Mathematical
Modelling and
Simulation

Introduction to System Dynamics

Overview

- Introduction
- General Information
- How to Build a System Dynamics Model
  - System Variables and Boundaries
  - Causal Loop Diagrams
  - Stock-and-Flow Diagrams
- Helpful Tools
- Analysis
- Simulators
- Conclusion
- Further Steps

General Information (1)

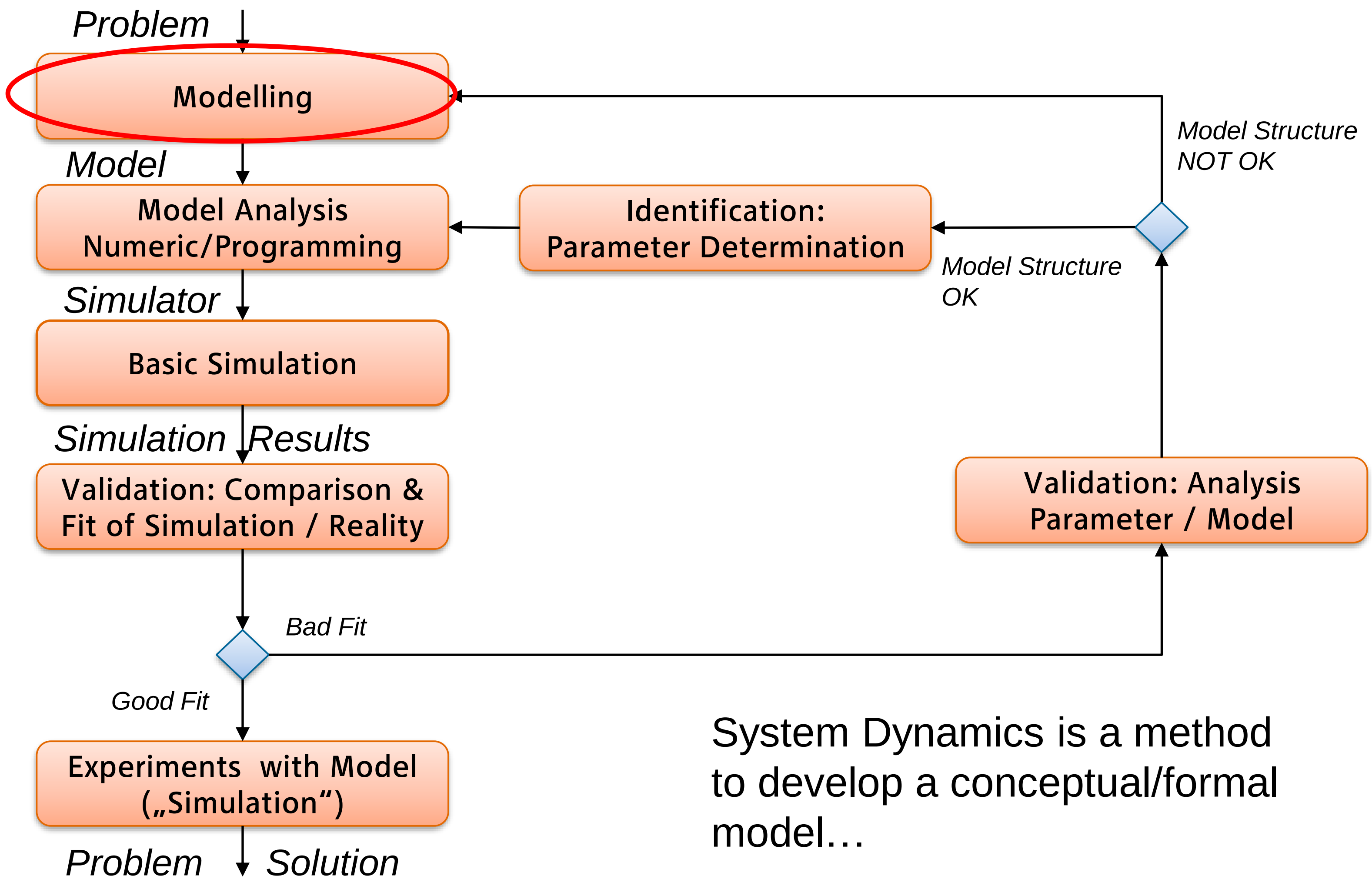
Mathematical Modelling and Simulation

- System Dynamics (short SD) is a modelling and simulation method developed by **Jay W. Forrester**.
- He adapted methods formerly used for system analysis of technological systems to social systems (MIT Sloan School of Management, 1956).
- Thus he was criticising mathematical models developed for management sciences.
- SD has roots on **control theory** and **nonlinear dynamics**
- SD is very intuitive, supported by graphics

General Information (2)

