(2) **Sufficient statistic and point estimator statistics**
Let $X_1, \dots X_n$ be a random sample from a population with pdf

$$f(x|\theta) = \begin{cases} \frac{\theta}{x^2}, & \theta \leq x \\ 0, & \text{otherwise} \end{cases}$$

with unknown $\theta > 0$. Use the Factorization theorem to obtain a sufficient statistic for θ.

The likelihood is given by $L(x|\theta) = \begin{cases} \theta^n \prod_{i=1}^{n} x_i^{-2} & , \text{ if } \theta \leq \min\{x_i \mid 1 \leq i \leq n\} \\ 0 & , \text{ otherwise} \end{cases}$

$T(x) := \min\{x_i \mid 1 \leq i \leq n\}$ $\qquad$ $h(x) := \prod_{i=1}^{n} x_i^{-2}$

$$g(t|\theta) := \begin{cases} \theta^n & , \text{ if } \theta \leq t \\ 0 & , \text{ otherwise} \end{cases}$$

We have $L(x,\theta) = g(T(x)|\theta)\, h(x)$, hence $T(x)$ is a sufficient statistic for θ by the factorization theorem.