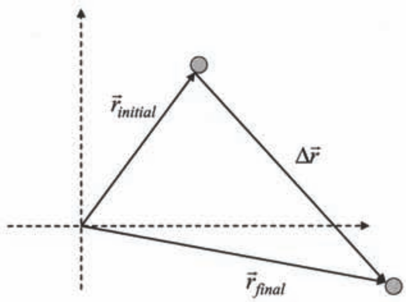

Figure 1.16

When a point particle moves along a **trajectory** (that is, its motion path), the position vector locating the particle is constantly changing. This change makes the position vector a function of time and, in this case, it is written $\vec{r}(t)$ to show the time dependence explicitly. Of course this means that the components of the vector are changing, so they are functions of time as well. Since the unit vectors $\hat{i}$ and $\hat{j}$ do not change with time, you can write the position vector showing the time dependence explicitly as

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

Figure 1.17 shows a trajectory and the position vector at two different times.

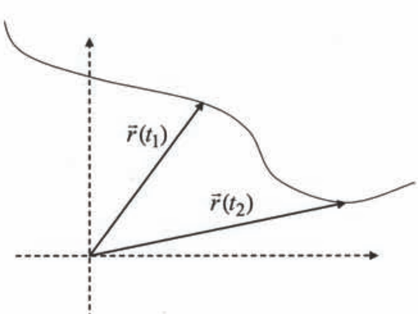

Figure 1.17

Example 3 A particle moves in two dimensions along a trajectory defined by the position vector

$$\vec{r}(t) = \frac{2}{(t+1)^2}\hat{i} - 3t\hat{j}$$

Find the displacement vector for the interval from $t = 1$ to $t = 2$.

Solution You need to calculate $\vec{r}(2) - \vec{r}(1)$.

$$\vec{r}(2) = 0.22\hat{i} - 6\hat{j} \qquad \vec{r}(1) = 0.5\hat{i} - 3\hat{j}$$
$$\Delta\vec{r} = -0.28\hat{i} - 3\hat{j}$$

1.7 Vector Addition with Components

From the rule for vector addition, it is clear that the magnitude of the sum of two vectors is not necessarily the sum of the individual magnitudes. For example, two equal forces of magnitude F applied to an object will produce a force of magnitude 0 if they are applied in opposite directions; a force of magnitude $2F$ if applied in the same direction; or one anywhere in between, depending on the angle between the two vectors. However, by the very nature of their definition, the components of the sum of two vectors are the sum of the individual components of each vector in the sum. This can be easily seen from Figure 1.18.

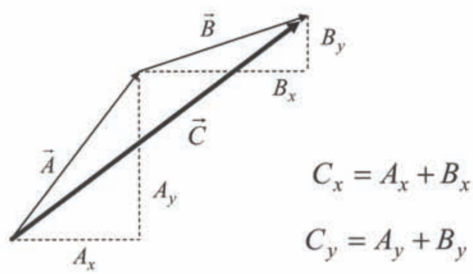

Figure 1.18 The components of the sum of two vectors are the sums of the individual components.

Once you know the components of two vectors, finding their sum is indeed a trivial matter.

Example 4 Consider the two force vectors

$$\vec{F}_1 = 10\hat{i} - 8\hat{j} \qquad \vec{F}_2 = -4\hat{i} + 16\hat{j}$$

Determine the magnitude and direction of the sum of the two forces.

Solution First find the components of the sum $\vec{F}$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = (10 - 4)\hat{i} + (-8 + 16\hat{j}) = 6\hat{i} + 8\hat{j}$$

This is a first quadrant vector because both components are positive.

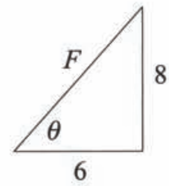

Figure 1.19

You find the magnitude using the Pythagorean theorem, and the direction from the tangent.

$$F = \sqrt{6^2 + 8^2} = 10 \qquad \theta = \tan^{-1}\frac{8}{6} = 53.13°$$

1.8 Dot Product

You have seen that vectors can be multiplied by a scalar and can be added and subtracted as well. Vectors can also be multiplied by other vectors! In fact, there are several ways to define multiplication of vectors, depending on whether you want the product to be a scalar, a vector, or something else.

