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Numerik von Differentialgleichungen - Kreuzlübung 10

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Exercise 46:
Write a program that plots the stability region of an arbitrary linear multi-step method. Test your implementation with some of the implicit and explicit methods you have encountered so far. In particular, you should test with Adams-Bashforth methods (Example 5.5), Adams-Moulton methods (Example 5.6) und BDF methods (Exercise 41) of different order.

Hint: You can use a computer algebra system. Alternatively, you can compute the roots of a polynomial numerically by computing the eigenvalues of the corresponding companion matrix.

Exercise 47:
Solve the Hamiltonian system from Example 6.4 of the lecture notes numerically with the RK4 method from Example 2.25, the implicit midpoint rule from Example 3.5, and a symplectic Euler method from Example 6.29. Look at the Hamiltonian of the numerical solution over long periods. Explain the results.

Exercise 48:
Prove that the N-body problem from Example 6.5 is, in fact, a Hamiltonian system. Furthermore, prove the statement of Exercise 6.6.

Exercise 49:
Let $H \in C^2(\mathbb{R}^{2d};\mathbb{R})$ be a Hamiltonian and $\Psi \in C^1(\mathbb{R}^{2d};\mathbb{R}^{2d})$ a diffeomorphism, i.e., the inverse mapping Ψ^{-1} is in $C^1(\mathbb{R}^{2d};\mathbb{R}^{2d})$ as well. Let further Ψ be symplectic.

Show the following statement: If $y \in C^1([0,T];\mathbb{R}^{2d})$ is a solution to the Hamiltonian system

$$y' = J^{-1}\nabla H(y) \tag{1}$$

like in Remark 6.2 of the lecture notes, then $u := \Psi \circ y$ is the solution of the Hamiltonian system with Hamiltonian $K := H \circ \Psi^{-1}$.

Exercise 50:
Show that all explicit, consistent and linear multi-step methods of the form (5.2) preserve linear invariants, if the initial values are chosen suitably.