This is the third homework assignment. The problems are to be presented on **April 13, 2021**. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions. The problems should be ticked and solution paths uploaded by **23:59 on April 12, 2021**.

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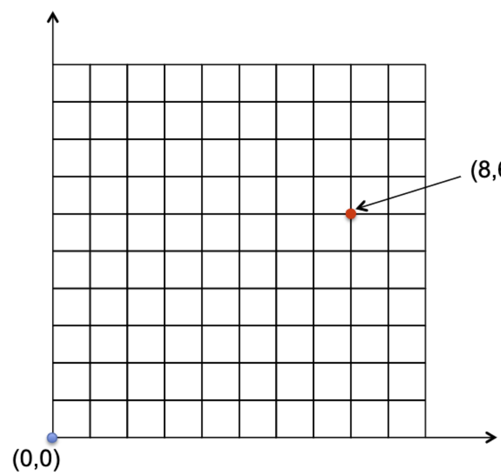
(1) **Random walk of a robot**
A robot is placed at the origin (the point $(0, 0)$) on a two-dimension integer grid (see the figure below). Denote the position of the robot by (x, y). The robot can either move right to $(x + 1, y)$ or move up to $(x, y + 1)$.



(a) Suppose each time the robot randomly moves right or up with equal chance. What is the probability that the robot will ever reach the point $(8, 6)$?

(b) Suppose another robot has a $\frac{2}{3}$ chance to move right and a $\frac{1}{3}$ chance to move up when $x + y$ is even, otherwise it has a $\frac{1}{4}$ chance to move right and a $\frac{3}{4}$ chance to move up. It stops whenever $|x - y| \geq 2$. Find the probability that $x - y = 2$ when it stops.

(2) **Continuous two-dimensional random variable**
The joint pdf of two random variables X and Y is defined by

$$f(x, y) = \begin{cases} c(x + 2y), & 0 < y < 1 \text{ and } 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find the value of c and the marginal distribution of Y.

(b) Find the joint cdf of X and Y.

(c) Find the marginal distribution of X and the pdf of $Z = \frac{9}{(X+1)^2}$.

(3) **Chi squared distribution**

Let X and Y be independent and identically distributed (i.i.d.) $\mathcal{N}(0,1)$ random variables. Define $Z = \min\{X, Y\}$. Show that $Z^2 \sim \chi_1^2$, i.e. show that the pdf of Z^2 is given by

$$f_{Z^2}(z) = \frac{1}{\sqrt{2\pi}} \cdot z^{-\frac{1}{2}} \cdot e^{-\frac{z}{2}} \cdot \mathbf{1}_{\{z>0\}},$$

(4) **Random variables on the unit disk**

Let (X, Y) be uniformly distributed on the unit disk $\{f(x;y) : x^2 + y^2 \leq 1\}$. Let

$$R = \sqrt{X^2 + Y^2}.$$

Find the cdf, pdf, and the expectation the random variable R.

(5) **Transformations**

Suppose X and Y are independent gamma distributed random variables with $X \sim Gamma(\alpha_1, \beta)$ and $Y \sim Gamma(\alpha_2, \beta)$. Consider the following two random variables

$$U = X + Y \qquad \text{and} \qquad V = \frac{X}{X+Y}.$$

(a) Show that $U \sim Gamma(\alpha_1 + \alpha_2, \beta)$.

(b) Show that U and V are also independent random variables.