

(2) **Coin game**

Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins. Assume player A flips first. Suppose that $P(\text{head}) = p$, not necessarily $\frac{1}{2}$. What is the probability that the player B wins?

$$P(\text{B wins}) = \sum_{n=0}^{\infty} (1-p)^{2n+1} p = p(1-p) \sum_{n=0}^{\infty} ((1-p)^2)^n = p(1-p) \frac{1}{1-(1-p)^2} = \frac{p(1-p)}{2p-p^2} = \frac{1-p}{2-p}$$