(2) **Basketball free throws**
Two professional basketball players, Tom and John, each throw ten free throws with a basketball. Tom makes 80% of the free throws he tries, while John makes 85% of the free throws he tries.

(a) What is the probability that the number of free throws that Tom will make is exactly 7?

(b) What is the probability that the number of free throws that John will make is at least 8?

(c) Player who achives the highest score wins the game. It is assumed that the two players do not influence each other when throwing. What is the probability that neither Tom or John will win the game?

Hint: Use R-function `dbinom()` to calculate the probability mass functions.

X ... number of throws Tom makes out of 10

Y ... number of throws John makes out of 10

a) $$\mathbb{P}(X=7) = \binom{10}{7}\left(\frac{8}{10}\right)^7\left(\frac{2}{10}\right)^3 = \frac{10!}{7!\cdot 3!}\left(\frac{8}{10}\right)^7\left(\frac{2}{10}\right)^3 = \frac{10\cdot 9\cdot 8}{3\cdot 2}\left(\frac{8}{10}\right)^7\left(\frac{2}{10}\right)^3 = 15\,\frac{8^8}{10^7}\,\frac{8}{10^3}$$

$$= \frac{15\cdot 8^9}{10^{10}} = \frac{15}{10}\left(\frac{8}{10}\right)^9 = \frac{3}{2}\left(\frac{4}{5}\right)^9 \approx 0{,}2$$

b) $$\mathbb{P}(X\geq 8) = \mathbb{P}(X=8) + \mathbb{P}(X=9) + \mathbb{P}(X=10) = \binom{10}{8}\left(\frac{85}{100}\right)^8\left(\frac{15}{100}\right)^2 + \binom{10}{9}\left(\frac{85}{100}\right)^9\left(\frac{15}{100}\right) + \binom{10}{10}\left(\frac{85}{100}\right)^{10}$$

$$= \frac{10\cdot 9}{2}\left(\frac{17}{20}\right)^8\left(\frac{3}{20}\right)^2 + 10\left(\frac{17}{20}\right)^9\frac{3}{20} + \left(\frac{17}{20}\right)^{10}$$

$$= \left(\frac{17}{20}\right)^8\left(5\cdot 9\cdot\frac{9}{20\cdot 20} + 10\cdot\frac{17}{20}\,\frac{3}{20} + \frac{17^2}{20^2}\right)$$

$$= \left(\frac{17}{20}\right)^8\left(\frac{81}{80} + \frac{51}{40} + \frac{17^2}{20^2}\right) \approx 0{,}82$$

c) $$\mathbb{P}(X=Y) = \sum_{k=0}^{10}\mathbb{P}(X=k)\,\mathbb{P}(Y=k) \approx 0{,}23$$