Mathematical Modelling and Simulation

Introduction to Modelling and Simulation

Methods and Algorithms

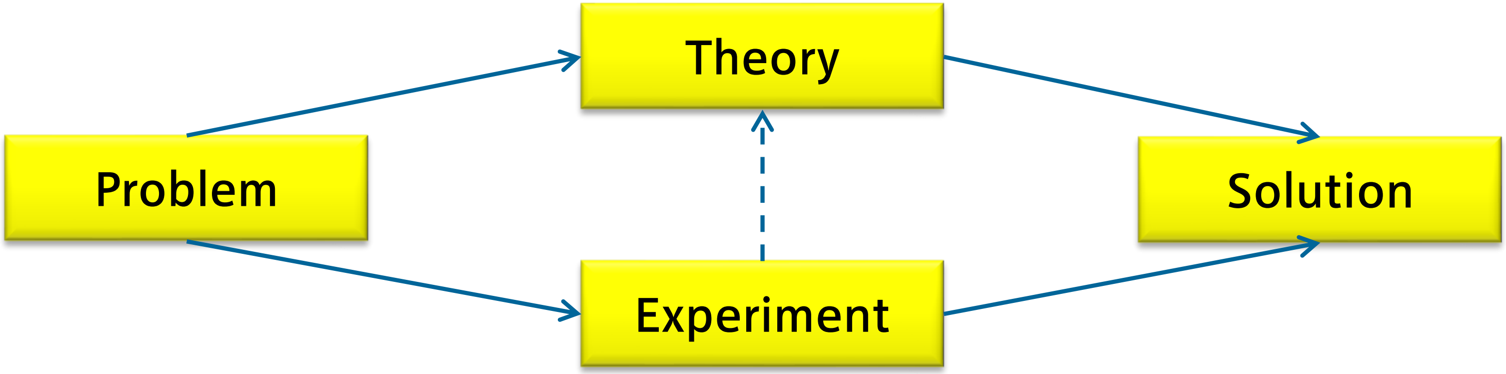
Classical Approach

Mathematical Modelling and Simulation

Classical Scientific Problem
- Application of Theories
- Execute Experiments

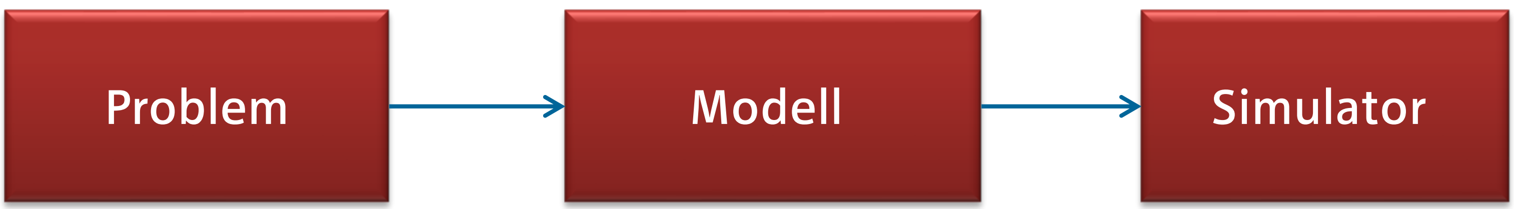
Classical Approach

Classical Scientific Problem
- Application of Theories
- Execute Experiments



Simulation

Simulation
- Experiments in virtual laboratory
- Experiments in the computer
- The third pillar of science beside theory and experiment



Scientific Research problem

Abstraction of Reality

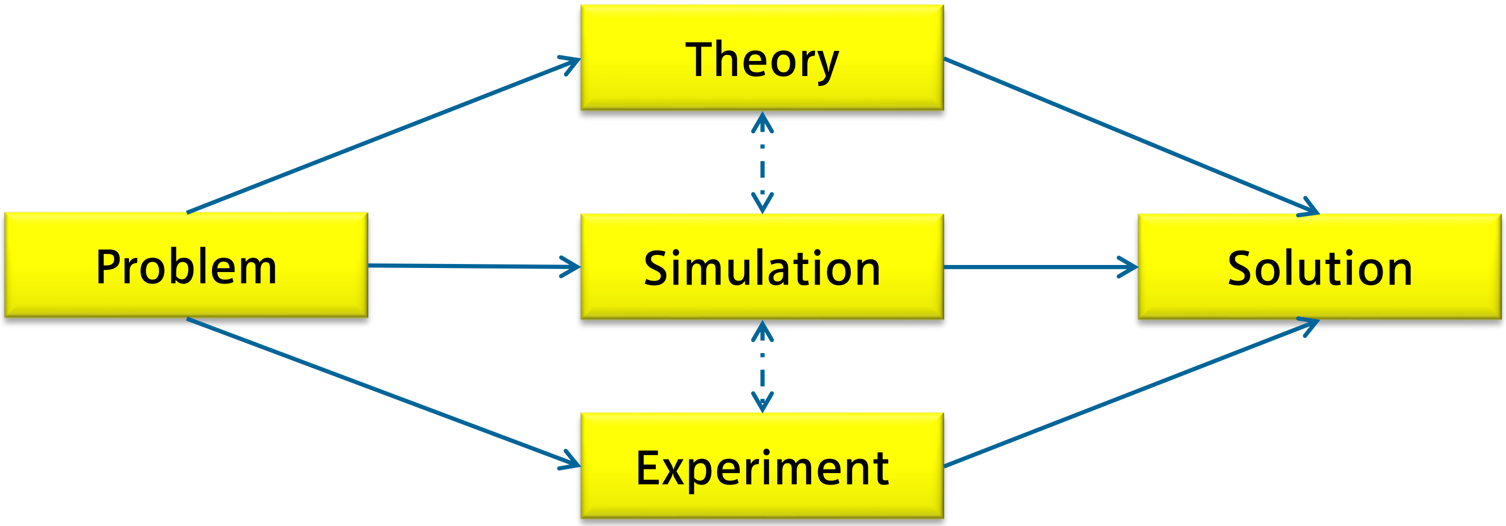
Experimental Environment

Simulation

Mathematical Modelling and Simulation

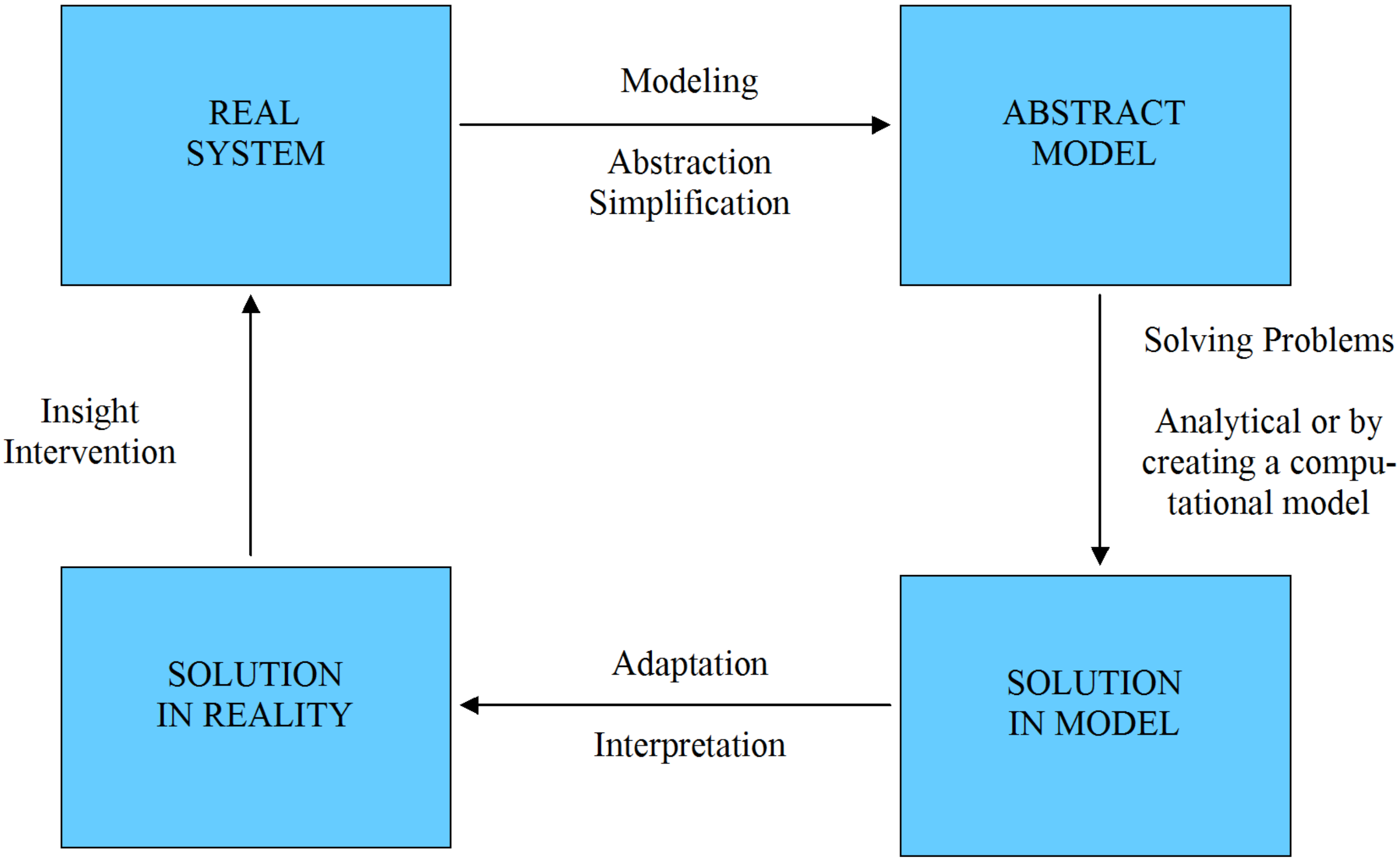
Simulation

- Experiments in virtual laboratory
- Experiments in the computer
- The third pillar of science beside theory and experiment



Solution finding

Mathematical Modelling and Simulation



What is Computer Simulation ?

Mathematical Modelling and Simulation

Definition (Shannon, 1975)

Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose either of understanding the behavior of the system and its underlying causes or of evaluating various designs of an artificial system or strategies for the operation of the system.

What is Computer Simulation ?

Mathematical Modelling and Simulation

Definition 2 (VDI-Richtlinie 3633)

Simulation is a (virtual) copy of a real system with its dynamic processes in a (virtual) model (computer model) and (virtual) experiments with experiments with this model, which allow interpretations for the real system.

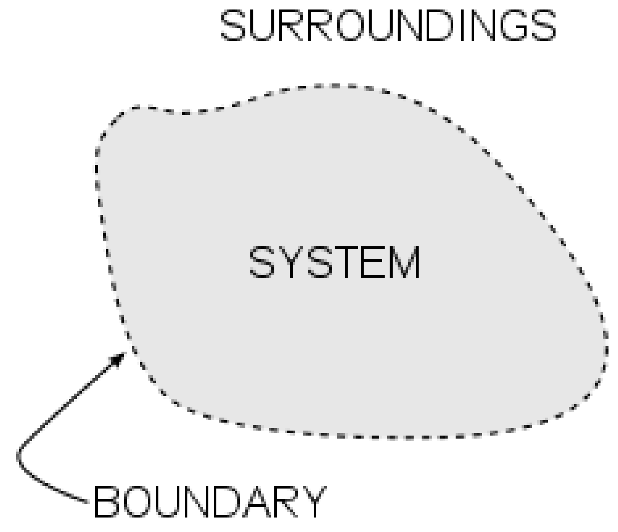
In a practical sense, simulation is i) preparing, ii) performing, and iii) evaluating experiments with a simulation model.

Simulation allows to study time-dependent behaviour of complex dynamical systems in a simulation model.

Dynamical Systems

A system is a set of interacting or interdependent components forming an integrated whole

A dynamic system is a set of dynamically interacting or interdependent components forming an integrated whole



Dynamical Systems

Mathematical Modelling and Simulation

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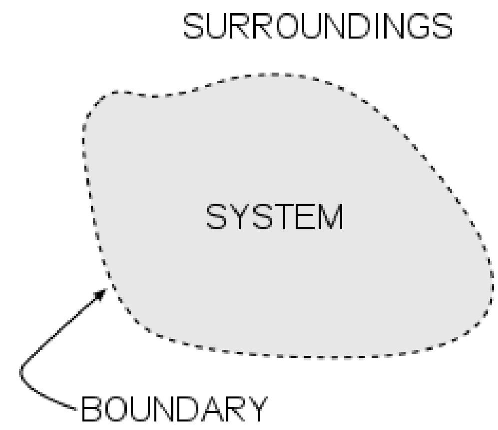

- Dynamical systems change their behaviour dependent on acting input signals, disturbances, and initial values
-
- The behaviour of a dynamical system is not direct proportional to input and disturbance change, it changes its behaviour on basis of its own dynamic and on inputs.

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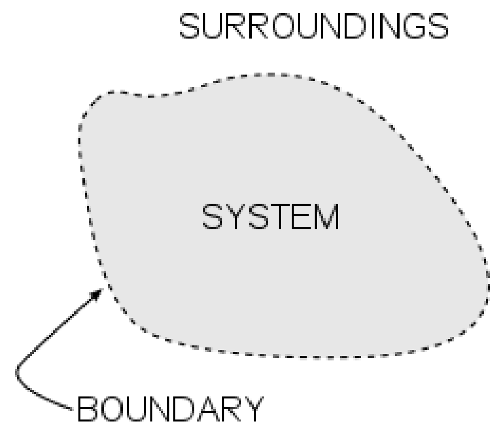

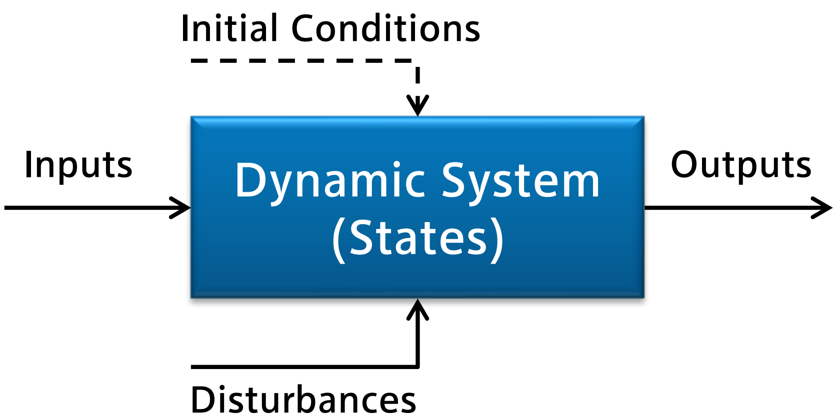

Elements of a Dynamical System

- States **x**(t)
- Inputs **u**(t)
- Disturbances **w**(t) = Inputs
- Outputs **y**(t)
- Fixed Parameters, Intial Conditions
- Time dependent Parameters (Inputs)

Dynamical Systems

Mathematical Modelling and Simulation

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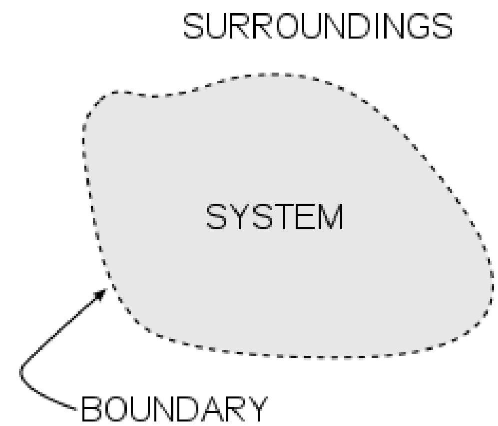

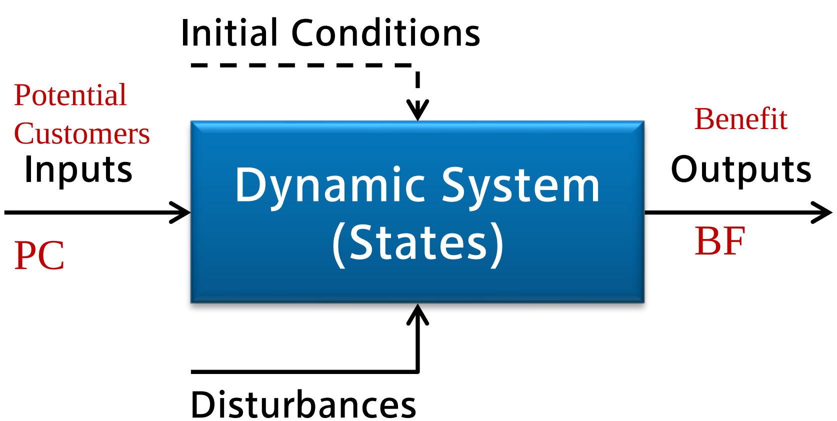

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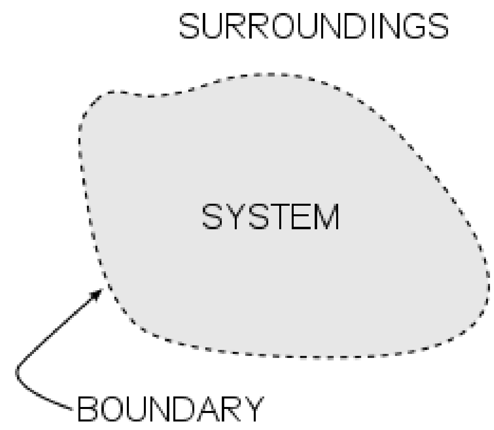

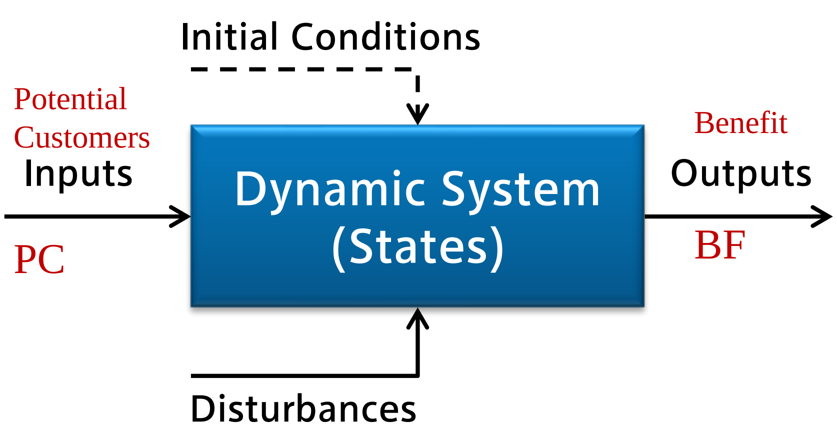

static

$$BF = Faktor \cdot PC$$

Dynamical Systems

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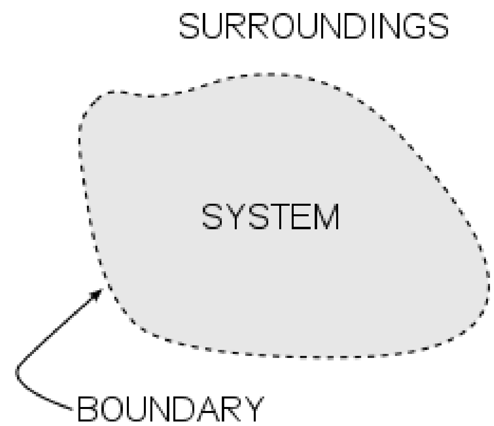

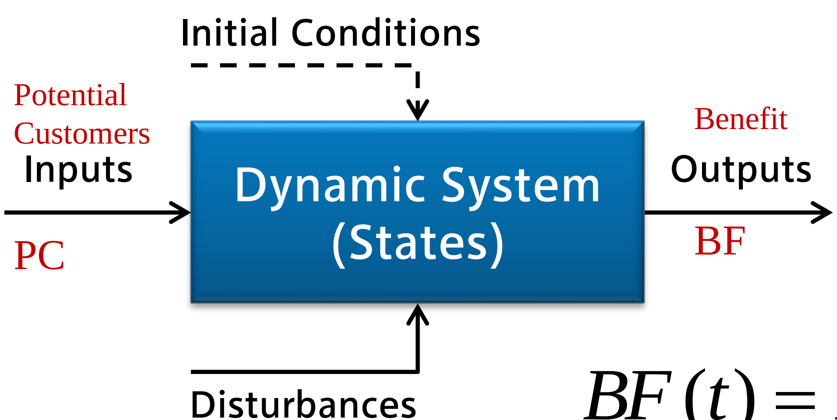

static

$$BF = Faktor \cdot PC$$

dynamic

$$BF(t) = Function(PC(t), t, Parameters)$$

Dynamical Systems

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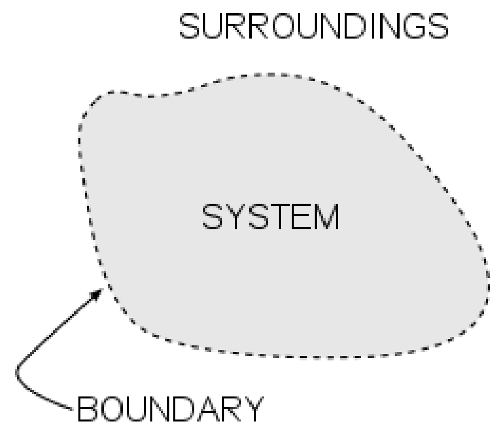

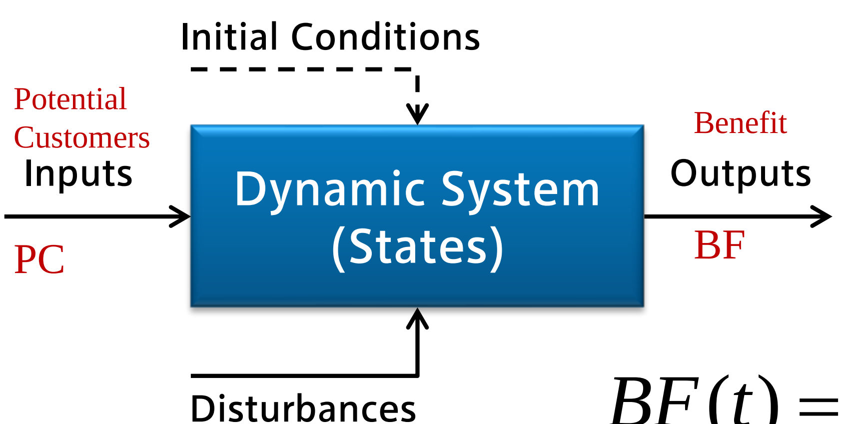

static

$$BF = Faktor \cdot PC$$

really dynamic

$$BF(t) = Function(PC(t), BF(t), t, Par)$$

Dynamical Systems

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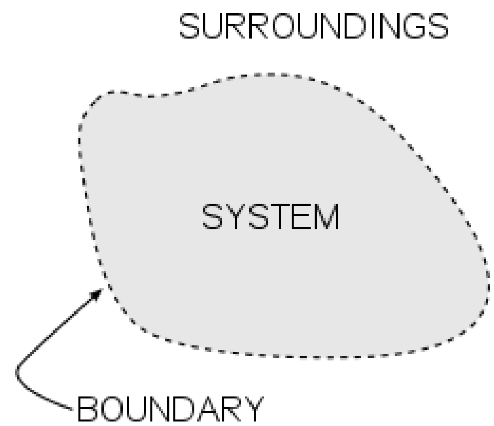
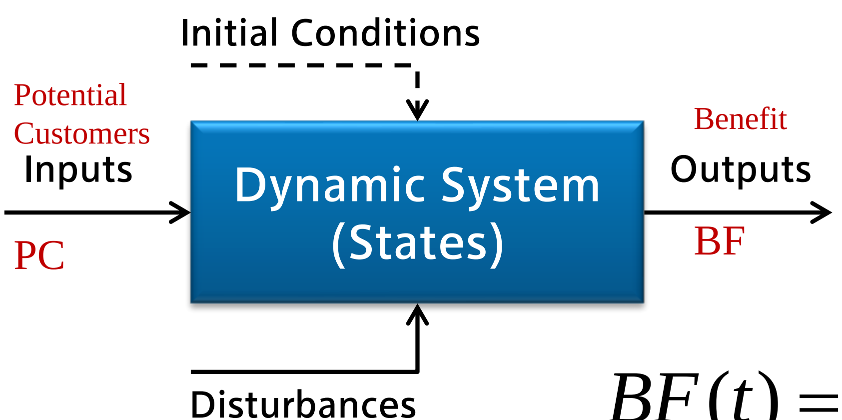




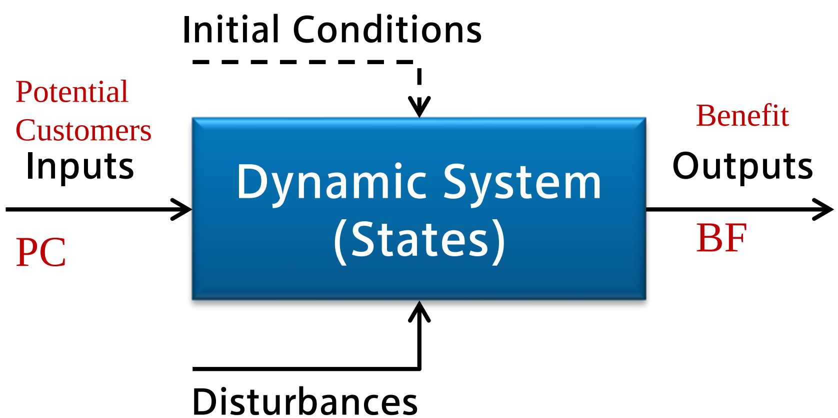
static formula

$$BF = Faktor \cdot PC$$

dynamic model

$$BF(t) = Function(PC(t), BF(t), t, Par)$$

Dynamical Systems



static formula

$$BF = Faktor \cdot PC$$

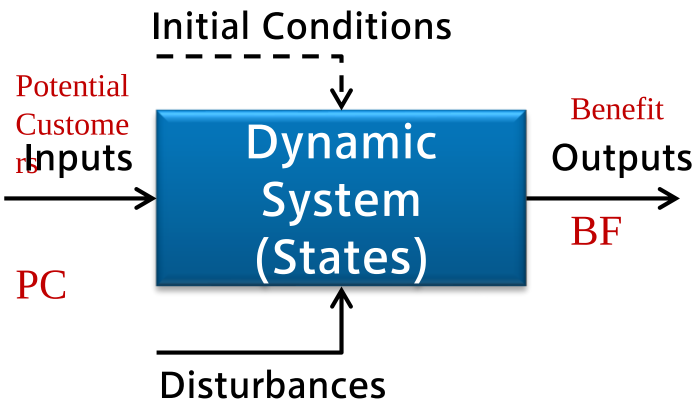
Calculation

dynamic model

$$BF(t) = Function(PC(t), BF(t), t, Par)$$

Simulation

Dynamical Systems

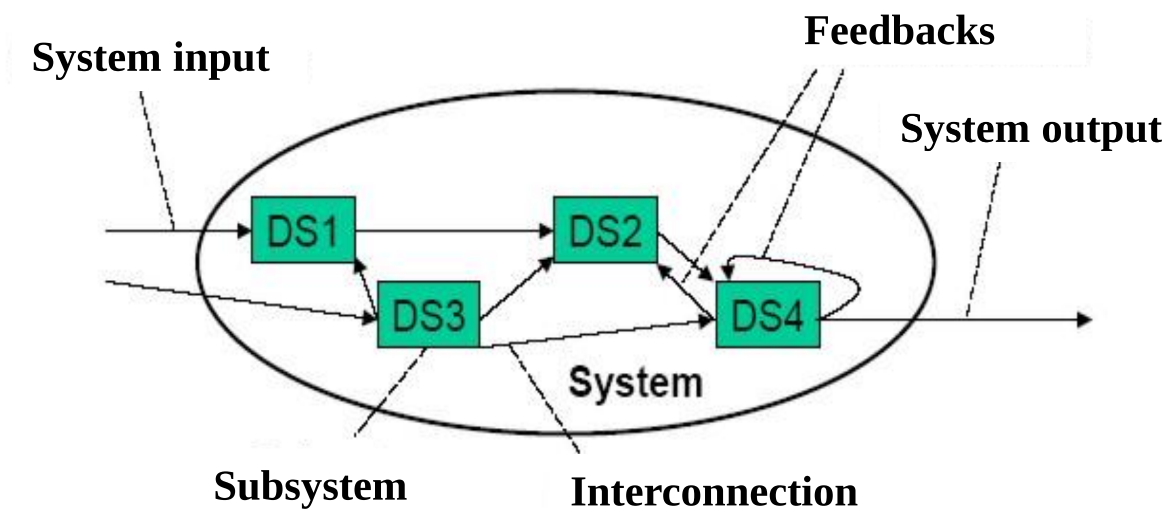


Dynamic mathematical model

$$BF(t) = Function(PC(t), BF(t), t, par)$$

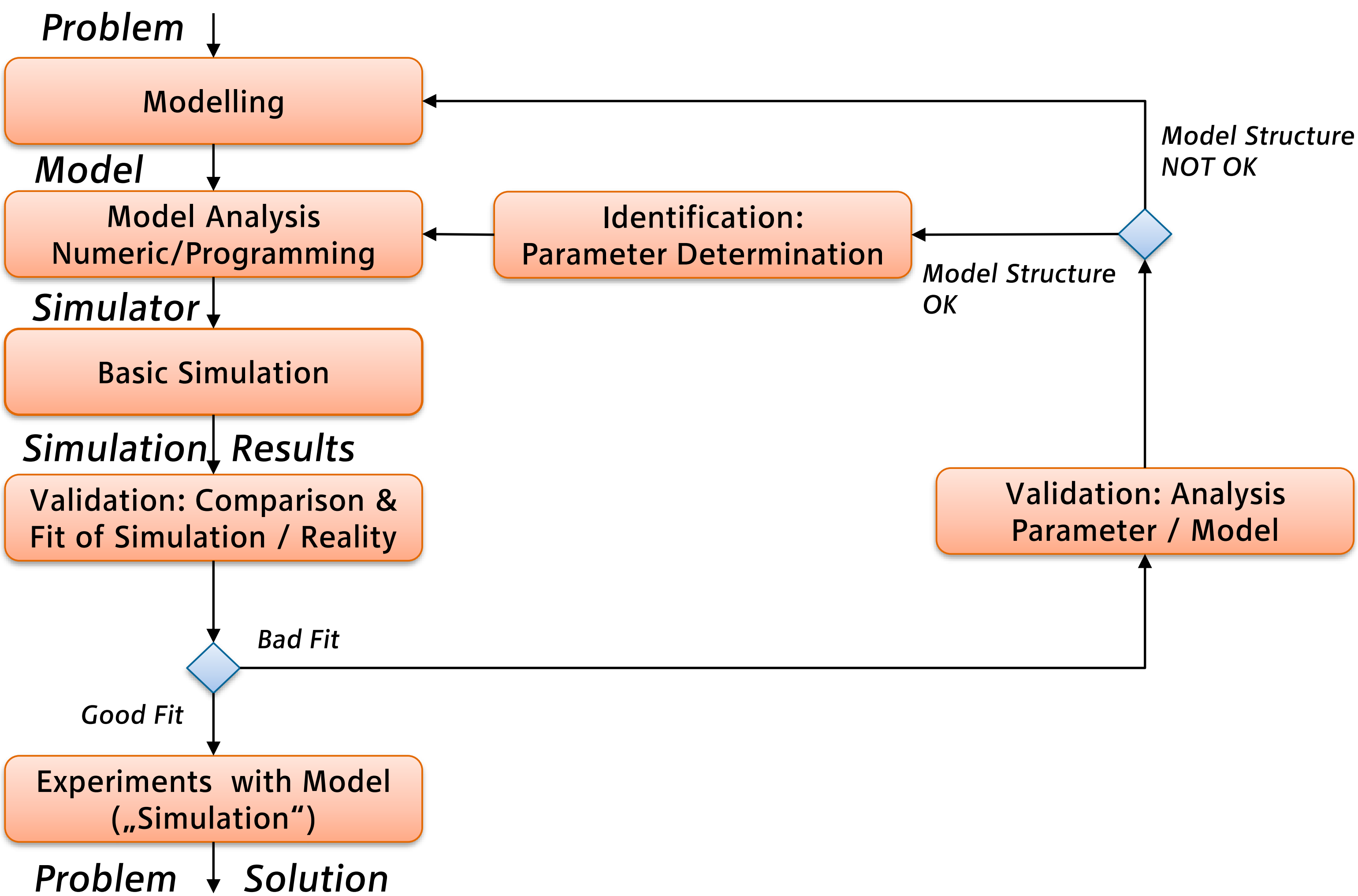
Simulation

A dynamical system may consist of a set of components, which themselves are dynamical subsystems and which influence each other



Simulation Circle

Mathematical Modelling and Simulation



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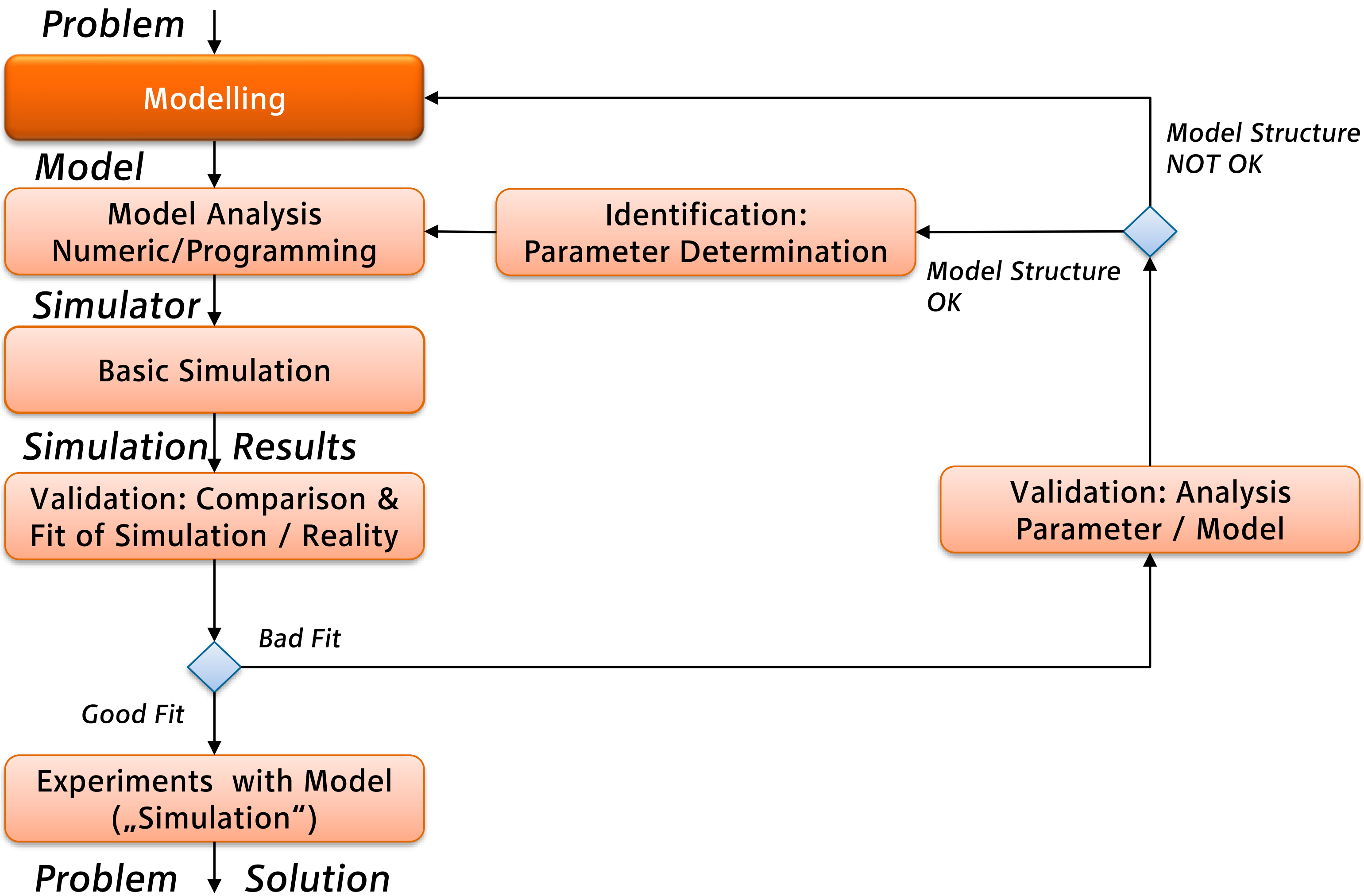
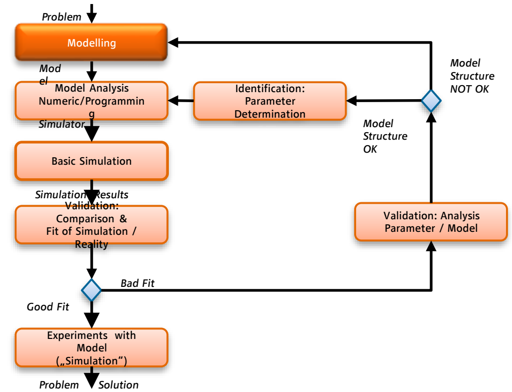


What is a Model?

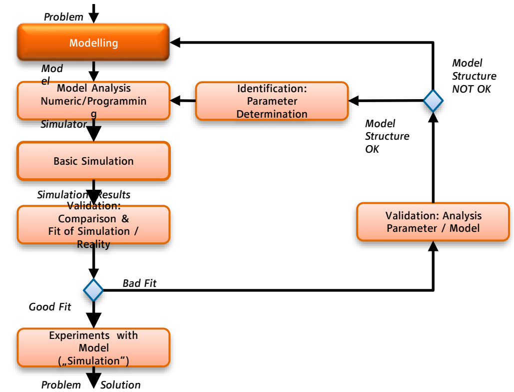
1. **<u>Mapping</u>** - A model is a representation of a natural or an artificial object.
2. **<u>Reduction</u>** - A model is usually simplified and does not have all attributes of the original object.
3. **<u>Pragmatism</u>** - A model is always created for a certain purpose, a certain subject and a certain time-span.

