

J.1 Table of Critical Values for the Wilcoxon Rank-Sum Test

The tables on the following pages provide critical values for the Wilcoxon rank-sum test for independent samples with sizes from 3 to 25. Column m is the sample size for the smaller sample and column n is the sample size for the larger sample. If the sample sizes are equal, either sample can be designated m . For each pair of sample sizes (m, n) there are two sets of critical values, one set for one-tail $\alpha = 0.025$ and two-tail $\alpha = 0.05$ and a second set for one-tail $\alpha = 0.05$ and two-tail $\alpha = 0.10$. Suppose for a two-tailed test at $\alpha = 0.05$ we have $m = 8$ and $n = 9$. In the appropriate row and column we find the following numbers 51 93 16 0.0232. The 51 and 93 are the lower and upper critical values for W_X , the statistic testing $H_0: M_X = M_Y$. If $W_X \leq 51$ or $W_X \geq 93$, H_0 would be rejected. The value 0.0232 is the exact P value for the critical values of 51 or 93. The 16 under the column heading d is called the depth. Basically d is the depth one must go into the rank-ordered elementary estimates from each end to find the confidence limit values. In this case, the 16th smallest elementary estimate and the 16th largest elementary estimate are the 95% confidence interval limits for $M_x - M_y$.

1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$			1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$		
<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>
3	3				6	15	1 .0500	5	10	23	57	9 .0200	26	54	12 .0496
3	4				6	18	1 .0286	5	11	24	61	10 .0190	27	58	13 .0449
3	5	6	21	1 .0179	7	20	2 .0357	5	12	26	64	12 .0242	28	62	14 .0409
3	6	7	23	2 .0238	8	22	3 .0476	5	13	27	68	13 .0230	30	65	16 .0473
3	7	7	26	2 .0167	8	25	3 .0333	5	14	28	72	14 .0218	31	69	17 .0435
3	8	8	28	3 .0242	9	27	4 .0424	5	15	29	76	15 .0209	33	72	19 .0491
3	9	8	31	3 .0182	10	29	5 .0500	5	16	30	80	16 .0201	34	76	20 .0455
3	10	9	33	4 .0245	10	32	5 .0385	5	17	32	83	18 .0238	35	80	21 .0425
3	11	9	36	4 .0192	11	34	6 .0440	5	18	33	87	19 .0229	37	83	23 .0472
3	12	10	38	5 .0242	11	37	6 .0352	5	19	34	91	20 .0220	38	87	24 .0442
3	13	10	41	5 .0196	12	39	7 .0411	5	20	35	95	21 .0212	40	90	26 .0485
3	14	11	43	6 .0235	13	41	8 .0456	5	21	37	98	23 .0243	41	94	27 .0457
3	15	11	46	6 .0196	13	44	8 .0380	5	22	38	102	24 .0234	43	97	29 .0496
3	16	12	48	7 .0237	14	46	9 .0423	5	23	39	106	25 .0226	44	101	30 .0469
3	17	12	51	7 .0202	15	48	10 .0465	5	24	40	110	26 .0219	45	105	31 .0445
3	18	13	53	8 .0233	15	51	10 .0398	5	25	42	113	28 .0246	47	108	33 .0480
3	19	13	56	8 .0201	16	53	11 .0435	6	6	26	52	6 .0206	28	50	8 .0465
3	20	14	58	9 .0232	17	55	12 .0469	6	7	27	57	7 .0175	29	55	9 .0367
3	21	14	61	9 .0203	17	58	12 .0410	6	8	29	61	9 .0213	31	59	11 .0406
3	22	15	63	10 .0230	18	60	13 .0443	6	9	31	65	11 .0248	33	63	13 .0440
3	23	15	66	10 .0204	19	62	14 .0473	6	10	32	70	12 .0210	35	67	15 .0467
