T11

$H_0: \xi \sim p_0(x) = 1 \, \{(0,1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \, \{(0,1)\}$

a) $n = 1, \alpha$

$l = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} e^{-x}}{1} \geq C$

$e^{-x} \geq B \to x \leq A$

$\underline{P(x \leq A | H_0)} = \alpha$

α_1

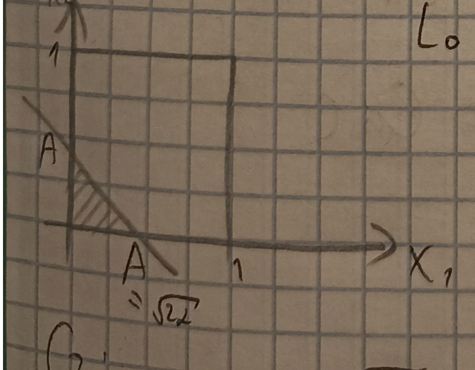
$\int_0^A 1 \, dx = A = \alpha$ $\qquad G: x \leq \alpha$

$\alpha_1 = \alpha$

$W = P(x \leq A | H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} \, dx = \frac{e}{e-1}(1 - e^{-\alpha})$

$\alpha_2 = 1 - W$

$P(x_1 + x_2 \leq A | H_0) = \alpha$



$L_0 = 1$

$$\iint_{x_1 + x_2 = A} 1 \, dx_1 \, dx_2 = \frac{A^2}{2} = \alpha$$

$A = \sqrt{2\alpha}$

$G: x_1 + x_2 \leq \sqrt{2\alpha}$ $\qquad \alpha_1 = \alpha$

$$W = P(x_1 + x_2 \le A \mid H_1) = \iint\limits_{x_1+x_2<A} \left(\frac{e}{e-1}\right)^2 e^{-x_1-x_2}\, dx_1 dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A dx_1 \int_0^{A-x_1} e^{-x_1} e^{-x_2}\, dx_2 = \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1}\left(1-e^{-(A-x_1)}\right) dx_1 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^{A} \left(e^{-x_1} - e^{-A}\right) dx_1 = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-A} - Ae^{A}\right)$$

$$A = \sqrt{2\alpha}$$

$$\alpha_2 = 1 - W$$

b) и c) $\ell = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \ge C$

$$\ln \ell = \sum_{i=1}^{n} \ln \frac{p_1(x_i)}{p_0(x_i)} \ge \ln C$$

$$\eta_i$$

$$\frac{\sum \eta_i - n M[\eta_1]}{\sqrt{n D[\eta_1]}} \rightsquigarrow N(0,1)$$

$$P(\ln \ell \ge \ln C \mid H_0) = \alpha$$

H_0: $\eta = \ln\left(\frac{e}{e-1} e^{-x}\right) = \ln \frac{e}{e-1} - x$

$$\ln \ell = \sum \ln \frac{e}{e-1} - \sum x_i \ge \ln C$$

$$G: \sum x_i \le A$$

$$P\left(\sum x_i \le A \mid H_0\right) = \alpha$$

$$P\left(\frac{\sum x_i - n M[x]}{\sqrt{n D[x]}} \le \frac{A - n M[x]}{\sqrt{n D[x]}} \,\Big|\, H_0\right) = \alpha$$

$$M[X] = \frac{1}{2}$$

$$D[X] = \frac{1}{12}(b-a)^2$$

$$\frac{A - n\cdot\frac{1}{2}}{\sqrt{n\cdot\frac{1}{12}}} = u_\alpha \qquad A = n\cdot\frac{1}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$G: \sum x_i \le \underbrace{n\cdot\frac{1}{2} + u_\alpha\sqrt{\frac{n}{12}}}_{A}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\sum x_i \le A \mid H_1\right) = P\left(\frac{\sum x_i - n\,M(X)}{\sqrt{n\,D(X)}} \le \frac{A - n\,M[X]}{\sqrt{n\,D(X)}} \,\middle|\, H_1\right)$$

