Python3 introduction: exercise sheet

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Disclaimer: These exercises are **NOT** mandatory! However, if you are new to Python3, we suggest doing Exercise 1.

Exercise 1: Take a look at the slides and install Python3 on your system.

Exercise 2: Consider a partition $0 = x_0 < x_1 < \ldots < x_N = 1$ of the interval $[0, 1]$. Let

$$h_0 := 0, \quad h_{N+1} := 0, \quad \text{and} \quad h_i := x_i - x_{i-1} \quad \text{for } i = 1, \ldots, N.$$

Consider the matrix $A \in \mathbb{R}^{(N+1)\times(N+1)}$, given by

$$A_{ij} := \begin{cases} h_i^{-1} + h_{i+1}^{-1} & \text{if } i = j, \\ -h_i^{-1} & \text{if } |i - j| = 1, \\ 0 & \text{else.} \end{cases}$$

Write a program that reads the points x_i from the file `mesh.csv` (there, $N = 2000$ and every line contains a pair (i, x_i)). Assemble the matrix A and check if the matrix is correct by checking whether $z^\top A z = 0$ for $z = (1, \ldots, 1)^\top$. Furthermore, compute the eigenvalues of A and write them to a file `eigenvalues.csv`.

Exercise 3: Let $f\colon \mathbb{R} \to \mathbb{R}\colon x \mapsto \sin(x)$. Plot f in the interval $[-3 - 3]$.

Exercise 4: Let $(f_n)_{n\in\mathbb{N}}$ and $(g_n)_{n\in\mathbb{N}}$ be the sequences given by

$$f_n := \sqrt{6 \sum_{k=0}^{n} \frac{1}{(k+1)^2}} \qquad \text{and} \qquad g_n := \sum_{k=0}^{n} \frac{1}{16^k} \Big(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \Big).$$

For $n = 0, \ldots, 10$ compute f_n and g_n and visualize the convergence of $|\pi - f_n|$ and $|\pi - g_n|$.

Exercise 5: An easy algorithm to find a root of a given function $f\colon \mathbb{R} \to \mathbb{R}$ is the so-called bisection algorithm. Starting with an interval $[a, b]$ containing a root (i.e., a value x such that $f(x) = 0$), we set $x_0^\ell = a, x_0^r = b$. For given $n \in \mathbb{N}$ and x_n^ℓ, x_n^r, we define $x_n^m := (x_n^\ell + x_n^r)/2$ and

$$x_{n+1}^\ell := \begin{cases} x_n^m & \text{if } \operatorname{sign}(x_n^\ell) = \operatorname{sign}(x_n^m), \\ x_n^\ell & \text{else,} \end{cases} \quad \text{and} \quad x_{n+1}^r := \begin{cases} x_n^m & \text{if } \operatorname{sign}(x_n^m) = \operatorname{sign}(x_n^r), \\ x_n^r & \text{else.} \end{cases}$$

Visualize the algorithm for $f(x) = \sin(x)$ and $[a, b] = [2, 4]$, as well as the convergence $|x_n^m - \pi|$.