(4) **Mechanics**

In order to compare the means of two populations, independent random samples of 400 observations are selected from each population, with the following results:

| Sample 1 | Sample 2 |
|---|---|
| $\bar{x}_1 = 5,275$ | $\bar{x}_2 = 5,240$ |
| $s_1 = 150$ | $s_2 = 200$ |

(a) Use a 95% confidence interval to estimate the difference between the population means $(\mu_1 - \mu_2)$. Interpret the confidence interval.

(b) Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 0$ versus the alternative hypothesis $H_1 : (\mu_1 - \mu_2) \neq 0$. Give the $p$-value of the test, and interpret the result.

(c) Suppose the test in the previous part were conducted with the alternative hypothesis $H_1 : (\mu_1 - \mu_2) > 0$. How would your answer change?

(d) Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 25$ versus the alternative $H_1 : (\mu_1 - \mu_2) \neq 25$. Give the $p$-value, and interpret the result. Compare your answer with that obtained from the test conducted in part (b).

(e) What assumptions are necessary to ensure the validity of the inferential procedures applied in parts (a)-(d)?

We approximate the distance by $\hat{\mu} := \overline{X_1 - X_2} = \bar{X}_1 - \bar{X}_2 = \frac{35}{100} = \frac{7}{20}$

Since $n_1 = n_2 = 400 \geq 30$ is large we use the procedure from slide 32 in lecture 11.

Assuming that $X_1$ is a sample of $N(\mu_1, \sigma_1^2)$ and $X_2$ a sample of $N(\mu_2, \sigma_2^2)$ random variables, we obtain approximately $Z_\mu := (\bar{X}_1 - \bar{X}_2 - \mu)\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{-1/2} \sim N(0,1)$

a) The formula for the confidence interval was derived on slide 32 of lecture 11.

The confidence interval is given by $\left[\bar{X}_1 - \bar{X}_2 + z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}},\ \bar{X}_1 - \bar{X}_2 - z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}\right)$

which is approximately $[10{,}5, 59{,}5)$, an interval centered around 35.

b) $2\,\mathbb{P}\left(Z_\mu < -\hat{\mu}\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{-1/2}\right) \approx 0.005$

c) $\mathbb{P}\left(Z_\mu < -\hat{\mu}\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{-1/2}\right) \approx 0.0025$

$\Big\}$ We reject $H_0$

d) $2\,\mathbb{P}\left(Z_\mu < -(\hat{\mu} - 25)\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{-1/2}\right) \approx 0.424 \rightarrow$ We do not reject $H_0$.

e) Assumptions were already stated in the beginning.