3	24	16	68	11 .0229	19	65	14 .0421	6	11	34	74	14 .0238	37	71	17 .0491
3	25	16	71	11 .0205	20	67	15 .0449	6	12	35	79	15 .0207	38	76	18 .0415
4	4	10	26	1 .0143	11	25	2 .0286	6	13	37	83	17 .0231	40	80	20 .0437
4	5	11	29	2 .0159	12	28	3 .0317	6	14	38	88	18 .0204	42	84	22 .0457
4	6	12	32	3 .0190	13	31	4 .0333	6	15	40	92	20 .0224	44	88	24 .0474
4	7	13	35	4 .0212	14	34	5 .0364	6	16	42	96	22 .0244	46	92	26 .0490
4	8	14	38	5 .0242	15	37	6 .0364	6	17	43	101	23 .0219	47	97	27 .0433
4	9	14	42	5 .0168	16	40	7 .0378	6	18	45	105	25 .0236	49	101	29 .0448
4	10	15	45	6 .0180	17	43	8 .0380	6	19	46	110	26 .0214	51	105	31 .0462
4	11	16	48	7 .0198	18	46	9 .0388	6	20	48	114	28 .0229	53	109	33 .0475
4	12	17	51	8 .0209	19	49	10 .0390	6	21	50	118	30 .0244	55	113	35 .0487
4	13	18	54	9 .0223	20	52	11 .0395	6	22	51	123	31 .0224	57	117	37 .0498
4	14	19	57	10 .0232	21	55	12 .0395	6	23	53	127	33 .0237	58	122	38 .0452
4	15	20	60	11 .0243	22	58	13 .0400	6	24	54	132	34 .0219	60	126	40 .0463
4	16	21	63	12 .0250	24	60	15 .0497	6	25	56	136	36 .0231	62	130	42 .0473
4	17	21	67	12 .0202	25	63	16 .0493	7	7	36	69	9 .0189	39	66	12 .0487
4	18	22	70	13 .0212	26	66	17 .0491	7	8	38	74	11 .0200	41	71	14 .0469
4	19	23	73	14 .0219	27	69	18 .0487	7	9	40	79	13 .0209	43	76	16 .0454
4	20	24	76	15 .0227	28	72	19 .0485	7	10	42	84	15 .0215	45	81	18 .0439
4	21	25	79	16 .0233	29	75	20 .0481	7	11	44	89	17 .0221	47	86	20 .0427
4	22	26	82	17 .0240	30	78	21 .0480	7	12	46	94	19 .0225	49	91	22 .0416
4	23	27	85	18 .0246	31	81	22 .0477	7	13	48	99	21 .0228	52	95	25 .0484
4	24	27	89	18 .0211	32	84	23 .0475	7	14	50	104	23 .0230	54	100	27 .0469
4	25	28	92	19 .0217	33	87	24 .0473	7	15	52	109	25 .0233	56	105	29 .0455
5	5	17	38	3 .0159	19	36	5 .0476	7	16	54	114	27 .0234	58	110	31 .0443
5	6	18	42	4 .0152	20	40	6 .0411	7	17	56	119	29 .0236	61	114	34 .0497
5	7	20	45	6 .0240	21	44	7 .0366	7	18	58	124	31 .0237	63	119	36 .0484
5	8	21	49	7 .0225	23	47	9 .0466	7	19	60	129	33 .0238	65	124	38 .0471
5	9	22	53	8 .0210	24	51	10 .0415	7	20	62	134	35 .0239	67	129	40 .0460

1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$			1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$		
<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>
7	21	64	139	37	.0240	69	134	42	.0449	10	20	110	200	56	.0498
7	22	66	144	39	.0240	72	138	45	.0492	10	21	113	207	59	.0478
7	23	68	149	41	.0241	74	143	47	.0481	10	22	116	214	62	.0459
7	24	70	154	43	.0241	76	148	49	.0470	10	23	119	221	65	.0482
7	25	72	159	45	.0242	78	153	51	.0461	10	24	122	228	68	.0465
8	8	49	87	14	.0249	51	85	16	.0415	10	25	126	234	72	.0486
8	9	51	93	16	.0232	54	90	19	.0464	11	11	96	157	31	.0440
8	10	53	99	18	.0217	56	96	21	.0416	11	12	99	165	34	.0454
8	11	55	105	20	.0204	59	101	24	.0454	11	13	103	172	38	.0467
8	12	58	110	23	.0237	62	106	27	.0489	11	14	106	180	41	.0477
8	13	60	116	25	.0223	64	112	29	.0445	11	15	110	187	45	.0486
8	14	62	122	27	.0211	67	117	32	.0475	11	16	113	195	48	.0494
