(4) **Box with coins**

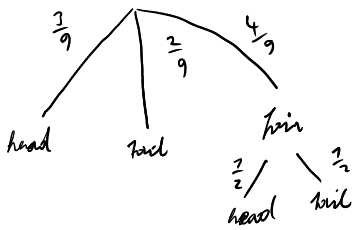
A box contains three coins with a head on each side, two coins with a tail on each side, and four fair coins.

(a) One of these nine coins is selected at random and tossed once. What is the probability of getting a tail?

(b) If we get a tail, what is the probability that the selected coin has a tail on both side? If we get a tail, what is the probability that it is a fair coin?

(c) If the first toss is tail, and another coin is selected at random from the remaining eight coins and tossed once, what is the probability of getting a tail again?

a) $\mathbb{P}(\text{tail}) = \frac{2}{9} + \frac{4}{9}\frac{1}{2} = \frac{4}{9}$



b) $\mathbb{P}(\text{tail on both sides} \mid \text{tail}) = \frac{\mathbb{P}(\text{tail on both sides} \cap \text{tail})}{\mathbb{P}(\text{tail})}$

$= \frac{\frac{2}{9}}{\frac{4}{9}} = \frac{1}{2}$

c) $\mathbb{P}(\text{second is tail} \mid \text{first was tail}) = \frac{\mathbb{P}(\text{first \& second tail})}{\mathbb{P}(\text{first tail})}$

$= \frac{\mathbb{P}(\text{first only tail})\,\mathbb{P}(\text{second tail} \mid \text{first only tail}) + \mathbb{P}(\text{first fair \& tail})\,\mathbb{P}(\text{second tail} \mid \text{first fair \& tail})}{\mathbb{P}(\text{first tail})}$

$= \frac{\frac{2}{9}\left(\frac{1}{8} + \frac{4}{8}\frac{1}{2}\right) + \frac{4}{9}\frac{1}{2}\left(\frac{2}{8} + \frac{3}{8}\frac{1}{2}\right)}{\frac{2}{9} + \frac{4}{9}\frac{1}{2}}$

$= \frac{\frac{2}{9}\frac{3}{8} + \frac{2}{9}\frac{7}{16}}{\frac{4}{9}} = \frac{\frac{2}{9}}{\frac{4}{9}}\left(\frac{6}{16} + \frac{7}{16}\right) = \frac{1}{2}\frac{13}{16} = \frac{13}{32}$