

Beendet am

Verbrauchte Zeit

Bewertung

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Status

Beendet

1 Stunde 29 Minuten

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Unfortunately you did not pass the exam. Best of luck next time!

Donnerstag, 17. Juni 2021, 13:34

Donnerstag, 17. Juni 2021, 15:04

Frage 1

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Student's one-sample 99%-confidence interval is evaluated on  $n$  data and it overlaps a claimed parameter  $\mu_0$ . Let  $q$  be the 99.5%-quantile of the  $t(n-1)$ -distribution. It holds that

Wählen Sie eine Antwort:

☐ a. the null hypothesis  $H_0 : \mu = \mu_0$  of Student's (two-sided one-sample)  $t$ -test is not rejected at 95% significance level

☒ b. the variance of the sum of two independent and  $t(n-1)$ -distributed random variables is larger than the sum of their variances

☐ c. the distance of the mean of the data and  $\mu_0$  is larger than  $q$  times the standard error of the mean

☐ d. the distance of the mean of the data and  $\mu_0$  is smaller or equals than  $q$  times the standard error of the mean

Frage 2

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Suppose that the distribution of  $Y$  conditional on  $X = x$  is  $\mathcal{N}(x, x)$  and that the marginal distribution of  $X$  is uniform on  $(-1, 1)$ . Compute the variance of  $Y$ .

Wählen Sie eine Antwort:

☒ a.  $\frac{2}{3}$

☐ b.  $\frac{5}{4}$

☐ c.  $\frac{1}{3}$

☐ d.  $\frac{5}{12}$

Frage 3

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Let  $X_1, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu = 10$  and variance  $\sigma^2 = 100$ . Let  $T = c \sum_{i=1}^n (X_i - 10)^2$  have a  $\chi^2$  distribution with  $d$  degrees of freedom. Then  $c$  and  $d$  are

Wählen Sie eine Antwort:

☐ a.  $c = 0.1, d = n$

☐ b. none of the rest

☐ c.  $c = 0.01, d = n - 1$

☒ d.  $c = 0.1, d = n - 1$

Frage 4

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

In the situation of a two-sided one-sample t-test we find  $\bar{x} = 10$ ,  $s^2 = 36$  and  $n = 9$ . For a given significance level we find the rejection region  $R = (-\infty, -2.2] \cup [2.2, \infty)$ . Then for the null hypothesis  $H_0 : \mu = 5$  it holds

Wählen Sie eine Antwort:

☐ a. we reject  $H_0$ , and we would also reject for any smaller significance level

☒ b. we do not reject  $H_0$ , but we would reject if only the significance level was chosen large enough

☐ c. we do not reject  $H_0$ , but we would reject if only the significance level was chosen small enough

☐ d. we reject  $H_0$ , and we would also reject for any larger significance level

Frage 5

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Two features of a novel operating system are compared using a two-sample  $t$ -test. The statistics for the first feature are  $\bar{x} = 15$ ,  $s_x^2 = 55$  and  $n_x = 5$  and those for the second feature are  $\bar{y} = 18$ ,  $s_y^2 = 10$  and  $n_y = 4$ . The rejection region is given through  $SR = (-\infty, -q] \cup [q, \infty)$ . Then it holds

Wählen Sie eine Antwort:

☐ a. we do neither reject for  $q = 2.5$  nor for  $q = 1.5$

☐ b. we reject for both  $q = 2.5$  and  $q = 1.5$

☒ c. we do not reject for  $q = 2.5$  but for  $q = 1.5$

☐ d. we reject for  $q = 2.5$  but not for  $q = 1.5$

Frage 6

Vollständig

Erreichte Punkte

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Frage markieren

A plumbing contractor obtains 60% of her boiler circulators from a company whose defect rate is 0.005, and the rest from a company whose defect rate is 0.01. What proportion of the circulators can be expected to be defective? If a circulator is defective, what is the probability that it came from the first company?

