Stat. 12. UE

(1.)

	μ_1, σ_1 Schizophrenia	μ_2, σ_2 Normal
Sample size	41	49
Mean time	104,23	62,24
Sd	62,24	16,34

a) Parameter of interest: $\mu_1 - \mu_2$

b) $H_0: \mu_1 = \mu_2$ $\quad H_1: \mu_1 - \mu_2 > 0$

c) $\alpha = 0,01$, $\quad p\text{-value} = 0,001 < \alpha \Rightarrow$ we can reject H_0

d) Find 99% confidence interval for the target parameter:

We have independent samples with size ≥ 30, so our CI reads

$$\bar{x}_1 - \bar{x}_2 \pm z_{1-\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \approx (16,2;\ 67,7) \subseteq \mathbb{R}^+.$$

Since $0 \notin CI$, this supports the conjecture that $\mu_1 - \mu_2 > 0$.

(2.) New method: 80, 76, 70, 80, 66, 85, 79, 71, 81, 76 $\quad \mu_1$

Old method: 79, 73, 72, 62, 76, 68, 70, 86, 75, 68, 73, 66 $\quad \mu_2$

a) Calculate 95% CI for $\mu_1 - \mu_2$: Independent samples, small sample size, we assume unequal variance.

We use R and get $CI \approx [-1,36;\ 9,49]$.

b) If we repeat the experiment and compute the 95% CI every time, the true value will be contained within the CI 95% of the time.

c) Small Sample ✓ Both populations are normally distributed (✓)

(3.)

	μ_1, σ_1 Blacks	μ_2, σ_2 Whites
Sample size	55	159
Mean pain intensity	8,2	6,9

(a) Why dangerous to draw statistical inference from the summarized data?

We don't have an information about the standard deviation, so we can't make a reasonable test or calculate a CI.

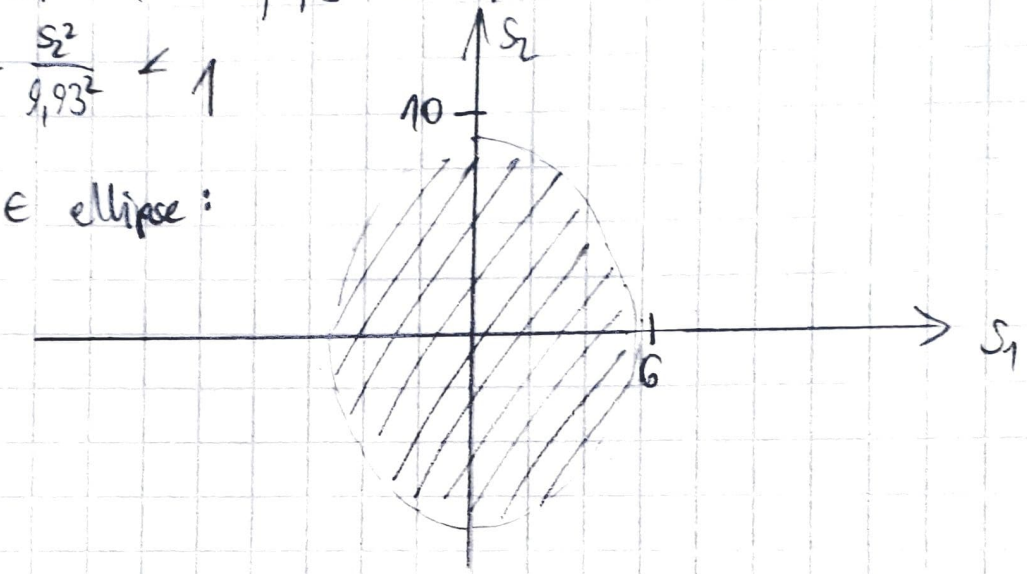
(b) What values of s_1/s_2 would lead to conclude that (at $\alpha = 0,05$) Blacks > Whites? $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 > \mu_2$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{55} + \frac{s_2^2}{159}}} = \frac{1,3}{\sqrt{\frac{s_1^2}{55} + \frac{s_2^2}{159}}} \approx_{H_0} \mathcal{N}(0,1) \rightsquigarrow \text{reject } H_0 \text{ if } z > z_{1-\alpha} \approx 1,64$$

$$\frac{1{,}3}{\sqrt{\dots}} > 1{,}64 \iff 0{,}79 > \sqrt{\dots} \overset{\approx}{\iff} 0{,}62 > \frac{S_1^2}{55} + \frac{S_2^2}{159} \overset{\wedge}{\iff} \frac{S_1^2}{34{,}1} + \frac{S_2^2}{98{,}58} < 1$$

$$\iff \frac{S_1^2}{5{,}84^2} + \frac{S_2^2}{9{,}93^2} < 1$$

$\iff (S_1, S_2) \in$ ellipse:



(c) What values of S_1/S_2 would lead to an inconclusive decision (at $\alpha = 0{,}05$)?

The values outside of the ellipse as $Z < z_{1-\alpha} \iff (S_1, S_2) \notin$ ellipse.

5.)

Subject	x_1 Before	x_2 After
1	10	18
2	16	19
3	7	11
4	4	3
5	7	5
6	2	3

Small, paired sample; we assume unequal variances.

$H_0: \mu_1 - \mu_2 = 0$ $\quad H_1: \mu_1 - \mu_2 < 0$

$\overset{R}{\rightsquigarrow} z = \frac{\bar{d}}{s_1/\sqrt{n}} \approx -0{,}242 > -2{,}02 = t_{n-1;\,0{,}05}$, so we cannot reject H_0 at level 0,05.

$[z \approx_{H_0} t_{n-1}]$

Possible issue: Very small sample size.