8	15	65	127	30	.0237	69	123	34	.0437	11	17	117	202	52	.0453
8	16	67	133	32	.0224	72	128	37	.0463	11	18	121	209	56	.0461
8	17	70	138	35	.0247	75	133	40	.0487	11	19	124	217	59	.0468
8	18	72	144	37	.0235	77	139	42	.0452	11	20	128	224	63	.0474
8	19	74	150	39	.0224	80	144	45	.0475	11	21	131	232	66	.0480
8	20	77	155	42	.0244	83	149	48	.0495	11	22	135	239	70	.0486
8	21	79	161	44	.0233	85	155	50	.0464	11	23	139	246	74	.0490
8	22	81	167	46	.0223	88	160	53	.0483	11	24	142	254	77	.0495
8	23	84	172	49	.0240	90	166	55	.0454	11	25	146	261	81	.0499
8	24	86	178	51	.0231	93	171	58	.0472	12	12	115	185	38	.0444
8	25	89	183	54	.0247	96	176	61	.0488	12	13	119	193	42	.0488
9	9	62	109	18	.0200	66	105	22	.0470	12	14	123	201	46	.0475
9	10	65	115	21	.0217	69	111	25	.0474	12	15	127	209	50	.0463
9	11	68	121	24	.0232	72	117	28	.0476	12	16	131	217	54	.0500
9	12	71	127	27	.0245	75	123	31	.0477	12	17	135	225	58	.0486
9	13	73	134	29	.0217	78	129	34	.0478	12	18	139	233	62	.0474
9	14	76	140	32	.0228	81	135	37	.0478	12	19	143	241	66	.0463
9	15	79	146	35	.0238	84	141	40	.0478	12	20	147	249	70	.0493
9	16	82	152	38	.0247	87	147	43	.0477	12	21	151	257	74	.0481
9	17	84	159	40	.0223	90	153	46	.0476	12	22	155	265	78	.0471
9	18	87	165	43	.0231	93	159	49	.0475	12	23	159	273	82	.0496
9	19	90	171	46	.0239	96	165	52	.0474	12	24	163	281	86	.0486
9	20	93	177	49	.0245	99	171	55	.0473	12	25	167	289	90	.0475
9	21	95	184	51	.0225	102	177	58	.0472	13	13	136	215	46	.0454
9	22	98	190	54	.0231	105	183	61	.0471	13	14	141	223	51	.0472
9	23	101	196	57	.0237	108	189	64	.0470	13	15	145	232	55	.0489
9	24	104	202	60	.0243	111	195	67	.0469	13	16	150	240	60	.0458
9	25	107	208	63	.0249	114	201	70	.0468	13	17	154	249	64	.0472
10	10	78	132	24	.0216	82	128	28	.0446	13	18	158	258	68	.0485
10	11	81	139	27	.0215	86	134	32	.0493	13	19	163	266	73	.0497
10	12	84	146	30	.0213	89	141	35	.0465	13	20	167	275	77	.0470
10	13	88	152	34	.0247	92	148	38	.0441	13	21	171	284	81	.0481
10	14	91	159	37	.0242	96	154	42	.0478	13	22	176	292	86	.0491
10	15	94	166	40	.0238	99	161	45	.0455	13	23	180	301	90	.0467
10	16	97	173	43	.0234	103	167	49	.0487	13	24	185	309	95	.0476
10	17	100	180	46	.0230	106	174	52	.0465	13	25	189	318	99	.0485
10	18	103	187	49	.0226	110	180	56	.0493	14	14	160	246	56	.0469
10	19	107	193	53	.0250	113	187	59	.0472	14	15	164	256	60	.0466

1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$			1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$			$\alpha = 0.05$ $\alpha = 0.10$		
<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>m</i>	<i>n</i>	<i>W</i>	<i>d</i>	<i>P</i>	<i>W</i>	<i>d</i>	<i>P</i>
14	16	169 265	65	.0236	176 258	72	.0463	17	24	282 432	130	.0239	294 420	141	.0492
14	17	174 274	70	.0242	182 266	78	.0500	17	25	288 443	136	.0238	300 431	147	.0480
14	18	179 283	75	.0247	187 275	83	.0495	18	18	270 396	100	.0235	280 386	109	.0485