☐ a. 0.007 and 0.571

☐ b. 0.034 and 0.118

☐ c. 0.034 and 0.882

☒ d. 0.007 and 0.429

Frage 7

Vollständig

Erreichte Punkte

5 von 5

Frage markieren

Let  $S_n$  denote the number of heads obtained in  $n$  independent tosses of a fair coin. Using the Chebyshev inequality, the smallest value of  $n$  such that  $P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| < 0.1\right) \geq \frac{3}{4}$  is

Wählen Sie eine Antwort:

☐ a. 400

☒ b. 100

☐ c. 200

☐ d. 300

Frage 8

Vollständig

Erreichte Punkte

5 von 5

Frage markieren

For a statistical test of significance level  $\alpha$  it holds

Wählen Sie eine Antwort:

☐ a. rejection at level  $\alpha$  implies rejection at level  $\alpha/2$

☐ b. the rejection area does not depend on  $\alpha$

☐ c. the rejection area does not depend on the distribution of the test statistic under the null hypothesis

☒ d. the rejection area shrinks when  $\alpha$  is decreased

Frage 9

Vollständig

Erreichte Punkte

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Frage markieren

Let  $X$  be a random variable with the probability density function

$$f_X(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{else} \end{cases}$$

If  $Z = -\ln X$  is a random variable, compute the median of  $Z$ .

The following table of approximate values of the natural logarithm is useful

x	1	1.5	2	4	8
$\approx \ln(x)$	0	0.4	0.7	1.4	2.1

Wählen Sie eine Antwort:

☒ a. It is smaller than 0.5

☐ b. It is bigger than 0.85

☐ c. It is smaller than 0.85 but bigger than 0.5

☐ d. It is not unique

Frage 10

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Three (fair six-sided) die are rolled. Which R-command approximates the expected number of outcome '6'?

Wählen Sie eine Antwort:

☒ a. `sum(rbinom(1000,3,1/6))/3000`

☐ b. `sum(rbinom(1000,3,1/6))/1000`

☐ c. `sum(rbinom(1000,3,1/6) + rbinom(1000,3,1/6) + rbinom(1000,3,1/6))/1000`

☐ d. `sum(rbinom(1000,3,1/2) + rbinom(1000,3,1/2) + rbinom(1000,3,1/2))/1000`

Frage 11

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

In a linear regression model ( $y_i$  modeled as a linear function of  $x_i$  plus error) the parameters are estimated via least squares. For the mean and the empirical standard deviation of the  $x$  and  $y$  values we obtain  $\bar{x} = 3$ ,  $s_x = 4$ ,  $\bar{y} = 7$  and  $s_y = 3$ . It holds that

Wählen Sie eine Antwort:

☒ a. the regression line goes through  $(3, 4)$

☐ b. the slope of the regression line is larger or equals  $-3/4$

☐ c. the regression line goes through  $(7, 3)$

☐ d. the slope of the regression line is smaller than  $-3/4$

Frage 12

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Regarding the data 21, 22, 9, 3, 5 it holds

Wählen Sie eine Antwort:

☐ a. the set of 50%-quantiles is [9, 21]

☐ b. the 1/5-quantile is not unique

☒ c. 22 is not a 5/6-quantile

☐ d. 8 is a median

Frage 13

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Five groups are compared with an ANOVA. The size of the  $j$ th group is 10 if  $j$  is even, and 20 if  $j$  is odd, for  $j = 1, 2, \dots, 5$ . Let  $f$  denote the Fisher-statistic calculated on the data. The following table shows the 99%-quantiles of the  $\mathcal{F}(df_1, df_2)$ -distribution.

	$df_1$					
	4	5	6	7	8	
$df_2$	55	3.68	3.37	3.15	2.98	2.85
	60	3.65	3.34	3.12	2.95	2.82
	65	3.62	3.31	3.09	2.93	2.80
	70	3.60	3.29	3.07	2.91	2.78
	75	3.58	3.27	3.05	2.89	2.76
	80	3.56	3.26	3.04	2.87	2.74

From the given information we conclude that

Wählen Sie eine Antwort:

☒ a. For  $f = 3.62$  we do not reject the null hypothesis on the 1%, but we do not know of whether we reject it on the 5%-level

☐ b. For  $f = 3.34$  we do not reject the null hypothesis on the 0.1%-level, and we know of whether we reject it on the 1%-level

☐ c. For  $f = 3.34$  we do not reject the null hypothesis on the 1%-level, but we do not know of whether we reject it on the 0.1%-level

☐ d. For  $f = 3.62$  we do reject the null hypothesis on the 1%-level, but we do not know of whether we reject it on the 5%-level

Frage 14

Vollständig

Erreichte Punkte

5 von 5

Frage markieren

Let  $X$  and  $Y$  be two independent random variables with moment generating functions  $M_X(t) = e^{t^2+2t}$  and  $M_Y(t) = e^{3t^2+t}$ . Then, the moment generating function of  $X + 2Y$  is

Wählen Sie eine Antwort:

☐ a.  $e^{\frac{1}{3}(t^2+t)}$

☐ b.  $e^{7t^2+4t}$

☒ c.  $e^{13t^2+4t}$

☐ d.  $2e^{4t^2+3t}$

Frage 15

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Let  $X_1, X_2, \dots, X_{81}$  be an i.i.d. sample from a population with population mean  $\mu = 5$  and population variance  $\sigma^2 = 4$  and let  $S = X_1 + X_2 + \dots + X_{81}$ . Approximate the probability  $P(S \notin [387, 423])$  using the Central limit theorem.

