

$$b) \lambda(x) = \frac{\sup \{L(\mu, \sigma^2; x) \mid (\mu, \sigma^2) \in \mathbb{R} \times \mathbb{R}^+\}}{\sup \{L(\mu, \sigma^2; x) \mid (\mu, \sigma^2) \in \mathbb{R} \times (0, \sigma_0^2]\}} = \begin{cases} 1, & \text{if } \hat{\sigma}^2 \leq \sigma_0^2 \\ \left(\frac{\hat{\sigma}^2}{\sigma_0^2}\right)^{-\frac{n}{2}} \exp\left(\left(\frac{1}{\sigma_0^2} - \frac{1}{\hat{\sigma}^2}\right)\frac{n}{2} \hat{\sigma}^2\right), & \text{if } \hat{\sigma}^2 > \sigma_0^2 \end{cases}$$

$$c) \lambda(x) = \frac{\sup \{L(\mu, \sigma^2; x) \mid (\mu, \sigma^2) \in \mathbb{R} \times \mathbb{R}^+\}}{\sup \{L(\mu, \sigma^2; x) \mid (\mu, \sigma^2) \in \mathbb{R} \times [\sigma_0, \infty)\}} = \begin{cases} 1, & \text{if } \hat{\sigma}^2 \geq \sigma_0^2 \\ \left(\frac{\hat{\sigma}^2}{\sigma_0^2}\right)^{-\frac{n}{2}} \exp\left(\underbrace{\left(\frac{1}{\sigma_0^2} - \frac{1}{\hat{\sigma}^2}\right)\frac{n}{2} \hat{\sigma}^2}_{= \left(\frac{\hat{\sigma}^2}{\sigma_0^2} - 1\right)\frac{n}{2}}\right), & \text{if } \hat{\sigma}^2 < \sigma_0^2 \end{cases}$$