(2) **Confidence interval 1**
In the June 1986 issue of Consumer Reports, some data on the calorie content of beef hot dogs is given. Here are the numbers of calories in 20 different hot dog brands:

$$186, 181, 176, 149, 184, 190, 158, 139, 175, 148, 152, 111, 141, 153, 190, 157, 131, 149, 135, 132.$$

Assume that the numbers are from a normal distribution with mean μ and variance σ^2, both unknown. Use `R` to obtain a 90% confidence interval for the mean number of calories μ.

We can estimate the mean $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$ and the variance $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

The confidence interval is given by $[\hat{\mu}-\delta, \hat{\mu}+\delta)$, where $\mathbb{P}(\hat{\mu}-\delta \leq X < \hat{\mu}+\delta) = \alpha$ with $X \sim \mathcal{N}(\hat{\mu}, \hat{\sigma}^2)$ and $1-\alpha = \frac{9}{10}$ or $\alpha = \frac{1}{10}$. We have

$$1-\alpha = \mathbb{P}(\hat{\mu}-\delta \leq X < \hat{\mu}+\delta) = 1-2\,\mathbb{P}(X < \hat{\mu}-\delta) = 1-2\,\mathbb{P}\left(\frac{(X-\hat{\mu})\sqrt{n}}{\hat{\sigma}} < -\frac{\delta\sqrt{n}}{\hat{\sigma}}\right)$$

$$\Leftrightarrow \phi\left(-\frac{\delta\sqrt{n}}{\hat{\sigma}}\right) = \frac{\alpha}{2} \Leftrightarrow -\frac{\delta\sqrt{n}}{\hat{\sigma}} = \phi^{-1}\left(\frac{\alpha}{2}\right) \Leftrightarrow \delta = -\frac{\hat{\sigma}}{\sqrt{n}}\phi^{-1}\left(\frac{\alpha}{2}\right)$$

Using R, we obtain $\hat{\mu} \approx 156,85$ and $\hat{\sigma} \approx 22,642$ and $\delta \approx 8,328$

Hence, $\hat{\mu}-\delta \approx 148,522$ and $\hat{\mu}+\delta \approx 165,178$