In this section you will learn how to multiply two vectors to form a scalar. This form is called the scalar product or **dot product.** For two vectors $\vec{A}$ and $\vec{B}$ that have an angle θ between them, the dot product is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Note that because this is a scalar, it has no direction. However, it can be positive or negative, depending on the angle. It is also easy to see that the dot product is 0 when the vectors are perpendicular, and at its maximum possible value when the two vectors are parallel. If you think in terms of components, the dot product has a ready interpretation. Divide both sides in the definition by B, the magnitude of $\vec{B}$.

$$\vec{A} \cdot \frac{\vec{B}}{B} = \frac{AB\cos\theta}{B}, \text{ so } \vec{A} \cdot \hat{B} = A\cos\theta$$

When you consider the resulting equation, $\vec{A} \cdot \hat{B} = A\cos\theta$, remember that $\hat{B}$ is a unit vector in the same direction as $\vec{B}$.

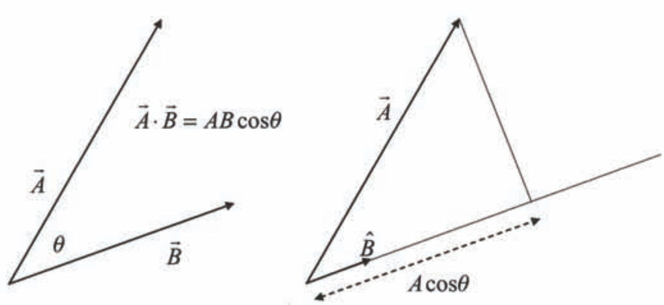

Figure 1.20

So, as can be seen from Figure 1.20, $A\cos\theta$ is the component of $\vec{A}$ that lies along the direction of $\vec{B}$. Thus the dot product of two vectors gives you information about how much of each vector lies along the direction of the other. This is consistent with the dot product being 0 when the two are perpendicular, because then neither has a component along the other.

If you know the components of two vectors in a rectangular coordinate system, it is very easy to calculate the dot product. Consider the two vectors shown in Figure 1.21.

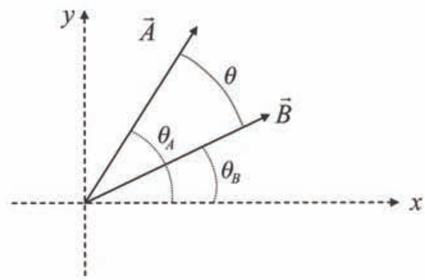

Figure 1.21

The vector $\vec{A}$ makes an angle θ_A with respect to the x-axis, and $\vec{B}$ makes the angle θ_B. It follows that the angle between them $\theta = \theta_A - \theta_B$. Then for the dot product, you have, using a trigonometric identity:

$$\begin{aligned} \vec{A}\cdot\vec{B} &= AB\cos(\theta_A - \theta_B) \\ &= AB(\cos\theta_A\,\cos\theta_B + \sin\theta_A\sin\theta_B) \\ &= (A\cos\theta_A)(B\cos\theta_B) + (A\sin\theta_A)(B\sin\theta_B) \\ \text{so, } \vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y \end{aligned}$$

Notice that the dot product of a vector with itself yields the Pythagorean theorem.

$$\vec{A}\cdot\vec{A} = A^2 = A_x^2 + A_y^2$$

Example 5 Find the dot product of the two vectors

$$\vec{F} = 4\hat{\imath} - 7\hat{\jmath} \qquad \vec{x} = -6\hat{\imath} - 2\hat{\jmath}$$

Solution Using the component form for the dot product, you have

$$\vec{F}\cdot\vec{x} = (4)(-6) + (-7)(-2) = -10$$

Another way to think about the dot product can be illustrated using the previous example. Recall that $\hat{\imath}$ and $\hat{\jmath}$ are perpendicular unit vectors. This means that

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = 1 \qquad \hat{\imath}\cdot\hat{\jmath} = 0$$

Then for Example 5 you could also write, using the distributive property of multiplication:

$$\vec{F}\cdot\vec{x} = (4\hat{\imath} - 7\hat{\jmath})\cdot(-6\hat{\imath} - 2\hat{\jmath}) = -24\hat{\imath}\cdot\hat{\imath} + 14\hat{\jmath}\cdot\hat{\jmath} + (34\hat{\imath}\cdot\hat{\jmath}) = -10$$

In three dimensions, the dot product in component form becomes

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Example 6 Find the angle θ_x that the vector $\vec{P} = 3\hat{i} - 5\hat{j} - 2\hat{k}$ makes with the positive x-axis.