(Stachoviak 1973)

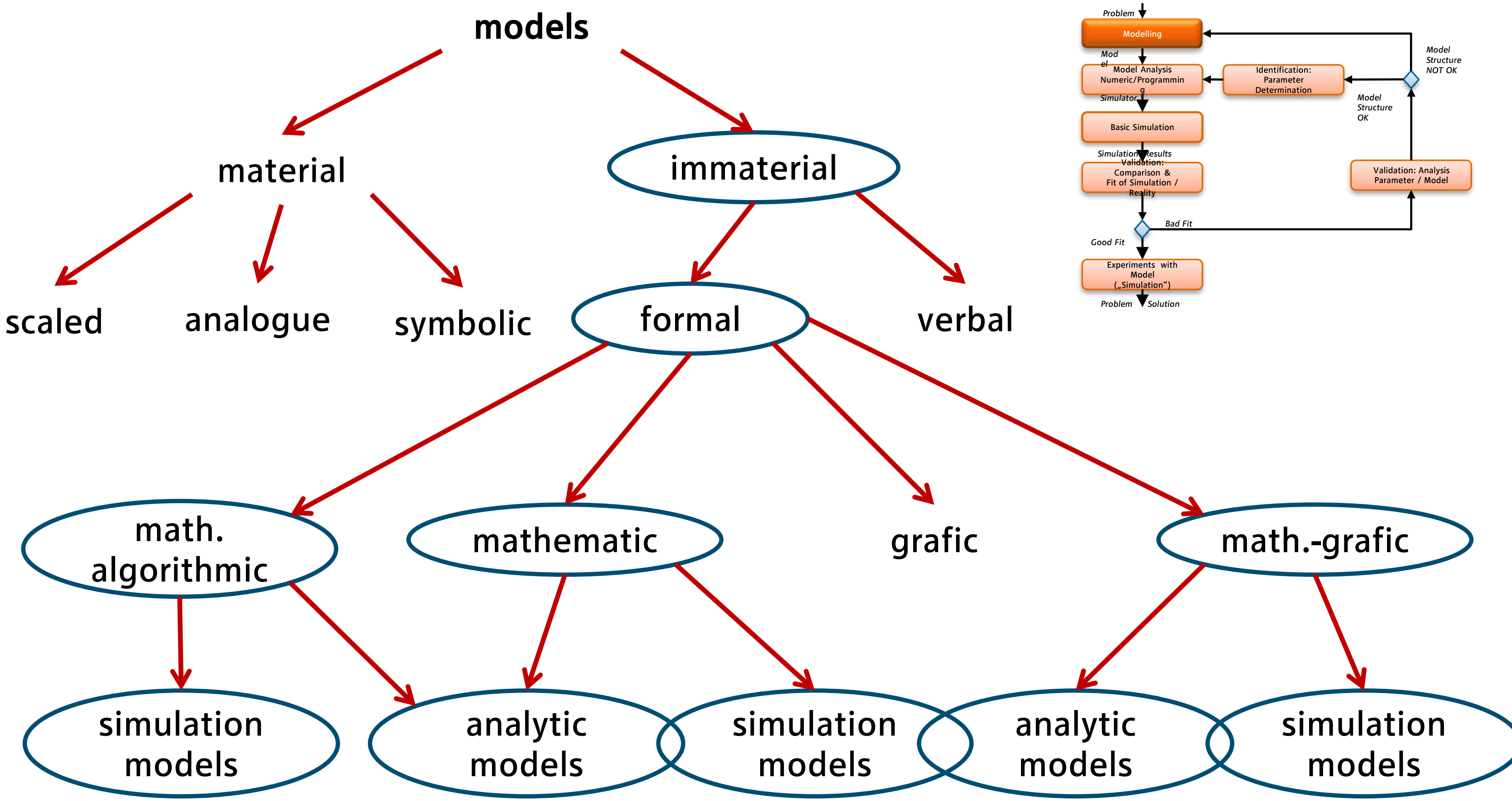


Model Classification

- models
  - material
    - scaled
    - analogue
    - symbolic
  - immaterial
    - formal
      - math. algorithmic
        - simulation models
        - analytic models
      - mathematic
        - analytic models
        - simulation models
      - grafic
      - math.-grafic
        - analytic models
        - simulation models
    - verbal



Model Classification



Two Steps of Abstraction

- Structural Abstraction – Qualitative Knowledge
  Identification of system borders and states

- Phenomenological Abstraction – Quantitative Knowledge
  quantisation of states, identification of physical, economic, biologic, ... interactions in and with subsystems

Modelling vs Model

Mathematical Modelling and Simulation

Modelling Approach	Model Type
• System Dynamics (SD) • Transfer Functions (TF) • Compartment Modelling • Math. Formula • Lagrange Formalism • Port-based physical Modelling • Difference Equation Modelling • Cellular Automata Modelling • Agent-based Modelling • Event Graphs • Process Flow	• Ordinary Differential Equations (ODEs) • Partial Differential Equations (PDEs) • Differential Algebraic Equations (DAEs) • Difference Equations (DEs) • Cellular Automata (CAs) • Agent-based Systems/Models (ABMs) • Discrete Event Systems (DES)

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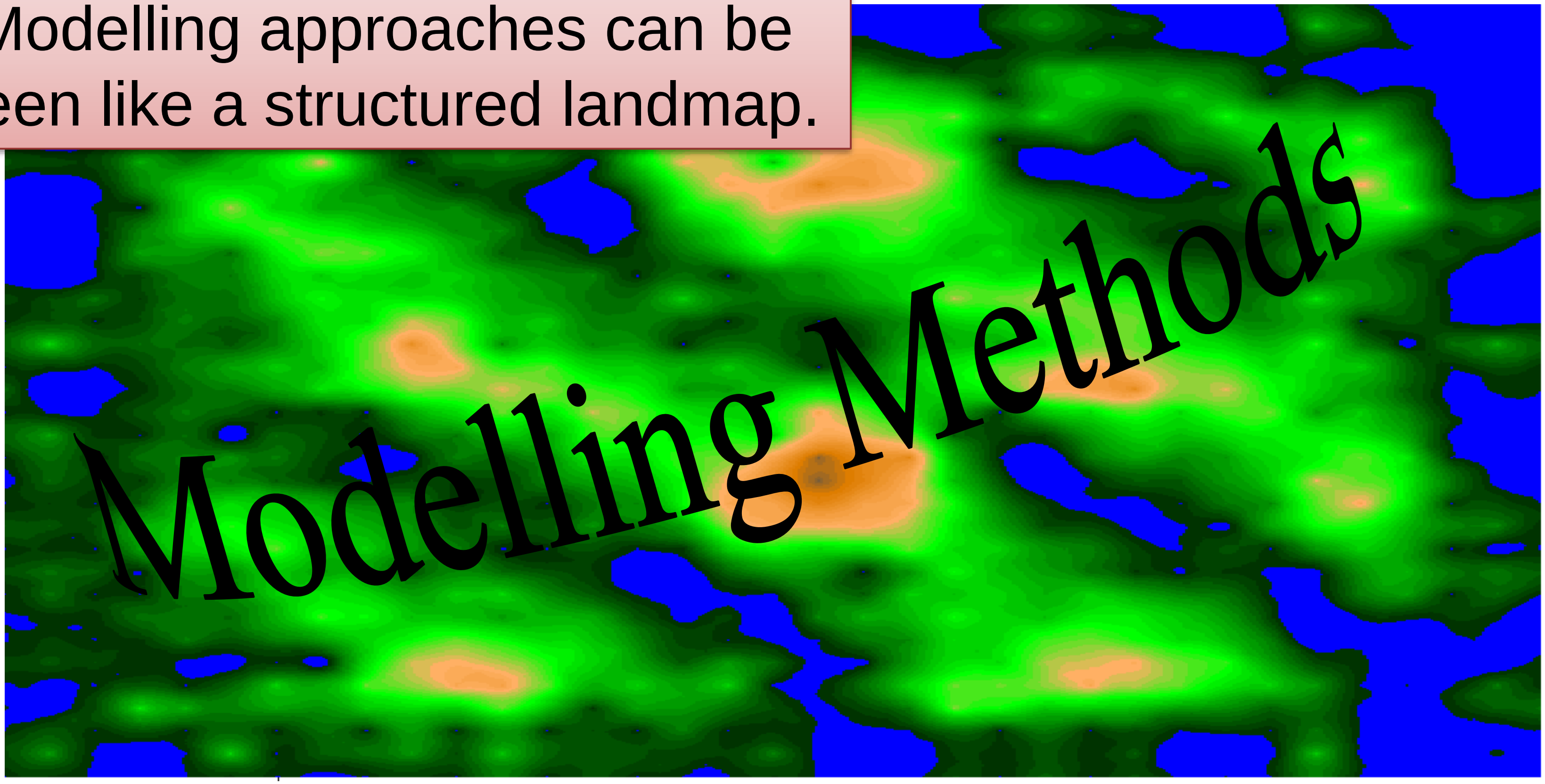
Landmap of Modelling Methods

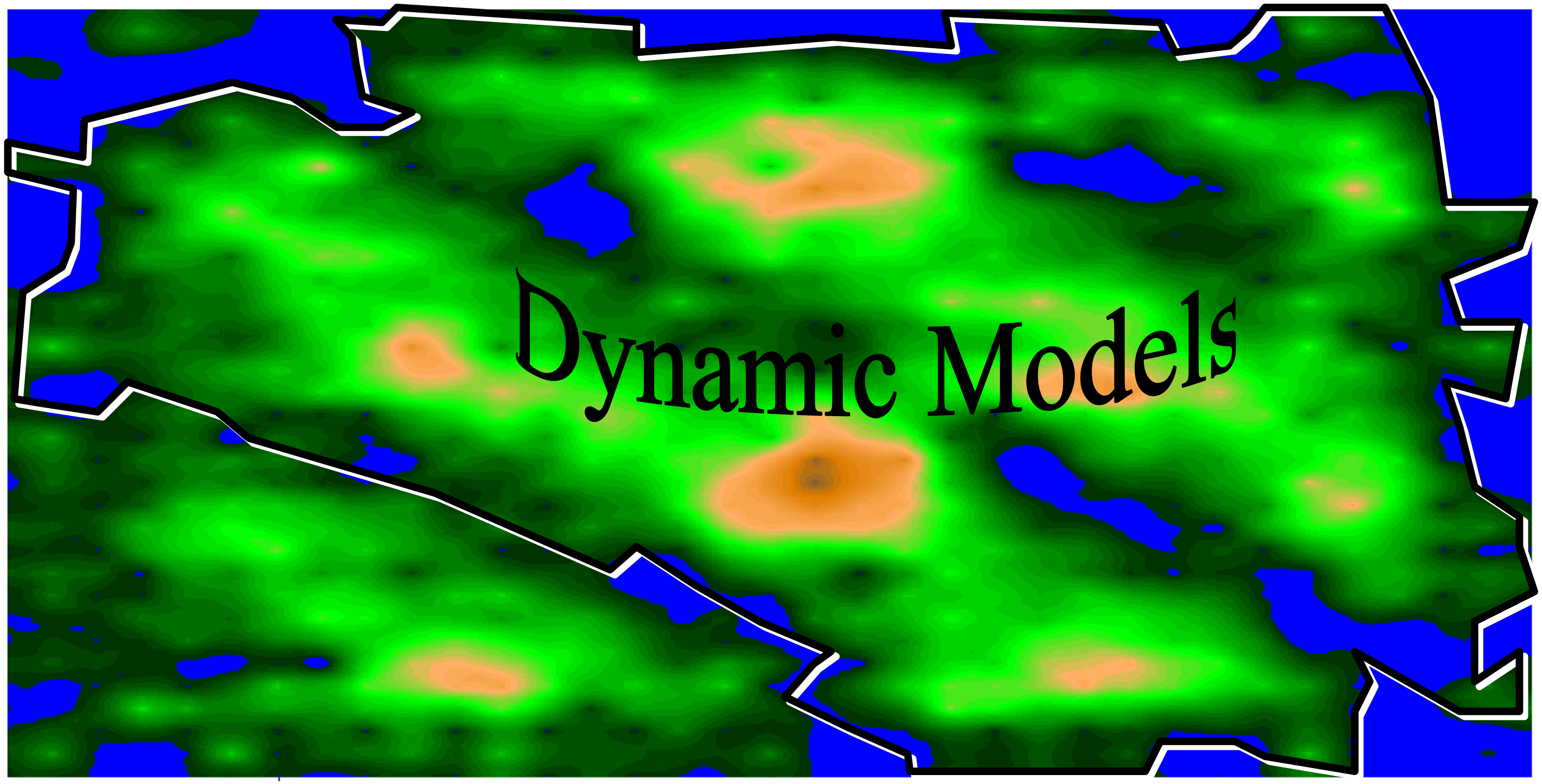
Mathematical Modelling and Simulation

The variety of different Modelling approaches can be seen like a structured landmap.



Landmap of Modelling Methods – Dynamic Models

Mathematical Modelling and Simulation



Dynamic Models – Time Discrete/Continuous

Mathematical Modelling and Simulation

Neglecting quantum-mechanics (space as well as) time can be seen to be a continuous number.

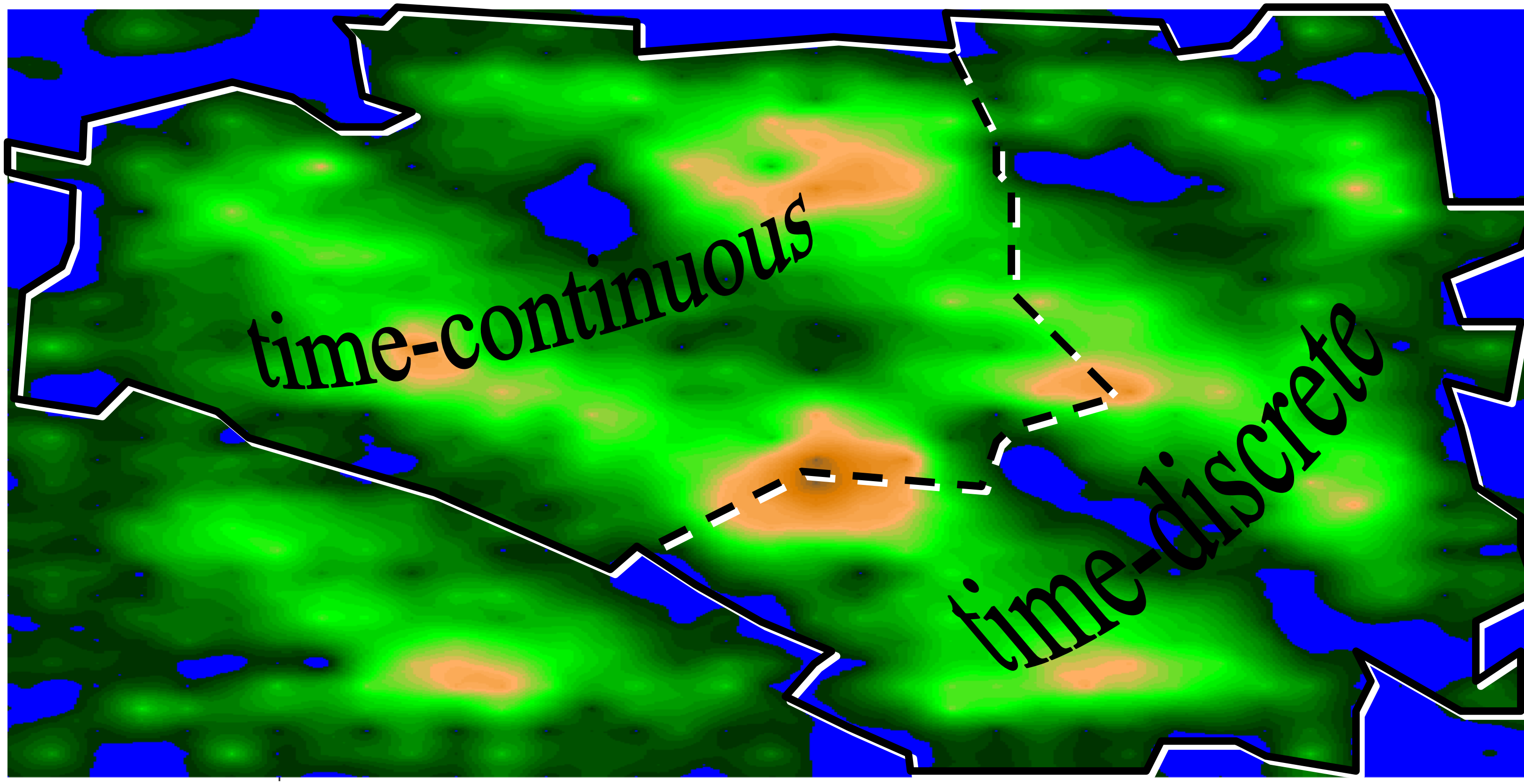
- A model is called time-continuous if the output value of the model can be calculated at any time ($\approx t \in \mathbb{R}$).
- In the opposite a model is called time-discrete if values are only calculated at a finite number of predefined timesteps ($\approx t \in \mathbb{N}$).

Dynamic Models – Time Discrete/Continuous

Mathematical Modelling and Simulation

- Usually time-continuous models are preferred to time-discrete models, but the simulation process is usually more difficult.
- Yet, there are processes in real world for which time continous models are not necessary or even dont make sense.
- Very often, time-continuous models cannot be simulated continously. So they need to be reformalised in a time-discrete manner – this process is called discretisation.

Landmap of Modelling Methods – Time Discrete / Continous



Dynamic Models – Value Discrete/Continuous

Mathematical Modelling and Simulation

Similar to time-discrete/continuous, also output values can be determined discrete or continuously.

- **Value-discrete**:
  - Number of passengers on a plane
  - Number of cars searching for a parking spot.
- **Value-continuous**:
  - Voltage/Current in an Electrical Circuit
  - Angular Velocity of a Pendulum

Dynamic Models – Value Discrete/Continuous

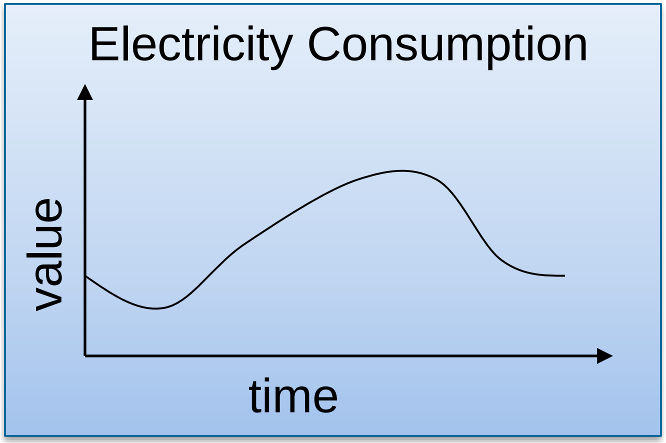
- Although simulation output is expected to be continuous/discrete, it is not necessarily modelled in a continuous/discrete way.

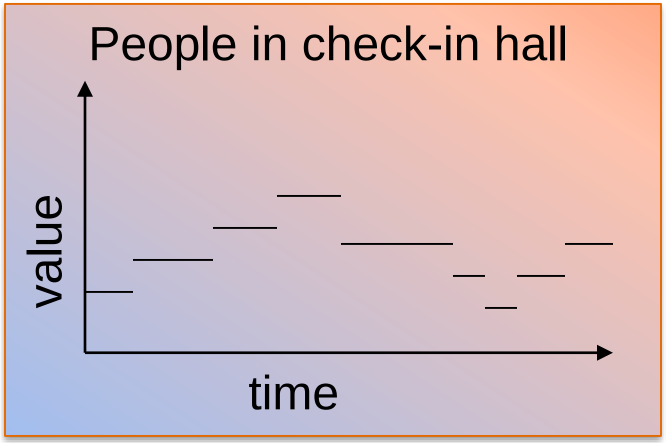
  E.g.:
  Population of a country is a discrete number…
  … yet it can be modelled by a continous model

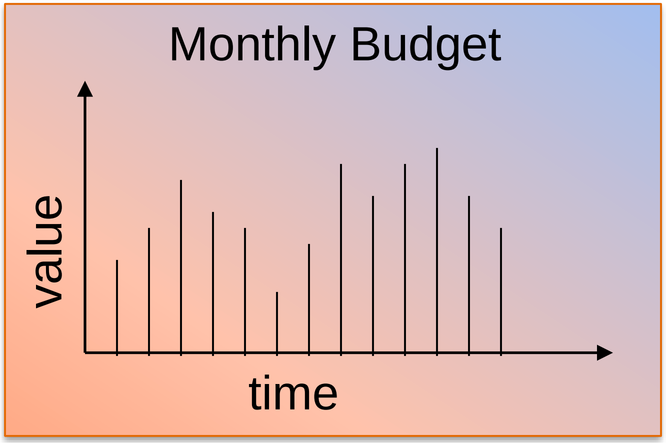
  It requires a correct result interpretation!

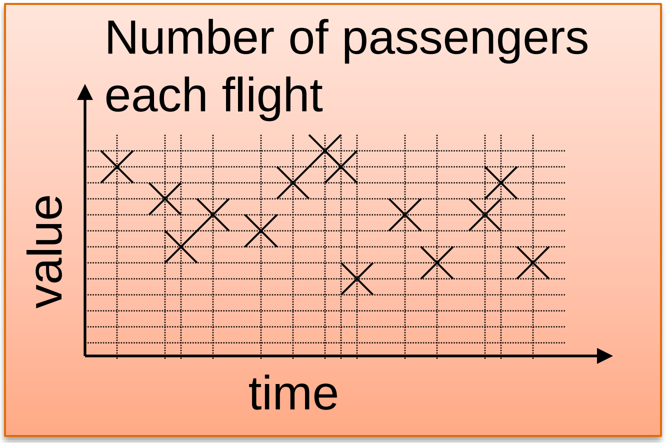
Examples –Discrete/Continuous

Mathematical Modelling and Simulation

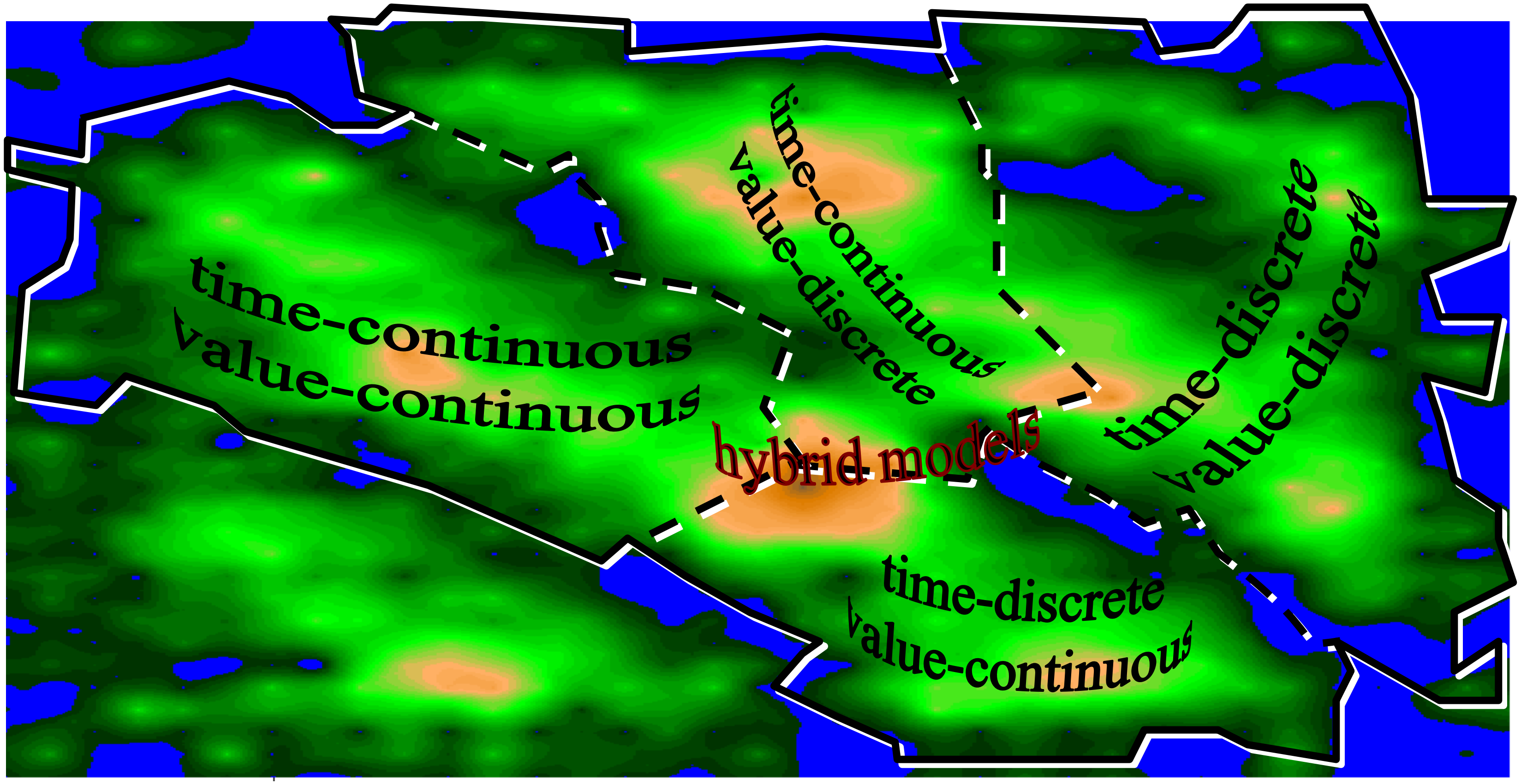








Landmap of Modelling Methods – Discrete / Continous



Model Procedures

System

Theoretical Modelling
Deductive Analysis

Modelling by Laws and Rules

White Box Modelling

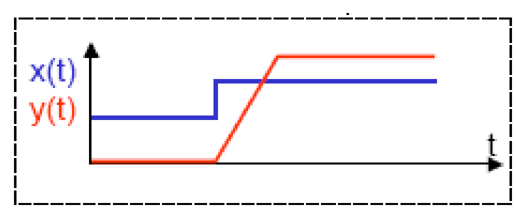
Experimental Modelling
Inductive Analysis

Modelling by using models with observed behaviour

Black Box Modelling

Ohms Law
Newton's Law
Supply/Demand Law

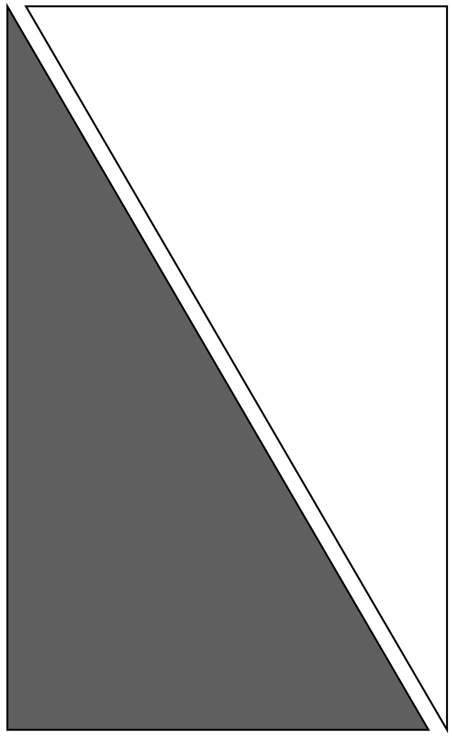
Mathematical Model



Application vs. Modelling Approach

White Box Modeling

- Electrotechnique
- Mechanics
- Environment
- Medicine
- Economy
- Sociology

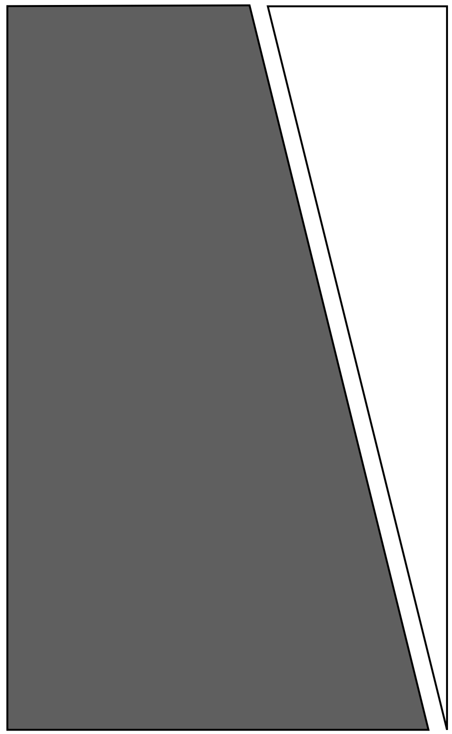


- Laws
- Laws and Observations
- Laws and Observations
- Observations and Characterisation
- Observations and Characterisation

Black Box Modeling

From Deduction to Induction

- Electrotechnique
- Mechanics
- Environment
- Medicine
- Economy
- Sociology



- Laws
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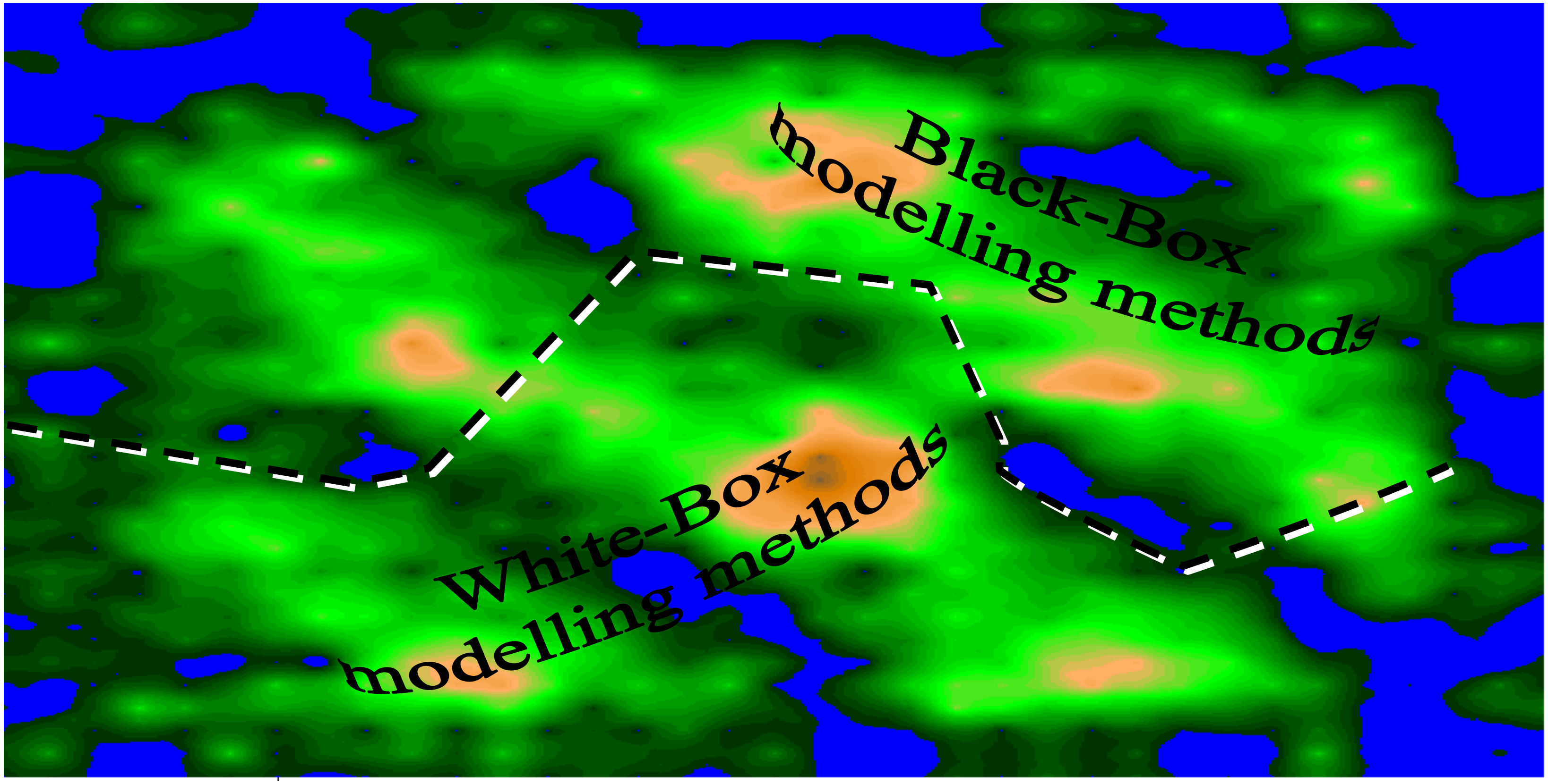
Deductive models may contain too many parameters – problems with identification

Landmap of Modelling Methods – Discrete / Continous

Mathematical Modelling and Simulation



Stochastic/Deterministic

- If the output of the simulation of a model is uniquely defined by input parameters, initial conditions and model parameters the model is called **deterministic**.
- Otherwise it is called **stochastic**.

Results:
14.2
14.2
14.2
14.2
…

$\Rightarrow$ deterministic

Results:
14.3
14.6
13.9
14.2
…

$\Rightarrow$ stochastic

Stochastic/Deterministic

Mathematical Modelling and Simulation

Stochastic models are necessary...

- ... if random effects are included in the system.
  $\rightarrow$ coin toss, rolling a dice, ...

- ... if emelents of the system are too complex to be described by deterministic rules.

$\rightarrow$ human behaviour, problems at system borders,...

Stochastic/Deterministic

Mathematical Modelling and Simulation

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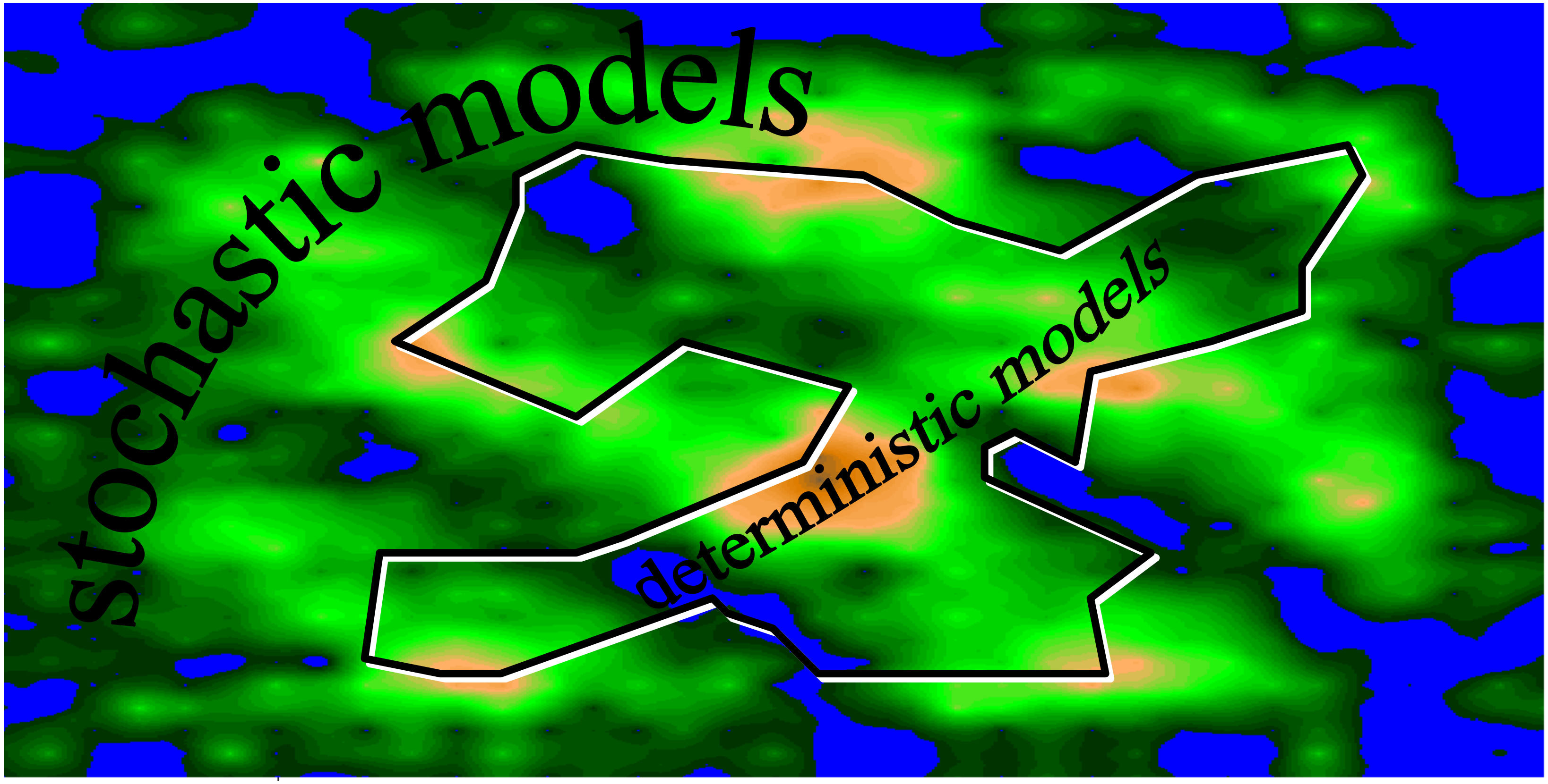
Landmap of Modelling Methods – Discrete / Continous

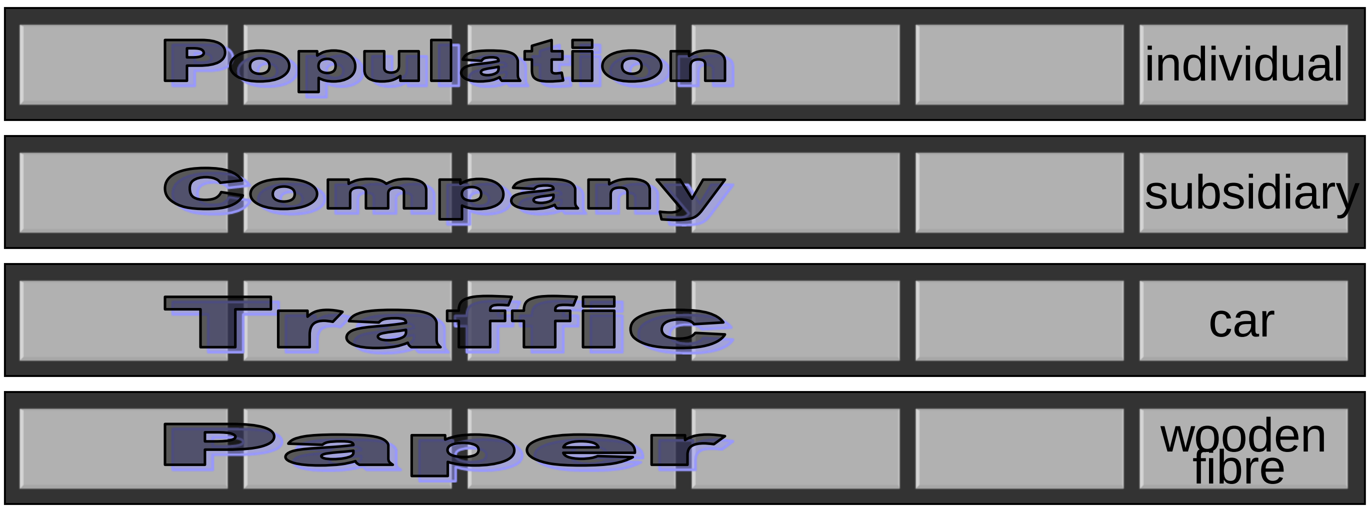
Mathematical Modelling and Simulation



Microscopic/Macroscopic Models

Mathematical Modelling and Simulation

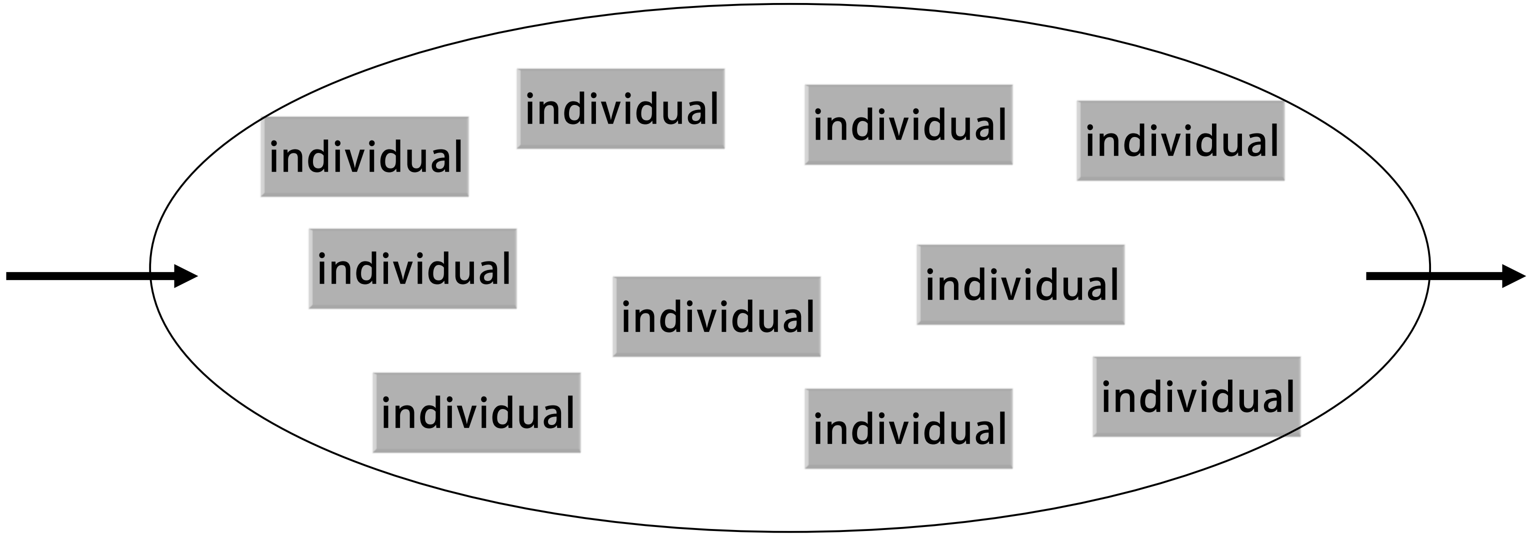
- If systems consist of a big set of similar subsystems…



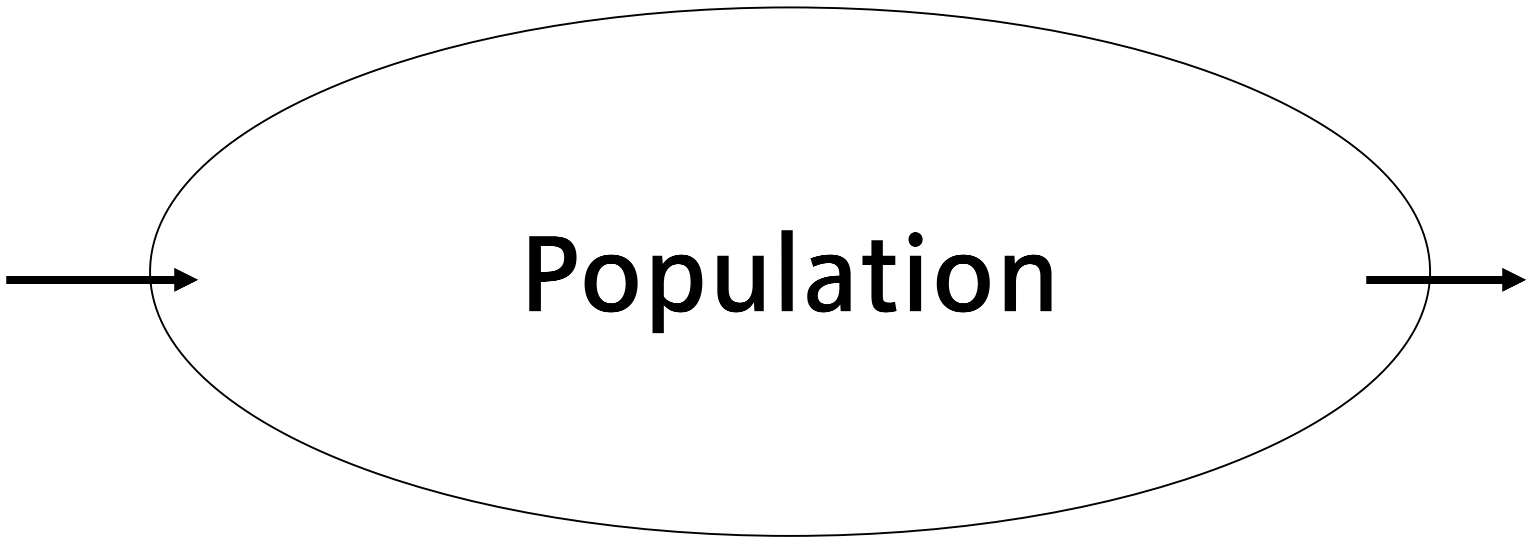
… the question arises whether a micro- meso- or macroscopic model should be used.

Microscopic/Macroscopic Models

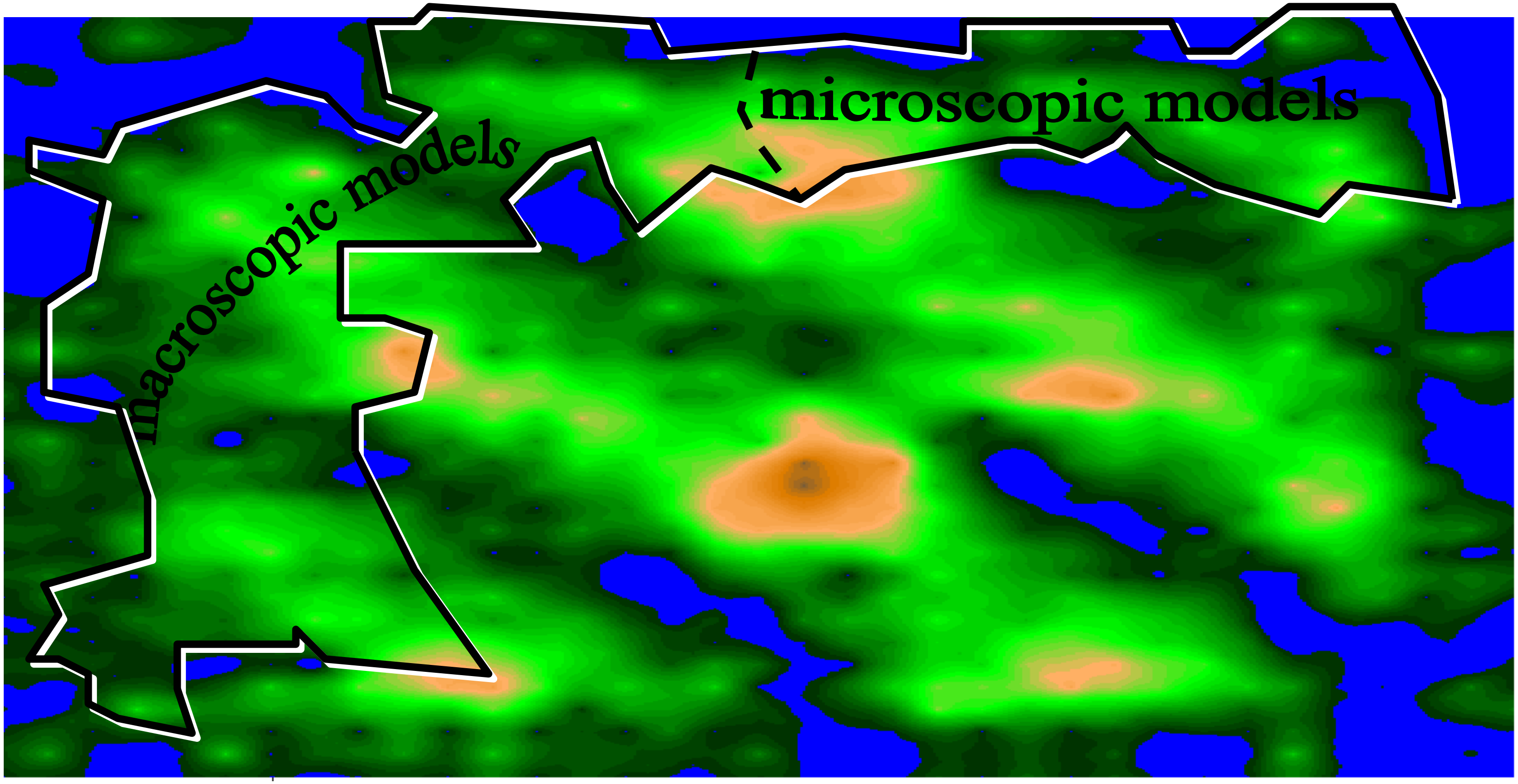
- Microscopic models treat each subsystem as an individual model. Finally they are linked in order to model the whole system.