14	19	183 293	79	.0230	192 284	88	.0489	18	19	277 407	107	.0246	287 397	116	.0490
14	20	188 302	84	.0235	197 293	93	.0484	18	20	283 419	113	.0238	294 408	123	.0495
14	21	193 311	89	.0239	202 302	98	.0480	18	21	290 430	120	.0247	301 419	130	.0499
14	22	198 320	94	.0243	207 311	103	.0475	18	22	296 442	126	.0240	307 431	136	.0474
14	23	203 329	99	.0247	212 320	108	.0471	18	23	303 453	133	.0248	314 442	143	.0478
14	24	207 339	103	.0233	218 328	114	.0498	18	24	309 465	139	.0240	321 453	150	.0481
14	25	212 348	108	.0236	223 337	119	.0492	18	25	316 476	146	.0248	328 464	157	.0484
15	15	184 281	65	.0227	192 273	73	.0488	19	19	303 438	114	.0248	313 428	123	.0482
15	16	190 290	71	.0247	197 283	78	.0466	19	20	309 451	120	.0234	320 440	130	.0474
15	17	195 300	76	.0243	203 292	84	.0485	19	21	316 463	127	.0236	328 451	138	.0494
15	18	200 310	81	.0239	208 302	89	.0465	19	22	323 475	134	.0238	335 463	145	.0486
15	19	205 320	86	.0235	214 311	95	.0482	19	23	330 487	141	.0240	342 475	152	.0478
15	20	210 330	91	.0232	220 320	101	.0497	19	24	337 499	148	.0241	350 486	160	.0496
15	21	216 339	97	.0247	225 330	106	.0478	19	25	344 511	155	.0243	357 498	167	.0488
15	22	221 349	102	.0243	231 339	112	.0492	20	20	337 483	128	.0245	348 472	138	.0482
15	23	226 359	107	.0239	236 349	117	.0474	20	21	344 496	135	.0241	356 484	146	.0490
15	24	231 369	112	.0235	242 358	123	.0486	20	22	351 509	142	.0236	364 496	154	.0497
15	25	237 378	118	.0248	248 367	129	.0499	20	23	359 521	150	.0246	371 509	161	.0478
16	16	211 317	76	.0234	219 309	84	.0469	20	24	366 534	157	.0242	379 521	169	.0484
16	17	217 327	82	.0243	225 319	90	.0471	20	25	373 547	164	.0237	387 533	177	.0490
16	18	222 338	87	.0231	231 329	96	.0473	21	21	373 530	143	.0245	385 518	154	.0486
16	19	228 348	93	.0239	237 339	102	.0474	21	22	381 543	151	.0249	393 531	162	.0482
16	20	234 358	99	.0247	243 349	108	.0475	21	23	388 557	158	.0238	401 544	170	.0478
16	21	239 369	104	.0235	249 359	114	.0475	21	24	396 570	166	.0242	410 556	179	.0497
16	22	245 379	110	.0242	255 369	120	.0476	21	25	404 583	174	.0245	418 569	187	.0492
16	23	251 389	116	.0248	261 379	126	.0476	22	22	411 579	159	.0247	424 566	171	.0491
16	24	256 400	121	.0238	267 389	132	.0476	22	23	419 593	167	.0244	432 580	179	.0477
16	25	262 410	127	.0243	273 399	138	.0476	22	24	427 607	175	.0242	441 593	188	.0486
17	17	240 355	88	.0243	249 346	97	.0493	22	25	435 621	183	.0240	450 606	197	.0494
17	18	246 366	94	.0243	255 357	103	.0479	23	23	451 630	176	.0249	465 616	189	.0499
17	19	252 377	100	.0243	262 367	110	.0499	23	24	459 645	184	.0242	474 630	198	.0497
17	20	258 388	106	.0242	268 378	116	.0485	23	25	468 659	193	.0246	483 644	207	.0495
17	21	264 399	112	.0242	274 389	122	.0473	24	24	492 684	193	.0241	507 669	207	.0486
17	22	270 410	118	.0241	281 399	129	.0490	24	25	501 699	202	.0241	517 683	217	.0496
17	23	276 421	124	.0240	287 410	135	.0477	25	25	536 739	212	.0247	552 723	227	.0497