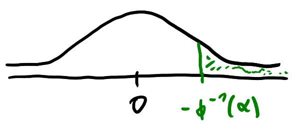
(1) **Test power in the z-test**
Let $X_1, \dots, X_n$ be i.i.d.random variables with $X_1 \sim N(\mu, \sigma^2)$, and $H_0 : \mu = \mu_0$.

(a) Compute the test power of the left-sided z-test. Express it through cdf of the $N(0,1)$-distribution, depending on μ_0, μ, σ, n and the significance level α.

(b) Comment on the impact of μ_0, μ, σ, n and α on the test power.

a) We test H_0 versus $H_1 : \mu > \mu_0$. Our test statistic is $T(x) = \frac{(\bar{x} - \mu_0)\sqrt{n}}{\sigma}$



The hypothesis H_0 is rejected, if and only if $T(x) \geq -\phi^{-1}(\alpha)$

The power is given by

$$\mathbb{P}(T(X) \geq -\phi^{-1}(\alpha) \mid \mu \neq \mu_0) = \mathbb{P}\left(\bar{X} \geq \mu_0 - \frac{\phi^{-1}(\alpha)\sigma}{\sqrt{n}}\right)$$

$$= \mathbb{P}\left(\frac{(\bar{X} - \mu)\sqrt{n}}{\sigma} \geq \frac{(\mu_0 - \mu)\sqrt{n}}{\sigma} - \frac{\sigma}{\sigma}\phi^{-1}(\alpha)\right)$$

$$= 1 - \mathbb{P}\left(\frac{(\bar{X} - \mu)\sqrt{n}}{\sigma} < \frac{(\mu_0 - \mu)\sqrt{n}}{\sigma} - \phi^{-1}(\alpha)\right)$$

$$= 1 - \phi\left(\frac{(\mu_0 - \mu)\sqrt{n}}{\sigma} - \phi^{-1}(\alpha)\right)$$

b) The power is monotonously decreasing in μ_0 and it is monotonously increasing in α and μ_0.
If $\mu_0 \leq \mu$, then the power is monotonously increasing in n and monotonously decreasing in σ
If $\mu > \mu_0$, then it is the other way round, hence our test is really only ~~good~~ if $\mu \geq \mu_0$.