- Macroscopic models treat the whole system, neglecting the fact, that it consists of subsystems.

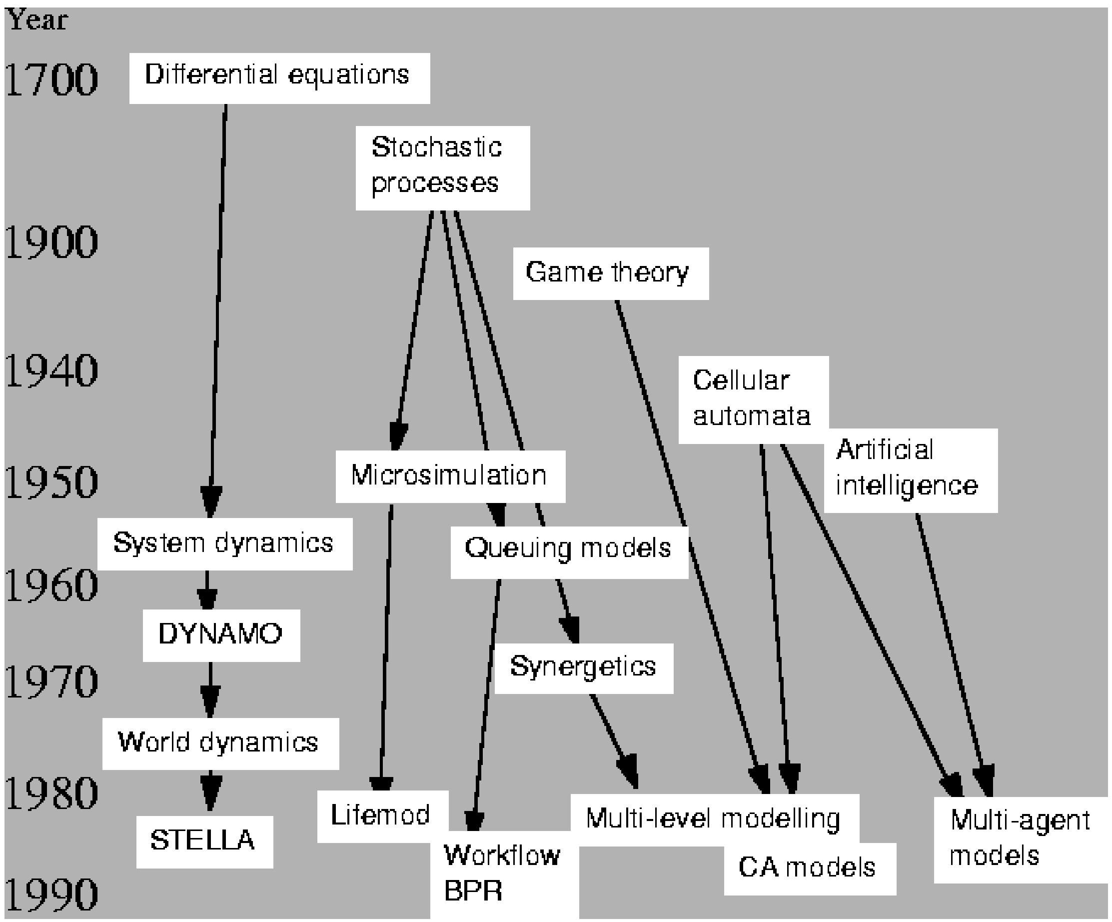


Landmap of Modelling Methods – Microscopic/Macroscopic



Approaches for Soft Sciences Simulation

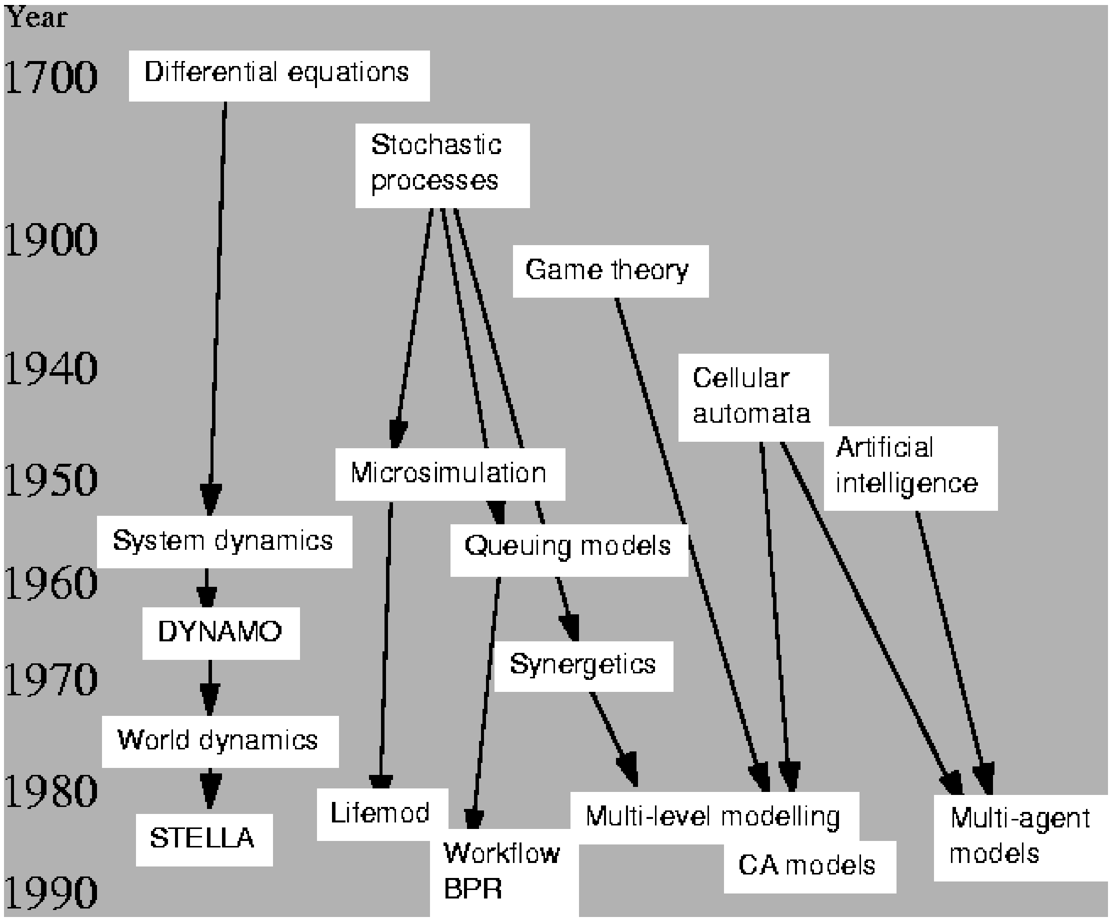
Mathematical Modelling and Simulation



(Troitzsch)

Approaches for Soft Sciences Simulation

Mathematical Modelling and Simulation

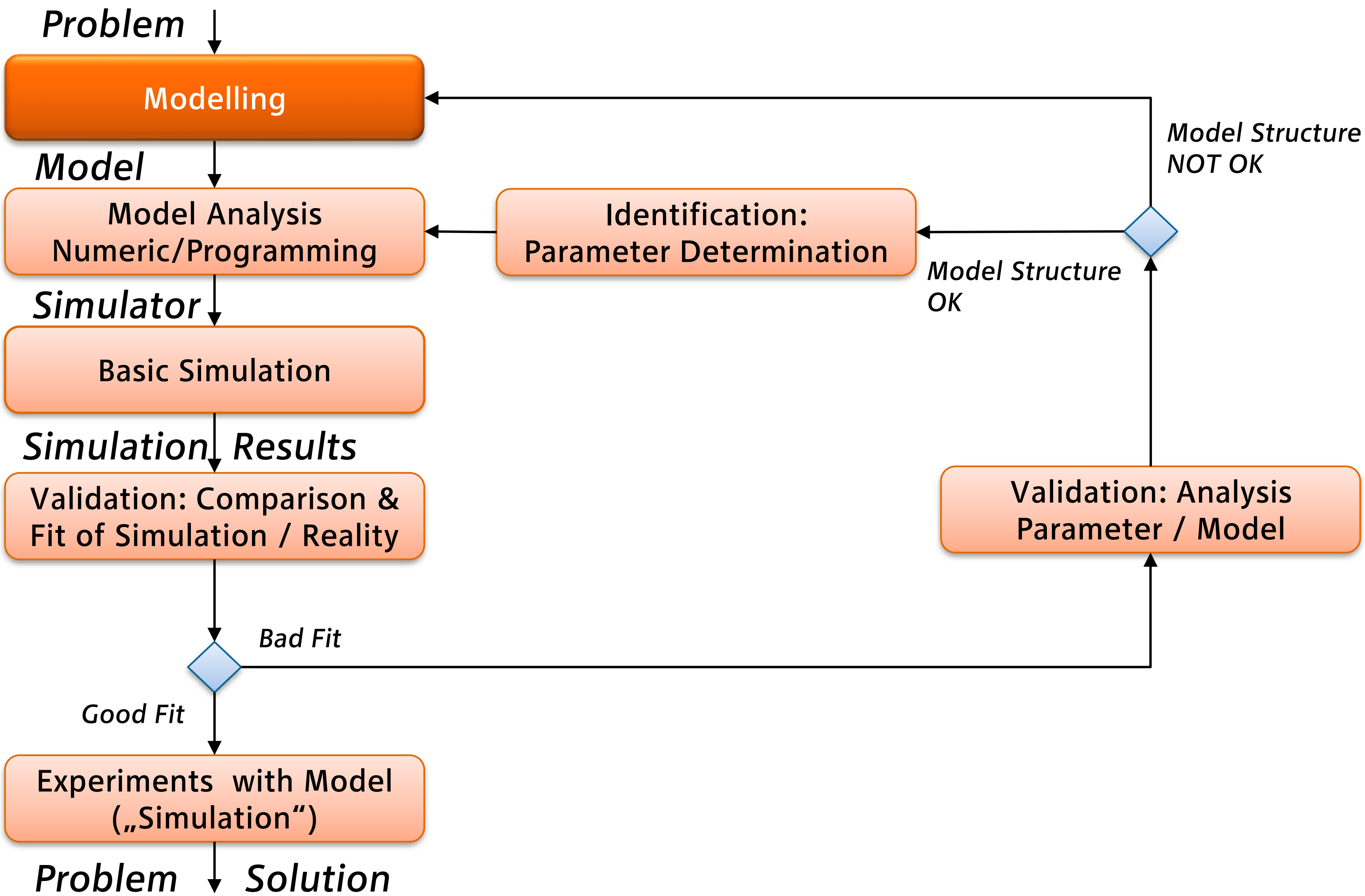


Aggregated ⟷ Distributed

(Troitzsch)

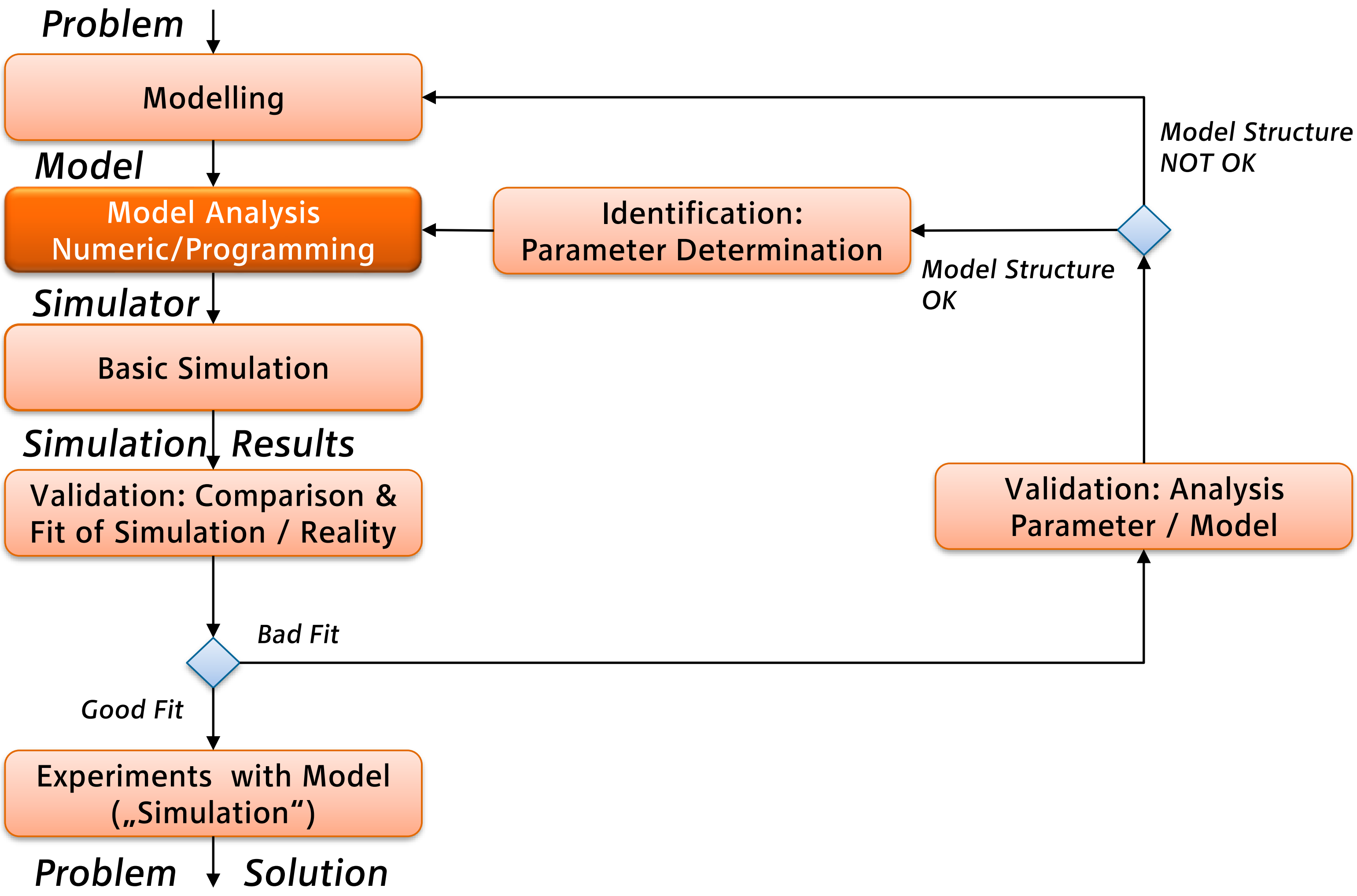
Simulation Circle

Mathematical Modelling and Simulation



Simulation Circle

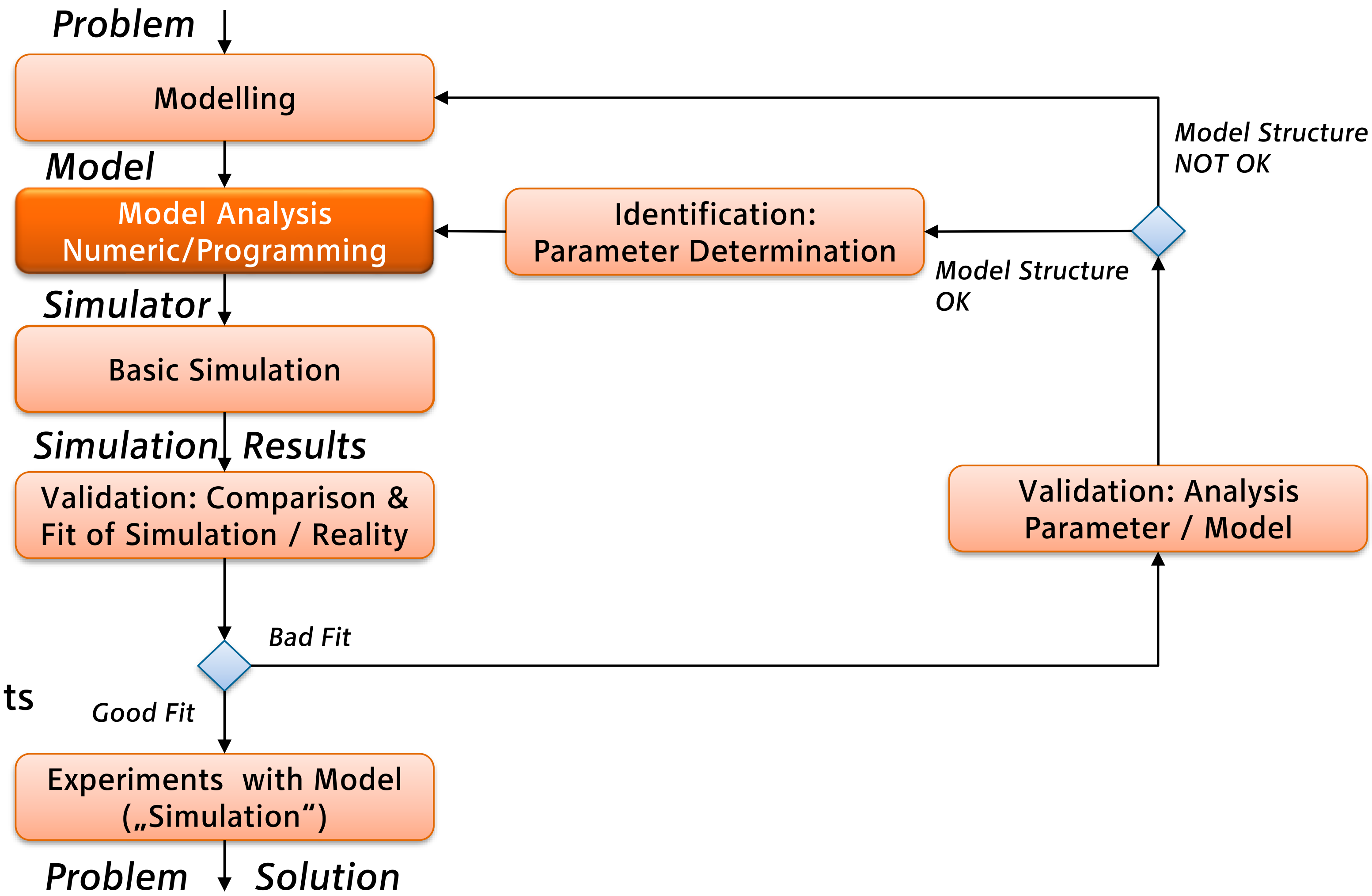
TU WIEN
Mathematical Modelling and Simulation



Simulation Circle

Mathematical Modelling and Simulation

- simulation software
- simulation languages
- Simulators
- Simulation systems
- Simulation Environments



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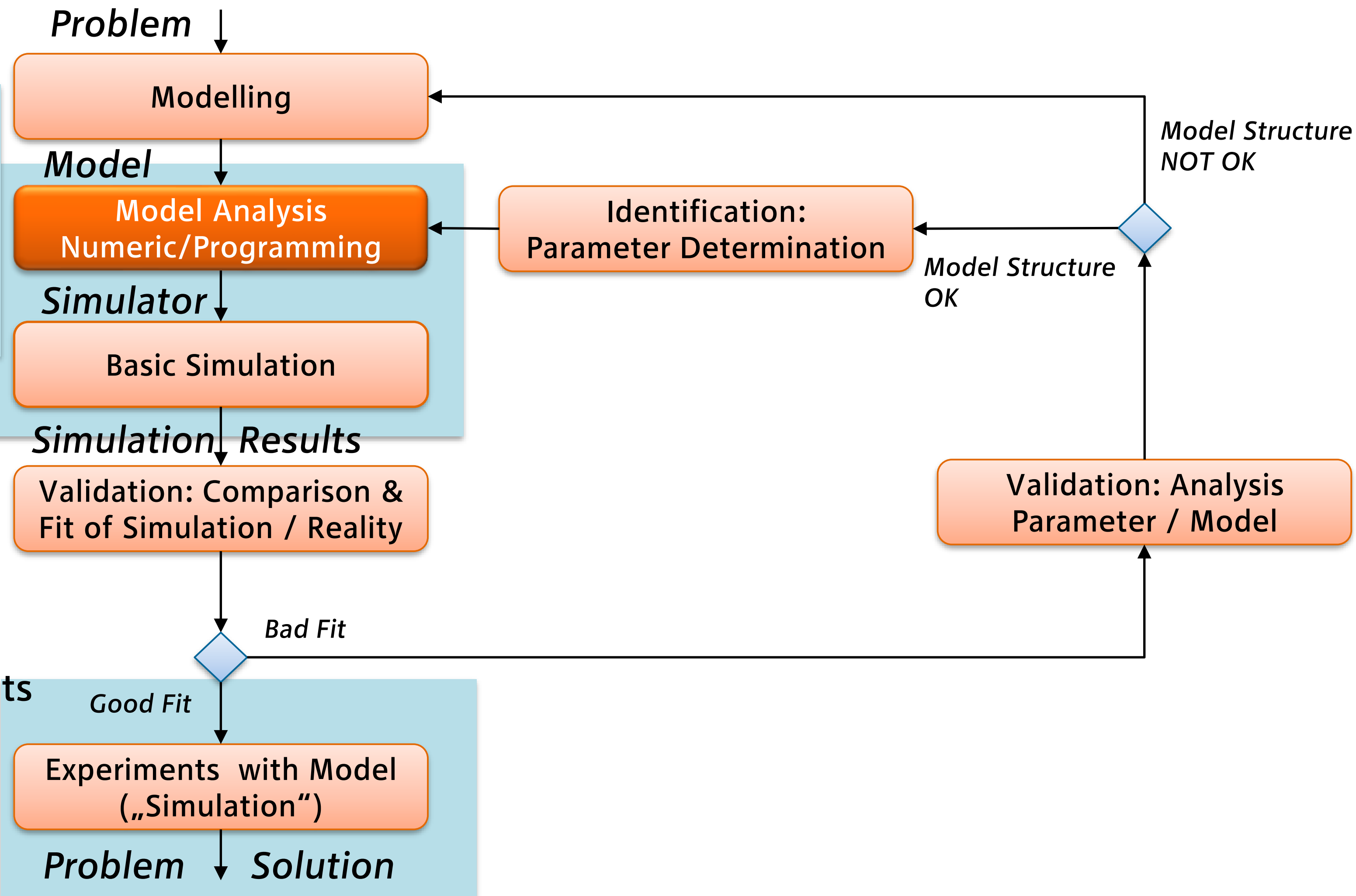

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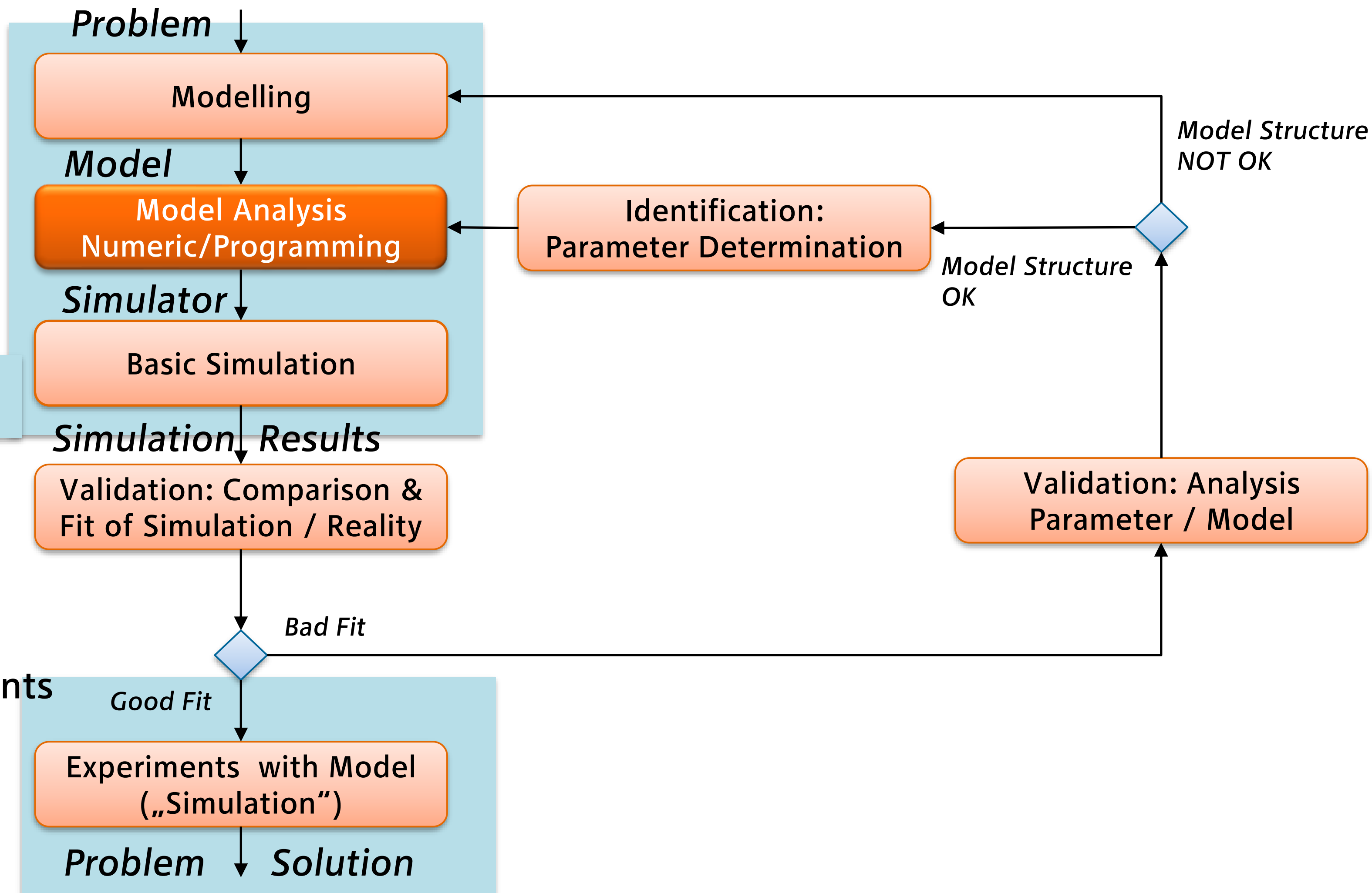

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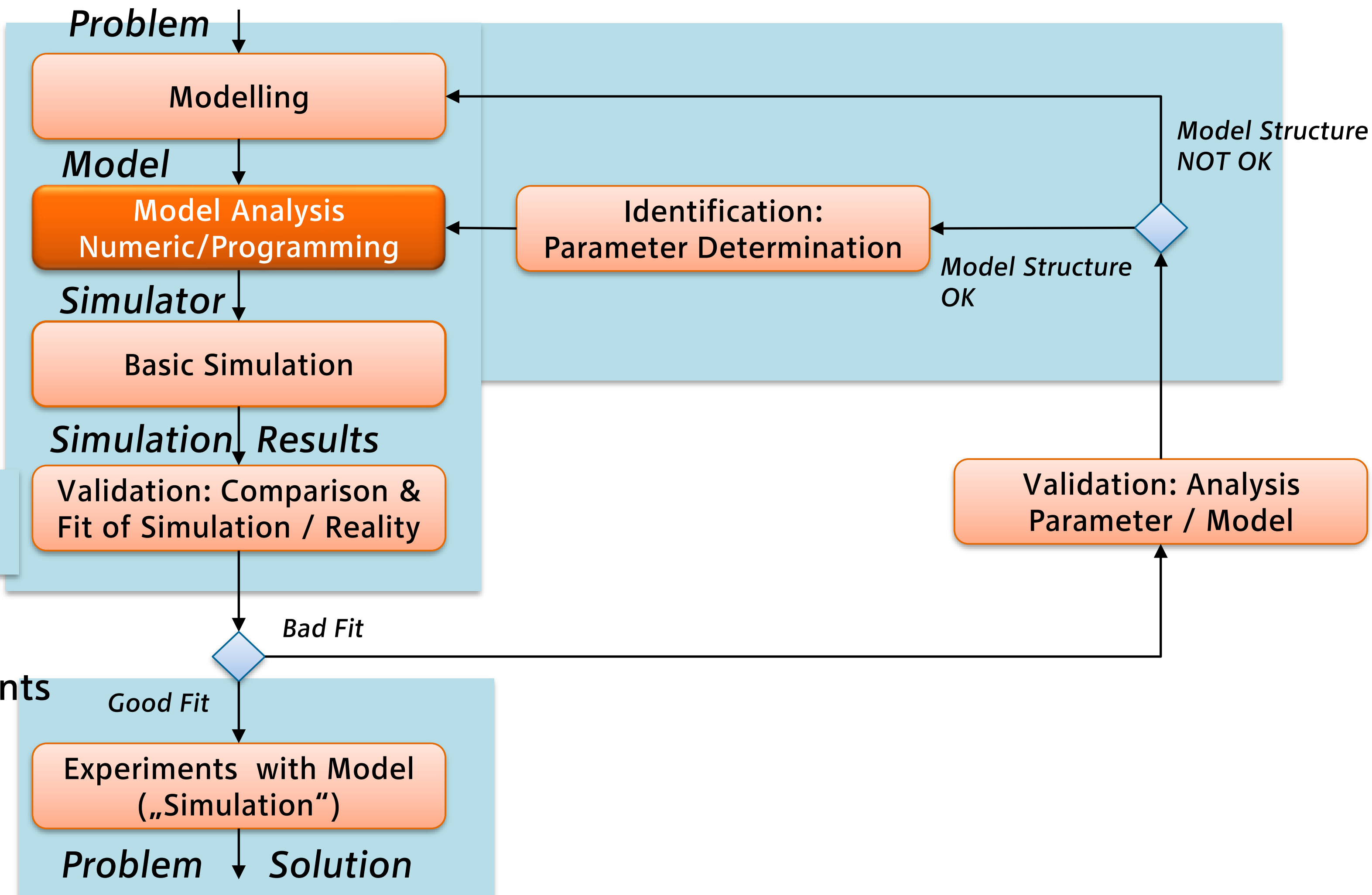

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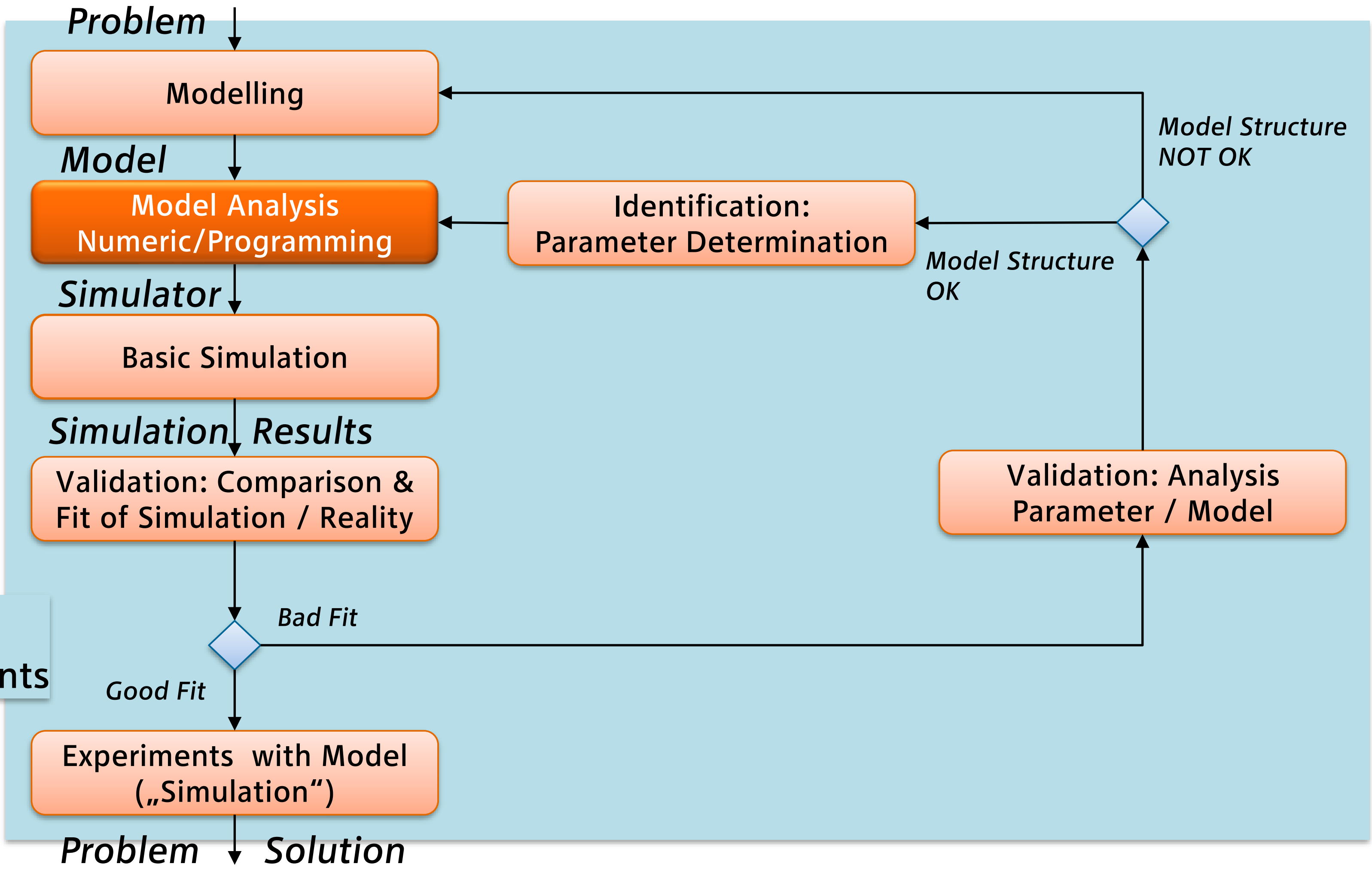

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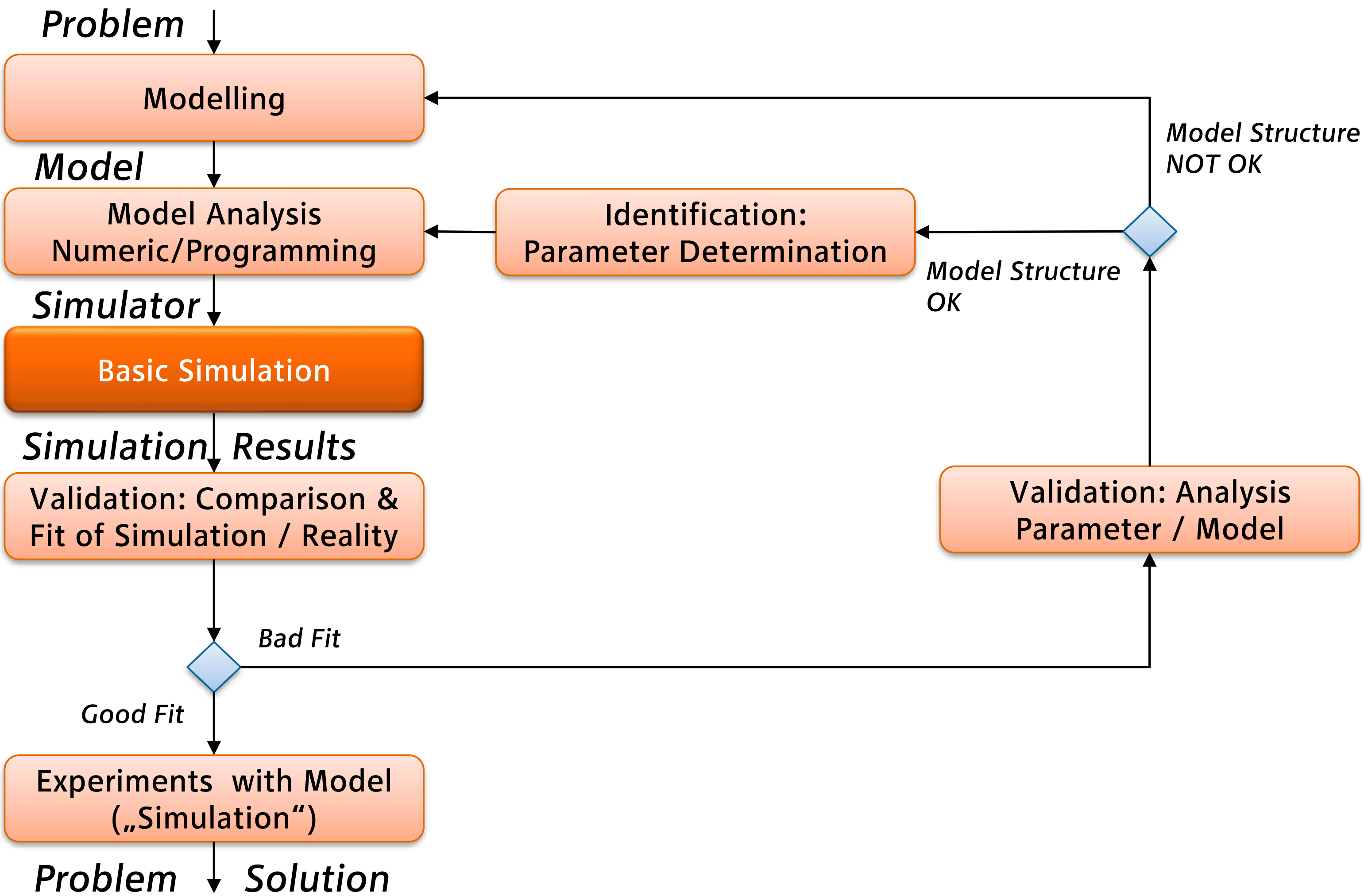

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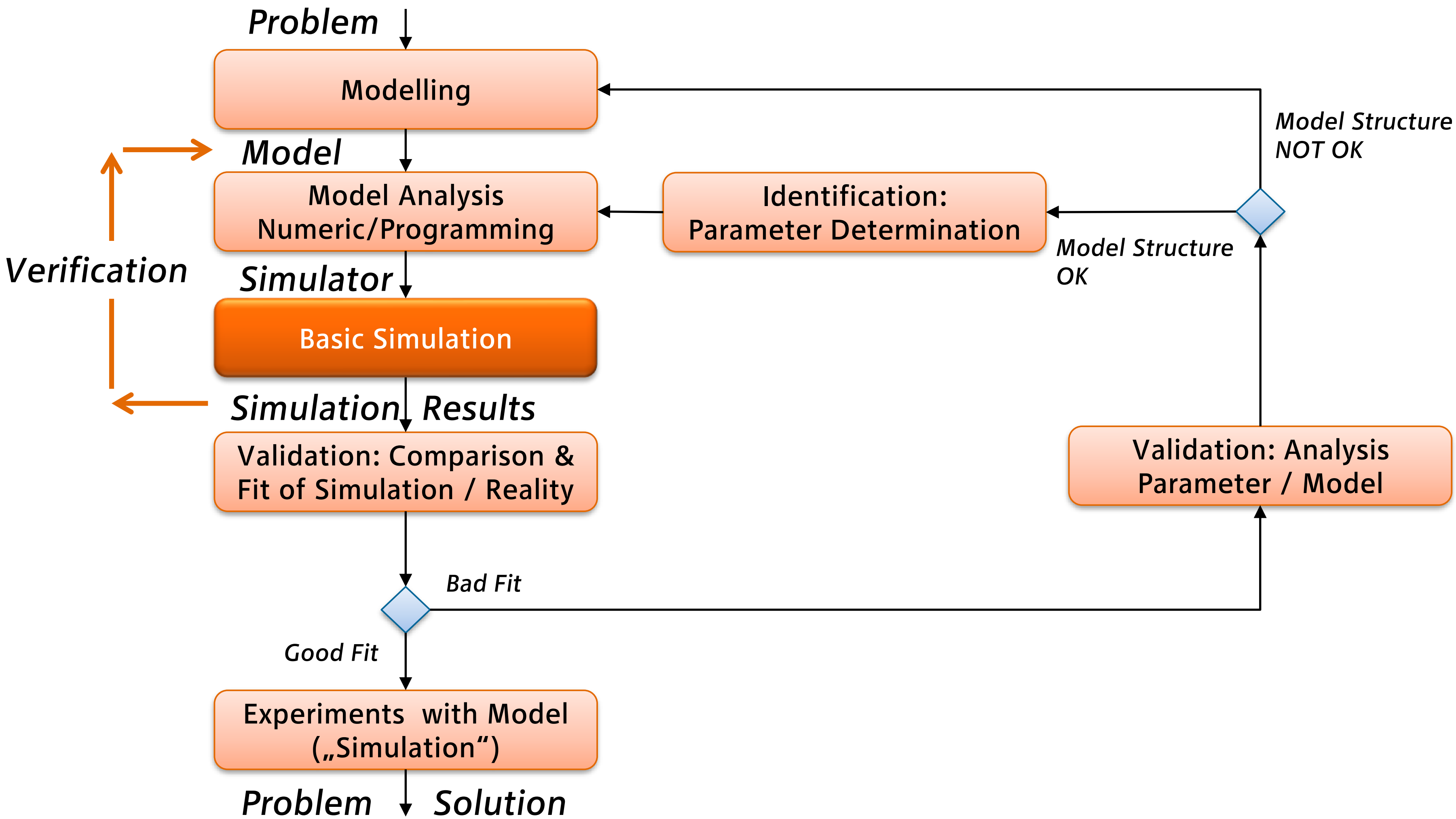