Solution From your understanding of the dot product and the fact that the magnitude of $\hat{i}$ is 1, this angle satisfies the relation

$$\vec{P} \cdot \hat{i} = P\cos\theta_x$$

Since $P = \sqrt{3^2 + (-5)^2 + (-2)^2} = 6.16$ and $\vec{P} \cdot \hat{i} = 3$, it follows that

$$3 = 6.16\cos\theta_x \Rightarrow \theta_x = 60.86°$$

The last example might have been quite difficult if you used a purely trigonometric approach. Use of the dot product yields the solution without even the need to draw a figure, although drawing figures to get an idea of the outcome is always a good check on your analytic approach.

1.9 Cross Product

It is possible to multiply two vectors $\vec{A}$ and $\vec{B}$ to form a new vector $\vec{C}$. This process is called forming the cross product of the two vectors and is written as

$$\vec{C} = \vec{A} \times \vec{B}$$

The magnitude of the cross product is given by

$$C = AB\sin\theta$$

where θ is the angle between the two vectors in the product. You may think of the magnitude of the cross product as the area of the parallelogram formed by the two vectors, as shown in Figure 1.22.

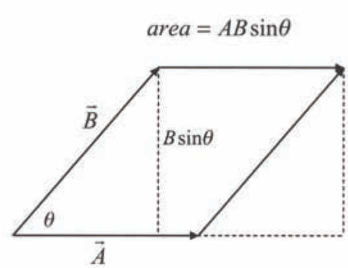

Figure 1.22

The magnitude of the cross product is 0 if the two vectors are parallel or antiparallel, and it is a maximum when the two vectors are perpendicular.

The direction of the cross product is perpendicular to both $\vec{A}$ and $\vec{B}$. Because two vectors define a geometric plane, as illustrated in Figure 1.23, there are two ways that a vector can be perpendicular to this plane.

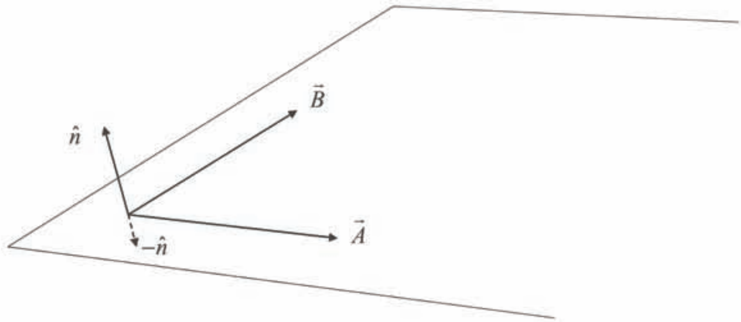

Figure 1.23

These two directions are indicated by the unit vectors $\hat{n}$ and $-\hat{n}$ shown in the figure. The direction of the cross product $\vec{C} = \vec{A} \times \vec{B}$ is defined to be in the direction of $\hat{n}$. This is in accordance with the "right-hand rule," which states:

> Place the two vectors tail to tail. Align your right hand along the first vector in the product, $\vec{A}$, so that the base of your palm is at the tail of the vector, and your fingertips are pointing toward the head. Then curl your fingers via the small angle toward the second vector, $\vec{B}$. If $\vec{B}$ is in a clockwise direction from $\vec{A}$, you will have to flip your hand over to make this work. In either case, the direction in which your thumb is pointing is the direction of $\vec{C}$.

This procedure is illustrated in Figure 1.24.

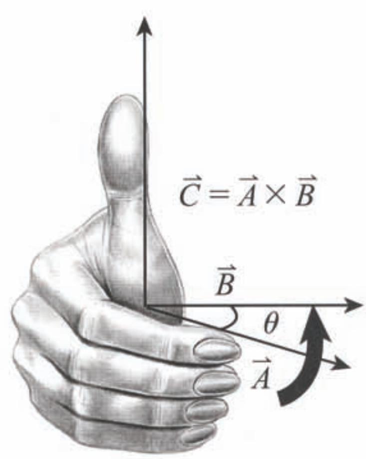

Figure 1.24

Note that because of the way the cross product is defined using the right-hand rule, the order of the product matters; that is, this kind of multiplication is *not* commutative.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

If you know the components of the two vectors, you can find the components of their cross product. This follows from the distributive rule and the facts that:

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0 \qquad \hat{\imath} \times \hat{\jmath} = \hat{k} \qquad \hat{\jmath} \times \hat{k} = \hat{\imath} \qquad \hat{k} \times \hat{\imath} = \hat{\jmath}$$

