### Assignment 8

**▶** Read the assignment carefully! Remember that the first line of a script must be the call to the script **preamble**.

#### A. Mandatory

A highly simplified model of population dynamics is the Verhulst-Pearl model:

$$y' = k y (1 - y/M),$$

where k is the growth rate, and M is the population limit (carrying capacity).

Write a Matlab script with the name Assignment08A\_IDxx.m<sup>1</sup> that solves the following problems, using dsolve and symbolic computation.

- 1. Find and display the general solution y(t) that is not a constant. No hard coding!
- 2. Find, display and plot the particular solution  $y_1(t)$  with k = 0.025, M = 12000 and  $y_1(0) = 600$  in the time interval  $0 \le t \le 300$ .
- 3. Compute the time  $t_1$  at which the population size  $y_1(t)$  surpasses q percent of the carrying capacity M, with q = 90. Show  $t_1$  converted to double in the command window and by a vertical line in the plot.
- 4. Compute the time  $t_2$  at which the instantaneous growth rate (first derivative) of  $y_1(t)$  attains its maximum. Show  $t_2$  converted to double in the command window and by a vertical line in the plot.

Add axis labels, a title, and a comprehensive legend to the plot. (3 pt)

# B. Mandatory

Write a MATLAB script with the name Assignment08B\_IDxx.m<sup>1</sup> that computes the numerical solutions to the second-order nonlinear ODE called *van der Pol's differential equation*:

$$y'' - \mu(1 - y^2)y' + y = 0,$$

with the initial conditions y(0) = 2, y'(0) = 0, for both  $\mu = 1$  and  $\mu = 10$ . Create a figure and plot the solution y(t) for  $\mu = 1$  in the interval  $0 \le t \le 20$  and for  $\mu = 10$  in the interval  $0 \le t \le 100$ , in two subplots of the figure. (2 pt)

# **▶** Continued on page 2!

<sup>&</sup>lt;sup>1</sup>xx is your (group's) ID number

#### C. Optional

Write a Matlab script with the name Assignment08C\_IDxx.m<sup>2</sup> that computes a numerical solution to the second-order linear ODE called *Mathieu's differential equation*:

$$y'' + [a - 2q\cos(2t)]y = 0,$$

in the interval  $0 \le t \le 100$ , for a = 1.25, q = 0.25 and the initial conditions y(0) = 0, y'(0) = 2. Plot the solution y(t) and its derivative y'(t) superimposed in the same figure.

(2 pt)

▶ Make sure that the relevant results and *only* those are shown in the output to the command window. Submit the script(s) until 5pm on May 26, 2021. Don't forget to put your partner in cc! Any violation of the naming convention will lead to the rejection of the submission.

<sup>&</sup>lt;sup>2</sup>xx is your (group's) ID number