This is the first homework assignment. The problems are to be presented on exercise session on **March 16, 2021**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions. The problems should be ticked and solution paths uploaded by **20:00 on March 15, 2021**.

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(1) **Card game**
A deck of 52 cards has 13 ranks $(2, 3, 4, 5, 6, 7, 8, 9, 10,$ J, Q, K, A) and 4 suits (♡, ♠, ♢, ♣). Three cards are drawn randomly without replacement from a deck of 52 cards.

(a) What is the probability that all three cards are in the same suit?

(b) What is the probability that all three cards have the same rank?

(c) What is the probability that the three cards contain exactly one pair (a pair means two cards with the same rank from two different suits)?

(2) **Coin game**
Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins. Assume player A flips first. Suppose that $P(\text{head}) = p$, not necessarily $\frac{1}{2}$. What is the probability that the player B wins?

(3) **Independence**
Let A and B be two independent events.

(a) Prove that A^c and B^c are also independent.

(b) If we additionally know that $P(A|B) = 0.6$ and $P(B|A) = 0.3$, compute the probabilities of the following two events

(i) at most one of A or B

(ii) either A or B but not both.

(4) **Box with coins**
A box contains three coins with a head on each side, two coins with a tail on each side, and four fair coins.

(a) One of these nine coins is selected at random and tossed once. What is the probability of getting a tail?

(b) If we get a tail, what is the probability that the selected coin has a tail on both side? If we get a tail, what is the probability that it is a fair coin?

(c) If the first toss is tail, and another coin is selected at random from the remaining eight coins and tossed once, what is the probability of getting a tail again?

(5) **Cumulative distribution function**
Let a cumulative distribution function (cdf) F of a continuous random variable Y be given by

$$F(y) = \begin{cases} 0, & y \leq 0 \\ \frac{2}{5}y, & 0 < y \leq 1 \\ ay - b, & 1 < y \leq 2 \\ 1, & y > 2 \end{cases},$$

where a and b are real constants.

(a) Find out the values of a and b.

(b) Write down the probability density function (pdf) of Y.

(c) What is the probability that an observed random variable Y is greater than 1.8, given that it is greater than 1?

(6) **A left-turn lane problem**
A civil engineer is studying a left-turn lane that is long enough to hold six cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(7 - x)$.

(a) Find the probability mass function (pmf) of X.

(b) Compute the probability that X will be at least 4.

(c) Calulate the expectation and standard deviation of X.
Note: `R` might be useful.