Simulation Circle

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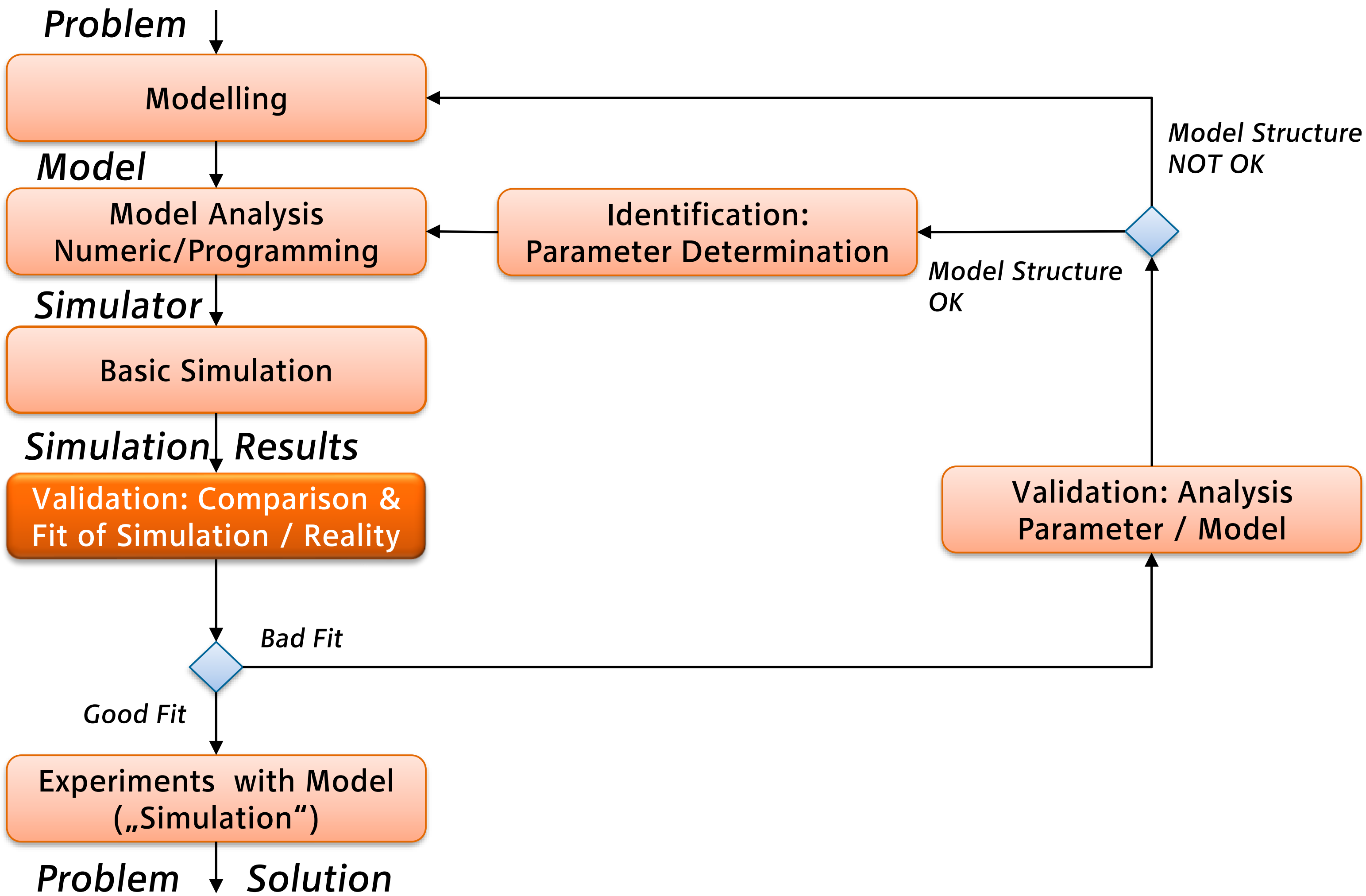

Simulation Circle

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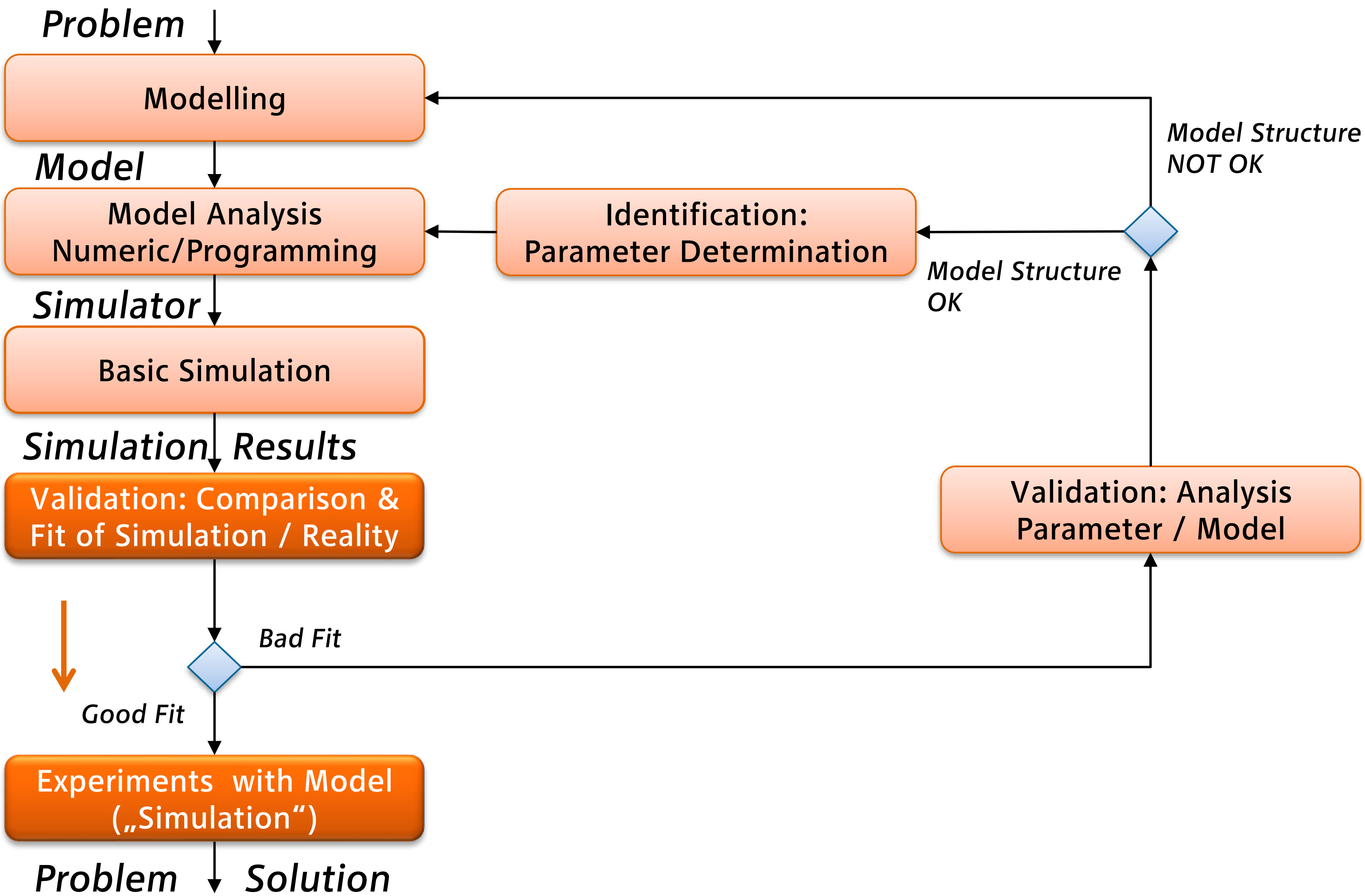

Simulation Circle

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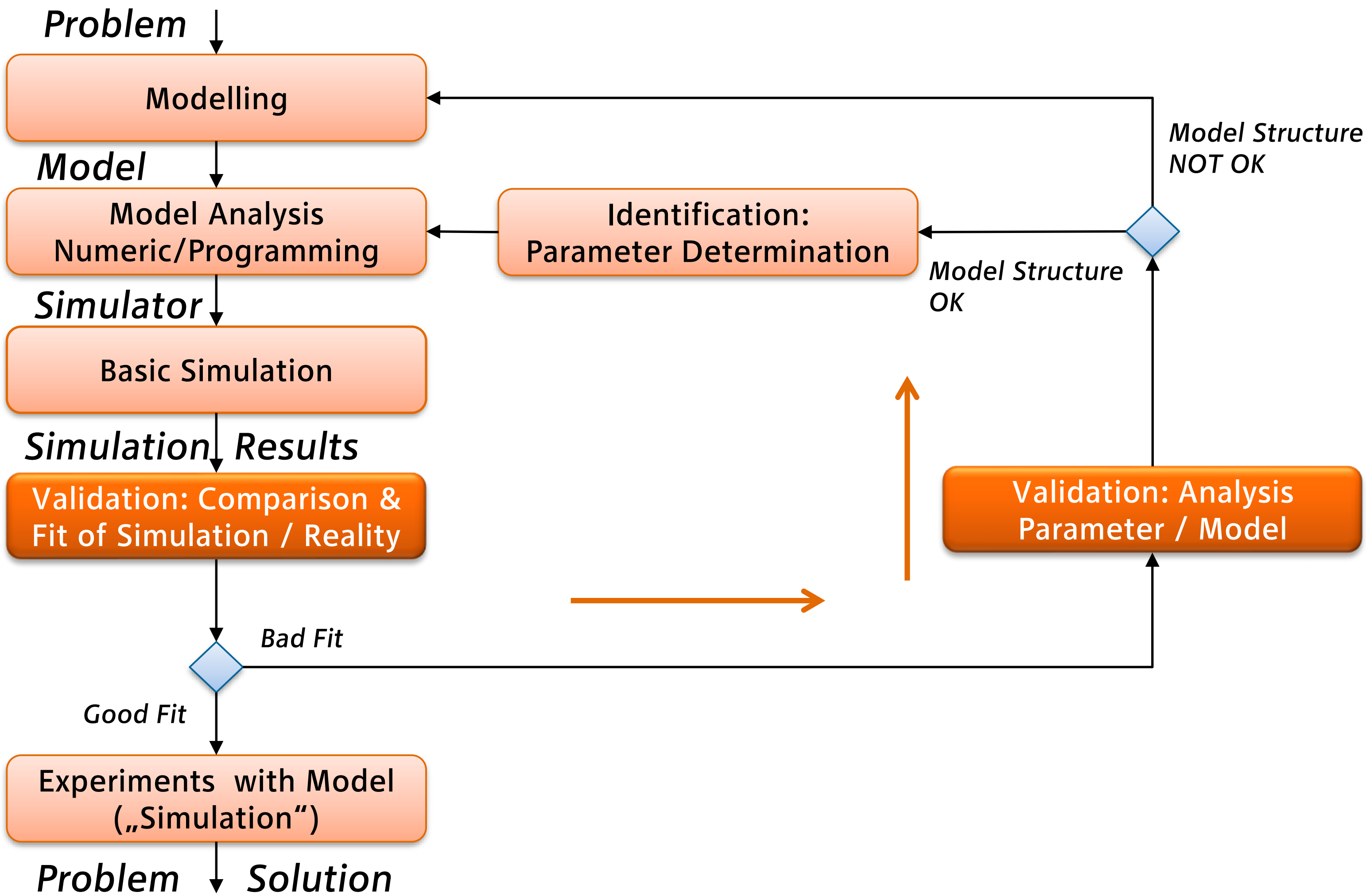

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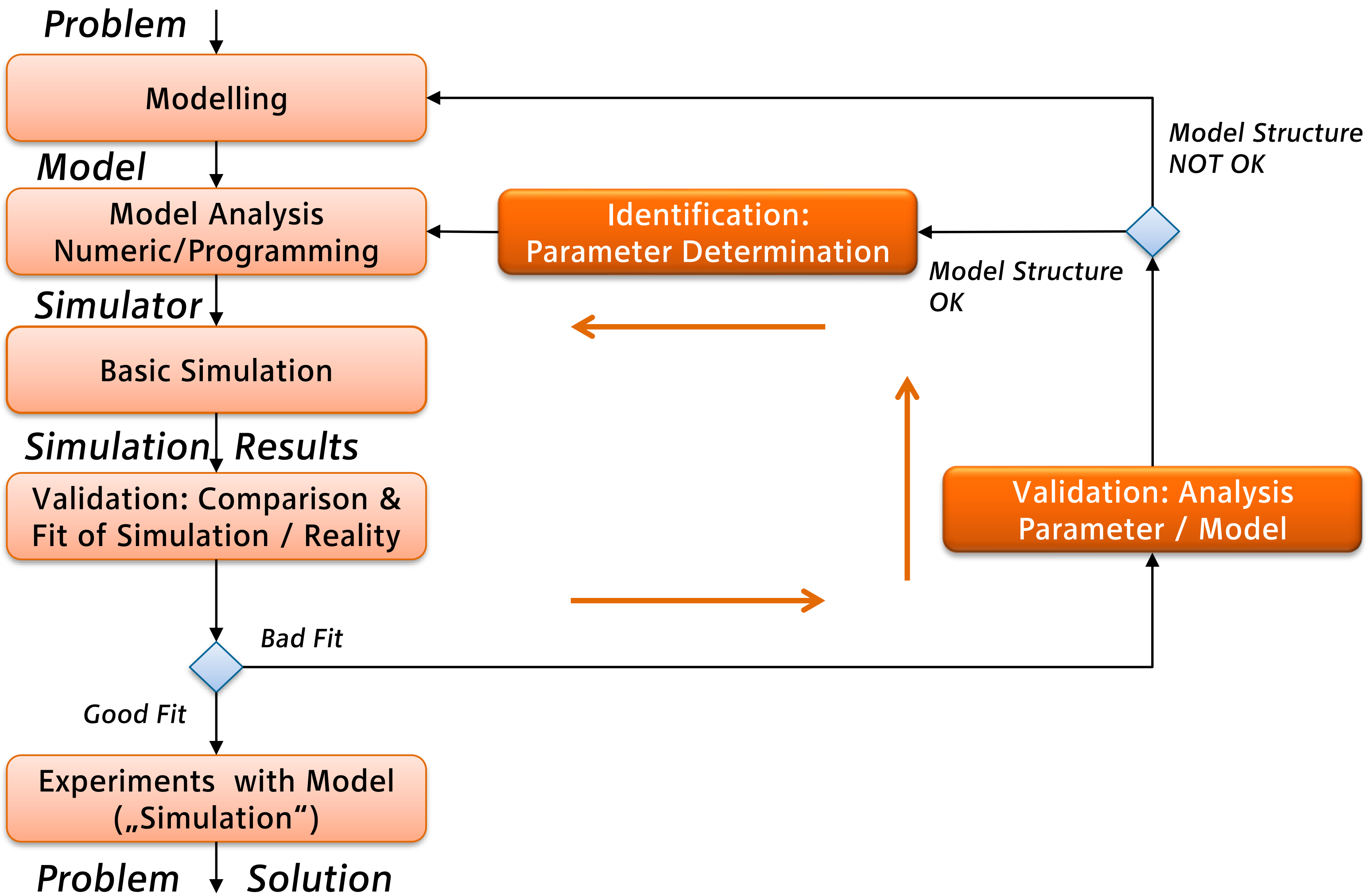
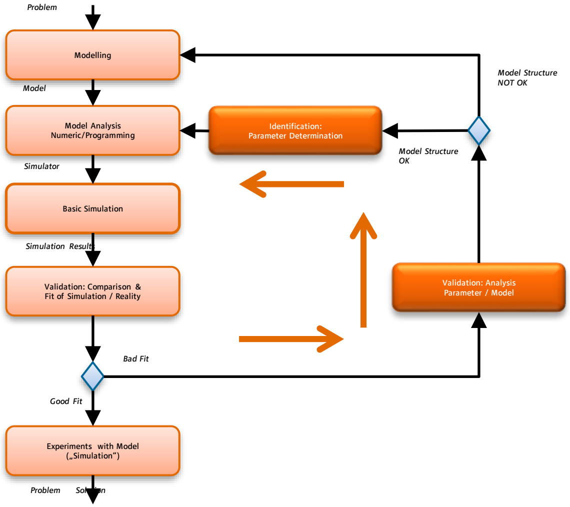
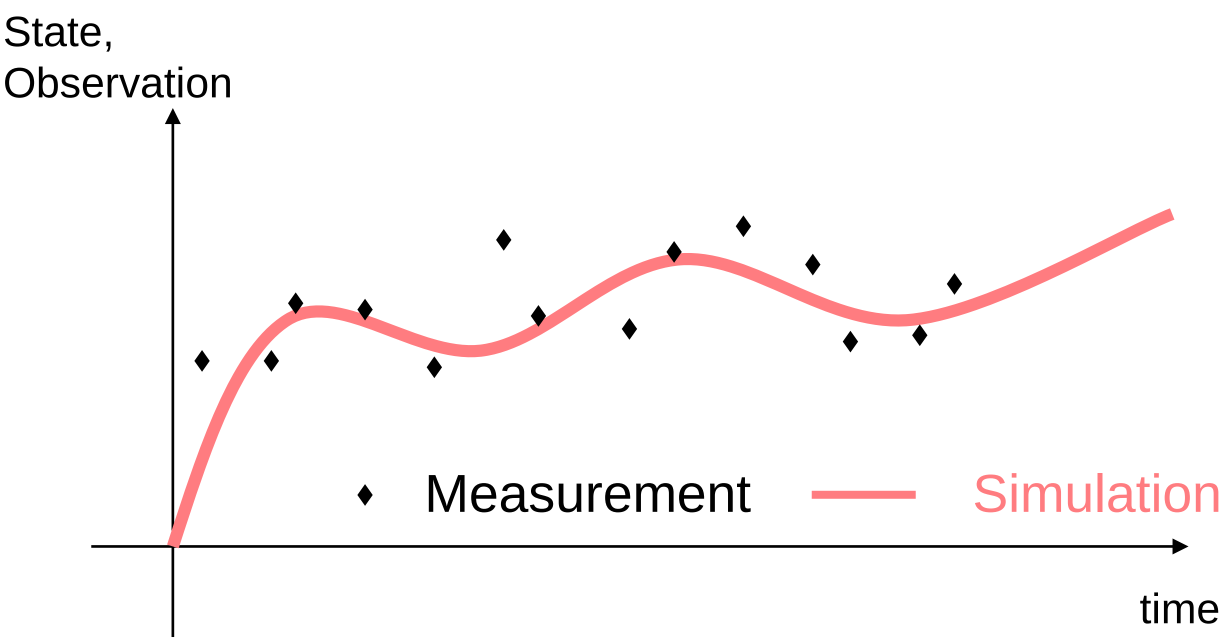


Simulation Circle





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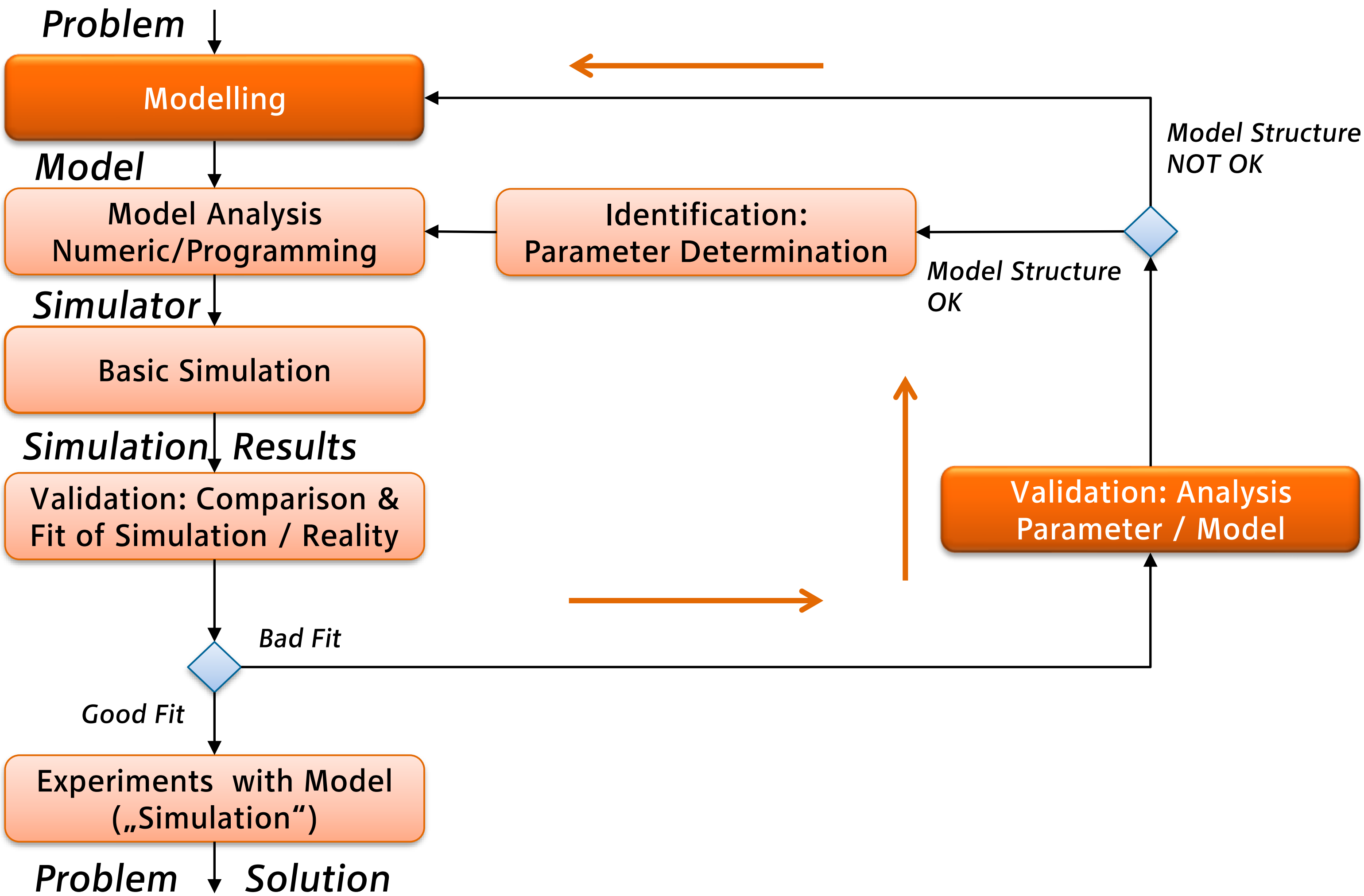

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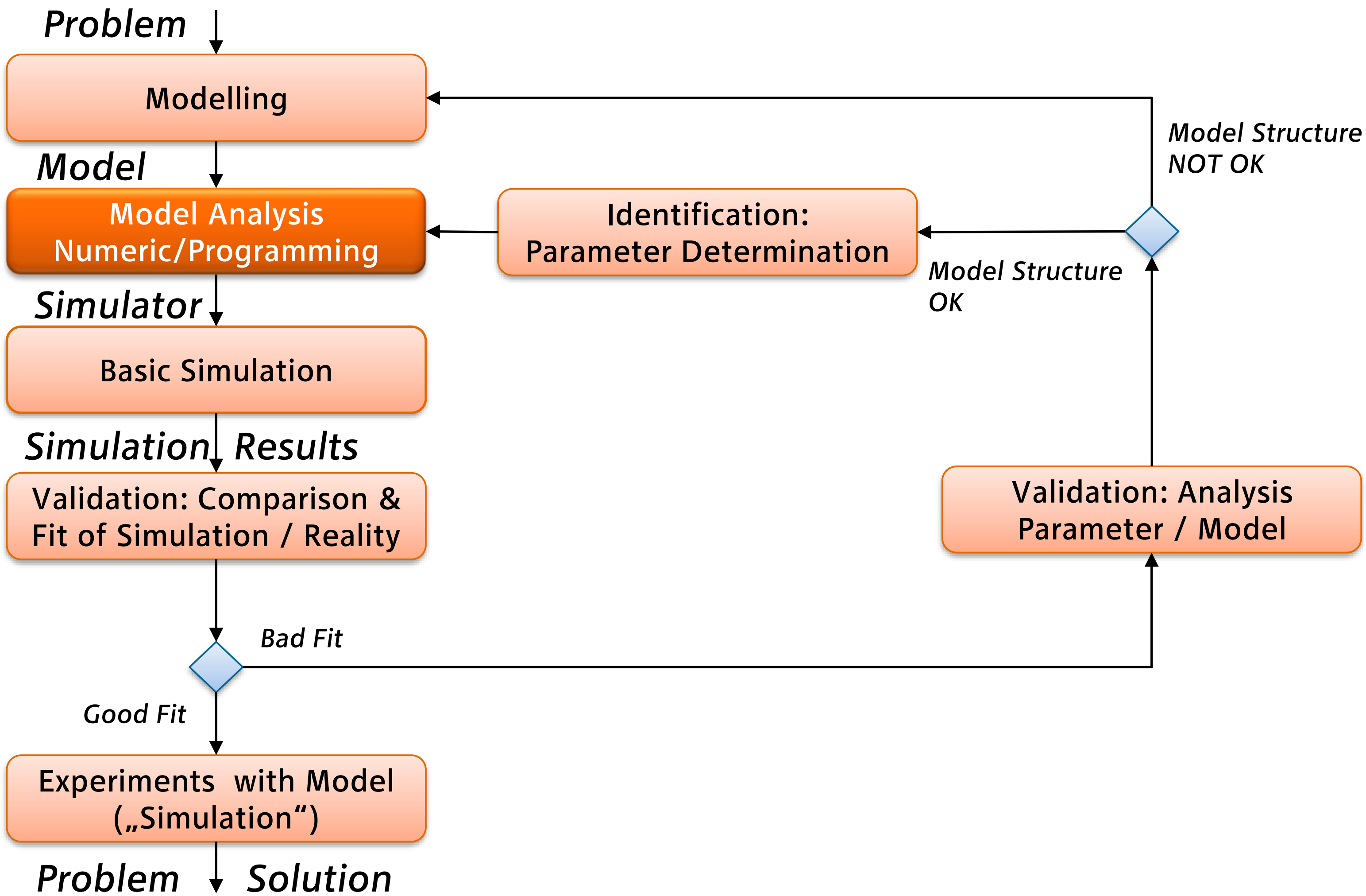

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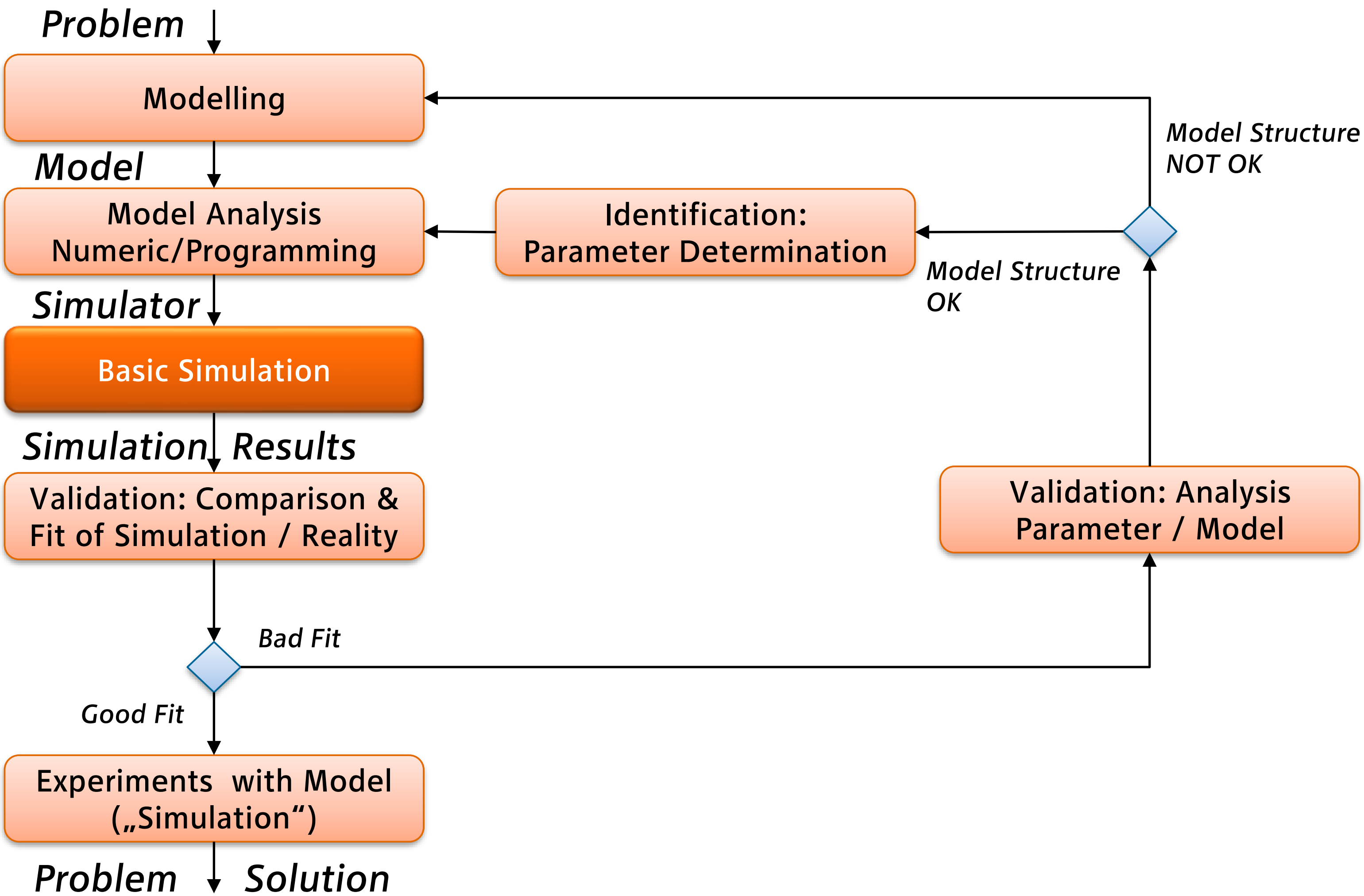

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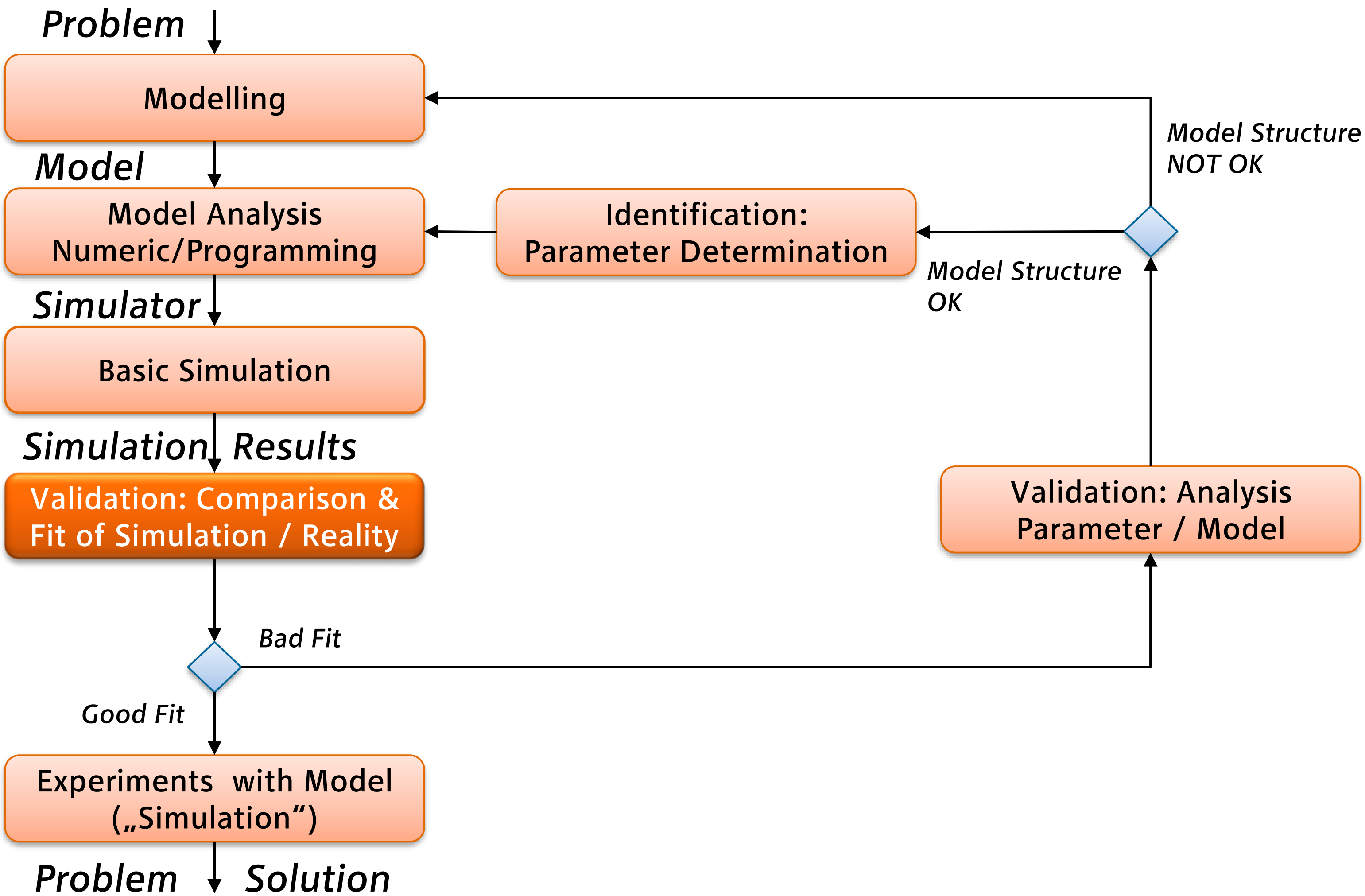

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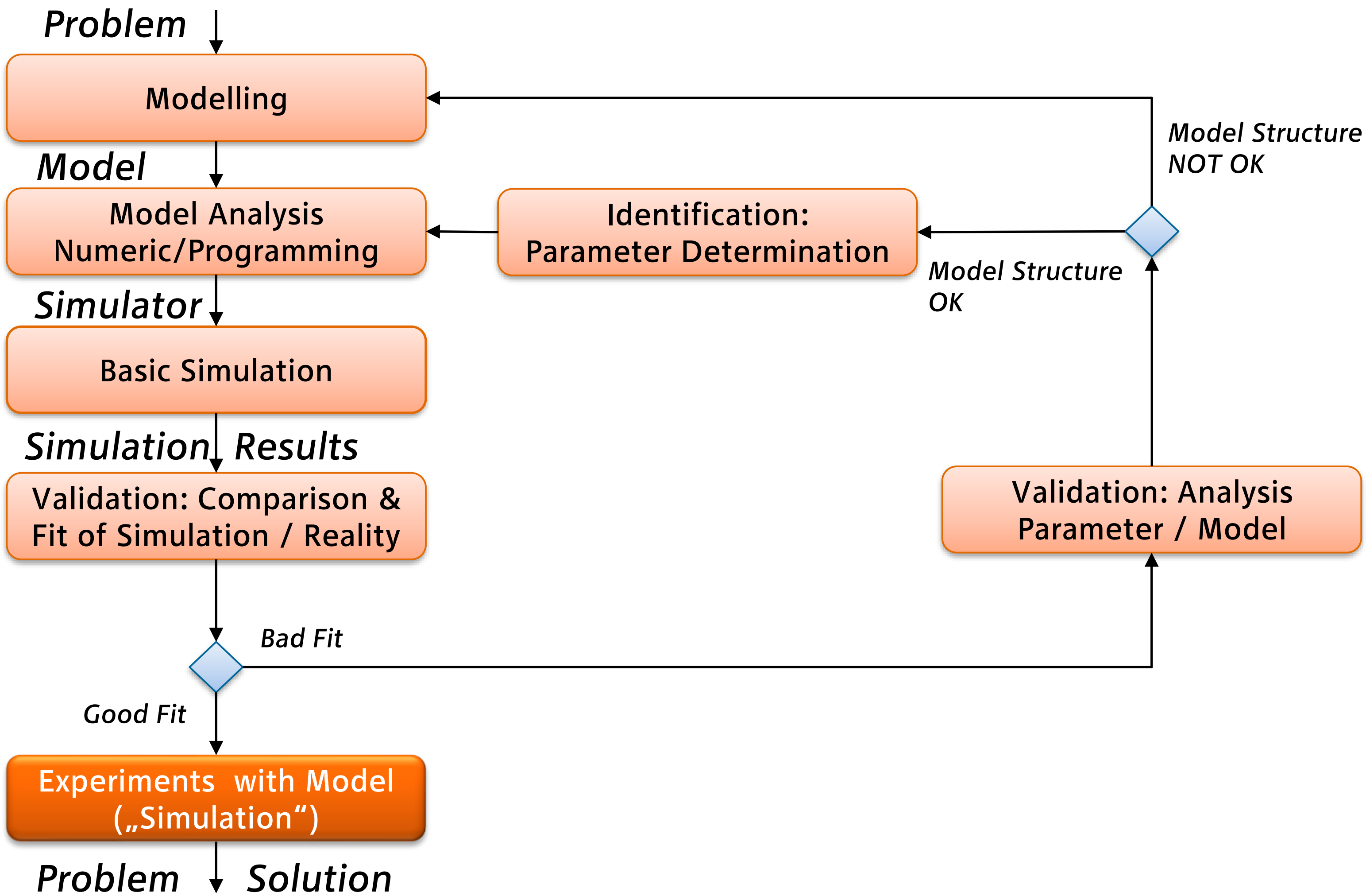

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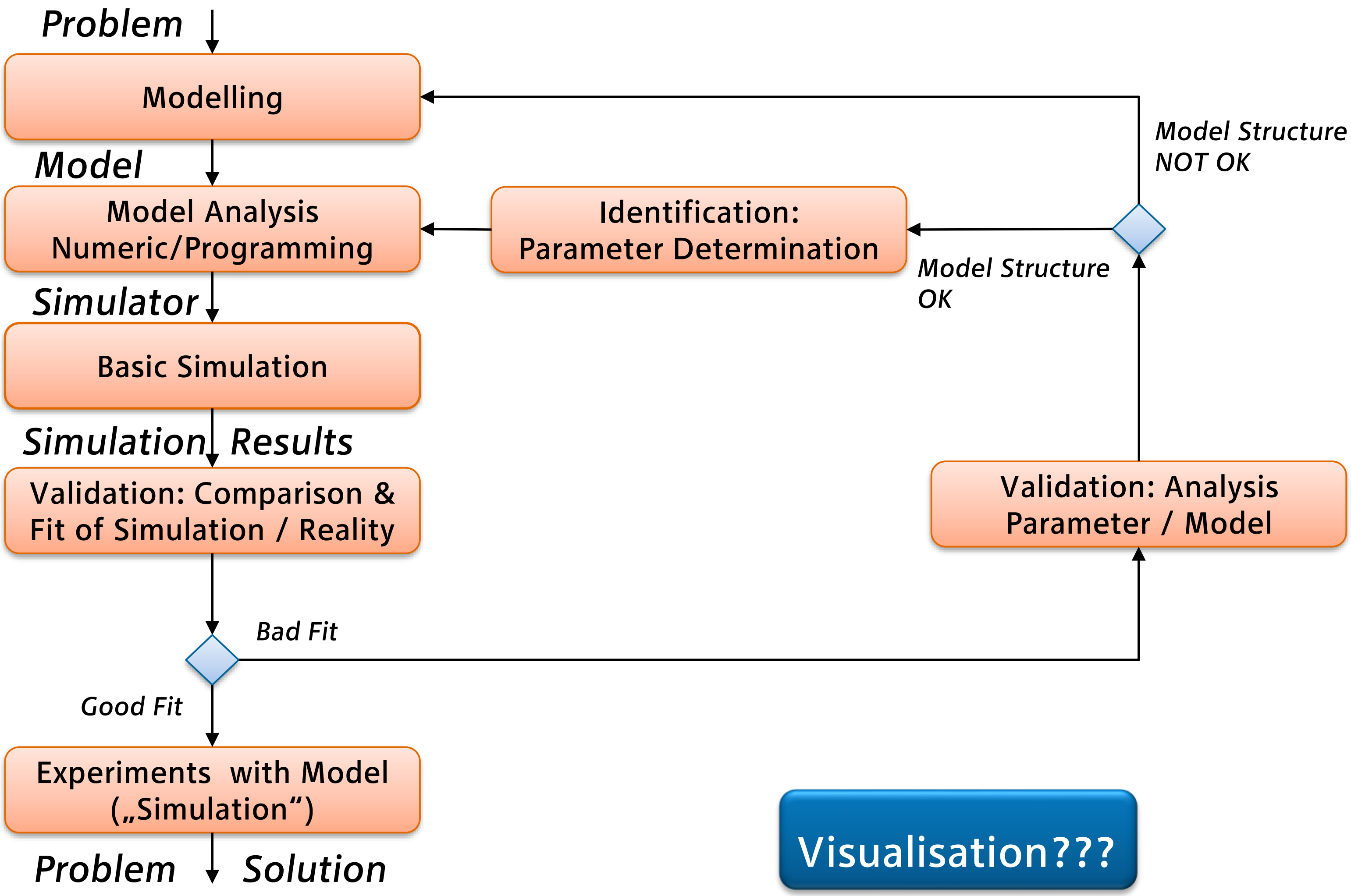
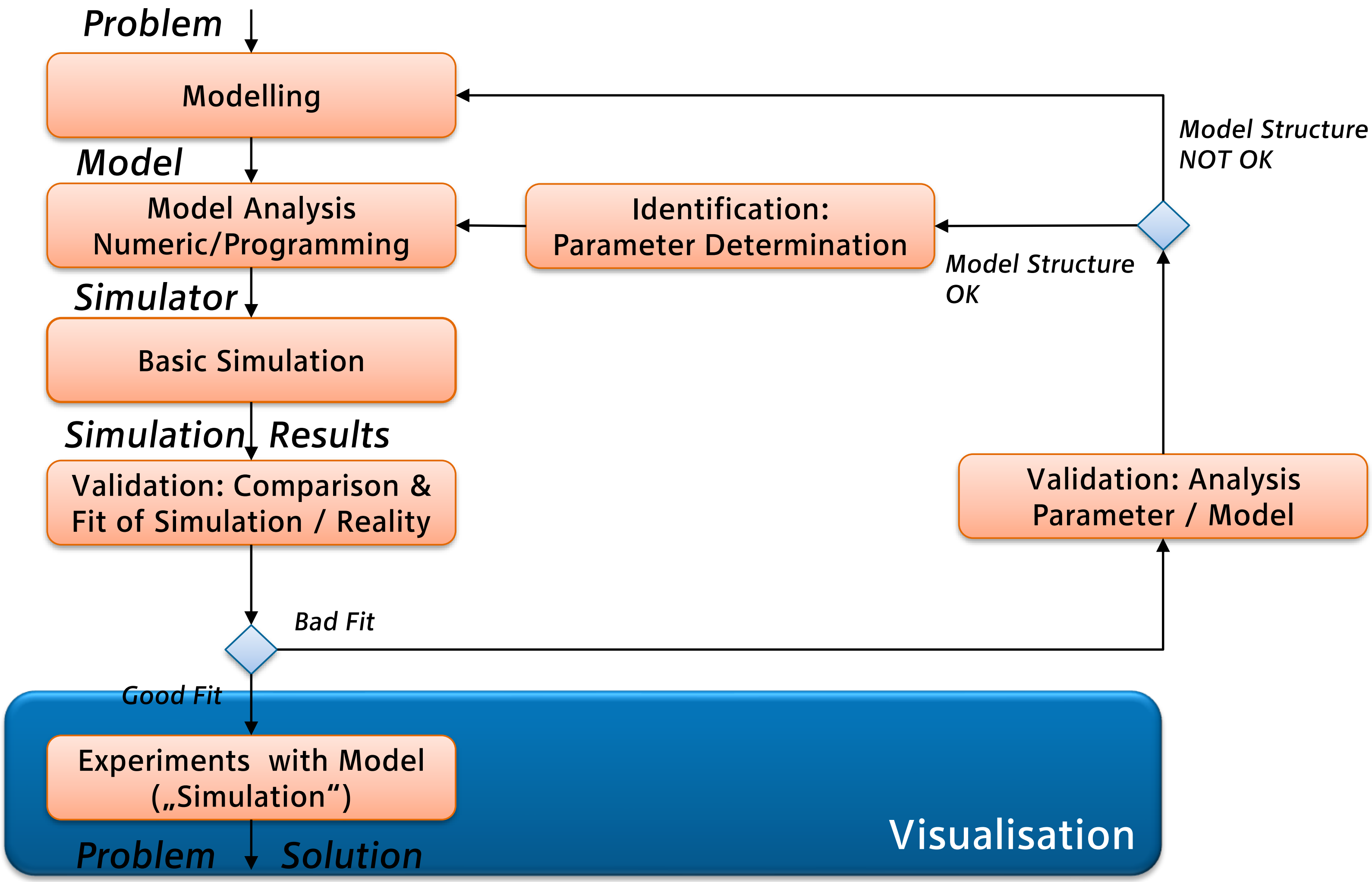


Visualisation???

