Stat. UE 11

(1.) $X_1,..,X_n$ iid $\sim \mathcal{N}(\mu, \sigma^2)$, $H_0: \mu = \mu_0$.

a) Power of the left-sided z-test: $H_1: \mu < \mu_0$.

$z = \frac{\overline{X_n} - \mu_0}{\sigma/\sqrt{n}} \approx_{H_0} \mathcal{N}(0,1)$. We reject H_0 at level α if $z < z_\alpha$.

Test power: $\mathbb{P}_\mu(\text{reject } H_0) = \mathbb{P}_\mu\left(\frac{\overline{X_n} - \mu_0}{\sigma/\sqrt{n}} < z_\alpha\right) = \mathbb{P}_\mu\left(\underbrace{\frac{\overline{X_n} - \mu}{\sigma/\sqrt{n}}}_{\sim \mathcal{N}(0,1)} < z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}\right)$

$= \Phi\left(z_\alpha + \overbrace{\frac{\mu_0 - \mu}{\sigma/\sqrt{n}}}^{>0}\right)$.

b) Impact on the test power: The test power is a monotonously increasing function of μ_0, n, α and a m. decreasing function of μ and σ.

(2.) Data 1: 8,8 10,5 12,5 9,7 9,6 13,2
Data 2: 8,4 10,1 12,0 9,3 9,0 13,0

We have paired samples and assume that the differences are $\sim \mathcal{N}(\mu_d, \sigma_d^2)$. $H_0: \mu_d = 0$.

We know that $t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} \approx_{H_0} t_{n-1}$; we choose $\alpha = 0{,}05$ and decide to reject H_0 if $|t| > t_{1-\alpha/2, n-1} \Leftrightarrow 7{,}68... > 2{,}57... \Leftrightarrow T$.

The p-value is $\mathbb{P}(|t| > 7{,}68...) = F_{t_{n-1}}(-7{,}68...) + (1 - F_{t_{n-1}}(7{,}68...)) \approx 0{,}0006$, which supports our conjecture that $\mu_d \neq 0$.

(4.) Independent samples: $\begin{cases} \overline{X_1} = 5.275 \\ S_1 = 150 \end{cases}$, $\begin{cases} \overline{X_2} = 5.240 \\ S_2 = 200 \end{cases}$, $n_1 = n_2 = 400$. [L 11/32]

(a) Use 95% CI to estimate $\mu_1 - \mu_2$: $\overline{X_1} - \overline{X_2} \pm z_{1-\alpha/2}\sqrt{\frac{S_1^2 + S_2^2}{n}} \approx [10{,}5; 59{,}4]$

Interpretation: If we repeat the experiment and calculate the CI every time, it will contain the true value in 95% of the experiments.

(b) Test $H_0: \mu_1 - \mu_2 = 0$ vs. $H_1: \mu_1 - \mu_2 \neq 0$:

$z = \frac{\overline{X_1} - \overline{X_2} - 0}{\sqrt{\frac{S_1^2 + S_2^2}{n}}} \approx_{H_0} \mathcal{N}(0,1)$. Reject if $|z| > z_{1-\alpha/2} \Leftrightarrow 2{,}8 > 1{,}96 \Leftrightarrow T$.

p-value $= \mathbb{P}_{H_0}(|z| > 2{,}8...) = 2\Phi(-2{,}8) = 0{,}005$.

Interpretation: If H_0 is true, the probability of obtaining a result at least as extreme as the result actually observed is $\approx 0{,}5\%$.

(c) Test H_0 vs H_1: $\mu_1 - \mu_2 > 0$:

The p-value is now $\mathbb{P}_{H_0}(z > 2{,}8) = 1 - \Phi(2{,}8) \approx 0{,}0026$ which is smaller, so we reject H_0.

(d) Test H_0: $\mu_1 - \mu_2 = 25$ vs. $\mu_1 - \mu_2 \neq 25$. Compare to (b).

$$z = \frac{\bar{x}_1 - \bar{x}_2 - 25}{\sqrt{\frac{s_1^2 + s_2^2}{n}}} = 0{,}8 \quad \leadsto \quad |z| = 0{,}8 \not> 1{,}57 \Rightarrow \text{we don't reject } H_0.$$

$$p\text{-value} = \mathbb{P}_{H_0}(|z| > 0{,}8) = \Phi(-0{,}8) + (1 - \Phi(0{,}8)) \approx 0{,}42 > 0{,}05.$$

(e) What assumptions where necessary for (a) − (d)?

- independent samples
- $n \geq 30$
- only one test per data set
- choose test before data is observed

(5) [L 11/134]

1: HS:	131	74	129	96	92
2: HF:	44	70	69	43	53
3: FB:	15	14	21	29	21

a) HS vs. HF: We have independent samples.
We have sample size $5 < 30$ and assume unequal variances.

The 95% CI is $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}(\nu)\sqrt{\frac{s_1^2 + s_2^2}{n}} \approx [18{,}0;\ 79{,}2] \subseteq \mathbb{R}^+$.

b) the 95% CI for $\mu_2 - \mu_3$ is $\approx [19{,}8;\ 51{,}8] \subseteq \mathbb{R}^+$.

c) I would recommend the fist bump, but based on common sense and not on the experiment which was carried out with questionable methods (high five for 3 seconds).