(3) **Simulations**

(a) By applying the `R`-function `replicate()` generate a sample $X_1, \ldots, X_{10}$ of size 10 from an exponential distribution with a rate parameter 0.2 and sum up its elements. Do this sum 10 000 times and make a histogram of the simulation. Can you say something about the shape of distribution?

(b) Use `R` to simulate 50 tosses of a fair coin (0 and 1). We call a *run* a sequence of all 1's or all 0's. Estimate the average length of the longest run in 10000 trials and report the result.

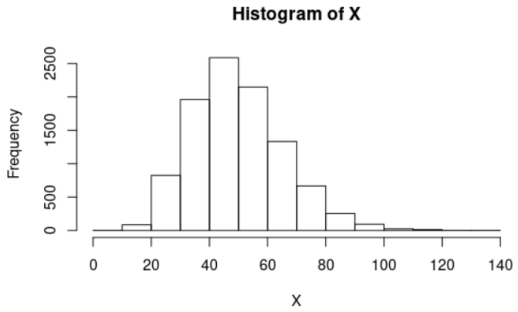
Hint: Use the commands `rbinom` and `rle`. The command `rle()` stands for run length encoding. For example,

```
rle(rbinom(5, 1, 0.5))$lengths
```

is a vector of the lengths of all the different runs in trial of 5 flips of a fair coin.

a) Defining $Y := \sum_{i=1}^{n} X_i$, $n = 10$, $\lambda := \frac{1}{5}$ we expect $Y \sim \text{Gamma}(n, \lambda)$,

where $E(Y) = \frac{n}{\lambda} = 5 \cdot 10 = 50$ and $\text{Var}(Y) = \frac{n}{\lambda^2} = 10 \cdot 25 = 250$



b) Estimation: longes run is approximately 5,94