Simulation Circle

Mathematical Modelling and Simulation



Simulation Circle

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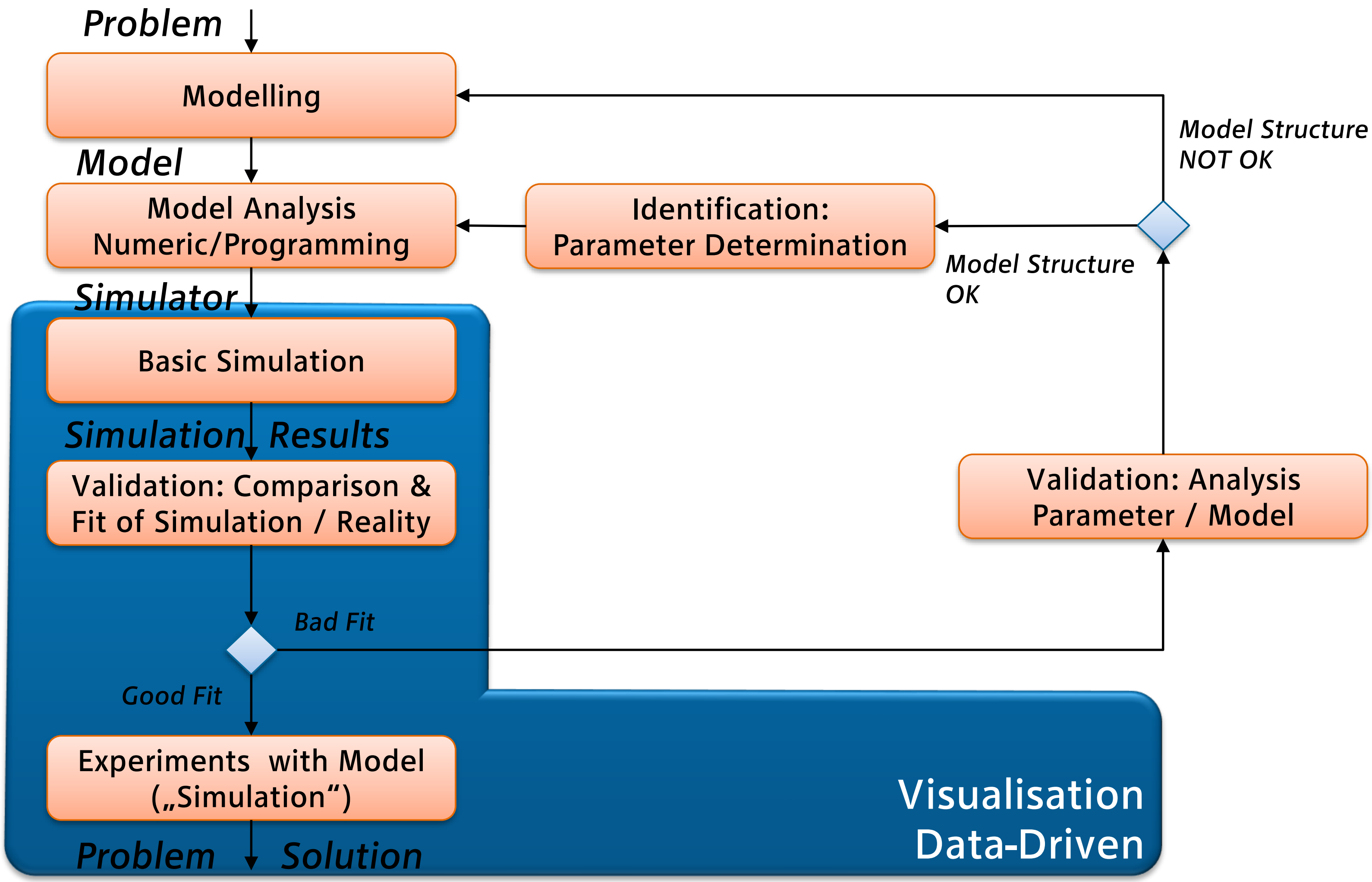

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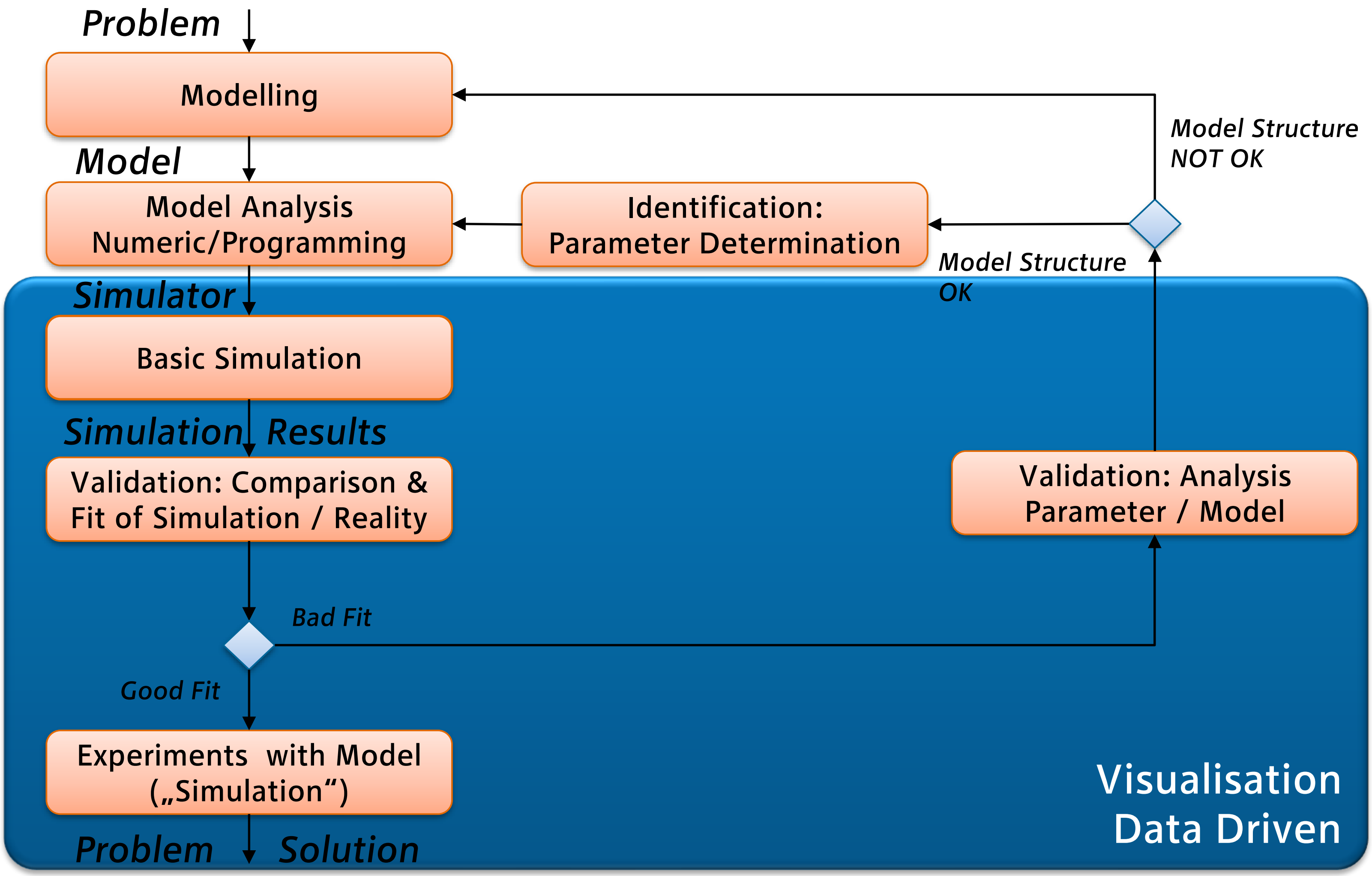

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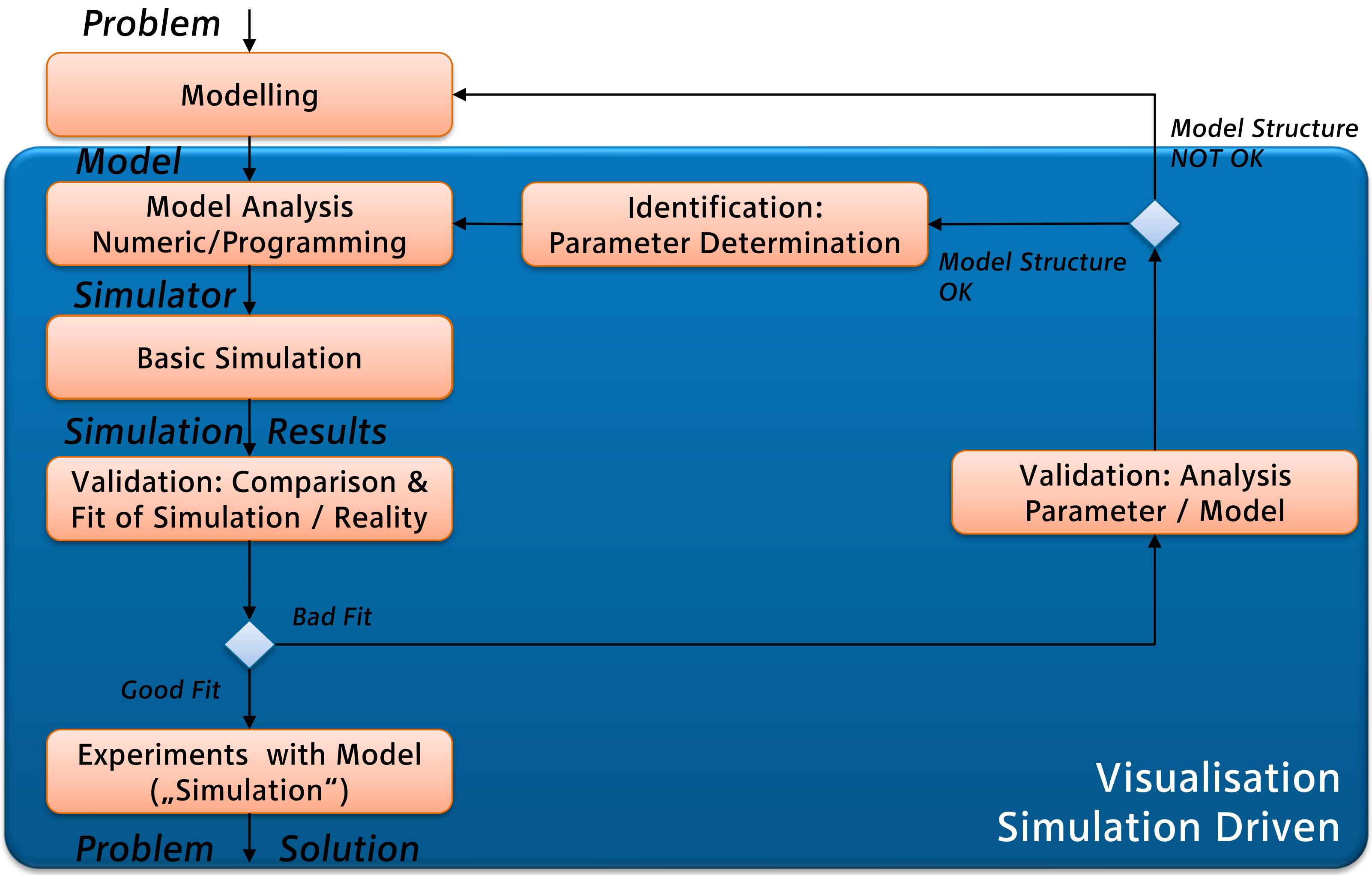

Simulation Circle

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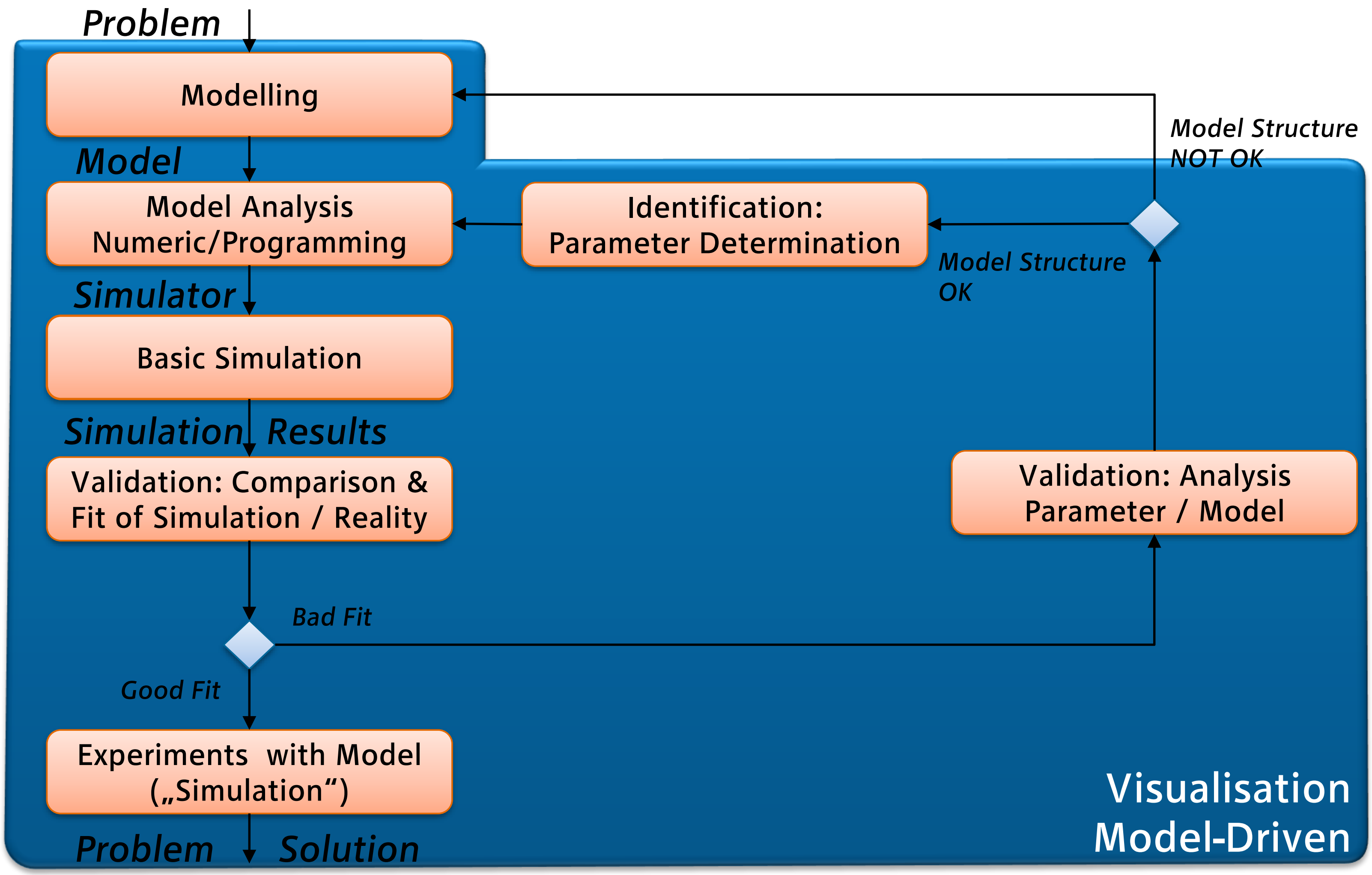
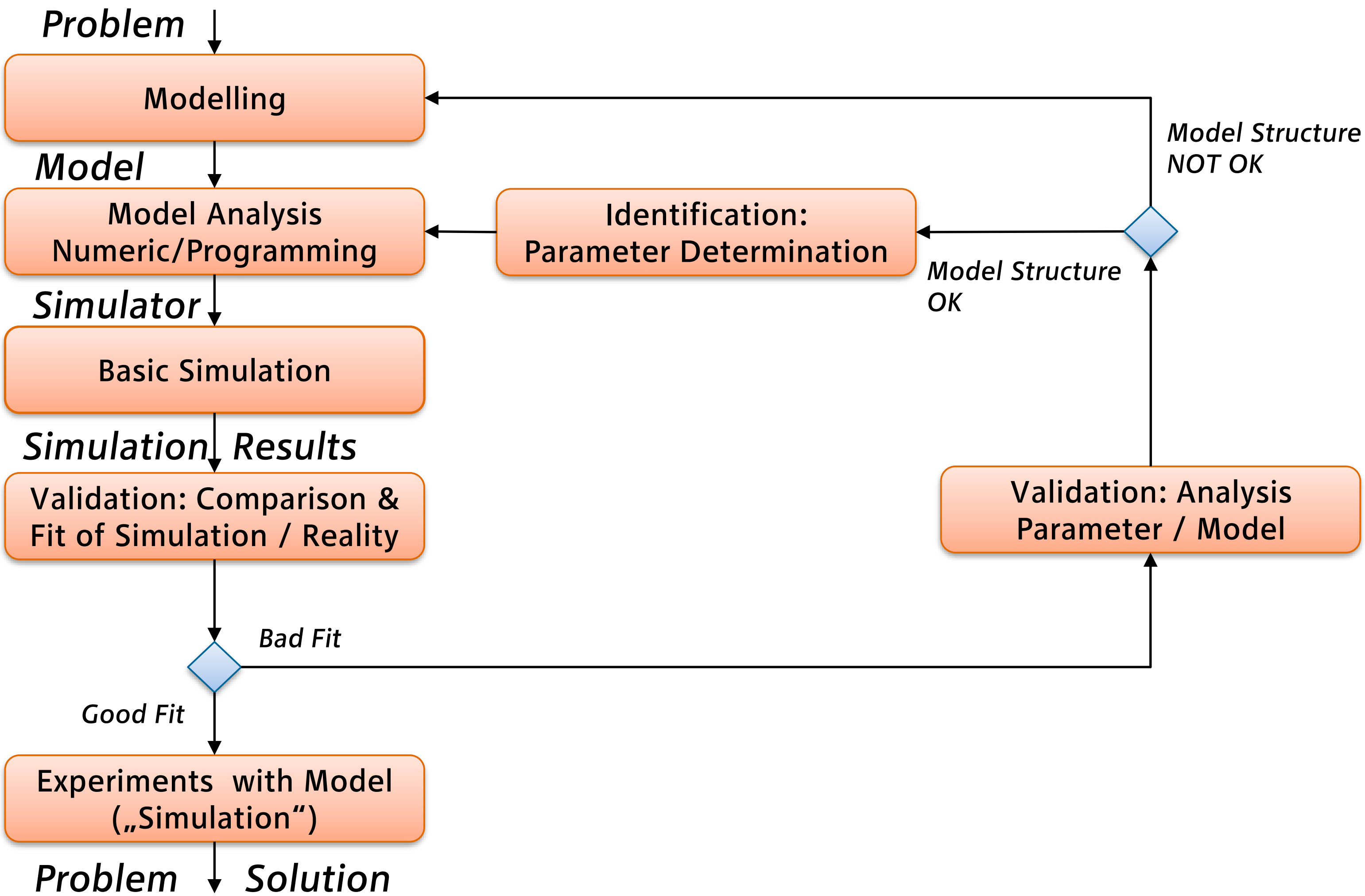


Mathematical Modelling and Simulation

Testcase: Predator-Prey

SIMULATION CIRCLE

Simulation Circle



What is System Dynamics

Mathematical Modelling and Simulation

Forrester, 1961

System Dynamics is a field that resulted from the pioneering efforts of Jay W. Forrester to apply the **engineering principles of feedback and control** to **social systems**.

System Dynamics generates **qualitative models based on causalities.**

By appropriate parameterisation, the qualitative models can be transformed into “quantitative” **computer models to simulate** the investigated system

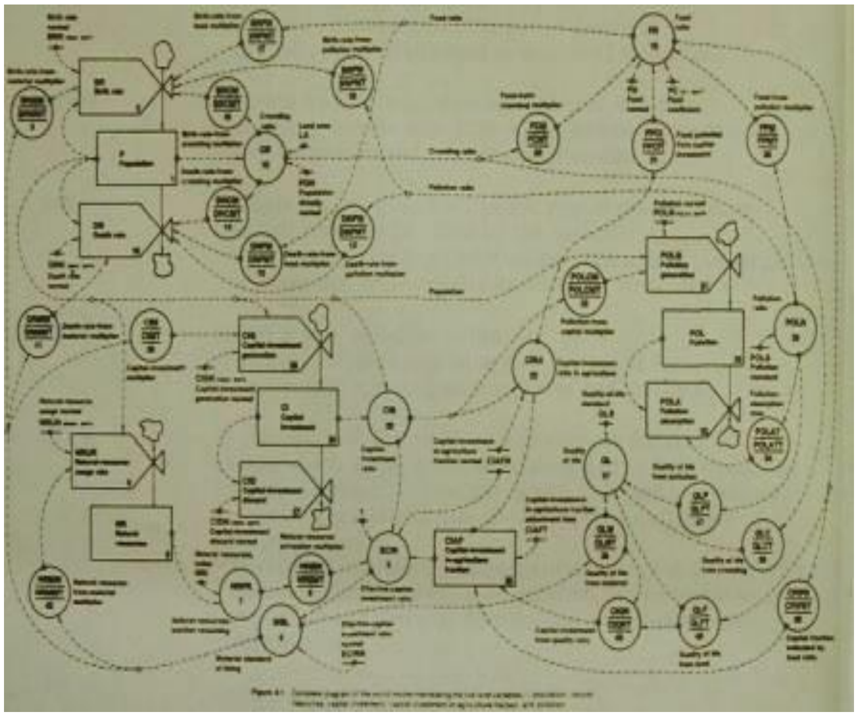
World Models

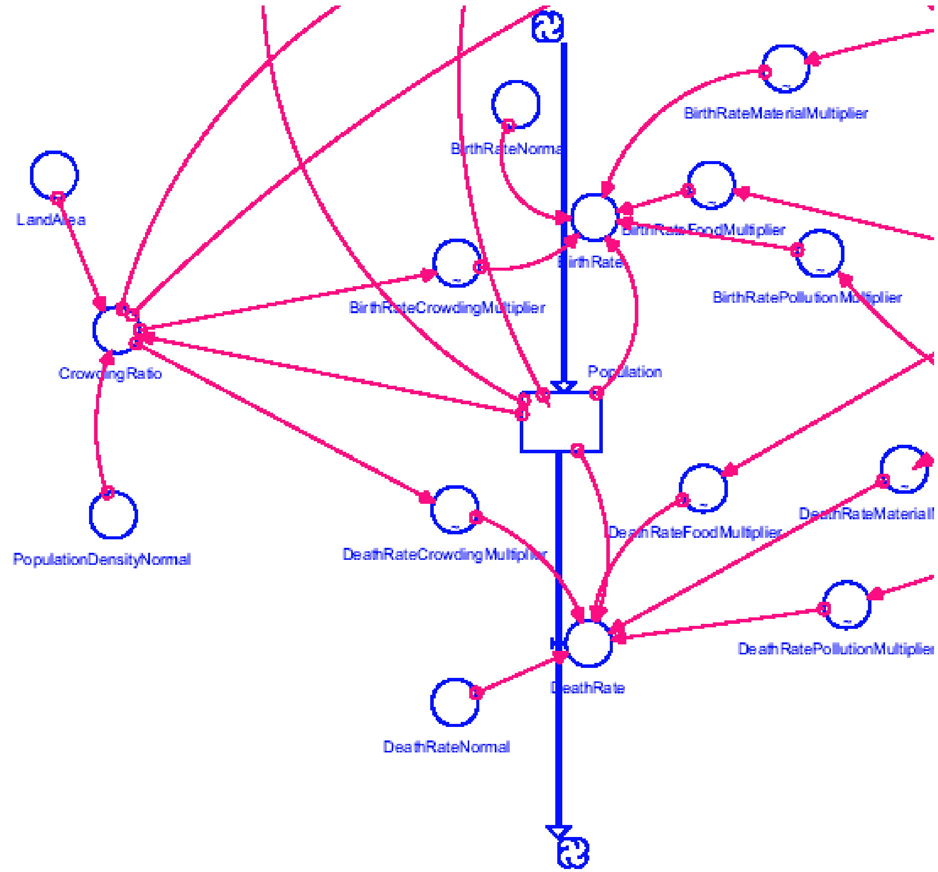
Systems Dynamics and DYNAMO received widespread interest mainly because they were used to build large world models such as

- WORLD2 (World Dynamics, Forrester1971);
- WORLD3 (The Dynamics of Growth in a Finite World, [Meadows]);
- and WORLD3 revisited (Beyond the Limits).

World Models

Mathematical Modelling and Simulation

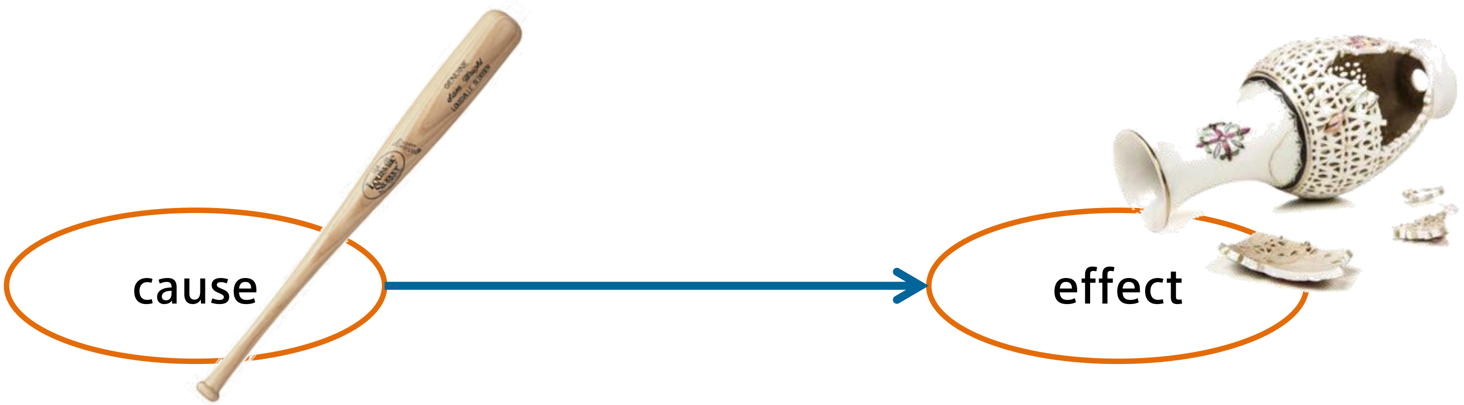




Key to develop SD Models

Causal thinking is the key to organizing ideas in a system dynamics study

(Roberts et al. 1983)



How to build a SD Model?

Mathematical Modelling and Simulation

1. Identify system variables and system boundaries
2. Capture **links** of variables in a **Causal Loop Diagram (CLD)**
3. Build a **Stock and Flow Diagram (SFD)**

- Implement the model in a simulator

1. System Variables and Boundaries

a. Analysis of the problem - Determining the purpose and the use of the model and defining a target for the simulation.

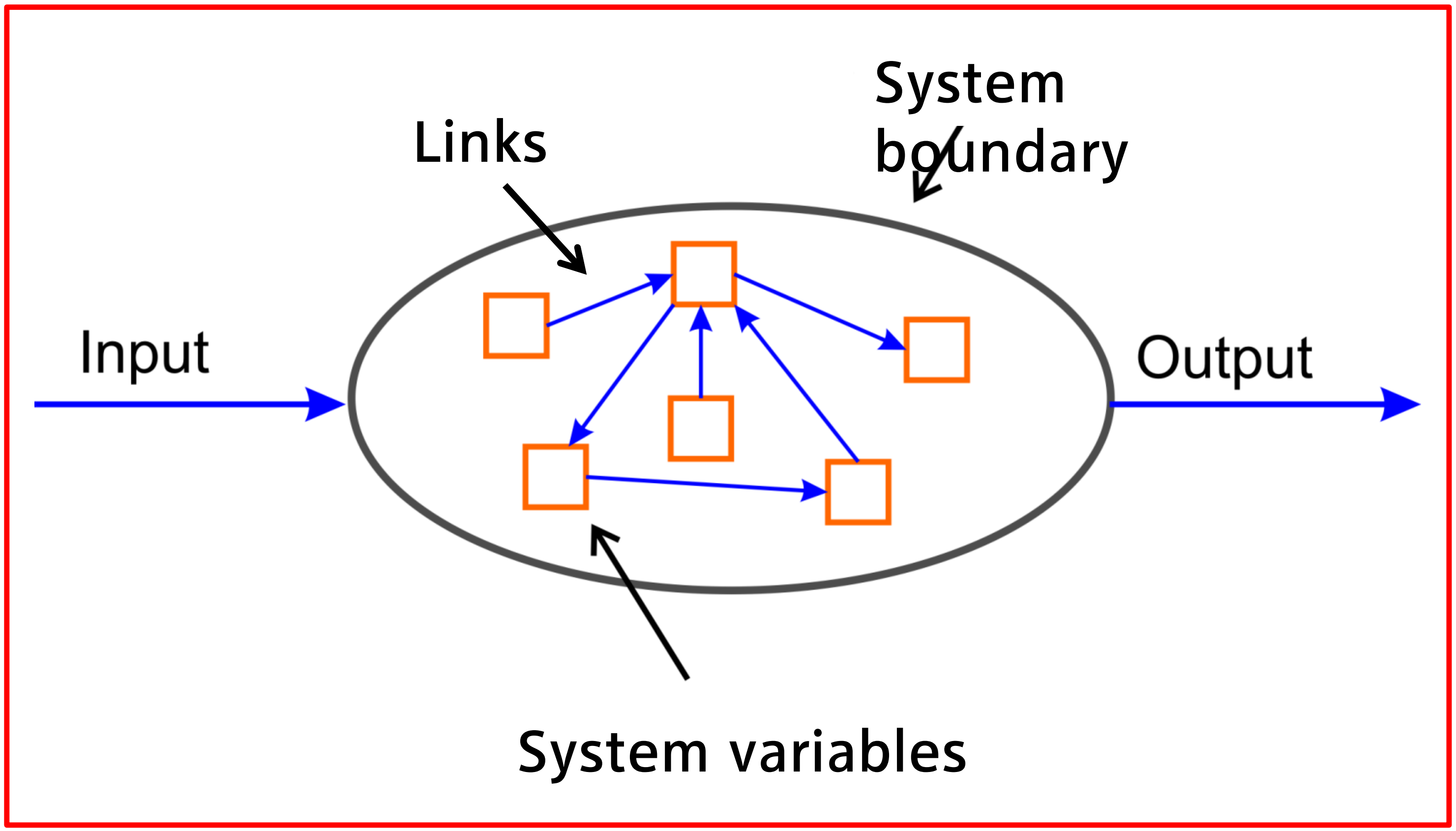
b. Start collecting information and data. Start developing hypothesis about the parts of the system.

c. Determine the elements of the system.

d. Determine causal relationships between the elements.

1. System Variables and Boundaries

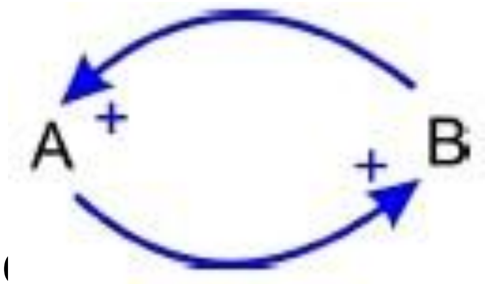
Mathematical Modelling and Simulation



2. Causal Loop Diagram

Mathematical Modelling and Simulation

Capture the **behavior** and **links** of and within the system by interlinking system variables that are related to each other



Behavior of syst

- Feedback Loops
- System memory (stocks)
- Delays in material and information delays

2. Causal Loop Diagram

Mathematical
Modelling and
Simulation

<u>Main components of CLDs:</u>

- **System variables:** names of elements
- **Link - positive:**

Represented by a plus-sign

Increase in variable *Eating* results in an increase in variable *Weight*

2. Causal Loop Diagram

<u>Main components of CLDs:</u>

- **Link – negative:**

Represented by minus-sign.

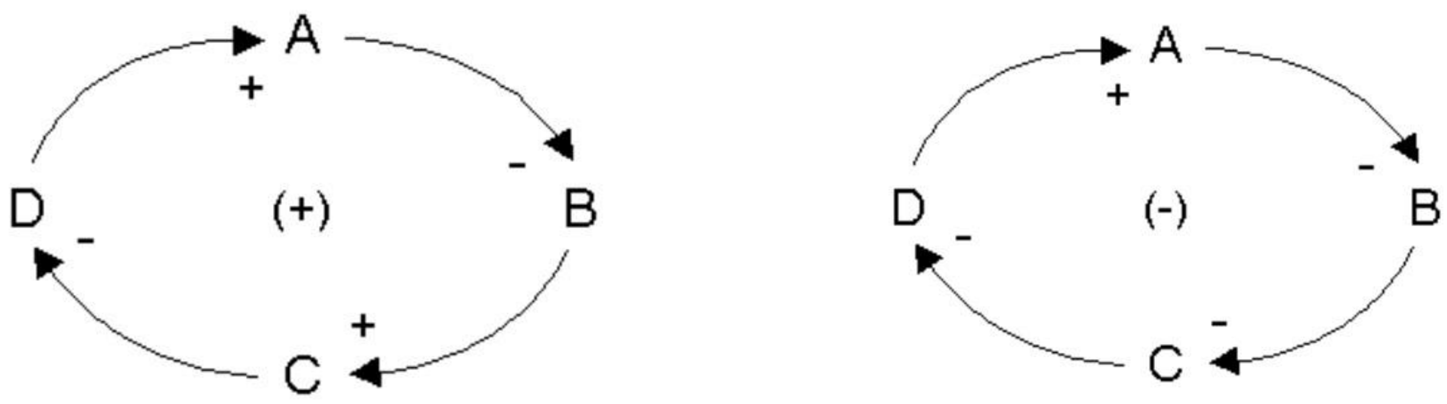
Increase in variable *Diet* results in a decrease in variable *Weight*

2. Causal Loop Diagram

Mathematical Modelling and Simulation

<u>Main components of CLDs:</u>

- **Feedback Loops:** are closed loops of arrows, represented by a:
"(+)" (or "(R)" for **reinforcing**) or
"(-)" (or "(B)" for **balancing**) sign in the middle.



2. Causal Loop Diagram

Mathematical Modelling and Simulation

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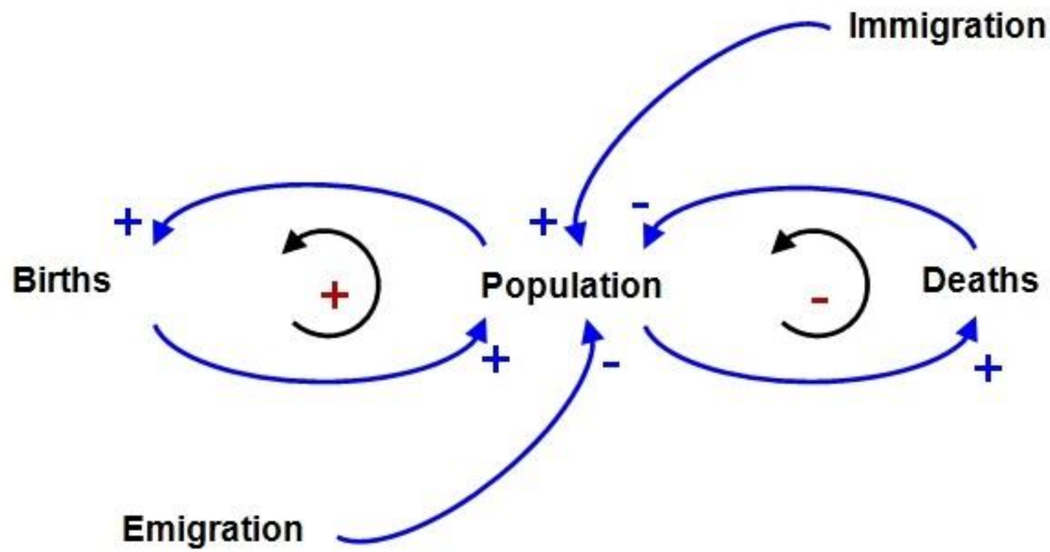


TU WIEN Mathematical Modelling and Simulation

Feedback Loops

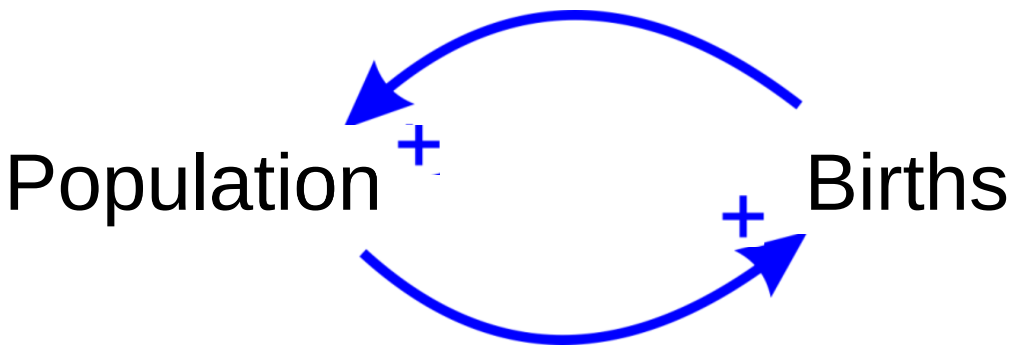
- Search to identify closed, causal feedback loops is one key element of System Dynamics
- The most important causal influences will be exactly those that are enclosed within feedback loops

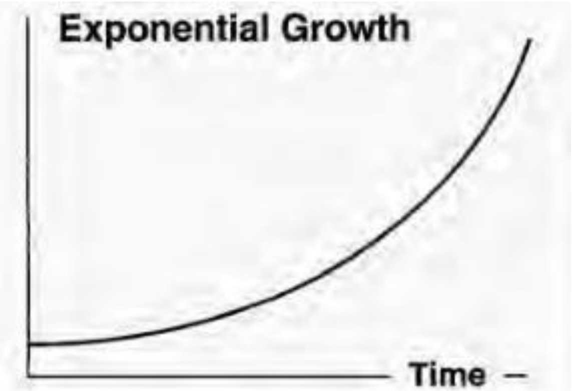
2. Causal Loop Diagram

Types of behavior due to loops:

- **Exponential Growth**: arises from **positive (reinforcing) feedback** loop.

Example:



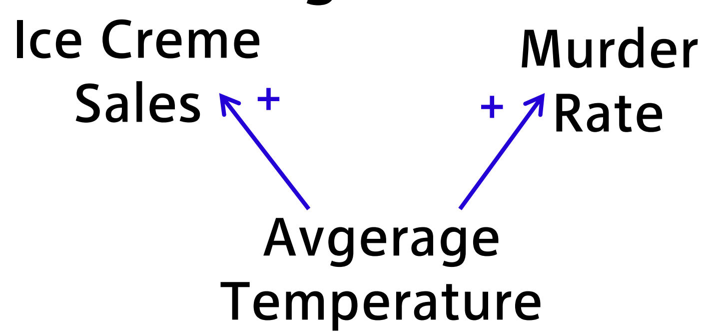


Causation vs. Correlation

- ***Correlation*** represents past behavior and not the structure of the system
- ***Causation*** represents the causal links of the structure

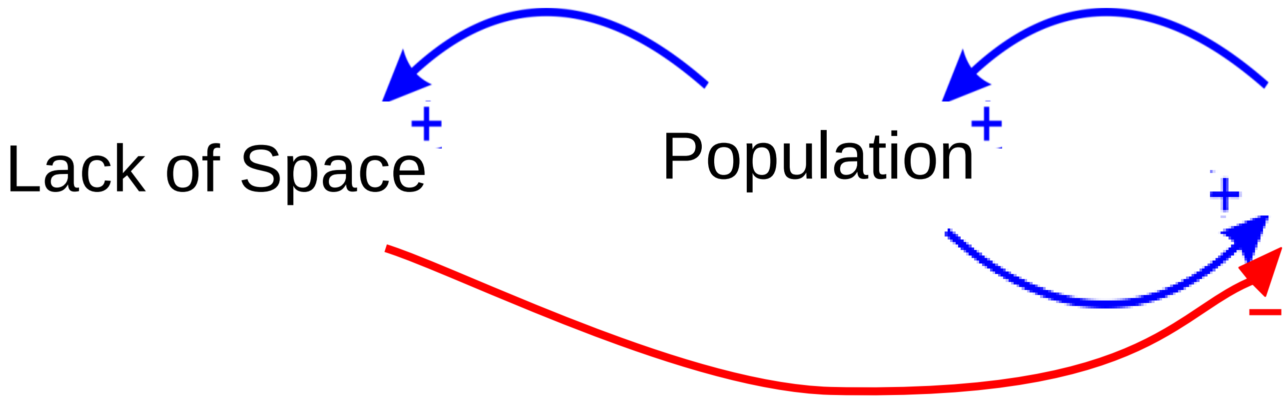
Wrong:

Right:



2. Causal Loop Diagram

At least one negative feedback loop is necessary to receive a stable system

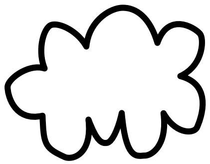


3. Stock and Flow Diagram

Problem: Not all system elements are system variables!