$$M[X] = \int_0^1 x\,\frac{e}{e-1}\,e^{-x}\,dx = \frac{e-2}{e-1}$$

$$M(X^2) = \frac{2e-5}{e-1}$$

$$D[X] = M(X^2] - M^2[X] = \frac{e^2-3e+1}{(e-1)^2}$$

$$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}}\,e^{-\frac{x^2}{2}}\,dx\,, \text{ где } B = \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n\,\frac{e^2-3e+1}{(e-1)^2}}}$$

$$\alpha_2 = 1 - W$$

$$\underset{u_\alpha<0}{B = \ldots} = \frac{\sqrt{n}\left(\frac{1}{2} - \frac{e-2}{e-1}\right) + u_\alpha\sqrt{\frac{n}{12}}}{\sqrt{\frac{e^2-3e+1}{(e-1)^2}}} \xrightarrow[n\to\infty]{} +\infty \Rightarrow$$

$$\Rightarrow W \xrightarrow[n\to\infty]{} 1 \Rightarrow \text{Критерий состоятелен}$$

$\alpha_2 = 1 - W$

$\alpha = \alpha_1 = 0{,}05$

$n = 1$	$x \leq 0{,}05$	$W = 0{,}077$	$\alpha_2 = 0{,}923$
$n = 2$	$x_1 + x_2 < 0{,}316$	$W = 0{,}102$	$\alpha_2 = 0{,}898$
$n = 10$	$\sum_{i=1}^{10} x_i < 3{,}5$	$W = \int_{-\infty}^{-0{,}78} \dots = 0{,}22$	$\alpha_2 = 0{,}78$

d) $G: x_{min} < C$

$P(\vec{X}_n \in G \mid H_0) = \alpha$

$P(X_{min} < C \mid H_0) = \alpha$

$H_0: \xi \sim R(0,1)$

$\xi \sim F_0(x) \Rightarrow \xi_{min} \sim 1 - (1 - F_0(x))^n$ $\quad \xi_1 \dots \xi_n$ nezav

$P(X_{min} < C) = 1 - (1 - F_0(C))^n = \alpha$, $F_0(C) = C$

$1 - \alpha = (1 - F_0(C))^n$; $1 - \alpha = (1 - C)^n$

$1 - C = \sqrt[n]{1-\alpha} \Rightarrow C = 1 - \sqrt[n]{1-\alpha}$

$G: x_{min} < 1 - \sqrt[n]{1-\alpha}$

$\alpha_1 = \alpha$

$W = P(\vec{X}_n \in G \mid H_1) = P(X_{min} < C \mid H_1) \;=$

$$\left\{ \begin{array}{l} H_1 = \xi \sim p(x) = \frac{e}{e-1} e^{-x} \{(0,1)\} \\ F_1(x) = \int_0^x \frac{e}{e-1} e^{-t} dt = \frac{-e}{e-1} e^{-t} \Big|_0^x = \frac{e(1-e^{-x})}{e-1} \end{array} \right\} =$$

$$= 1 - \left(1 - \left(\frac{e}{e-1}\right)\left(1 - e^{-c}\right)\right)^n$$

$\alpha_2 = 1 - W$, $\quad W \xrightarrow[n \to \infty]{} 1 \Rightarrow$ состоят.

$$c = 1 - (1-\alpha)^{\frac{1}{n}} = 1 - e^{\frac{1}{n}\ln(1-\alpha)} =$$

$$= 1 - 1 - \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)$$

$$e^{-c} = e^{\frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)} = 1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)$$

$$W = 1 - \left(1 + \frac{e}{e-1}\ln(1-\alpha)\frac{1}{n} + o\left(\frac{1}{n}\right)\right)^n \xrightarrow[n\to\infty]{}$$

$$\xrightarrow[n\to\infty]{} 1 - e^{\frac{e\ln(1-\alpha)}{e-1}} = 1 - (1-\alpha)^{\frac{e}{e-1}} \neq 1 \Rightarrow \text{не состоят}$$

при $\alpha = 0{,}05 \Rightarrow W = 0{,}0779$