T1 $\xi \sim R(0,\theta)$ $\theta > 0$ вер. модель

$\vec{X}_n$ - выборка объема n

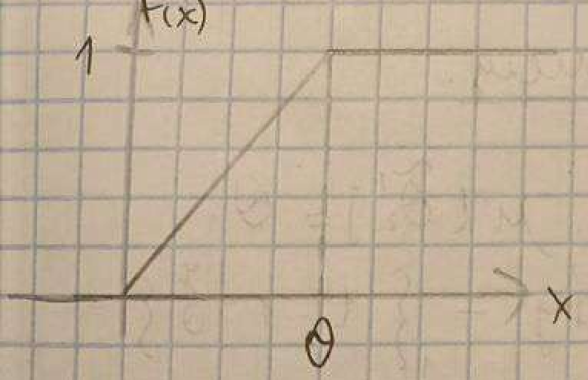
$$\tilde{\theta}_1 = 2\bar{x} = 2\frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\tilde{\theta}_2 = \min x_i$$

$$\tilde{\theta}_3 = \max x_i$$

$$\tilde{\theta}_4 = x_1 + \frac{1}{n-1}\sum_{i=2}^{n} x_i$$

$$M[\xi] = \int_{-\infty}^{\infty} x\, p(x)\, dx = \int_0^{\theta} \frac{x}{\theta}\, dx = \frac{\theta}{2}$$



$$p(x) = \frac{1}{\theta}\{(0,\theta)\}$$

$$M[\xi^2] = \int_{-\infty}^{\infty} x^2 p(x)\, dx = \int_0^{\theta} \frac{x^2}{\theta}\, dx = \frac{\theta^2}{3}$$

$$D[\xi] = \frac{\theta^2}{3} - \frac{\theta^2}{4} = \frac{\theta^2}{12}$$

1) $\tilde{\theta}_1 = 2\bar{x}$

$\forall \theta > 0$ $M[\tilde{\theta}] = \theta$

$$M\Big[\frac{2}{n}\sum_{i=1}^{n} x_i\Big] = \frac{2}{n}\sum_{i=1}^{n} M[x_i] = 2M[\xi] = \theta$$ несмещ.

$$D[\tilde{\theta}_1] = D\Big[\frac{2}{n}\sum_{i=1}^{n} x_i\Big] = \frac{4}{n^2}\sum D[x_i] = \frac{4}{n} D[\xi] =$$

$$= \frac{\theta^2}{3n} \xrightarrow[n\to\infty]{} 0$$ => по дост. условию состоят.

2) $\tilde{\theta}_2 = \min x_i$

$$M[\tilde{\theta}_2] = \int_{-\infty}^{\infty} y\, \varphi(y)\, dy$$

$$\Phi(y) = 1 - (1 - F(x))^n$$

$$\varphi(y) = \Phi'(y) = n(1 - F(y))^{n-1} \underbrace{p(y)}_{= \frac{1}{\theta}\{(0,\theta)\}}$$

$$M[\tilde{\theta}_2] = \int_0^{\theta} n\left(1 - \frac{y}{\theta}\right)^{n-1} \frac{1}{\theta}\, y\, dy = \left\{ t = 1 - \frac{y}{\theta} \right\} =$$

$$= -\int_1^0 n t^{n-1}(1-t)\theta\, dt = \int_0^1 n\theta t^{n-1} dt - \int_0^1 n\theta t^n dt =$$

$$= n\theta\left[\frac{1}{n} - \frac{1}{n+1}\right] = \frac{\theta}{n+1} - \underline{\text{смещ.}}$$

$$\tilde{\theta}_2' = (n+1)\underbrace{x_{min}}_{= \tilde{\theta}_2} - \text{несмещ. } M[\tilde{\theta}_2'] = \theta$$

$$M[\tilde{\theta}_2^2] = \int_0^{\theta} n\left(1 - \frac{y}{\theta}\right)^{n-1} \frac{1}{\theta} y^2 dy = \left\{ t = 1 - \frac{y}{\theta} \right\} =$$

$$= -\int_1^0 n t^{n-1} \theta^2 (1-t)^2 dt = n\theta^2\left[\int_0^1 (t^{n-1} - 2t^n + t^{n+1}) dt\right] =$$

$$= n\theta^2\left[\frac{1}{n} - 2\frac{1}{n+1} + \frac{1}{n+2}\right] = \cancel{n}\theta^2\left[\frac{(n+1)(n+2) - 2n(n+2) + n(n+1)}{\cancel{n}(n+1)(n+2)}\right] =$$

$$= \theta^2 \frac{n^2 + 3n + 2 - 2n^2 - 4n + n^2 + n}{(n+1)(n+2)} = \frac{2\theta^2}{(n+1)(n+2)}$$