Solution: distinguish between

- Sources/Sinks
- Levels/Stocks
- Flows
- Auxiliaries
- Paramters
- Links



Sources/Sinks:

Source represents systems of levels and rates outside the boundary of the model

Sink is where flows terminate outside the system

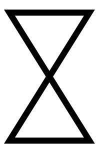
E.g.: Raw Material (Source for „Construction“ Flow), Graveyard (Sink for „Dying“ Flow)

Levels/Stocks/System variables:

A quantity that accumulates over time and changes its value continuously.

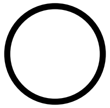
E.g.: Size of a population, Number of people waiting in a queue, Number of goods waiting to be transported, etc.

Flow/Rate/Activity/Movement:



Changes the values of levels. Every level has at least to be connected to one flow in order to change its value.

E.g.: Birth (Changes the value of the stock „population“), Eating (Changes the value of the stock „amount of food“), etc.



Auxiliary:

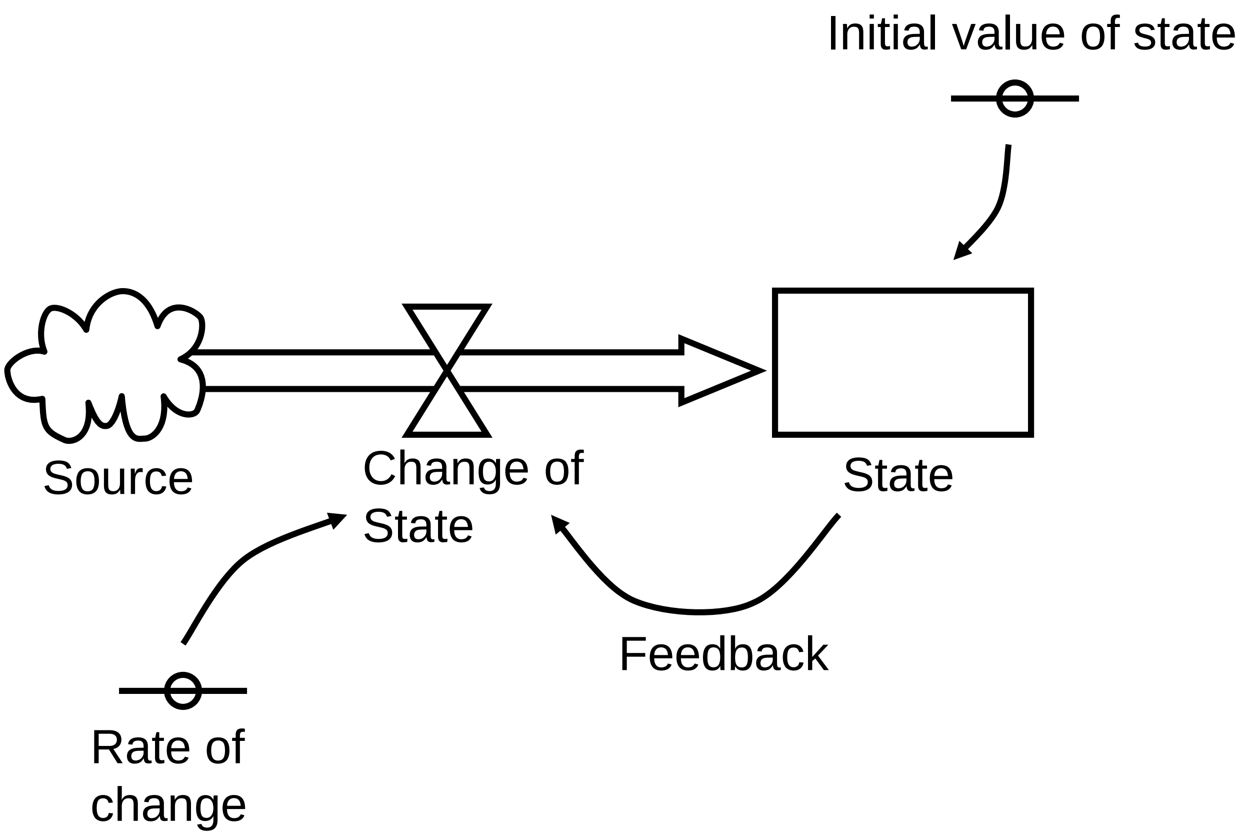
Everything that can directly/analytically be calculated out of stocks and constants.
Often useful, to avoid confusing models.

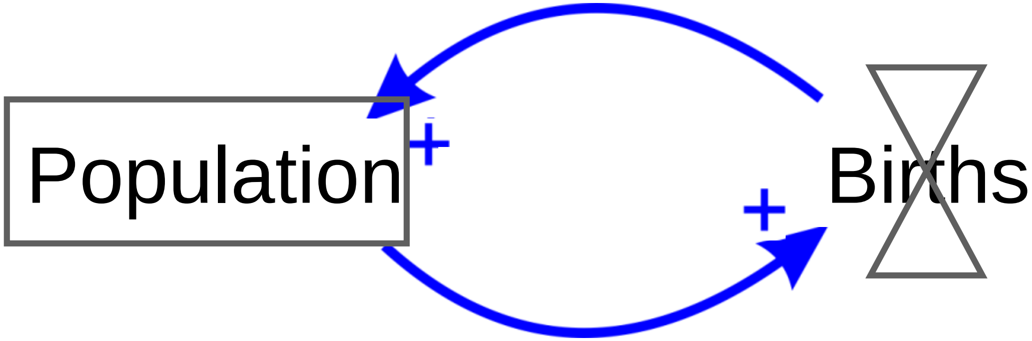
E.g.: Density (can directly be calculated by the stocks/constants „mass“ and „volume“), Quelength (calculated by stock „people in queue“ and constant „average size of one person“), etc.

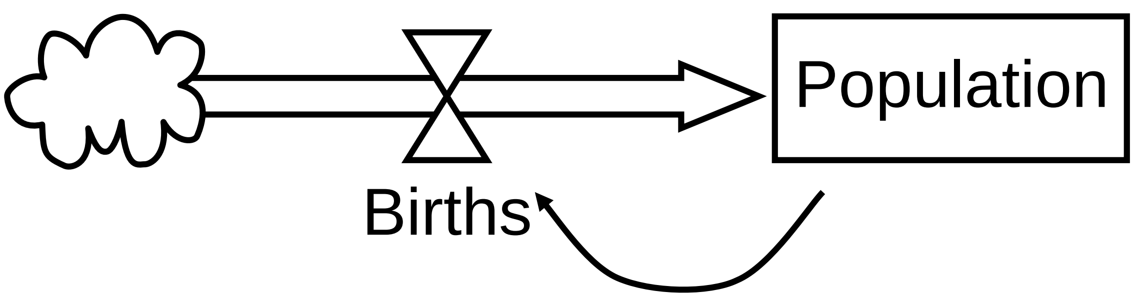
Parameter /Constant

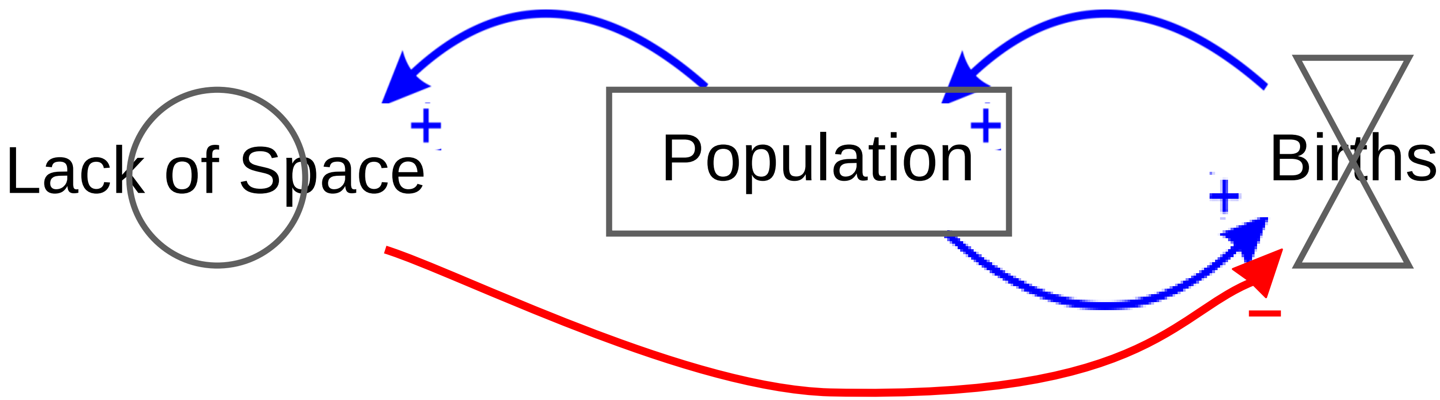
Everything that is predefined for the whole simulation – usually it is a constant but can be a function too.

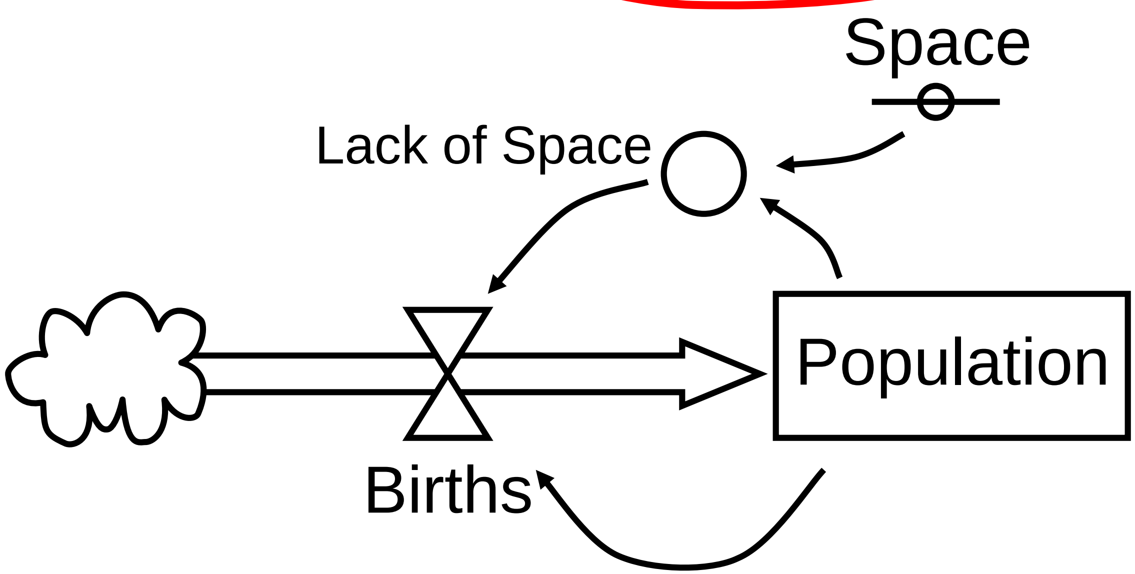
E.g.: Average Temperature, Number of Cash Desks (In a supermarket), Birth Rate, Maximum capacity of a Room, etc.


Initial value of state
Source
Change of State
State
Feedback
Rate of change


Population
+
+
Births


Population
Births


Lack of Space
Population
Births
+
+
+
−

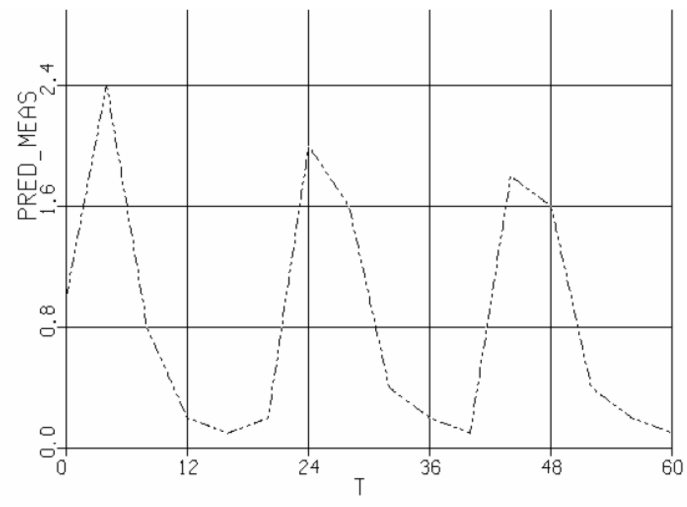

Space
Lack of Space
Population
Births

Predator – Prey System

Problem

Modelling

Model



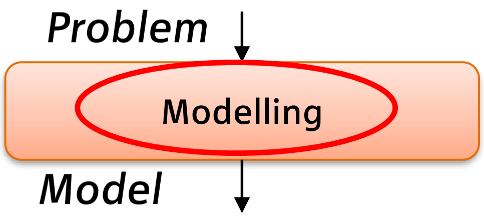
Dynamics: Predator eats Prey
Predator / Prey births, deaths

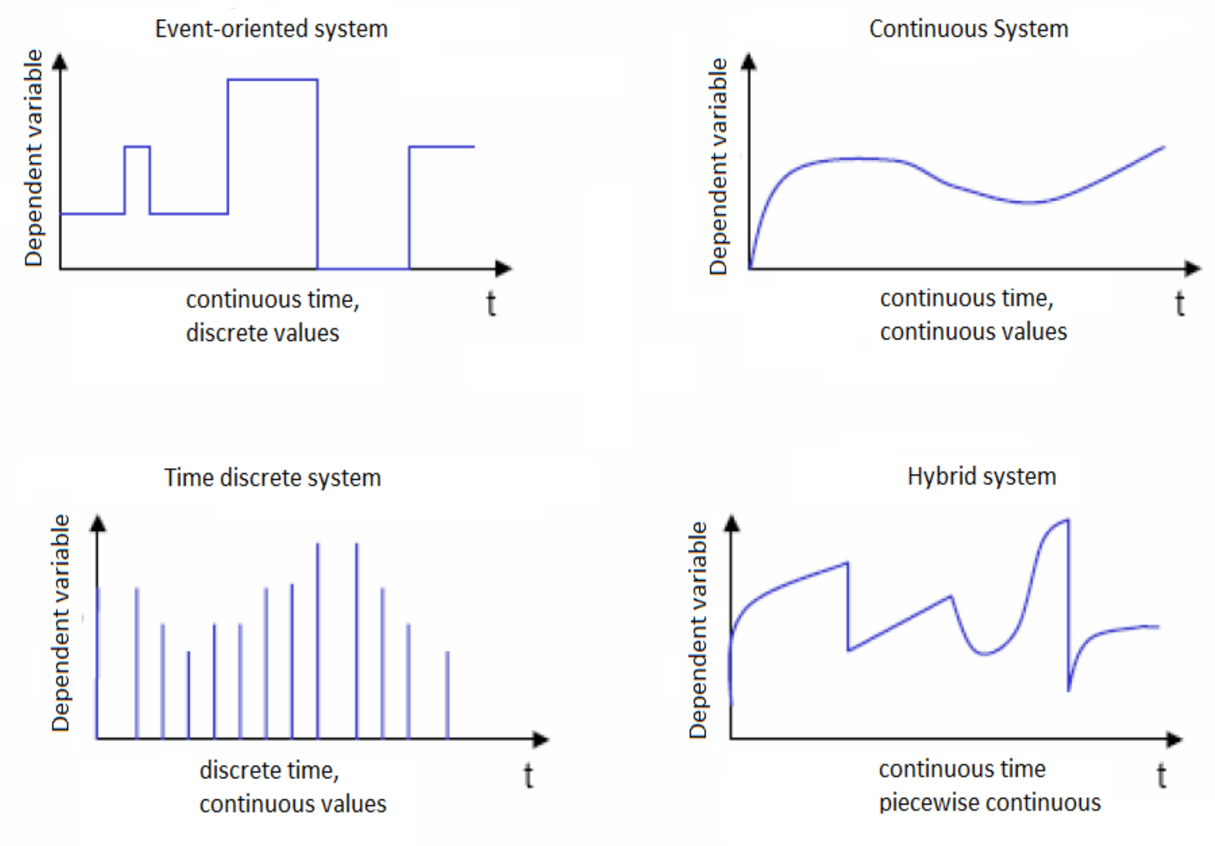
Environment: isolated

Measurement: Predator Population
5 Years = 60 months, quarterly

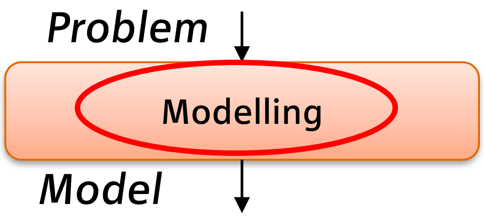
Problem: When is a reasonable time to use chemical pesticides to reduce number of predators?

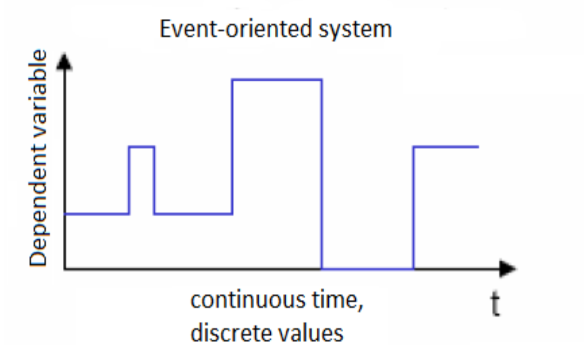
Predator – Prey System

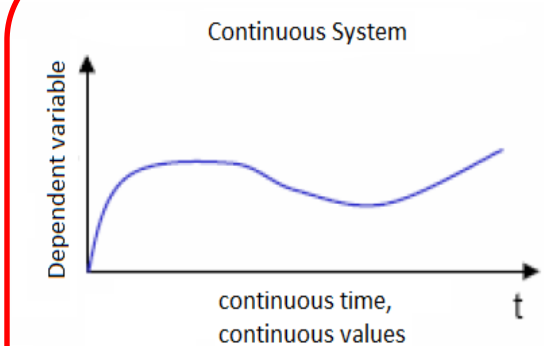




Predator – Prey System





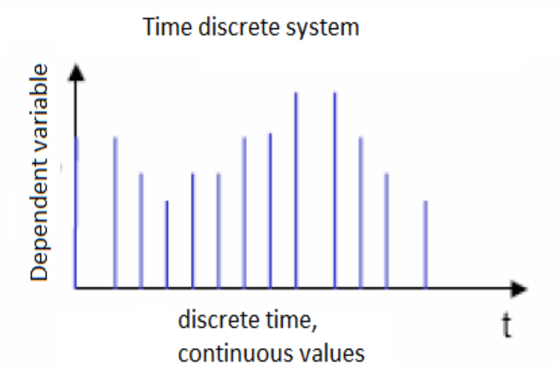


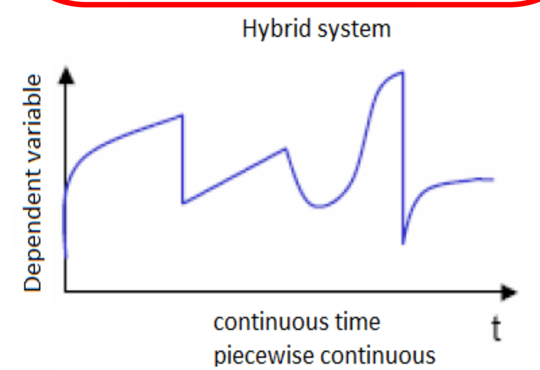
$Y(t)$ ... Prey

$X(t)$ ... Predators

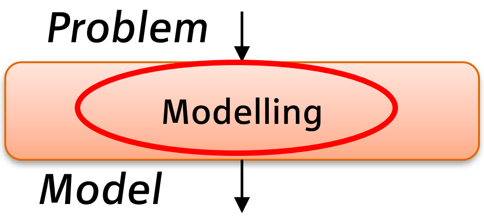
Separation –
Isolated environment

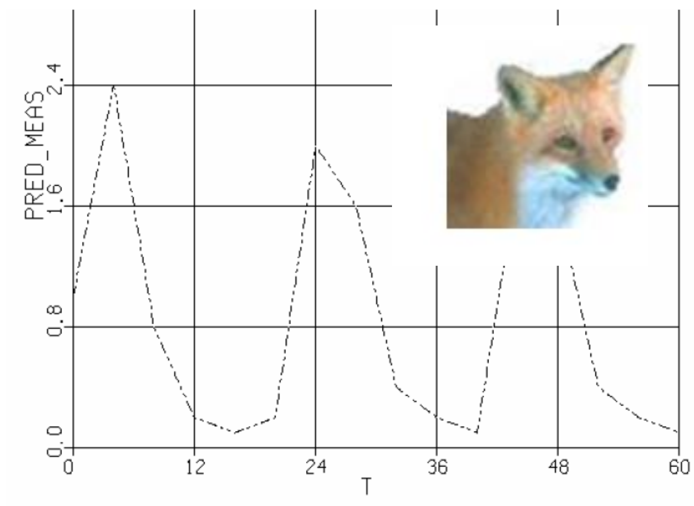
Choice -
2 variables = 2 states





Modelling





Separation –

Isolated environment

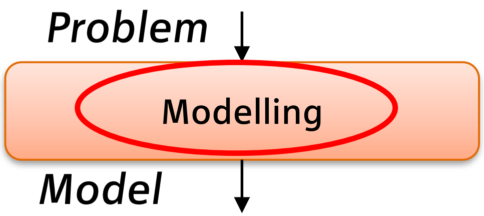
Choice -

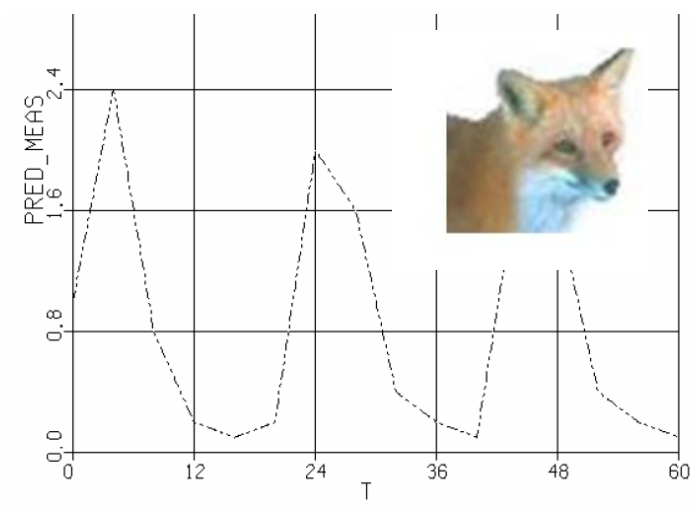
2 variables = 2 states

Causality –

Predator – Prey – Model

Modelling





Separation –

Isolated environment

Choice -

2 variables = 2 states

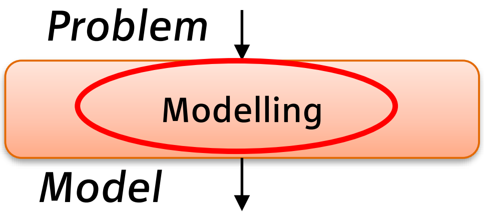
Causality –

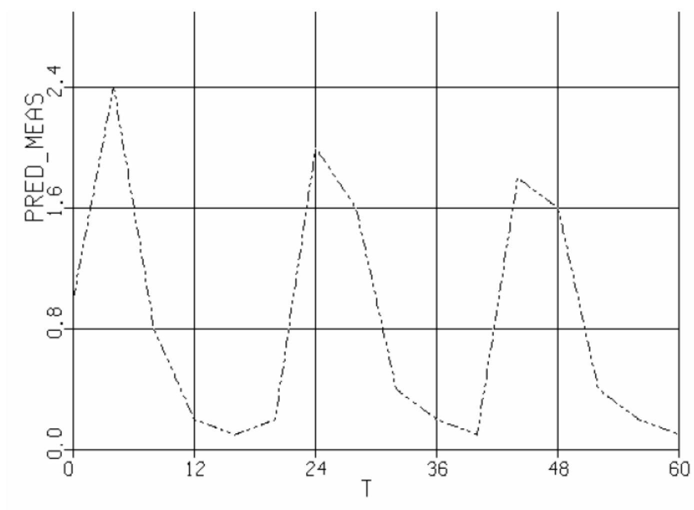
Predator – Prey – Model

Y(t) .. Prey Population

X(t) .. Predator Population

Modelling





Causality –

Predator – Prey – Model

Y(t) .. Prey Population

X(t) .. Predator Population

System Dynamics –

Population Interaction

Modelling

Problem

Modelling

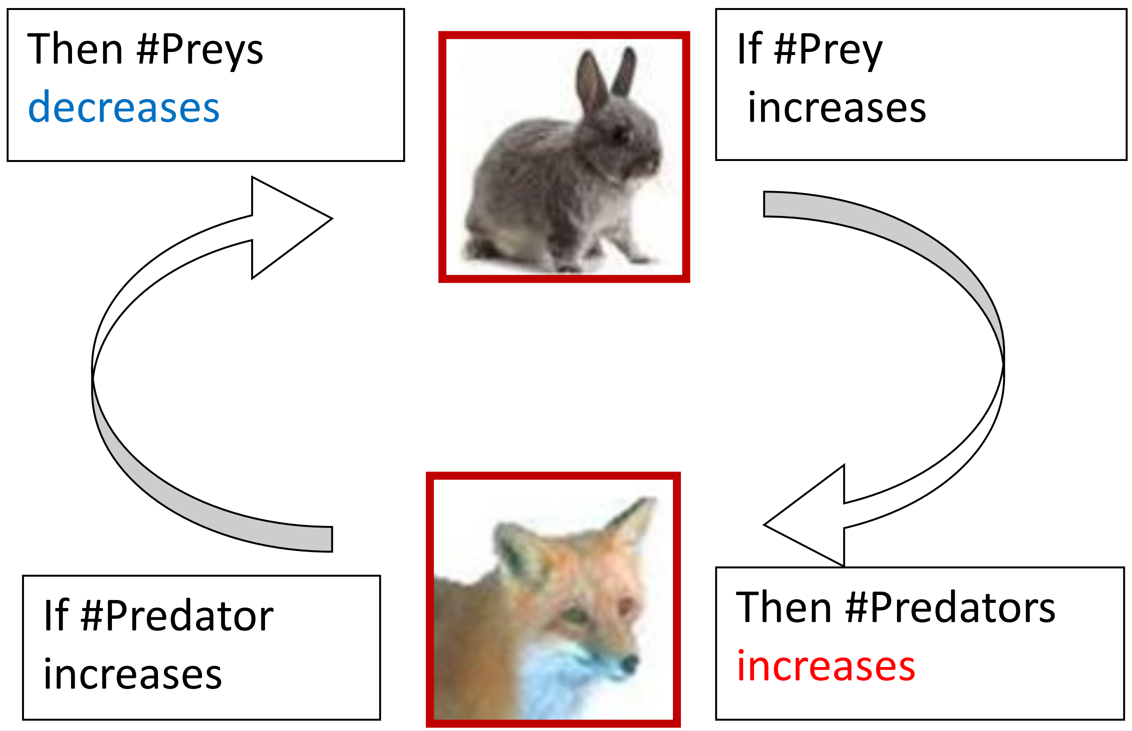
Model

Causality – Predator – Prey – Model

Y(t) .. Prey,

X(t) .. Predator

System Dynamics - Population interaction



Modelling

Mathematical Modelling and Simulation

Causality – Predator – Prey – Model

Y(t) .. Prey,

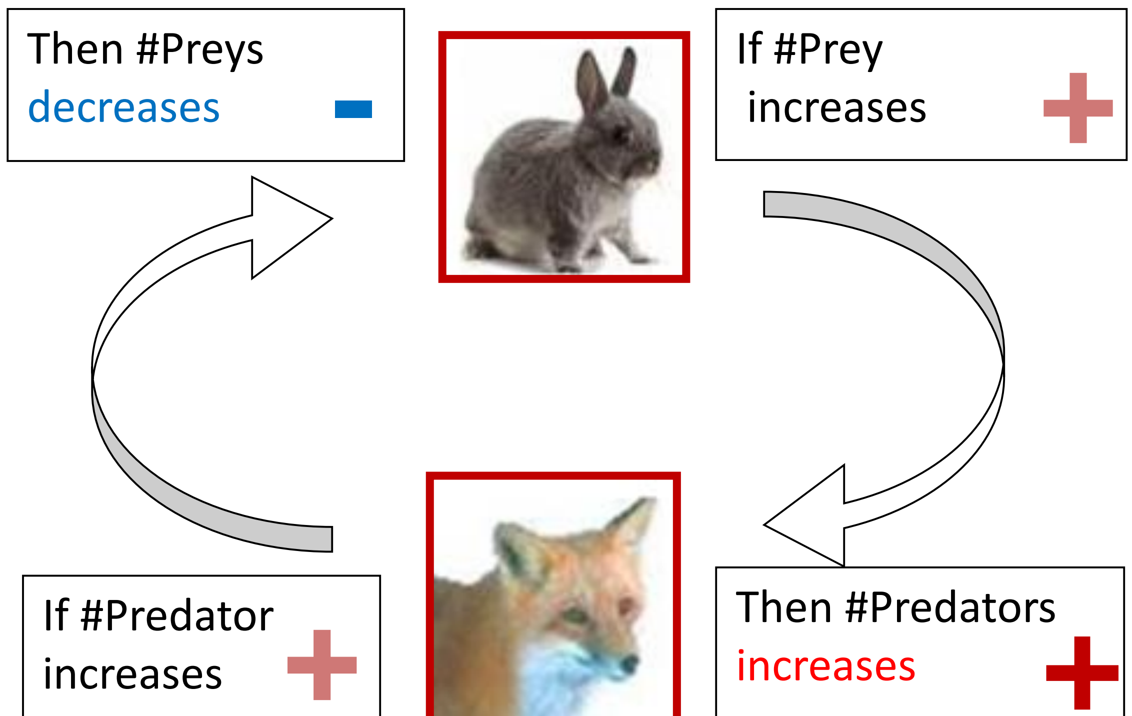
X(t) .. Predator

System Dynamics - Population interaction

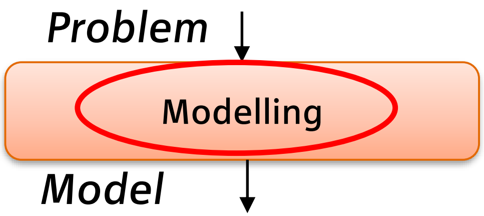
Problem

Modelling

Model

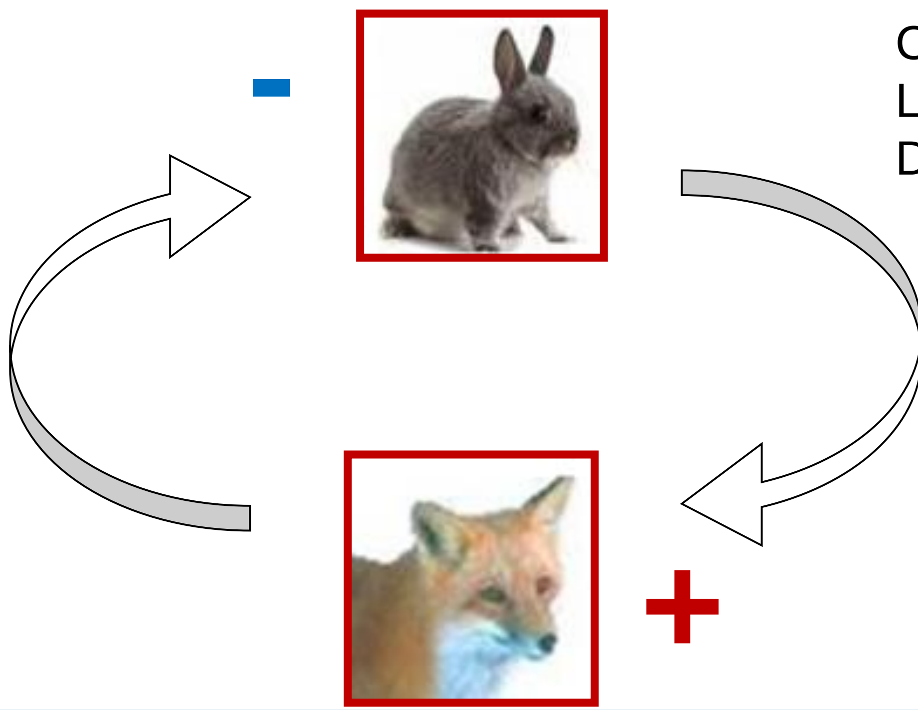


Modelling



Causality – Predator – Prey – Model
Y(t) .. Prey,
X(t) .. Predator

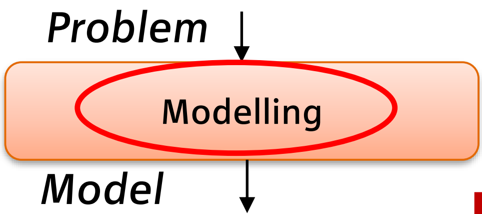
System Dynamics - Population interaction

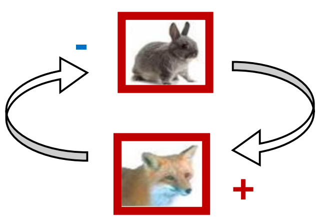


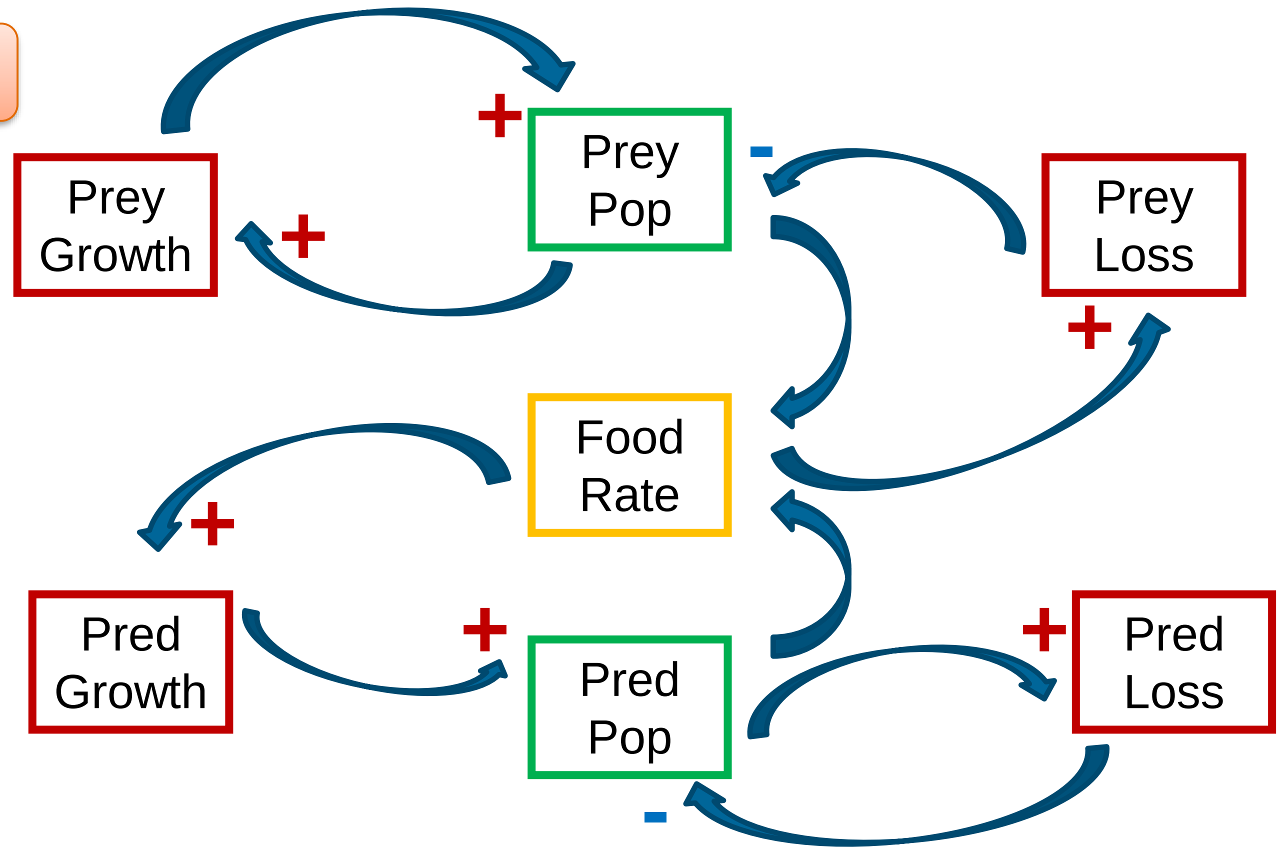
Causal Loop Diagram

Mathematical Modelling and Simulation

System Dynamics - Population interaction



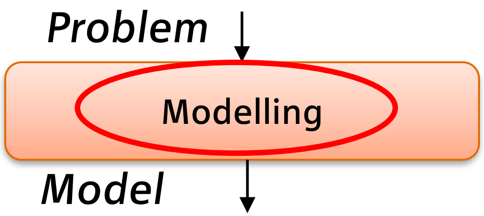


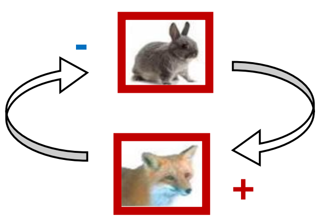


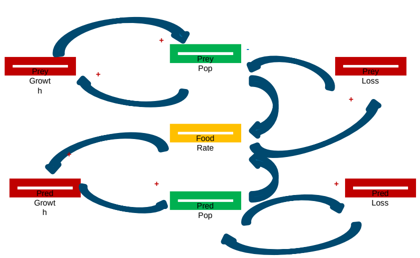
Causal Loop Diagram

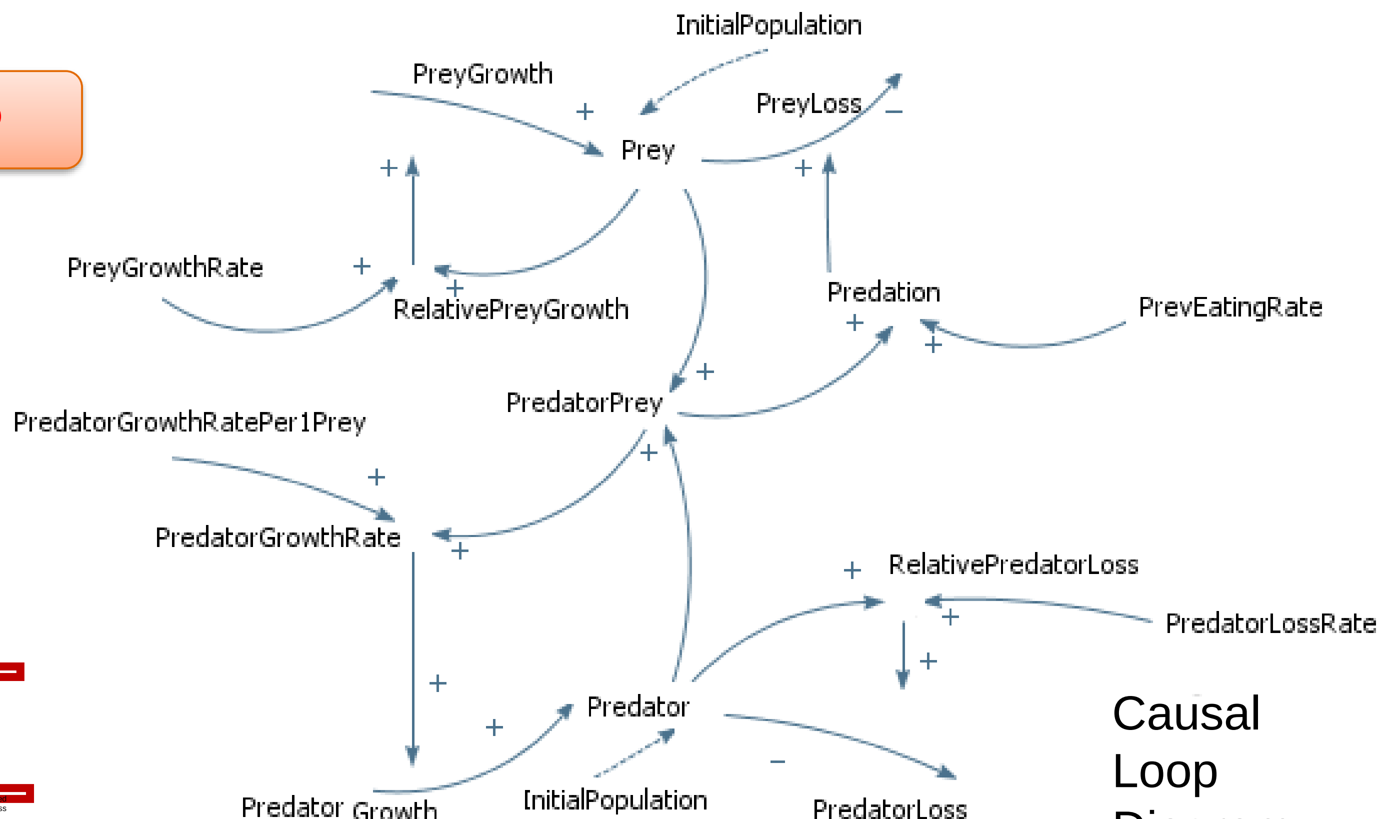
TU WIEN Mathematical Modelling and Simulation