For example, suppose the two vectors are in the x-y plane.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \qquad \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) \times (B_x\hat{\imath} + B_y\hat{\jmath}) = (A_xB_x)\hat{\imath} \times \hat{\imath} + (A_yB_y)\hat{\jmath} \times \hat{\jmath} + (A_xB_y)\hat{\imath} \times \hat{\jmath} + (A_yB_x)\hat{\jmath} \times \hat{\imath}$$

$$\vec{A} \times \vec{B} = (A_xB_y - A_yB_x)\hat{k}$$

It is left as an exercise for you to prove that in three dimensions

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Example 7 Find the cross product of the two vectors

$$\vec{v} = 5\hat{\imath} + 10\hat{\jmath} \qquad \hat{B} = 2\hat{k}$$

Solution From the formula above you can write

$$\vec{v} \times \vec{B} = (10 \cdot 2)\hat{\imath} + (-5 \cdot 2)\hat{\jmath} + 0\hat{k} = 20\hat{\imath} - 10\hat{\jmath}$$

Let's verify that this vector is indeed perpendicular to both of the vectors in the product.

$$(\vec{v} \times \vec{B}) \cdot \vec{v} = (20\hat{\imath} - 10\hat{\jmath}) \cdot (5\hat{\imath} + 10\hat{\jmath}) = 50 - 50 = 0$$
$$(\vec{v} \times \vec{B}) \cdot \vec{B} = (20\hat{\imath} - 10\hat{\jmath}) \cdot 2\hat{k} = 0$$

Practice Problems

1. Find the x and y components of the following two-dimensional vectors.
   a. A force of magnitude 100 N directed at 130° above the $+x$-axis
   b. A velocity directed at 20° below the $+x$-axis with a magnitude of 30 m/s
   c. An electric field of magnitude 2×10^3 V/m directed at 40° below the $-x$-axis
   d. A momentum vector of magnitude $50 \ \frac{\text{kg} \cdot \text{m}}{\text{s}}$ directed at 62° above the $+x$-axis

2. Find the magnitude and direction relative to the $+x$-axis for these four two-dimensional vectors.
   a. $\vec{R} = 13\hat{\imath} + 10\hat{\jmath}$
   b. $\vec{E} = -120\hat{\imath} + 300\hat{\jmath}$
   c. $\vec{a} = 9\hat{\imath} - 15\hat{\jmath}$
   d. $\vec{p} = -250\hat{\imath} - 100\hat{\jmath}$

3. The position vector of a projectile currently has a magnitude of 20 m and is directed at 40° below the $+x$-axis. Find the coordinates of the projectile.

4. Determine the unit vectors that point in the same direction as the following vectors. Express your answers in the $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ unit vector notation.
   a. $\vec{B} = 5\hat{\imath} - 8\hat{\jmath}$
   b. $\vec{r} = -4\hat{\imath} - 8\hat{\jmath} + 6\hat{k}$
   c. $\vec{V} = 14\hat{\jmath} - 9\hat{k}$
   d. $\vec{F} = 9\hat{\jmath} + 12\hat{\jmath} - 3\hat{k}$

5. Consider the following three vectors:
$$\vec{a} = 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k} \qquad \vec{b} = 3\hat{\imath} + 2\hat{k}$$
$$\vec{c} = -4\hat{\jmath} + \hat{k}$$
   Calculate the following vectors and express them in unit vector notation.
   a. $\vec{a} + \vec{b}$
   b. $\vec{a} - \vec{c}$
   c. $\vec{a} - \vec{b} + \vec{c}$
   d. $-4\vec{b} + 3(\vec{a} + \vec{c})$
   e. $5\vec{a} - 3\vec{b} - 2\vec{c}$

6. Two forces act on an object in the x-y plane. $\vec{F}_1$ has a magnitude of 30 N and is directed at 40° above the $+x$-axis whereas $\vec{F}_2$ has a magnitude of 60 N and is directed at 25° above the $-x$-axis. Find the magnitude and direction with respect to the $+x$-axis of the net force, the sum of the two forces.

