

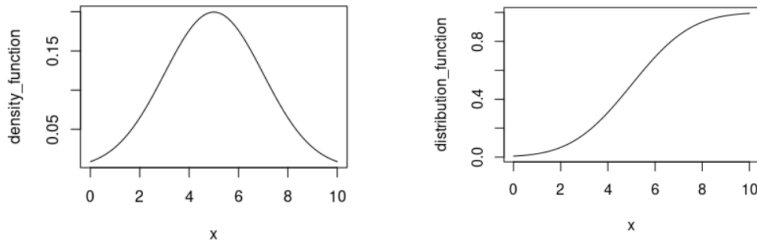
(4) **Sum and average**

Let X be a random variable with $\mathcal{N}(5, 2^2)$. Let X_1, X_2, \dots, X_{50} be independent identically distributed copies of X . Let S be their sum and \bar{X} their average, i.e.

$$S = X_1 + \dots + X_{50} \quad \text{and} \quad \bar{X} = \frac{1}{50}(X_1 + \dots + X_{50}).$$

- (a) Plot the density and the distribution function for X using R.
- (b) What are the expectation and the standard deviation of S and of \bar{X} ?
- (c) Generate a sample of 50 numbers from $\mathcal{N}(5, 2^2)$. Plot the histogram for this sample.
Do the same for a sample of 500 numbers from $\mathcal{N}(5, 2^2)$.

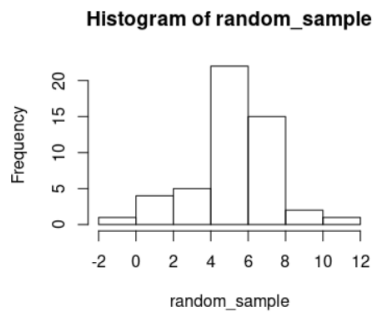
d)



b) By problem 1 we have $\bar{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n})$ and $S = n\bar{X} \sim \mathcal{N}(n\mu, n\sigma^2)$

Hence $E(\bar{X}) = \mu$, $\sqrt{\text{Var}(\bar{X})} = \frac{\sigma}{\sqrt{n}}$, $E(S) = n\mu$, $\sqrt{\text{Var}(S)} = \sqrt{n} \sigma$

2) $n=50$



$n=500$