System Dynamics - Population interaction



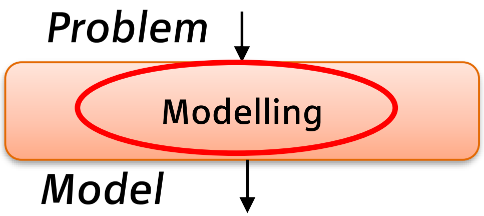


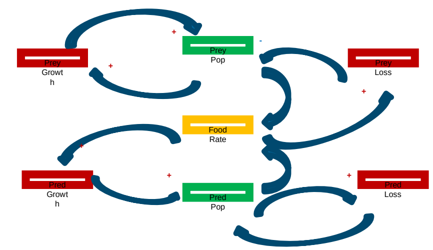


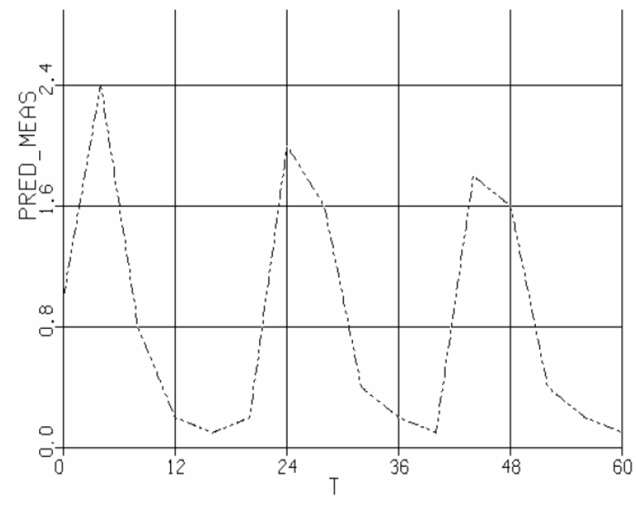


Causal Loop Diagram

Modelling







Causality –

Predator – Prey – Model

x(t) .. Prey

y(t) .. Predator

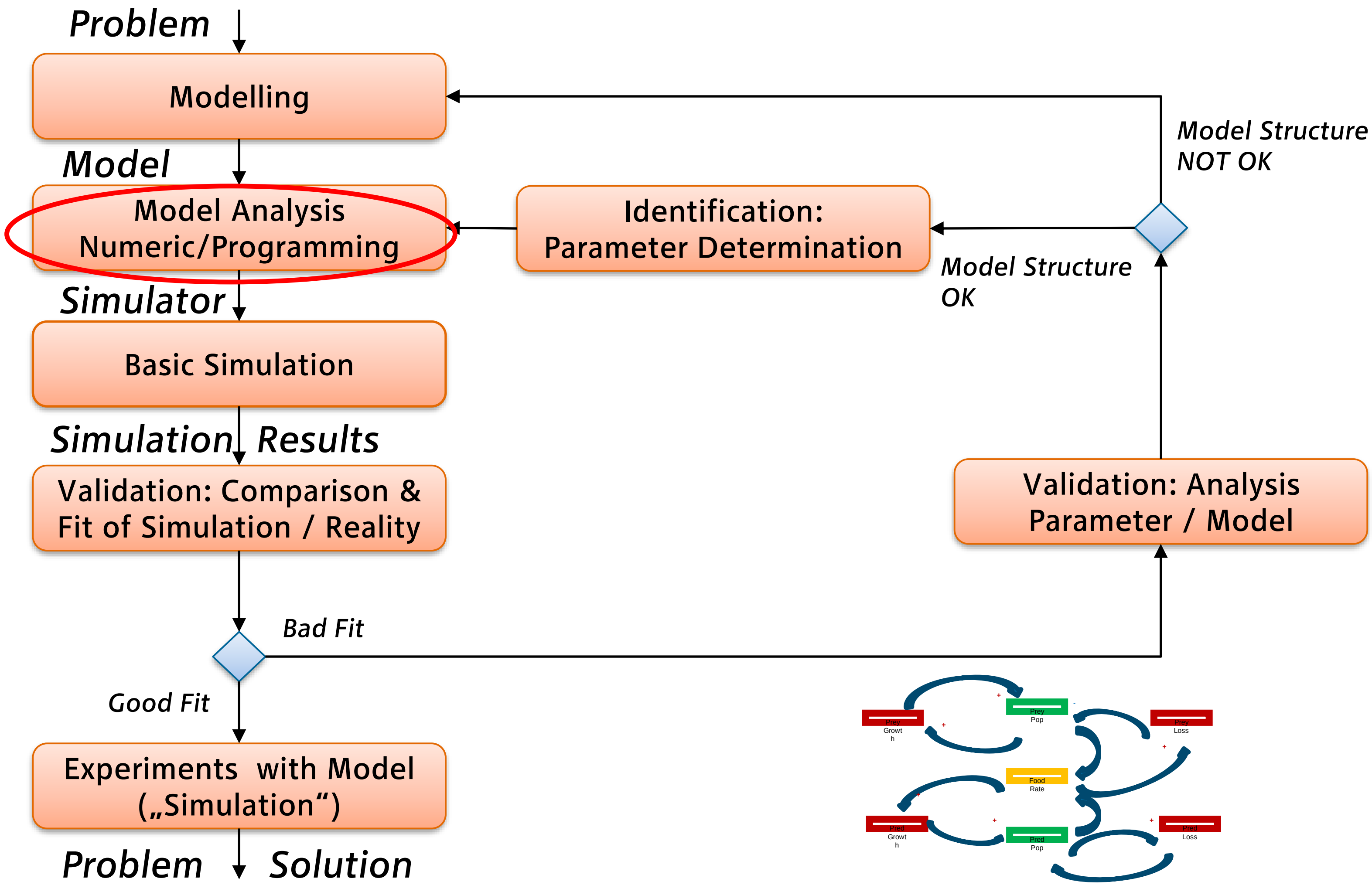
Logistic Growth -

$$\dot{x} = ax - bxy$$
$$\dot{y} = -dy + exy$$

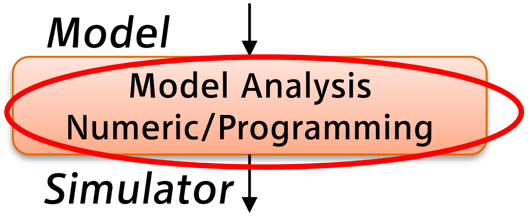
Population rate = Growth rate + food rate

Simulation Circle: Predator - Prey

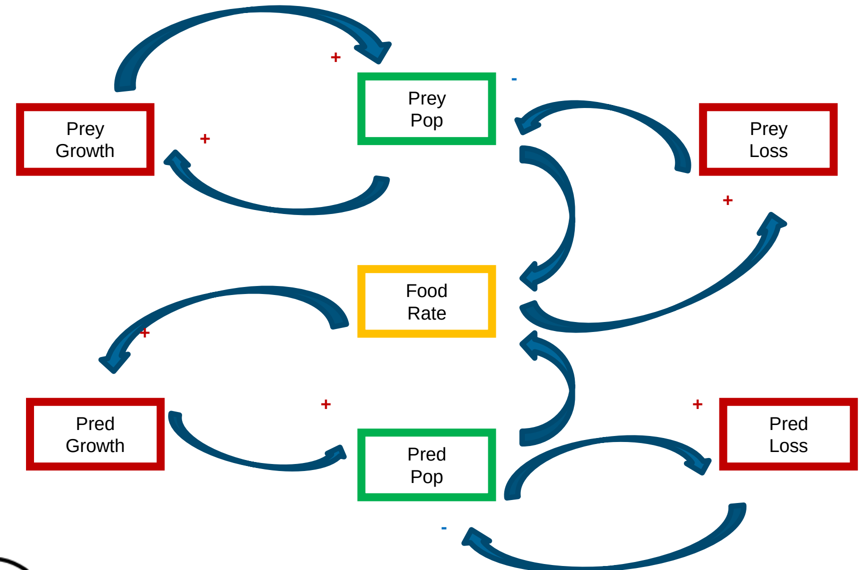
Mathematical Modelling and Simulation



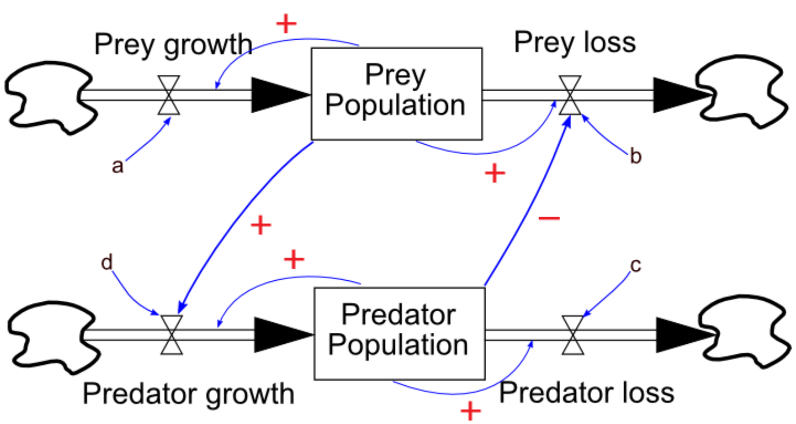
Model Analysis



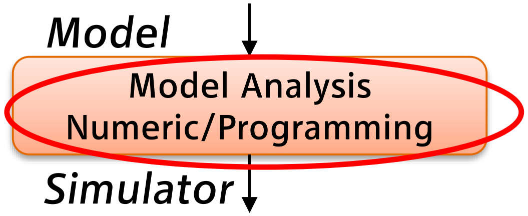
Causal Loop Diagram



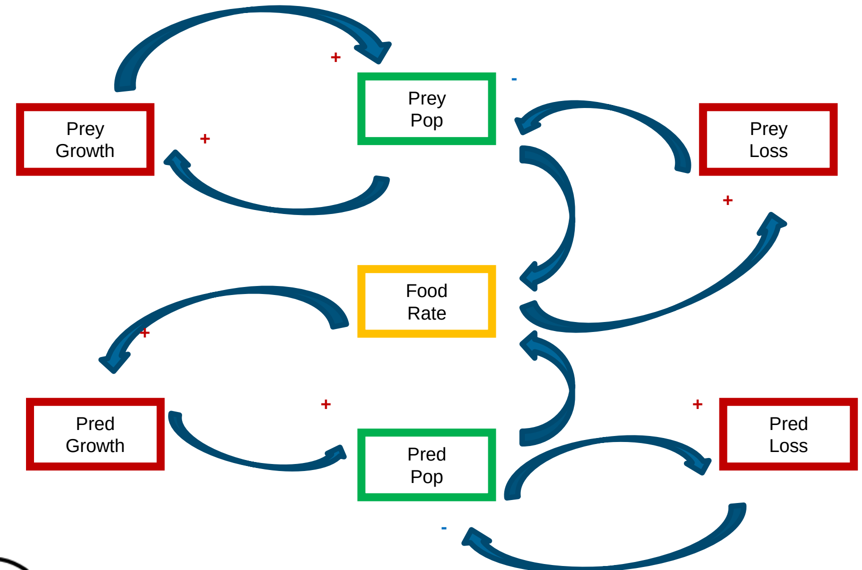
Stock and Flow Diagram

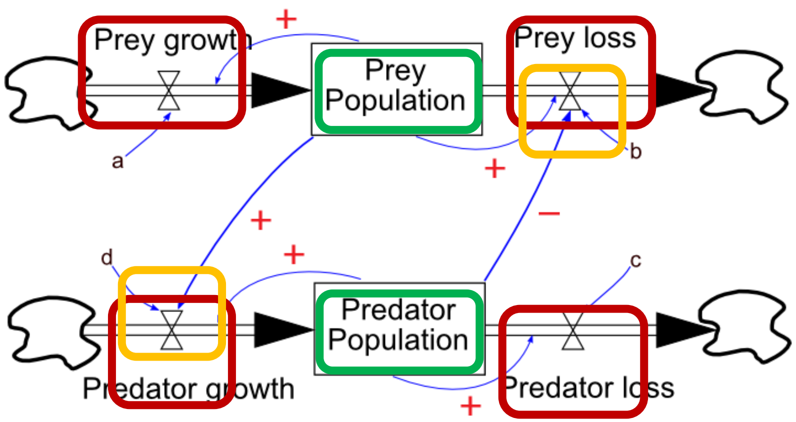


Model Analysis



Causal Loop Diagram





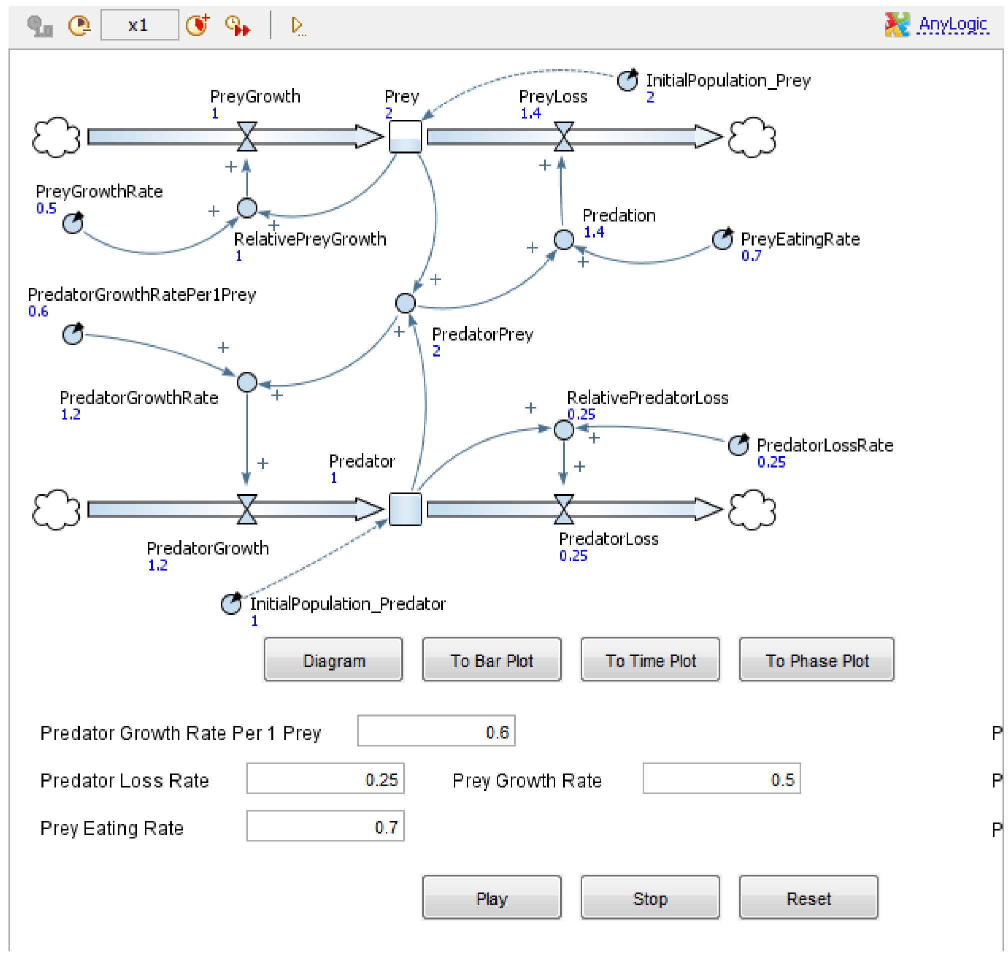
Stock and Flow Diagram

Implementation

Model

Model Analysis
Numeric/Programming

Simulator



Simulation Circle: Predator - Prey

Mathematical Modelling and Simulation

Problem

Modelling

Model

Model Analysis Numeric/Programming

Identification: Parameter Determination

Model Structure NOT OK

Model Structure OK

Simulator

Basic Simulation

Simulation Results

Validation: Comparison & Fit of Simulation / Reality

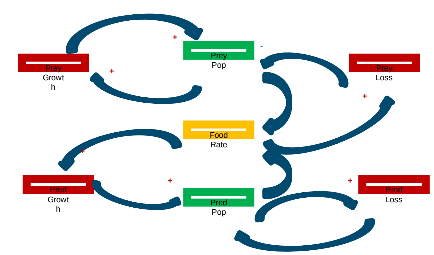
Validation: Analysis Parameter / Model

Bad Fit

Good Fit

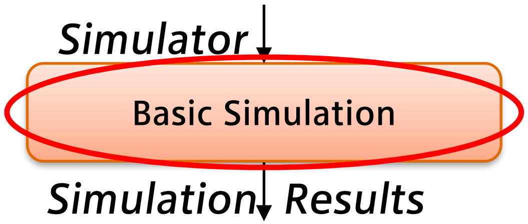
Experiments with Model („Simulation“)

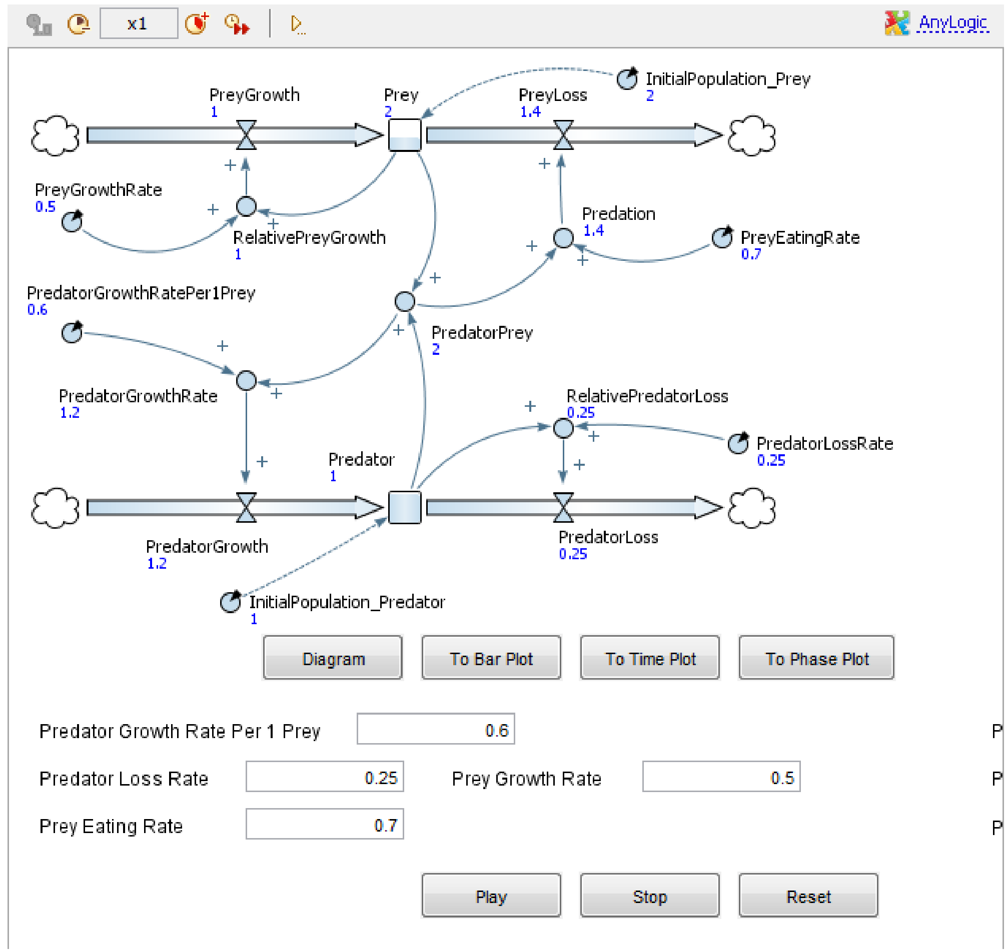
Problem Solution



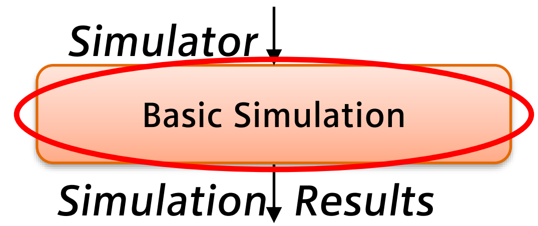
Implementation

Mathematical Modelling and Simulation

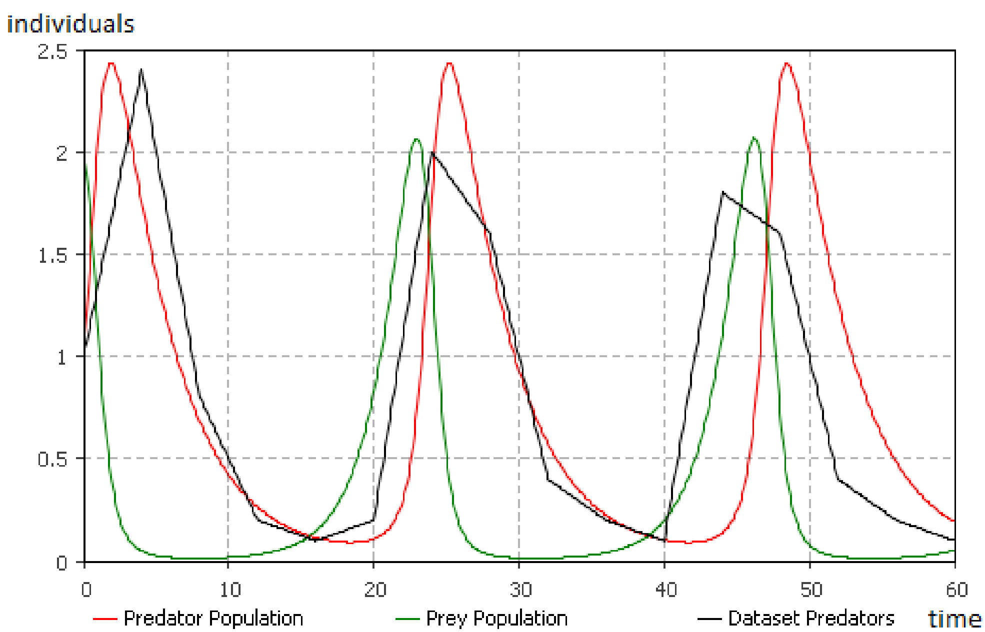




Implementation



Population development over time:



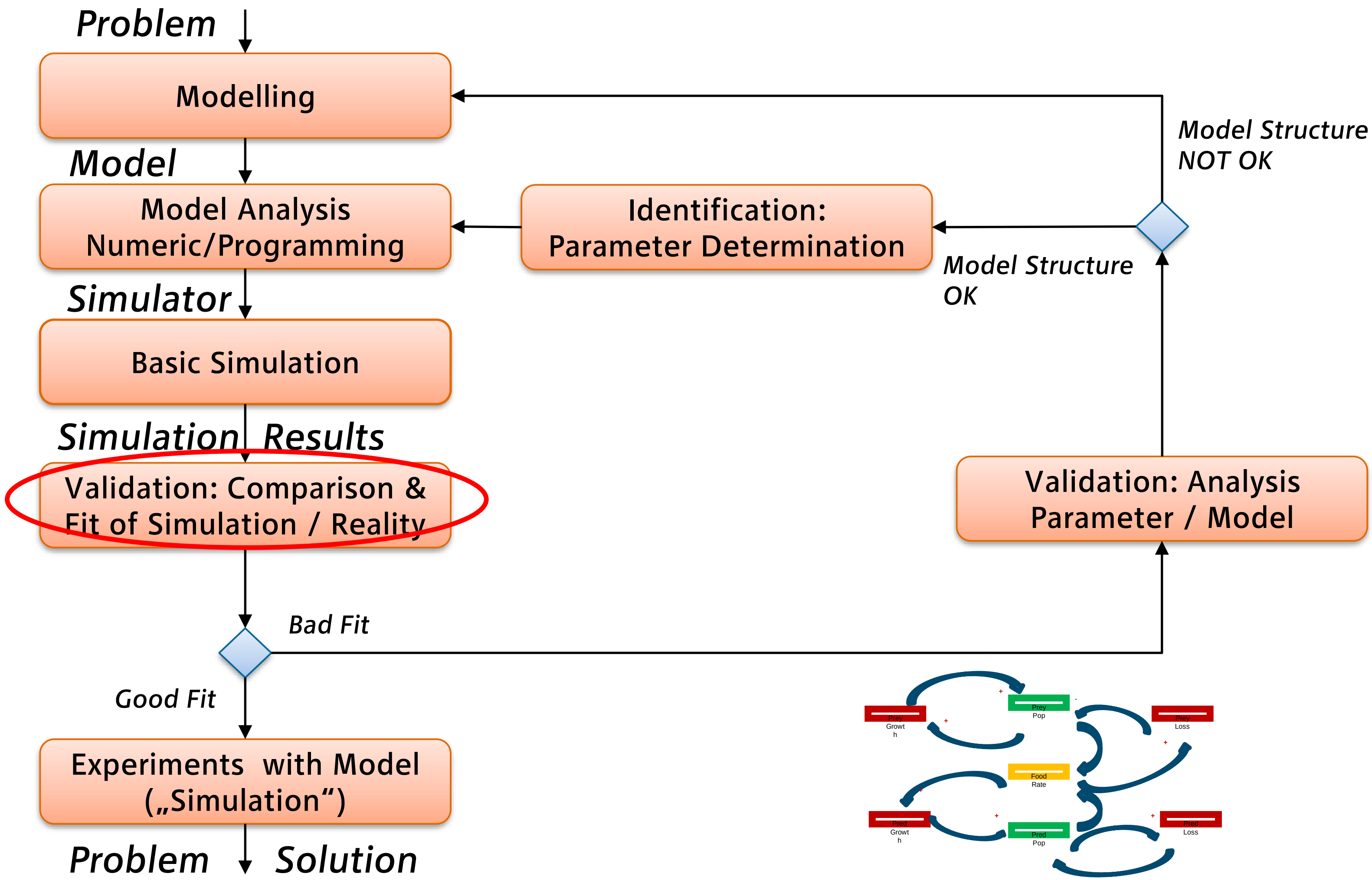
Parameters:

Predator Growth Rate Per 1 Prey	0.6
Predator Loss Rate	0.25
Prey Eating Rate	0.7
Prey Growth Rate	0.5

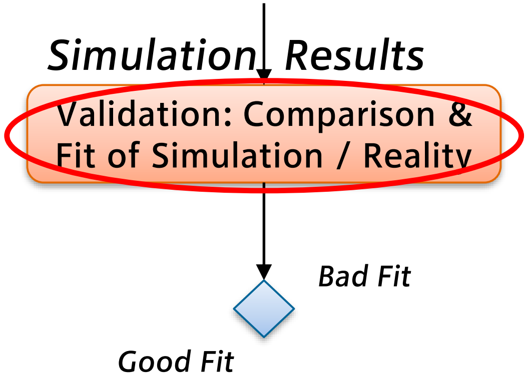
$$\dot{x} = (a - b \cdot y)x$$
$$\dot{y} = (-c + d \cdot x)y$$

Simulation Circle: Predator - Prey

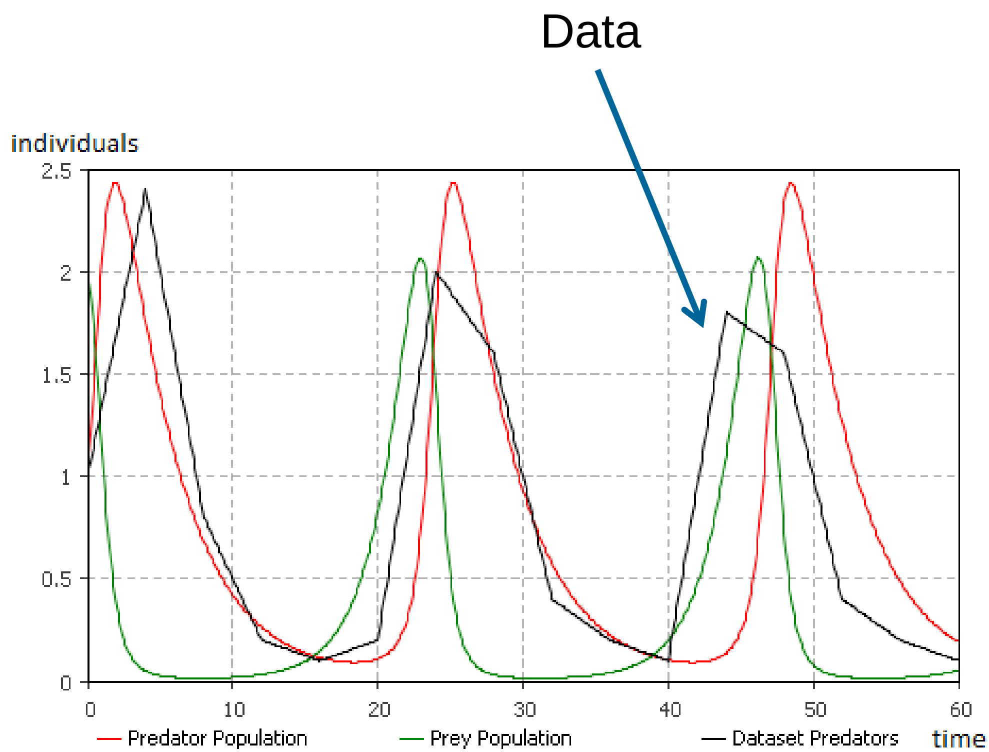
Mathematical Modelling and Simulation



Validation



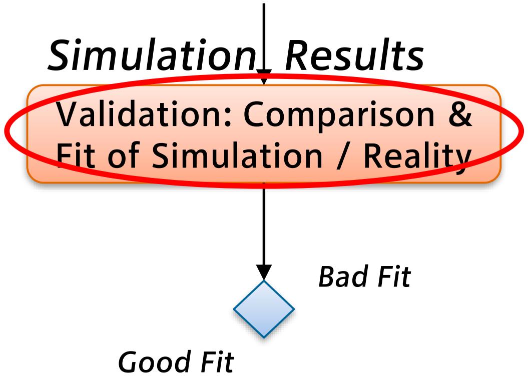
Simulation Results
Validation: Comparison & Fit of Simulation / Reality
Bad Fit
Good Fit

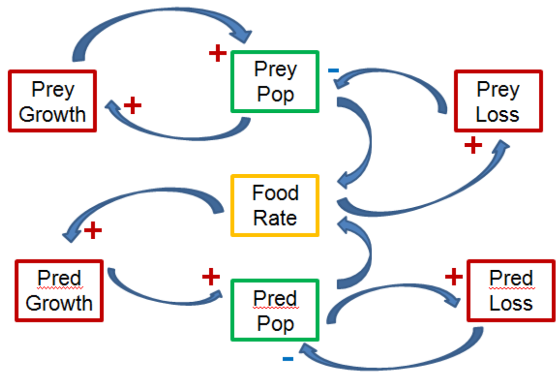


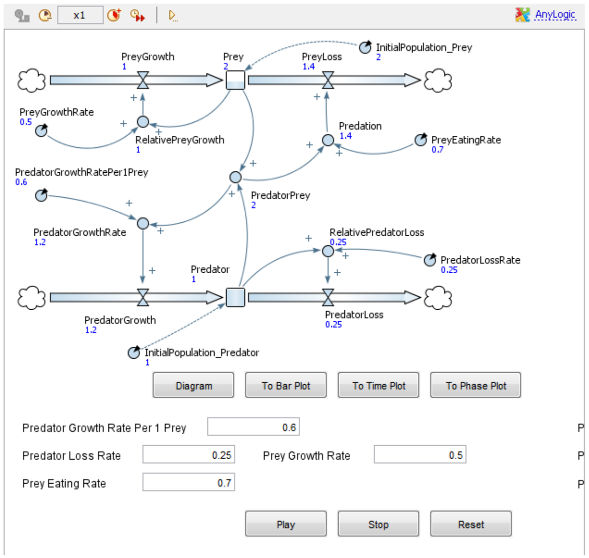
Data
individuals
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
Predator Population
Prey Population
Dataset Predators
time

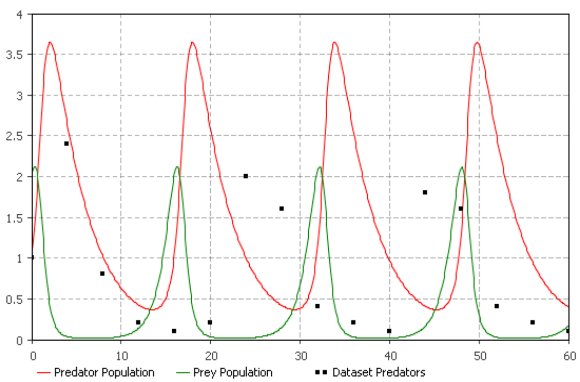
Data & Simulation Results

Mathematical Modelling and Simulation

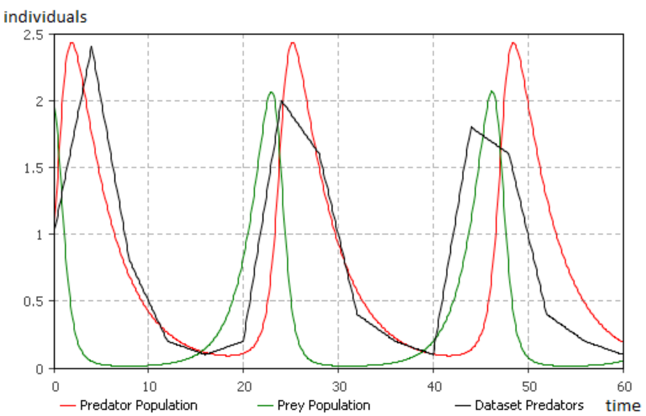






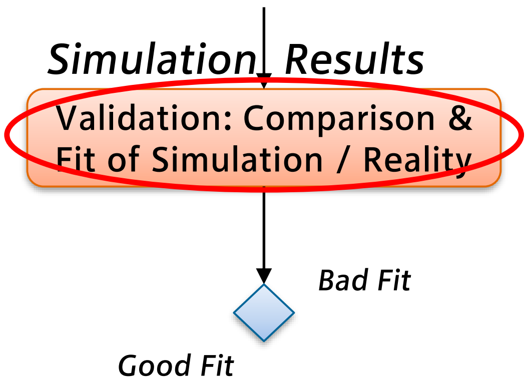


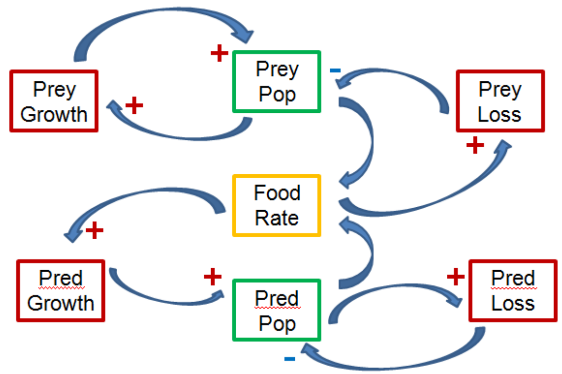
Search for convenient parameters

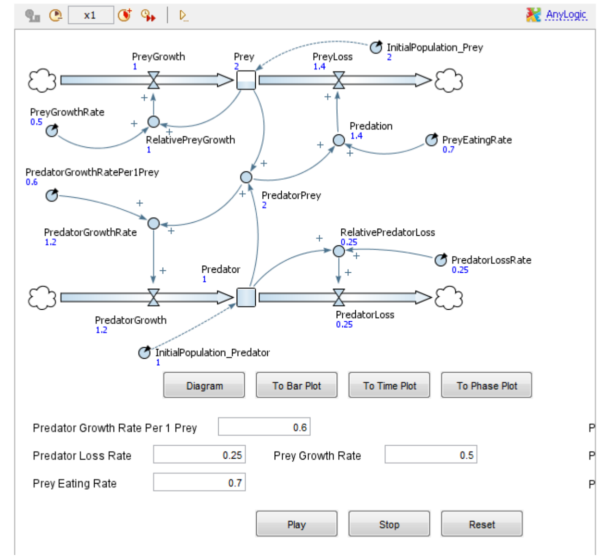


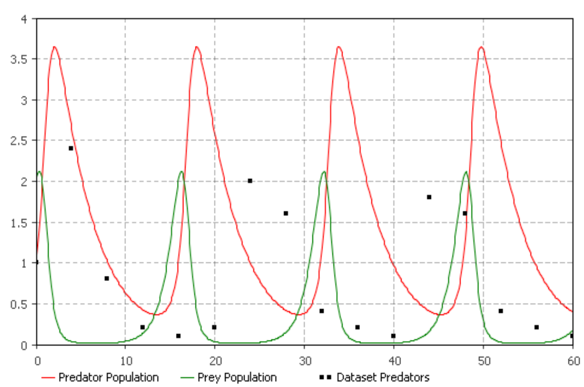
Data & Simulation Results

Mathematical Modelling and Simulation



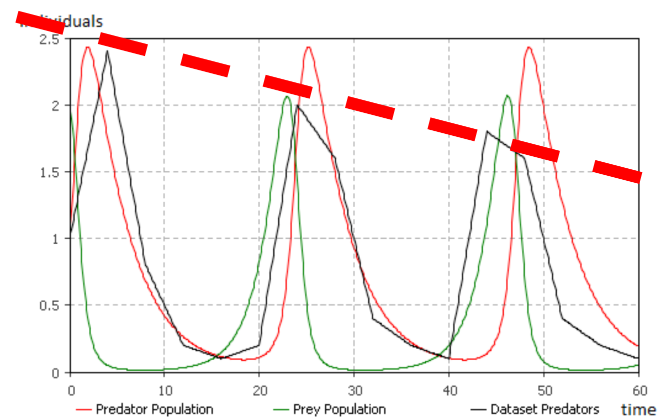




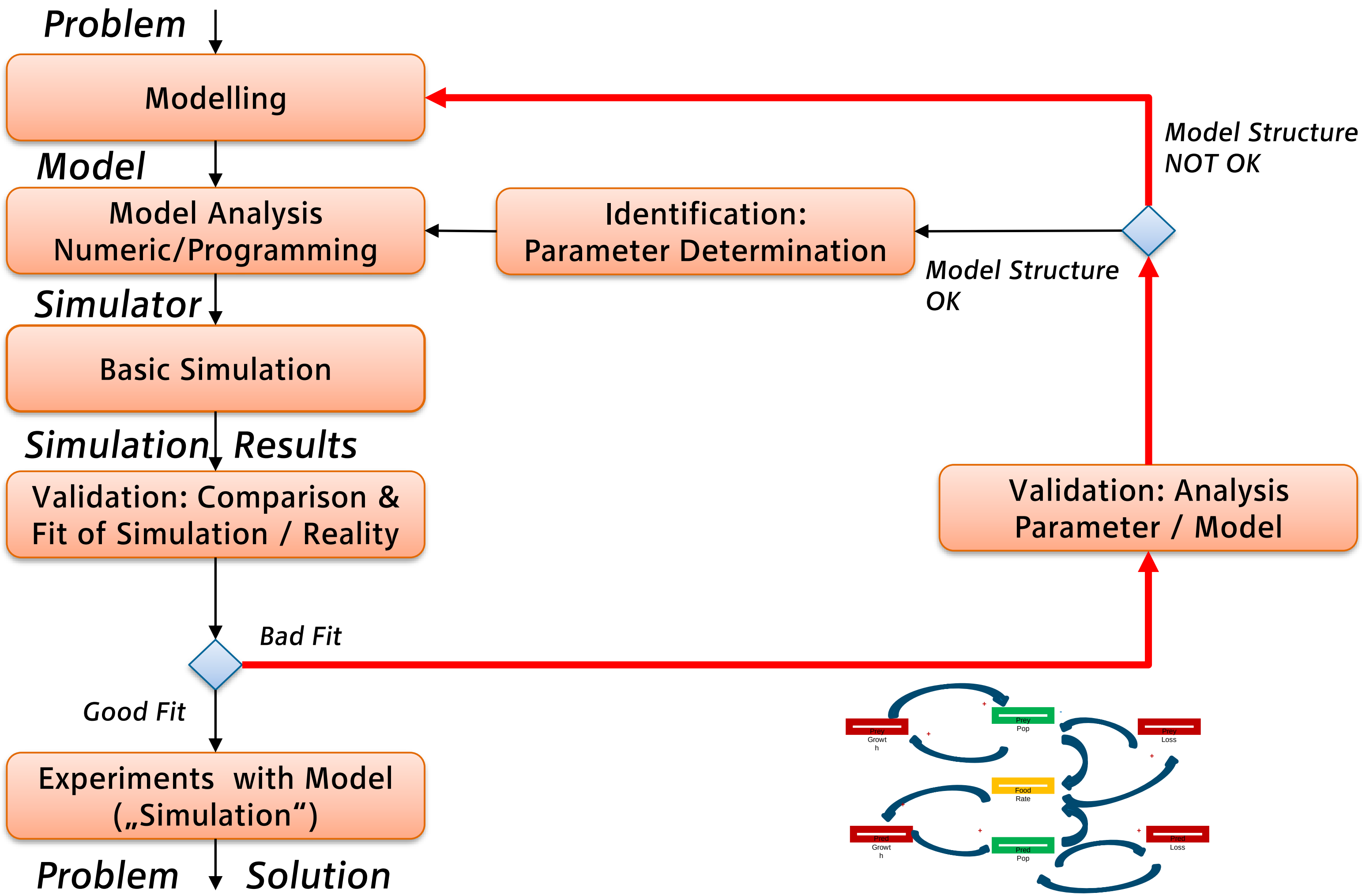


Search for convenient parameters

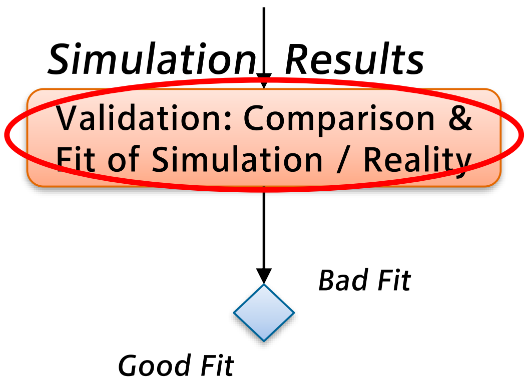
No Damping in Model

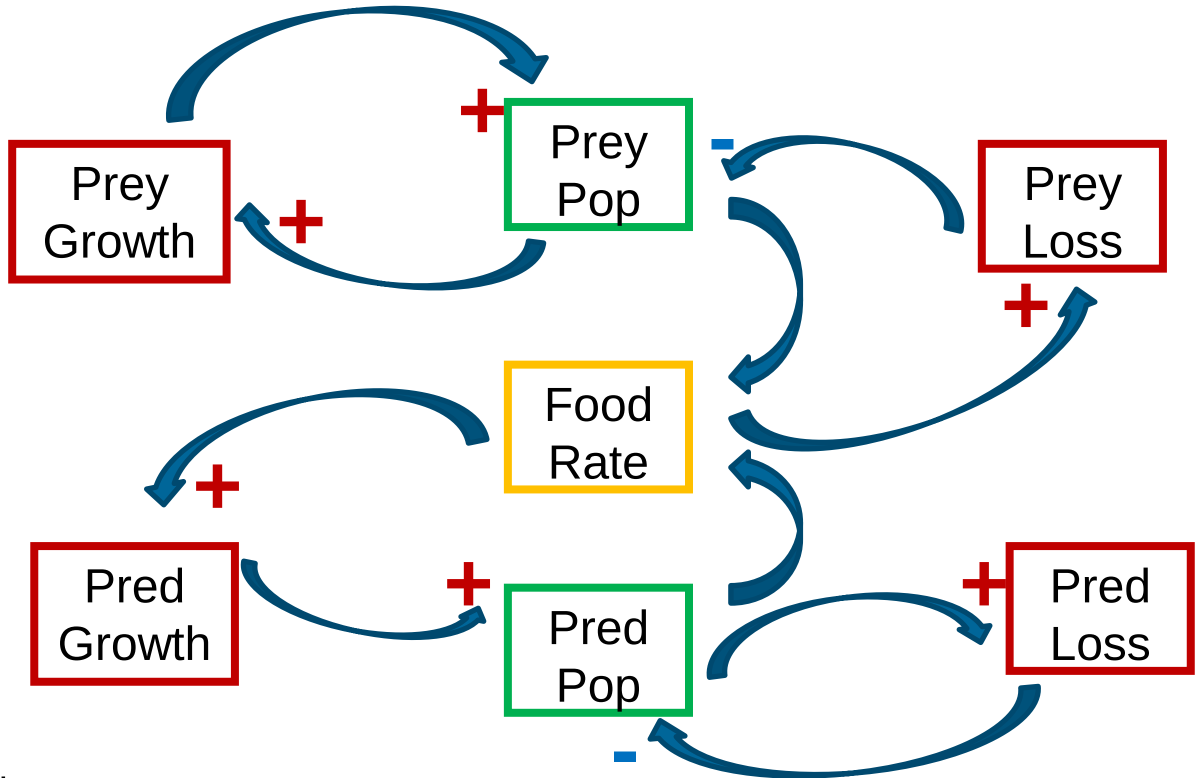


Simulation Circle: Predator - Prey



Model Modification



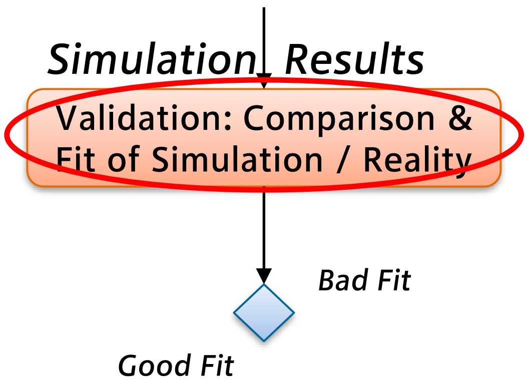


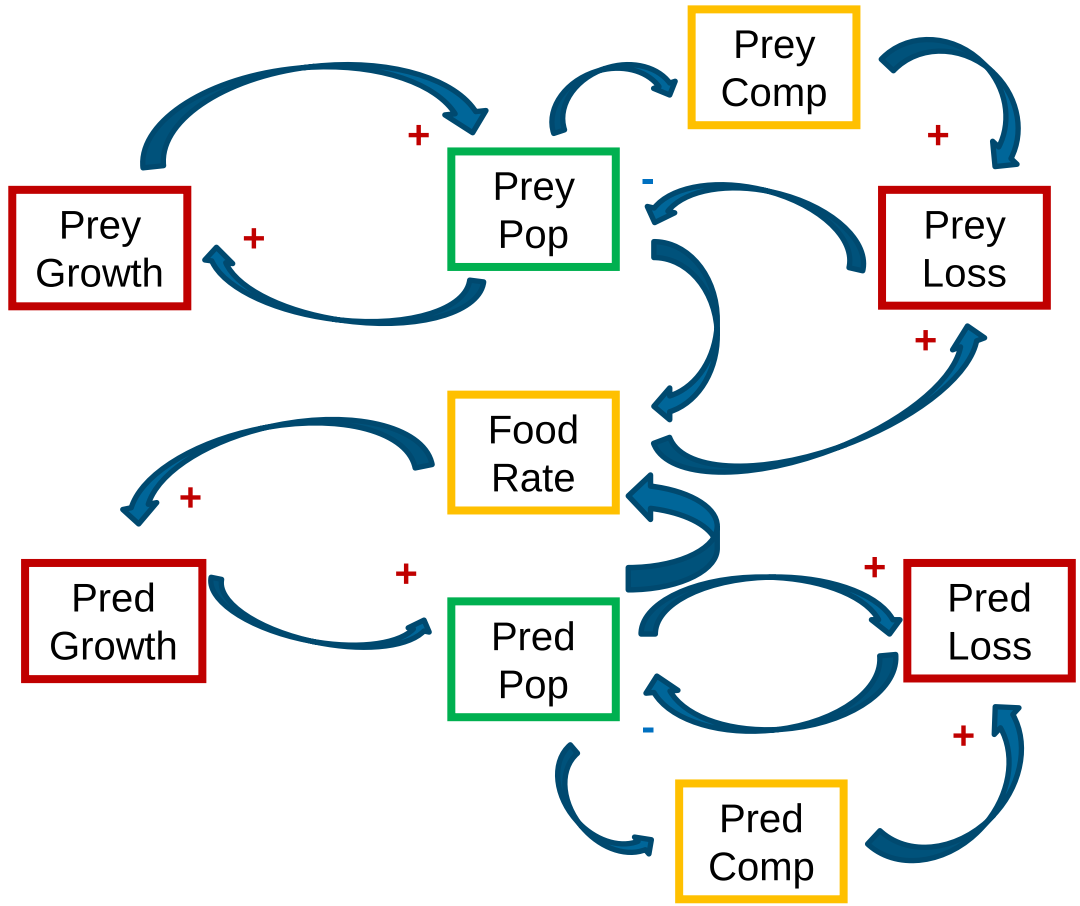
Model Extension:

- Both the predator and the prey compete for food and shelter in the forest.