7. An object moves in a circle of radius R, as illustrated in Figure 1.25.

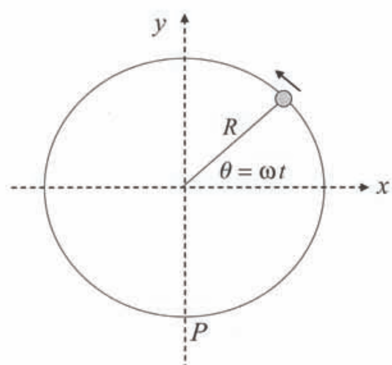

Figure 1.25

a. Suppose you choose to describe this motion using a coordinate system with origin at the center of the circle. You start your time clock at $t = 0$ when the object is passing through the x-axis, and the angle the radius vector makes at any later time t is given by $\theta = \omega t$ where ω is a constant. Write the position vector of the object for an arbitrary time t.

b. Write the position vector for an arbitrary time using an origin of coordinates located at P. Assume the object starts in the same place and moves in the same way.

8. Vectors $\vec{R}$, $\vec{S}$, $\vec{T}$ are related by the equation $\vec{R} + \vec{S} = \vec{T}$. The vector $\vec{R}$ is given in unit vector notation $\vec{R} = 4\hat{i} + 10\hat{j}$. The vector $\vec{T}$ has a magnitude of 15 and is directed at 60° above the $+x$-axis. Find the components of the vector $\vec{S}$.

9. A body is said to be in translational equilibrium if the sum of all the forces acting on it is 0. Suppose the following two forces act on a body.

$$\vec{F}_1 = 8\hat{i} - 12\hat{j} \qquad \vec{F}_2 = -15\hat{i} - 10\hat{j}$$

What third force could be applied to put the body in equilibrium?

10. Calculate the dot product of the following vectors. In each case, find the angle between the two vectors.

a. $\vec{F} = -3\hat{i} + 5\hat{j}$ $\qquad$ $\vec{v} = 6\hat{i} + 14\hat{j}$
b. $\vec{B} = 6\hat{i} + 2\hat{j}$ $\qquad$ $\vec{I} = 2\hat{i} - 6\hat{j}$
c. $\vec{f} = -10\hat{i} - 8\hat{j} + 3\hat{k}$ $\qquad$ $\Delta\vec{r} = 6\hat{i} - 4\hat{j} - 2\hat{k}$

11. The work done by a constant force on an ob-ject during a displacement of an object i defined as the dot product of the force witl the displacement vector. Suppose a mas moves with a trajectory defined by the posi-tion vector

$$\vec{r}(t) = (te^{-\frac{t}{10}})\hat{i} - (t - 3t^2)\hat{j}.$$

Find the work done by the force $\vec{F} = 10\hat{i} - 4$ over the interval from $t = 1$ to $t = 2$.

12. The instantaneous power delivered to an ob-ject by a force is defined as the dot product o the force with the velocity of the object. Finc the instantaneous power delivered at $t =$ by the force $\vec{F} = 20\hat{i} + 35\hat{j}$ to an object witl velocity vector

$$\vec{v} = (4t + t^2)\hat{i} + \left(\frac{t}{t^2 + 2}\right)\hat{j}$$

13. Find the cross product of the following vec-tors ($1^{st} \times 2^{nd}$) and express them in unit vec tor notation. In each case, also find the angl between the two vectors.

a. $\vec{v} = 3\hat{i} - 2\hat{j}$ $\qquad$ $\vec{B} = 6\hat{i} + \hat{k}$
b. $\vec{r} = 10\hat{i} + 7\hat{k}$ $\qquad$ $\vec{F} = -4\hat{i} - 8\hat{j}$
c. $\vec{E} = 6\hat{i} + 3\hat{j} - 2\hat{k}$ $\qquad$ $\vec{B} = -4\hat{i} + 4\hat{j} + 3\hat{k}$

14. The torque exerted by a force $\vec{F}$ on an object is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is the position vector that locates the point on the object where the force is applied. Because $\vec{r}$ depends on where the origin is defined, for a given force the torque will take on different values for different origins. This exercise illustrates this idea. Let the applied force be

$$\vec{F} = 10\hat{i} + 10\hat{j}$$

a. Suppose the origin is chosen so that the position vector to the point of application of the force is

$$\vec{r}_1 = 4\hat{i} + 6\hat{j}$$

Find the torque $\vec{\tau}_1$ exerted by the force about this origin.

b. Now suppose the same situation is described from an origin placed at the point (2, 12) in the old coordinate system. Find the torque $\vec{\tau}_2$ exerted about this origin.

c. What would the torque be about an origin placed at the point of application of the force?