(3) **Missing Information**

An investigation of ethnic differences in reports of pain perception was presented at the annual meeting of the American Psychosomatic Society (Mar. 2001). A sample of 55 blacks and 159 whites participated in the study. Subjects rated (on a 13-point scale) the intensity and unpleasantness of pain felt when a bag of ice was placed on their foreheads for two minutes. (Higher ratings correspond to higher pain intensity.) A summary of the results is provided in the following table.

	Blacks	Whites
Sample Size	55	159
Mean pain intensity	8.2	6.9

(a) Why is it dangerous to draw a statistical inference from the summarized data? Explain.

(b) What values of the missing sample standard deviations would lead you to conclude (at $\alpha = 0.05$) that blacks, on average, have a higher pain intensity rating than whites?

(c) What values of the missing sample standard deviations would lead you to an inconclusive decision (at $\alpha = 0.05$) regarding whether blacks or whites have a higher mean intensity rating?

a) It is dangerous, because we were not provided the standard deviation of the sample. If the standard deviation was high, then it might be difficult to draw a statistical inference.

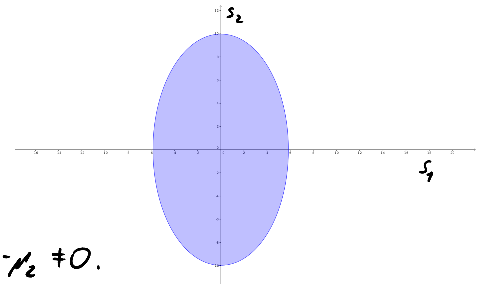
b) Since the sample sizes $n_1 = 55 \geq 30$ and $n_2 = 159 \geq 30$ are large, we choose the test on slide 37 of lecture 11. We test $H_0: \mu_1 - \mu_2 = 0$ vs. $H_1: \mu_1 - \mu_2 > 0$

$$\alpha \geq \mathbb{P}(\bar{X}_1 - \bar{X}_2 \geq \bar{x}_1 - \bar{x}_2) = 1 - \mathbb{P}\left(\frac{\bar{X}_1 - \bar{X}_2}{s} < \frac{\bar{x}_1 - \bar{x}_2}{s}\right) = 1 - \Phi\left(\frac{\bar{x}_1 - \bar{x}_2}{s}\right)$$

$$\Leftrightarrow \Phi\left(\frac{\bar{x}_1 - \bar{x}_2}{s}\right) \geq 1 - \alpha \Leftrightarrow \frac{\bar{x}_1 - \bar{x}_2}{s} \geq \Phi^{-1}(1-\alpha) \Leftrightarrow s \leq \frac{\bar{x}_1 - \bar{x}_2}{\Phi^{-1}(1-\alpha)} \approx 0.7903$$

where $s := \sqrt{s_1^2/n_1 + s_2^2/n_2}$

Since s_1 and s_2 are positive, only these values in the ellipse are possible for s_1 and s_2.



c) Here it seems more reasonable to test $H_0: \mu_1 - \mu_2 = 0$ vs. $H_1: \mu_1 - \mu_2 \neq 0$.

We want

$$\alpha = \mathbb{P}(|\bar{X}_1 - \bar{X}_2| \geq \bar{x}_1 - \bar{x}_2) = \mathbb{P}(\bar{X}_1 - \bar{X}_2 < -(\bar{x}_1 - \bar{x}_2)) + \mathbb{P}(\bar{X}_1 - \bar{X}_2 \geq \bar{x}_1 - \bar{x}_2) = 2\,\Phi\left(\frac{-(\bar{x}_1 - \bar{x}_2)}{s}\right)$$

$$\Leftrightarrow \frac{-(\bar{x}_1 - \bar{x}_2)}{s} = \Phi^{-1}\left(\frac{\alpha}{2}\right) \Leftrightarrow s = \frac{-(\bar{x}_1 - \bar{x}_2)}{\Phi^{-1}(\frac{\alpha}{2})} \approx 0.663$$