(2) **Shock absorbers**

A manufacturer of automobile shock absorbers was interested in comparing the durability of its shocks with that of the shocks produced by its biggest competitor. To make the comparison, one of the manufacturer's and one of the competitor's shocks were randomly selected and installed on the rear wheel of each of six cars. After the cars had been driven 20000 miles, the strength of each test shock was measured, coded, and recorded. Results are shown in the table.

Car number	Manufacturer's shock	Competitor's shock	$x-y=:d$
1	8.8	8.4	0.4
2	10.5	10.1	0.4
3	12.5	12.0	0.5
4	9.7	9.3	0.4
5	9.6	9.0	0.6
6	13.2	13.0	0.2
			mean: $25/60$

Do these data present sufficient evidence to conclude there is a difference in the mean strength of the two types of shocks after 20000 miles of use?

The samples are paired and the sample size $n=6<30$ is small. We use the test on slide 31 of lecture 11. We assume that $D_i := X_i - Y_i \sim \mathcal{N}(\mu_d, \sigma_d^2)$.

We have to use approximations $\hat{\mu}_d := \overline{x-y} = {}^{25}/_{60} = {}^{5}/_{12}$ and $\hat{\sigma}_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left((x_i-y_i)-{}^{5}/_{12}\right)^2 \approx 0{,}133$

We have $\bar{D} \sim \mathcal{N}\left(\mu_d, \frac{\sigma_d^2}{n}\right)$. We test the Hypothesis $H_0: \mu_d = 0$ versus $H_1: \mu_d > 0$.

Our test statistic is $Z_{\mu_d} = \frac{(\bar{D}-\mu_d)\sqrt{n}}{\hat{\sigma}_d} \sim t(n-1)$

We obtain

$$\begin{aligned}
\mathbb{P}(\bar{D} \geq \hat{\mu}_d \mid \mu_d = 0) &= \mathbb{P}\left(\frac{\bar{D}\sqrt{n}}{\hat{\sigma}_d} \geq \frac{\hat{\mu}_d\sqrt{n}}{\hat{\sigma}_d} \,\middle|\, \mu_d = 0\right) \\
&= 1 - \mathbb{P}\left(\frac{\bar{D}\sqrt{n}}{\hat{\sigma}_d} < \frac{\hat{\mu}_d\sqrt{n}}{\hat{\sigma}_d} \,\middle|\, \mu_d = 0\right) \\
&= 1 - F_{Z_0}\left(\frac{\hat{\mu}_d\sqrt{n}}{\hat{\sigma}_d}\right) \approx 0{,}000299
\end{aligned}$$

Since this is a very small value, we reject H_0, hence we claim that the manufacturers shocks are more durable than the competitors shocks.