- Competition sets in and the population of each species tends to control itself via a negative effect, that is the population decreases with a rate directly proportional to the present population of that species.

Model Modification





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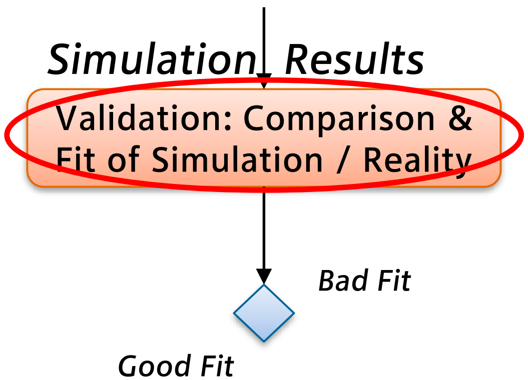
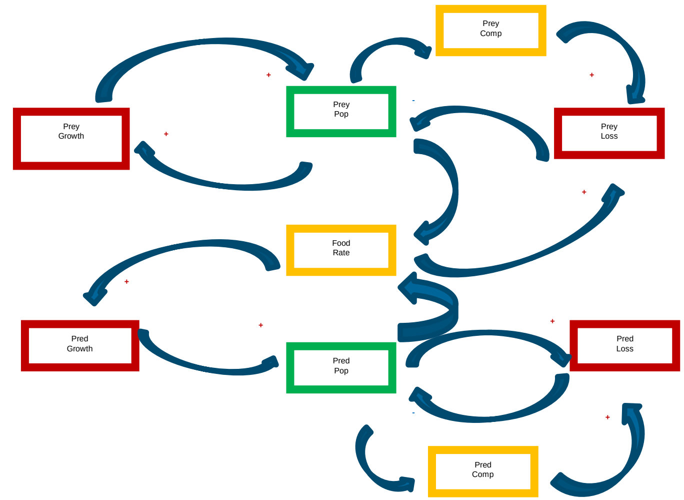
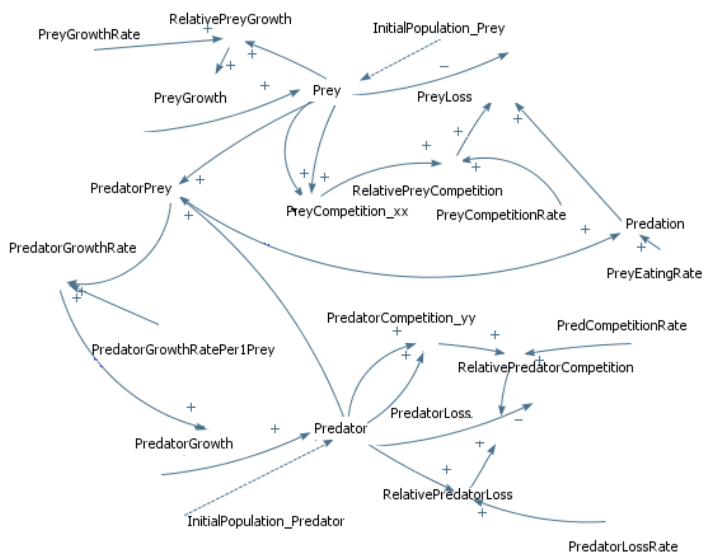
Model Modification

Mathematical Modelling and Simulation



Causal Loop Diagram

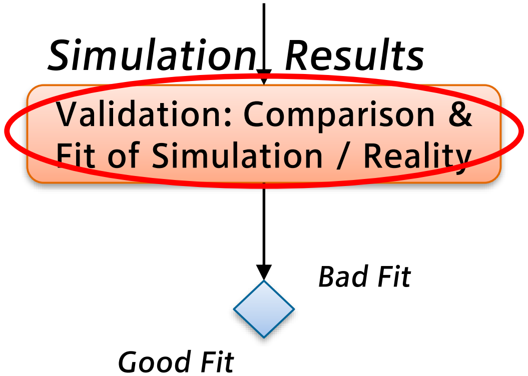




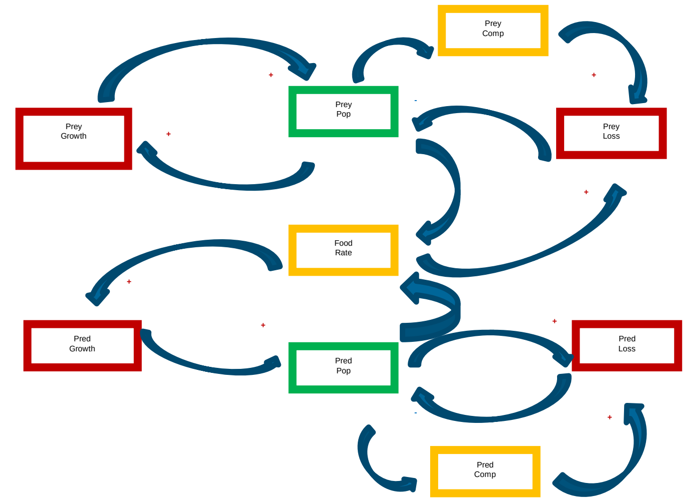
Model Extension:

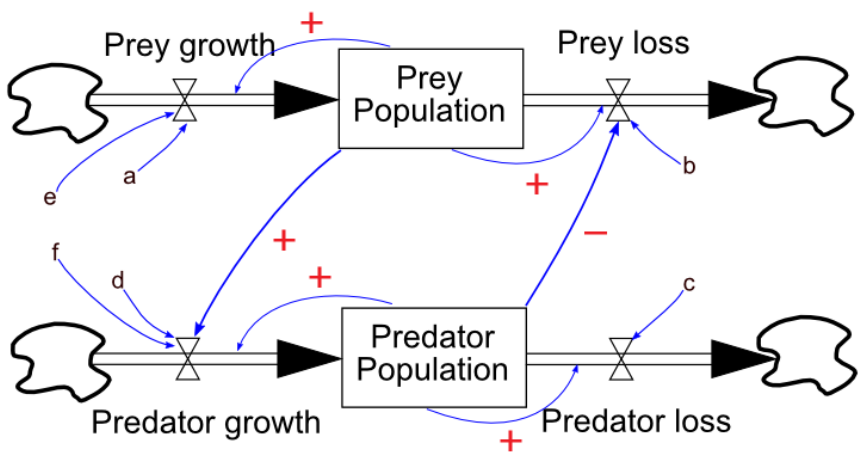
- Competition Feedback

Model Modification



Causal Loop Diagram

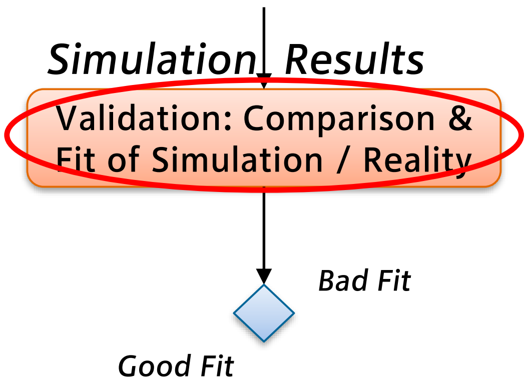




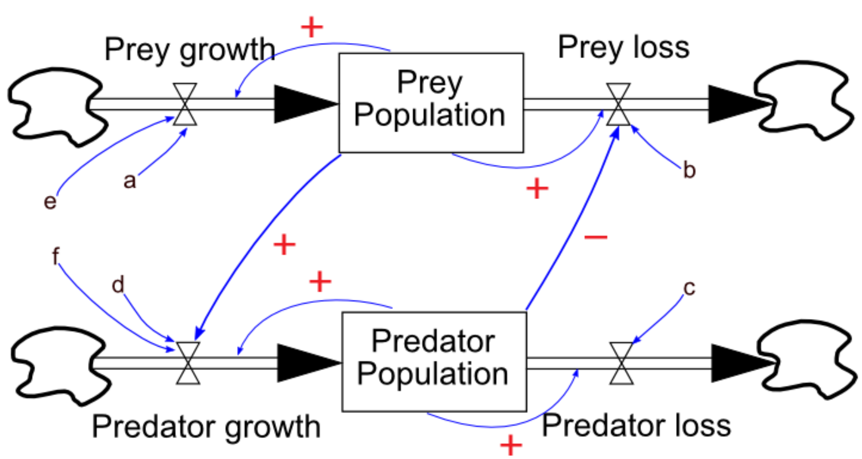
Stock and Flow Diagram

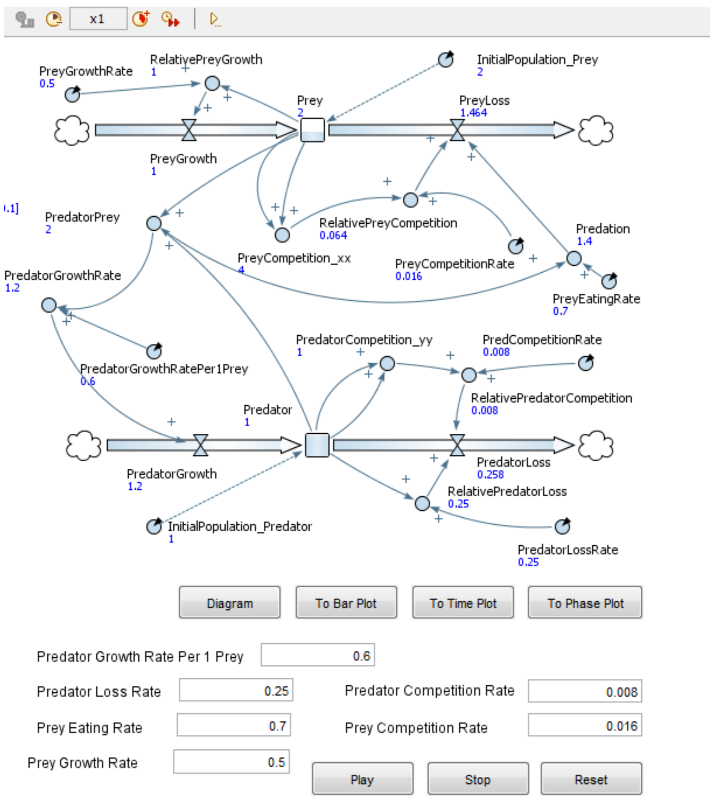
Model Modification

Mathematical Modelling and Simulation

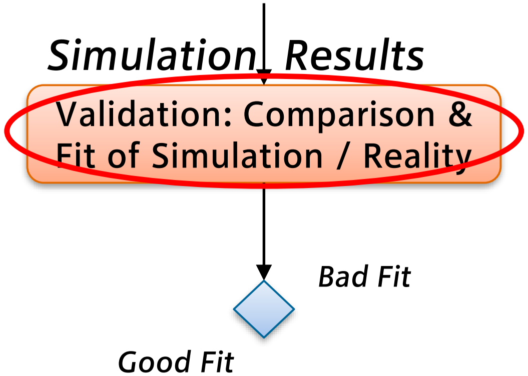


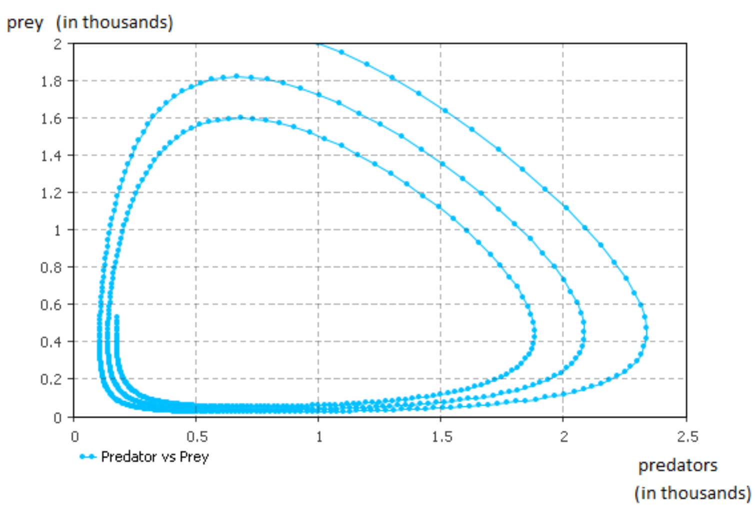
Stock and Flow Diagram





Model Modification





Parameters:

Predator Growth Rate Per 1 Prey	0.6
Predator Loss Rate	0.25
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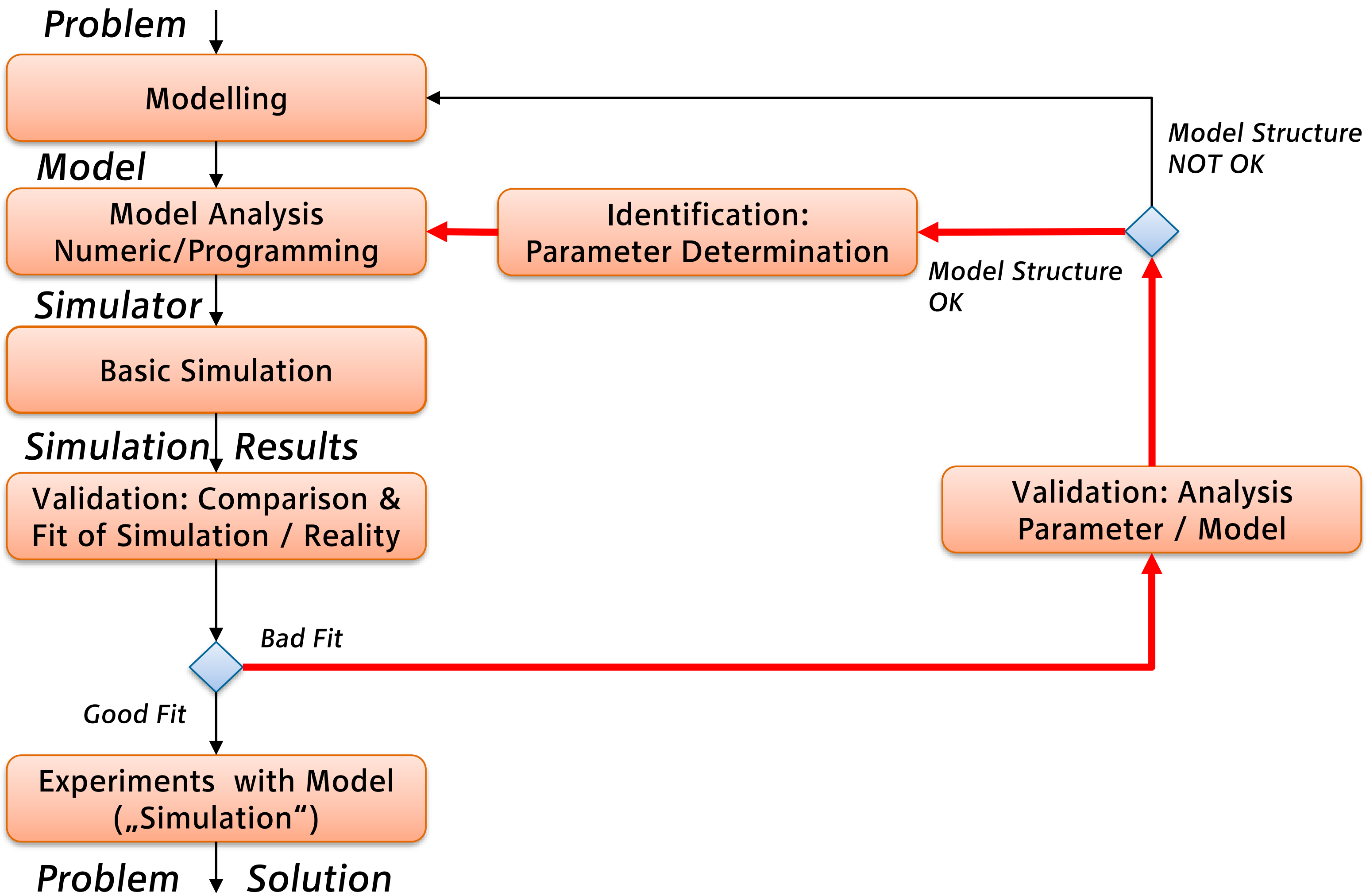
Prey Growth Rate	0.5
Predator Competition Rate	0.0080
Prey Competition Rate	0.016

$$\dot{x} = (a - b \cdot y)x - e \cdot x^2 = (a - e \cdot x - b \cdot y)x$$

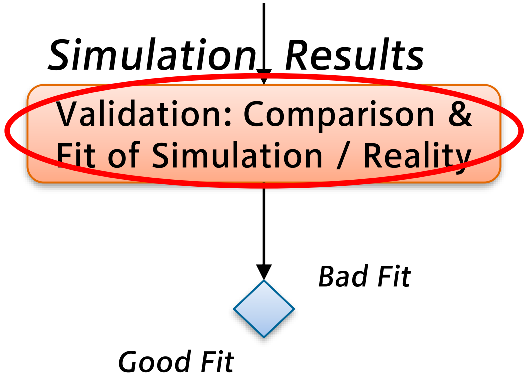
$$\dot{y} = (-c + d \cdot x)y - f \cdot y^2 = (-c - f \cdot y + d \cdot x)y$$

Simulation Circle: Predator - Prey

Mathematical Modelling and Simulation



Model Modification



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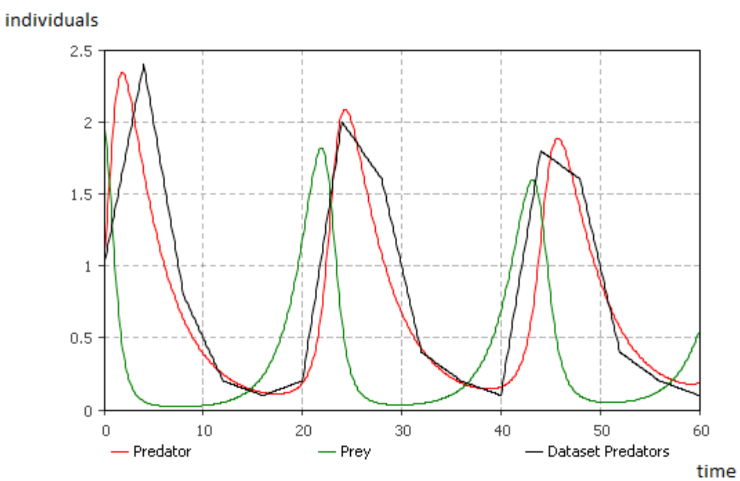

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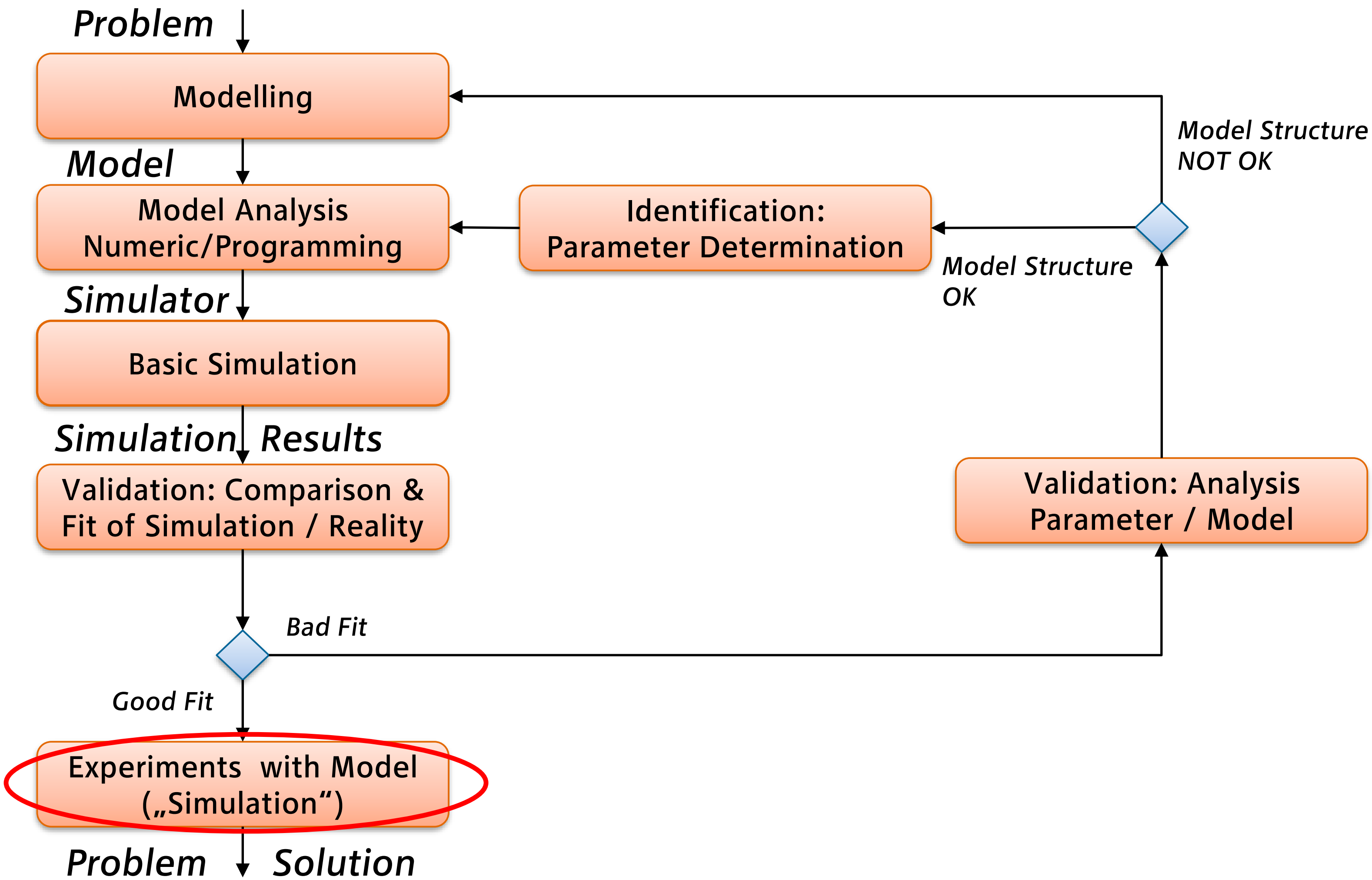
Simulation Circle: Predator - Prey

Mathematical Modelling and Simulation



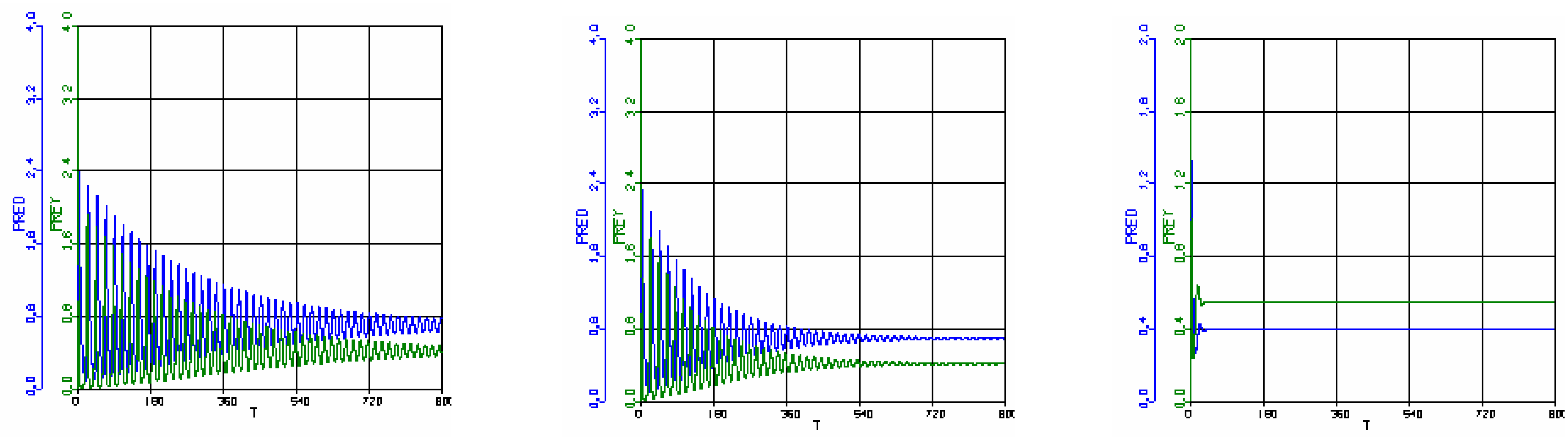
Results Interpretation / Analysis

Good Fit

Experiments with Model („Simulation“)

Problem *Solution*

- Determination of long time behavior / stationary solutions (equilibria)



Results Interpretation / Analysis

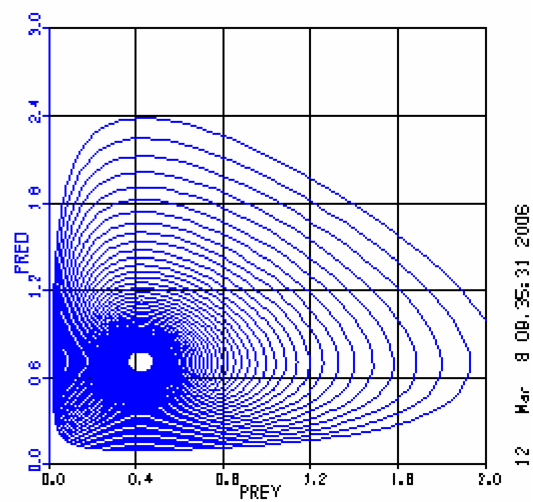
Mathematical Modelling and Simulation

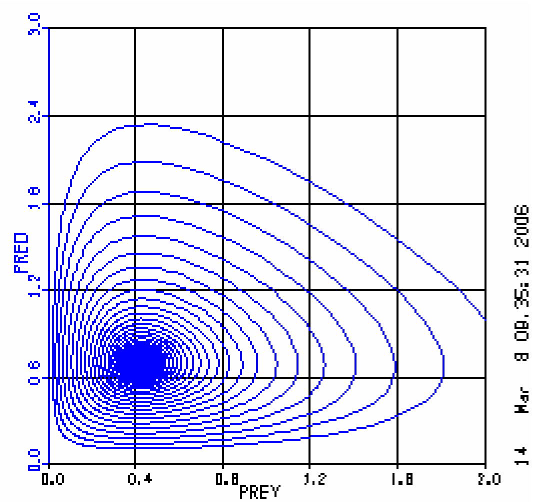
Good Fit

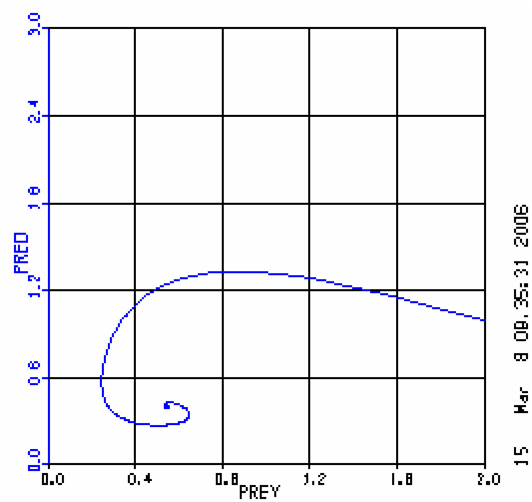
Experiments with Model („Simulation“)

Problem *Solution*

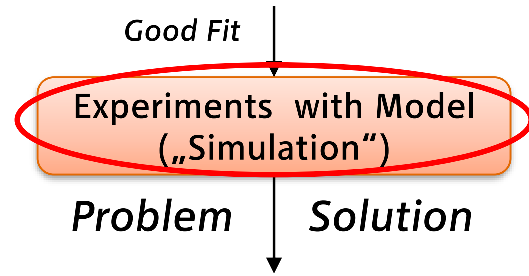
- Determination of long time behavior / stationary solutions (equilibria)







Use of Pesticide



Modification of Predator-prey model with intraspecific competition

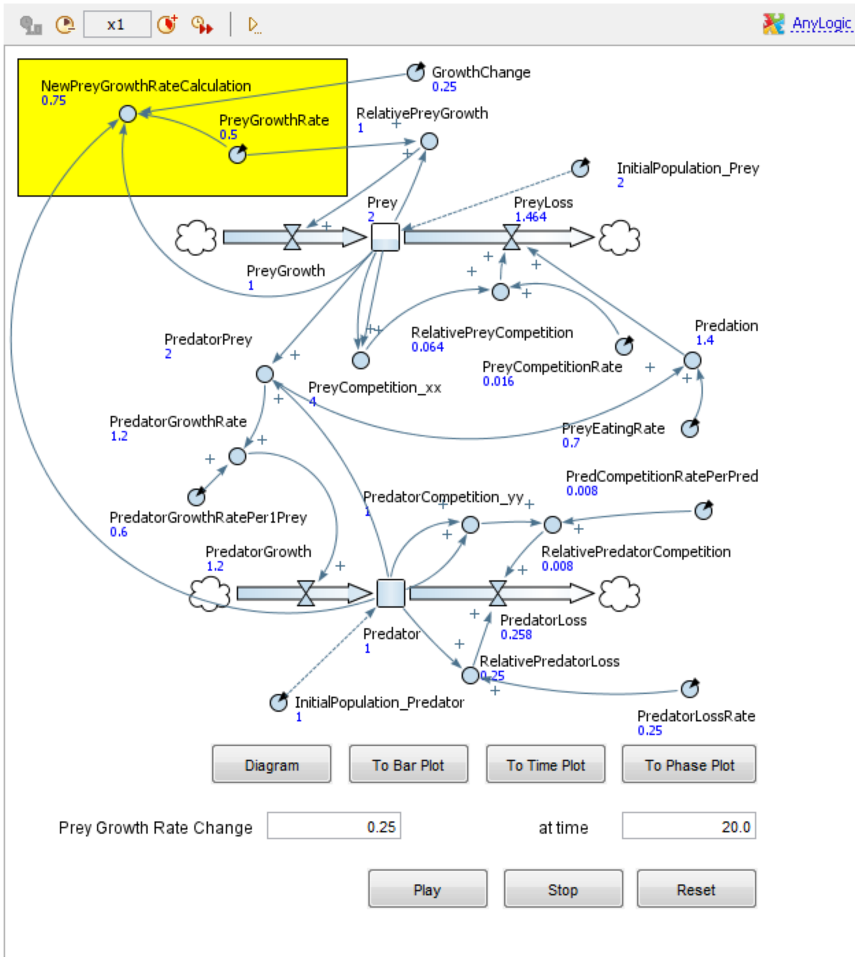
- Assume, that at a specific time poison is released into the system, e.g. some of predators are removed
  from the population by hunting.

- The growth rate **a** of prey is changed to:
  where K is growth rate change.

- This change occurs at the specific time point.

- The new growth rate **a** depends on the difference between populations at this specific time point and stays constant after that.

Use of Pesticide



Adequate time instant

Good Fit

Experiments with Model („Simulation“)

Problem | *Solution*

$$t_c : \quad d_{old} \rightarrow d_{new}, \quad f_{old} \rightarrow f_{new}$$

$$\dot{x} = ax - bxy - ex^2$$
$$\dot{y} = -cy + dxy - fy^2$$

$$d_{neu} = d_{alt} + d_c(x(t_c) - y(t_c))$$
$$f_{neu} = f_c \, f_{alt}$$

Modification of Predator-prey model with intraspecific competition

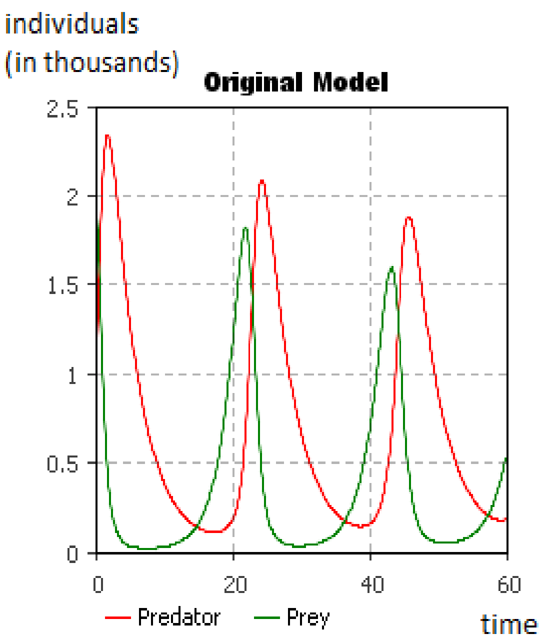
Mathematical Modelling and Simulation

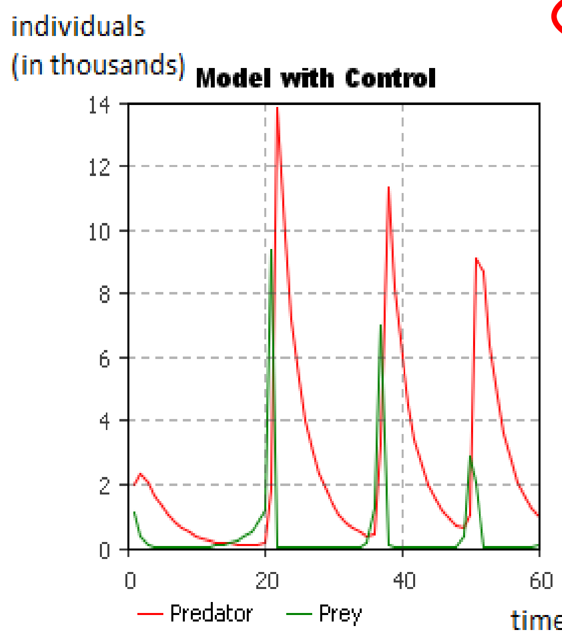
Good Fit

Experiments with Model („Simulation")

Problem ***Solution***

Population development over time:





Parameters:

Pred Loss Rate Change 2.0

at time 20.0

Note: Please note the different scaling of the plots.

$$d_{neu} = d_{alt} + d_c(x(t_c) - y(t_c))$$

$$f_{neu} = f_c\, f_{alt}$$

Modification of Predator-prey model with intraspecific competition

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Good Fit

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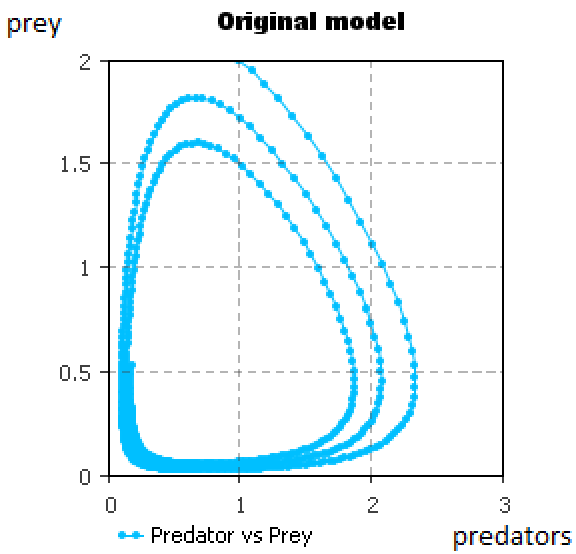
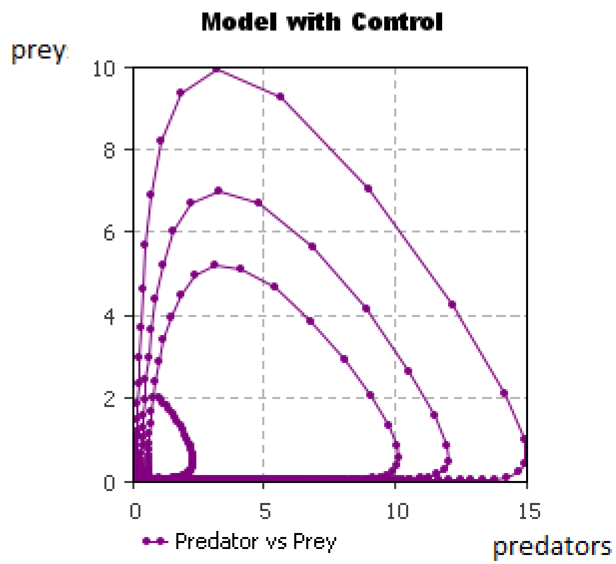
Problem | Solution

Population development over time:





Parameters:

Pred Loss Rate Change 2.0

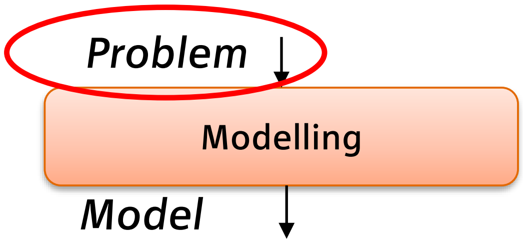
at time 20.0

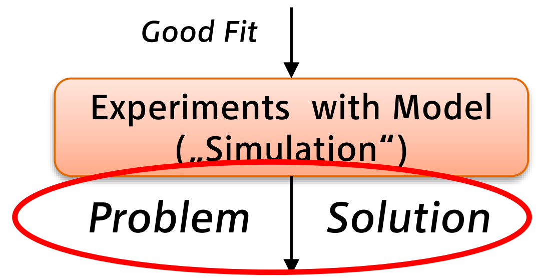
$$d_{neu} = d_{alt} + d_c(x(t_c) - y(t_c))$$

$$f_{neu} = f_c \, f_{alt}$$

Modification of Predator-prey model with intraspecific competition

Mathematical Modelling and Simulation





Dynamics: Prey – Predators

Environment: isolated

Measurement: natural enemies

5 Years = 60 months

quarterly

Problem: When is a reasonable time to use chemical pesticides?

Assignment: short time, changes the growth of preys, damping parameter

Approach: optimal time point t_c is dependent on the population difference

Result: **The assignment is not conducive**

The DON'Ts of Mathematical Modelling

Mathematical Modelling and Simulation

(S. W. Golomb, Simulation 14 (1970), 197-198)

- DON'T believe that the model is the reality
- DON'T extrapolate beyond the region of fit
- DON'T distort reality to fit the model
- DON'T retain a discredited model
- DON'T fall in love with your model