Wählen Sie eine Antwort:

☐ a. 32%

☐ b. 68%

☒ c. 5%

☐ d. 95%

Frage 16

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Let  $X_1, \dots, X_n$  be a random sample from  $\mathcal{N}(0, \theta^2)$  distribution, where  $\theta > 0$  is unknown. Let  $T_1 = \sum_{i=1}^n X_i$  and  $T_2 = \sum_{i=1}^n X_i^2$ .

Which one of the following statements for the statistics  $Y_1 = \frac{T_1^2}{n^3}$  and  $Y_2 = \frac{T_2}{2n}$  is correct?

Wählen Sie eine Antwort:

☐ a.  $Y_1$  is not an unbiased estimator of  $\theta^2$  and  $Y_2$  is an unbiased estimator of  $\theta^2$

☐ b. both  $Y_1$  and  $Y_2$  are unbiased estimators of  $\theta^2$

☐ c. neither  $Y_1$  nor  $Y_2$  are unbiased estimators of  $\theta^2$

☒ d.  $Y_1$  is an unbiased estimator of  $\theta^2$  and  $Y_2$  is not an unbiased estimator of  $\theta^2$

Frage 17

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Let  $X_1, \dots, X_n$  be a random sample from a Poisson  $Poi(\lambda)$  population, where  $\lambda > 0$  is unknown. The Cramer-Rao lower bound for the variance of any unbiased estimator  $g(\lambda) = \lambda e^{-\lambda}$  equals

Wählen Sie eine Antwort:

☐ a.  $\lambda(1-\lambda)e^{-2\lambda}$

☐ b.  $\frac{1}{n}\lambda(1-\lambda)e^{-2\lambda}$

☒ c.  $\lambda(1-\lambda)^2e^{-\lambda}$

☐ d.  $\frac{1}{n}\lambda(1-\lambda)^2e^{-2\lambda}$

Frage 18

Vollständig

Erreichte Punkte

5 von 5

Frage markieren

Let  $\mathcal{X} = (X_1, X_2) \sim mult(2, p)$  with  $p = (1, 0)$ . Which statement is **not** correct?

Wählen Sie eine Antwort:

☐ a.  $\mathbb{E}[X_1] = 2$  and  $\mathbb{Var}(X_1) = 0$

☐ b.  $P(X_1 = 2) = 1$

☒ c.  $P(\mathcal{X} = (1, 1)) = \binom{2}{1,1}(1/2)^1 \cdot (1/2)^1$

☐ d.  $X_1 \sim b(2, 1)$

Frage 19

Vollständig

Erreichte Punkte

5 von 5

Frage markieren

Let  $X_1, \dots, X_n$  be a random sample from a population with pdf

$$f(x|\theta) = \begin{cases} 3\theta x^2 e^{-\theta x^3}, & x > 0 \\ 0, & \text{otherwise} \end{cases},$$

with unknown  $\theta > 0$ . Then the maximum likelihood estimator of  $\theta$  is

Wählen Sie eine Antwort:

☐ a.  $\frac{\sum_{i=1}^n X_i^3}{n}$

☐ b.  $\frac{n}{\sum_{i=1}^n X_i^3}$

☐ c.  $\frac{\sum_{i=1}^n X_i^3}{n}$

☒ d.  $\frac{n}{\sum_{i=1}^n X_i^3}$

Frage 20

Vollständig

Erreichte Punkte

0 von 5

Frage markieren

Let  $X$  and  $Z$  be independent random variables and both  $\mathcal{N}(0, 1)$ -distributed. Let  $Y = \frac{1}{2}(X + Z)$ . Then the correlation between  $X$  and  $Y$  is

Wählen Sie eine Antwort:

☐ a.  $1/4$

☐ b.  $\sqrt{2}/2$

☒ c.  $1/2$

☐ d. 0