

(2) **Coin throws**

An unfair coin is thrown 600 times. The probability of getting a tail in each throw is  $\frac{1}{4}$ .

- (a) Use a Binomial distribution to compute the probability that the number of heads obtained does not differ more than 10 from 450.
- (b) Use a Normal approximation without a continuity correction to calculate the probability in (a). How does the result change if the approximation is provided with a continuity correction?

a)  $Y$  ... number of tails after 600 tosses,  $Y \sim \text{bin}(n, p)$ ,  $n = 600$ ,  $p = \frac{1}{4}$

$$P(|600 - Y| \leq 10) = P(|150 - Y| \leq 10) = P(140 \leq Y \leq 160) \stackrel{R}{\approx} 0,68$$

b)  $Y \approx Z \sim \mathcal{N}(np, np(1-p))$

*Symmetry of normal.*

$$\begin{aligned} P(140 \leq Z \leq 160) &= 1 - P(Z < 140) - P(Z > 160) \stackrel{!}{=} 1 - P(Z < 140) - P(Z < 140) \\ &= 1 - 2P(Z < 140) \stackrel{R}{\approx} 0,65 \end{aligned}$$

$$np = \frac{600}{4} = 150; \quad np(1-p) = \frac{600}{4} \cdot \frac{3}{4} = \frac{3 \cdot 150}{4} = \frac{450}{4} = \frac{225}{2}$$

with continuity correction:

$$P(140 \leq Y \leq 160) \approx P(140 - \frac{1}{2} < Z < 160 + \frac{1}{2}) = 1 - 2P(Z < 140 - \frac{1}{2}) \stackrel{R}{\approx} 0,68$$