Stat. UE 11

(1.) $X_1, \dots, X_n$ iid $\sim \mathcal{N}(\mu, \sigma^2)$, $\quad H_0: \mu = \mu_0$.

(a) Power of the left-sided z-test: $\quad H_1: \mu < \mu_0$.

$z = \frac{\overline{X}_n - \mu_0}{\sigma/n} \approx_{H_0} \mathcal{N}(0,1)$. $\quad$ We reject H_0 at level α if $z < -z_{1-\alpha}$.

Test power: $\mathbb{P}_\mu(\text{reject } H_0) = \mathbb{P}_\mu\left(\frac{\overline{X}_n - \mu_0}{\sigma/n} < -z_{1-\alpha}\right) = \mathbb{P}_\mu\left(\underbrace{\frac{\overline{X}_n - \mu}{\sigma/n}}_{\sim \mathcal{N}(0,1)} < -z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma/n}\right)$

$= Z\left(-z_{1-\alpha} + \overbrace{\frac{\mu_0 - \mu}{\sigma/n}}^{>0}\right)$.

(b) Impact on the test power: The test power is a monotonously increasing function of μ_0, n and a m. decreasing function of α, μ and σ.

(2.)

Data 1:	8,8	10,5	12,5	9,7	9,6	13,2
Data 2:	8,4	10,1	12,0	9,3	9,0	13,0

We have paired samples and assume that the differences are $\sim \mathcal{N}(\mu_d, \sigma_d^2)$. $\quad H_0: \mu_d = 0$.

We know that $t = \frac{\overline{d} - 0}{s_d/\sqrt{n}} \approx_{H_0} t_{n-1}$; we choose $\alpha = 0{,}05$ and decide to reject H_0 if $|t| > t_{1-\alpha/2, n-1} \Leftrightarrow 7{,}68\ldots > 2{,}57\ldots \Leftrightarrow T$.

The p-value is $P(|t| > 7{,}68\ldots) = F_t(-7{,}68\ldots) + (1 - F_t(7{,}68\ldots)) \approx 0{,}0006$, which supports our conjecture that $\mu_d \neq 0$.

(4.) Independent samples: $\begin{cases} \overline{X}_1 = 5{,}275 \\ S_1 = 150 \end{cases}$, $\begin{cases} \overline{X}_2 = 5{,}240 \\ S_2 = 200 \end{cases}$, $\quad n_1 = n_2 = 400$.

(a) Use 95% CI to estimate $\mu_1 - \mu_2$: $\overline{X}_1 - \overline{X}_2 \pm z_{1-\alpha/2}\sqrt{\frac{S_1^2 + S_2^2}{n}} \approx [-24{,}5;\ 24{,}5]$

Interpretation: If we repeat the experiment and calculate the CI every time, it will contain the true value in 95% of the experiments.

(b) Test $H_0: \mu_1 - \mu_2 = 0$ vs. $H_1: \mu_1 - \mu_2 \neq 0$:

$z = \frac{\overline{X}_1 - \overline{X}_2 - 0}{\sqrt{\frac{S_1^2 + S_2^2}{n}}} \approx_{H_0} \mathcal{N}(0,1)$. $\quad$ Reject if $|z| > z_{1-\alpha/2} \Leftrightarrow 0{,}028 > 1{,}9599 \Leftrightarrow \perp$.

p-value $= \mathbb{P}_{H_0}(|z| > 0{,}028\ldots) = 2\Phi(-0{,}028) = 0{,}998$.

Interpretation: If H_0 is true, the probability of obtaining a result at least as extreme as the result actually observed is $\approx 99{,}8\%$.

(c) Test H_0 vs $H_1: \mu_1 - \mu_2 > 0$:

The p-value is now $P_{H_0}(z > 0{,}028) = 1 - \Phi(0{,}028) \approx 0{,}499$, which is smaller. Nevertheless, we would reject H_0.

(d) Test $H_0: \mu_1 - \mu_2 = 25$ vs. $\mu_1 - \mu_2 \neq 25$. Compare to (b).

$$z = \frac{\bar{x}_1 - \bar{x}_2 - 25}{\sqrt{\frac{s_1^2 + s_2^2}{n}}} = -1{,}9972 \rightsquigarrow |z| = 1{,}9972 > 1{,}9599 \Rightarrow \text{reject.}$$

$$p\text{-value} = P_{H_0}(|z| > 1{,}9972) = 2\Phi(-1{,}9972) \approx 0{,}046 < 0{,}05.$$

(e) What assumptions where necessary for (a) - (d)?

- independent samples
- $n \geq 30$
- only one test per data set
- choose test before data is observed

(5)

1: HS:	131	74	129	96	92
2: HF:	44	70	69	43	53
3: FB:	15	14	21	29	21

a) HS vs. HF: We have independent samples. $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$.
We have sample size $5 < 30$ and assume unequal variances.

The 95% CI is $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}(\nu)\sqrt{\frac{s_1^2 + s_2^2}{n}} \approx [18{,}0\,;\ 79{,}2] \not\ni 0.$

b) the 95% CI for $\mu_2 - \mu_3$ is $\approx [19{,}7;\ 51{,}8] \not\ni 0.$

c) I would recommend the fist bumps, but based on common sense and not on the experiment which was carried out with questionable methods (high five for 3 seconds).