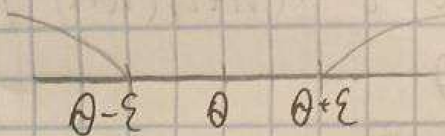
$$D[\tilde{\theta}_2] = \frac{2\theta^2}{(n+1)(n+2)} - \frac{\theta^2}{(n+1)^2} = \theta^2\left[\frac{2(n+1) - (n+2)}{(n+1)^2(n+2)}\right] =$$

$$= \theta^2\left[\frac{n}{(n+1)^2(n+2)}\right] \xrightarrow[n\to\infty]{} 0$$

$$D[\tilde{\theta}_2'] = (n+1)^2 D[\tilde{\theta}_2] = \frac{\theta^2 n}{n+2} \not\to 0$$

θ_2' по определению

$$\forall \theta > 0 \ \forall \varepsilon > 0 \ P(|\tilde{\theta}_2' - \theta| > \varepsilon) \xrightarrow[n\to\infty]{} 0$$



$$P(|\tilde{\theta}_2' - \theta| \geq \varepsilon) \geq P(\tilde{\theta}_2' \geq \theta + \varepsilon) = P((n+1)X_{min} \geq \theta + \varepsilon) =$$

$$= P\left(X_{min} \geq \frac{\theta+\varepsilon}{n+1}\right) = 1 - \underbrace{P\left(X_{min} < \frac{\theta+\varepsilon}{n+1}\right)}_{\varphi\left(\frac{\theta+\varepsilon}{n+1}\right)} =$$

$$= 1 - \left(1 - \left(1 - F\left(\frac{\theta+\varepsilon}{n+1}\right)\right)^n\right) =$$

$$= \left(1 - \left(\frac{\theta+\varepsilon}{\theta(n+1)}\right)\right)^n \xrightarrow[n\to\infty]{} e^{-\frac{\theta+\varepsilon}{\theta}} > 0$$ не явл. состоят.

$$\tilde{\theta}_2 : P(|\tilde{\theta}_2 - \theta| \geq \varepsilon) \xrightarrow[n\to\infty]{} 0$$

$\forall \theta > 0$

$\forall \varepsilon > 0$ $\quad P(\tilde{\theta}_2 < \theta - \varepsilon) + \underbrace{P(\underset{X_{min}}{\underline{\tilde{\theta}_2}} > \theta + \varepsilon)}_{\to 0}$

$$P(X_{min} < \theta - \varepsilon) = \varphi(\theta - \varepsilon) \underset{\varepsilon < \theta}{=} 1 - \left(1 - \frac{\theta - \varepsilon}{\theta}\right)^n = 1 - \left(\frac{\varepsilon}{\theta}\right)^n \xrightarrow[n\to\infty]{} 1$$

3) $\tilde{\theta}_3 = X_{max}$

$$M[\tilde{\theta}_3] = \int_{-\infty}^{\infty} z \psi(z) dz = \frac{n}{n+1}\theta$$ — смещ.

$$\Psi(z) = (F(z))^n$$

$$\Psi(z) = \Psi'(z) = n\,(F(z))^{n-1}\, p(z) = n\left(\frac{z}{\theta}\right)^{n-1}\frac{1}{\theta}\{[0,\theta]\}$$

$p(z) = \frac{1}{\theta}\{[0,\theta]\}$

$\tilde{\theta}_3' = \frac{n+1}{n} X_{max}$ – несмещ.

$$D[\tilde{\theta}_3] = \frac{n}{n+2}\theta^2 - \frac{n^2}{(n+1)^2}\theta^2 = \theta^2\left[\frac{n(n+1)^2 - n^2(n+2)}{(n+2)(n+1)^2}\right] =$$

$$= \theta^2\,\frac{\cancel{n^3} + \cancel{2n^2} + n - \cancel{n^3} - \cancel{2n^2}}{(n+2)(n+1)^2} = \frac{n\,\theta}{(n+2)(n+1)^2}$$

$$D[\tilde{\theta}_3'] = \frac{(n+1)^2}{n^2} D[\tilde{\theta}_3] = \frac{\theta^2}{n(n+2)} \xrightarrow[n\to\infty]{} 0 \text{ – состоят.}$$

$\tilde{\theta}_3$ по определению:

$\forall \theta > 0$
$\forall \varepsilon > 0$

$$P(|\tilde{\theta}_3 - \theta| \geq \varepsilon) = P(X_{max} < \theta - \varepsilon) + \underbrace{P(X_{max} > \theta + \varepsilon)}_{=0} = (F(\theta-\varepsilon))^n$$

$$0 < \varepsilon < \theta: \quad \left(\frac{\theta-\varepsilon}{\theta}\right)^n \xrightarrow[n\to\infty]{} 0$$

$$\varepsilon \geq \theta: \quad (0)^n \xrightarrow[n\to\infty]{} 0$$

$\tilde{\theta}_3'$ по определению:

$\forall \theta > 0$
$\forall \varepsilon > 0$

$$P(|\tilde{\theta}_3' - \theta| \geq \varepsilon) = P\left(\frac{n+1}{n} X_{max} < \theta - \varepsilon\right) + P\left(X_{max}\cdot\frac{n+1}{n} > \theta + \varepsilon\right) =$$

$$= P\left(X_{max} < \frac{n}{n+1}(\theta-\varepsilon)\right) + P\left(X_{max} > \frac{n}{n+1}(\theta+\varepsilon)\right) =$$

$$= \left(F\left(\frac{n}{n+1}(\theta-\varepsilon)\right)\right)^n + 1 - P\left(X_{max} \le \frac{n}{n+1}(\theta+\varepsilon)\right) =$$

$$= \left(\frac{n(\theta-\varepsilon)}{\theta(n+1)}\right)^n + 1 - \left(\frac{n(\theta+\varepsilon)}{\theta(n+1)}\right)^n = \left(\frac{n\theta - n\varepsilon}{n\theta+\theta}\right)^n + 1 -$$

$$- \left(\frac{n\theta+n\varepsilon}{n\theta+\theta}\right)^n = \underbrace{\left(\frac{\theta-\varepsilon}{\theta+\frac{\theta}{n}}\right)^n}_{\to 0} + 1 - \underbrace{\left(\frac{\theta+\varepsilon}{\theta+\frac{\theta}{n}}\right)^n}_{\to 1} \xrightarrow[n\to\infty]{} 0 \Rightarrow$$ состоят.

4) $\tilde{\theta}_4 = X_1 + \frac{1}{n-1}\sum_{i=2}^{n} X_i$

$$M[\tilde{\theta}_4] = M\left[X_1 + \frac{1}{n-1}\sum_{i=2}^{n} X_i\right] = M[X_1] + \frac{1}{n-1}\sum_{i=2}^{n} M[X_i] =$$

$$= \frac{\theta}{2} + \frac{\theta}{2} = \theta$$

$\xi \sim R(0,\theta),\ \theta > 0$ – несмещ.

$M[\xi] = \frac{\theta}{2}$

$$D[\theta_4] = D\left[X_1 + \frac{1}{n-1}\sum_{i=2}^{n} X_i\right] = D[\xi] + \frac{1}{(n-1)^2}(n-1)D[\xi] =$$

$$= \frac{\theta^2}{12} + \frac{\theta^2}{12(n-1)}$$

$D[\xi] = \frac{\theta^2}{12}$ $\quad \frac{\theta^2}{12}\frac{n}{n-1} \underset{n\to\infty}{\not\to} 0$ дост. усл. не работает

$\tilde{\theta}_4 \xrightarrow{P} \theta$

$\xi_n \to \xi,\ \eta_n \to \eta \Rightarrow \xi_n + \eta_n \xrightarrow{P} \xi + \eta$

воспользуемся

$X_1 \xrightarrow{P} X_1$

$\frac{1}{n-1}\sum_{i=2}^{n} X_i \xrightarrow{P} M[\xi] = \frac{\theta}{2}$ ~~[illegible]~~

ЗБЧ Хинчина: $\xi_1, \ldots, \xi_n$ независ, одинак. распред,

тогда $\frac{1}{n} \sum_{i=1}^{n} \xi_i \xrightarrow{P} M[\xi]$

$\hat{\theta}_1 = 2\bar{x}$

$\hat{\theta}_3' = \frac{n+1}{n} x_{max}$

$D[\hat{\theta}_1] = \frac{\theta^2}{3n}$

$D[\hat{\theta}_3'] = \frac{\theta^2}{n(n+2)}$

$\forall \theta > 0 \quad \frac{\theta^2}{n(n+2)} < \frac{\theta^2}{3n}$

$3n < n^2 + 2n$

$n^2 \geq n$

$n > 1$