Mathematical Modelling and Simulation

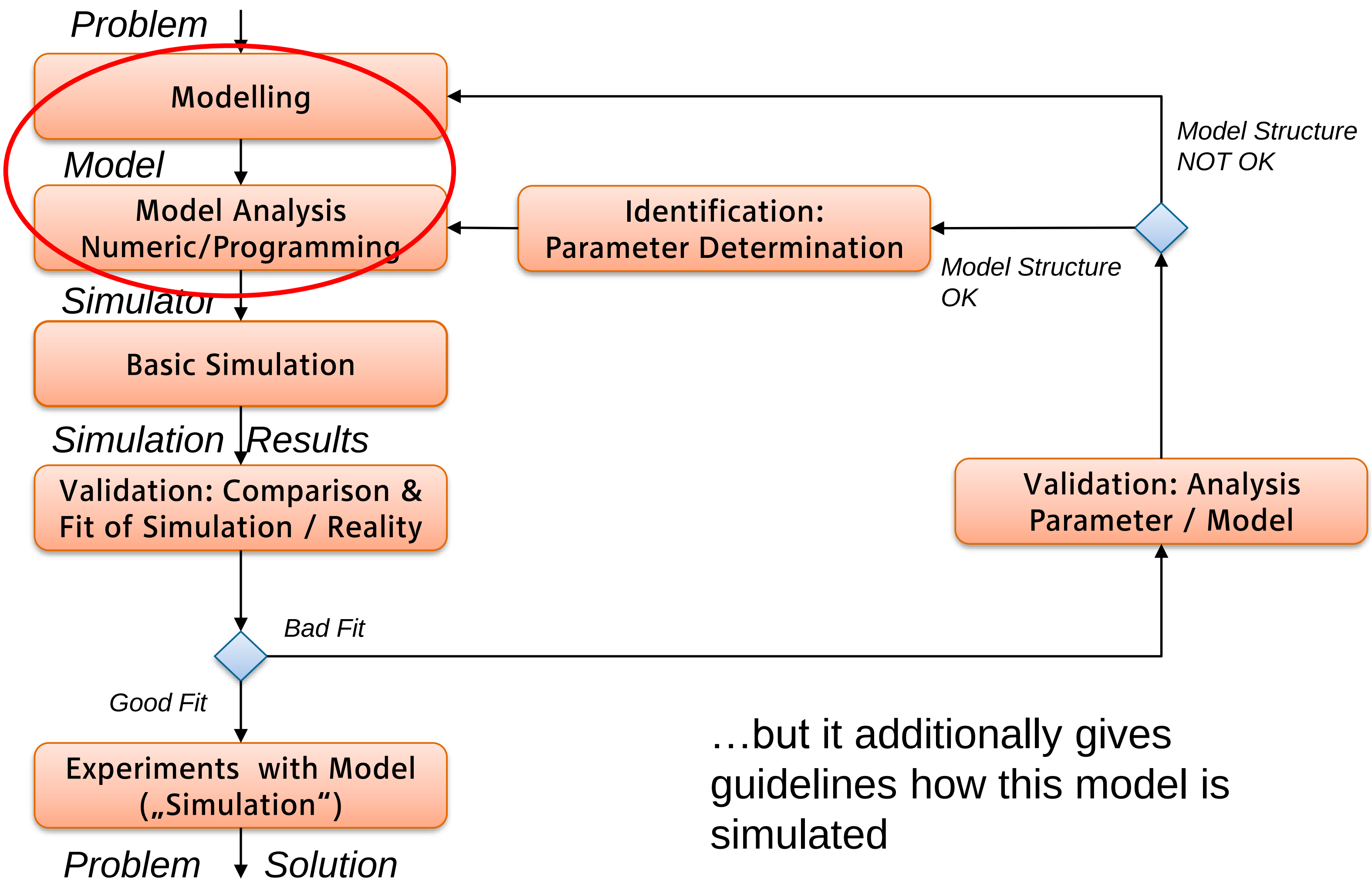
Simulation Circle



System Dynamics is a method to develop a conceptual/formal model…

General Information (2)

Mathematical Modelling and Simulation



Simulation Circle

…but it additionally gives guidelines how this model is simulated

General Information (3)

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Hypothesis:

- Manager usually know very good about processes and their causal relationships within their companies (system).
- The behaviour of a system is mostly predetermined by its (complex) structure.
- Practically useful models can usually not be simulated by analytic calculations.

General Information (4)

Mathematical Modelling and Simulation

Literature:

- 1961: *Industrial Dynamics* (Forrester)
- 1969: *Urban Dynamics* (Forrester), first use of System Dynamics apart from economic businesses.
- 1970: *World Dynamics* (Forrester), superwised by Club of Rome, use of System Dynamics for development of a so called „World Model“. Similar:
- 1972: Meadows et al.: *The Limits to Growth*

General Information (5)

Relationship: SD & Differential Equations Modelling

- Each System Dynamics model is equvalent to exactly one differential-equation (DE) system. It can be seen to be a graphical way for development of DE models.
- Advantages:
  - Picturesque
  - Optimized to understand dynamics and causal relationships of the system.
  - Finally calculated like a DE model.

General Information (5)

Mathematical Modelling and Simulation

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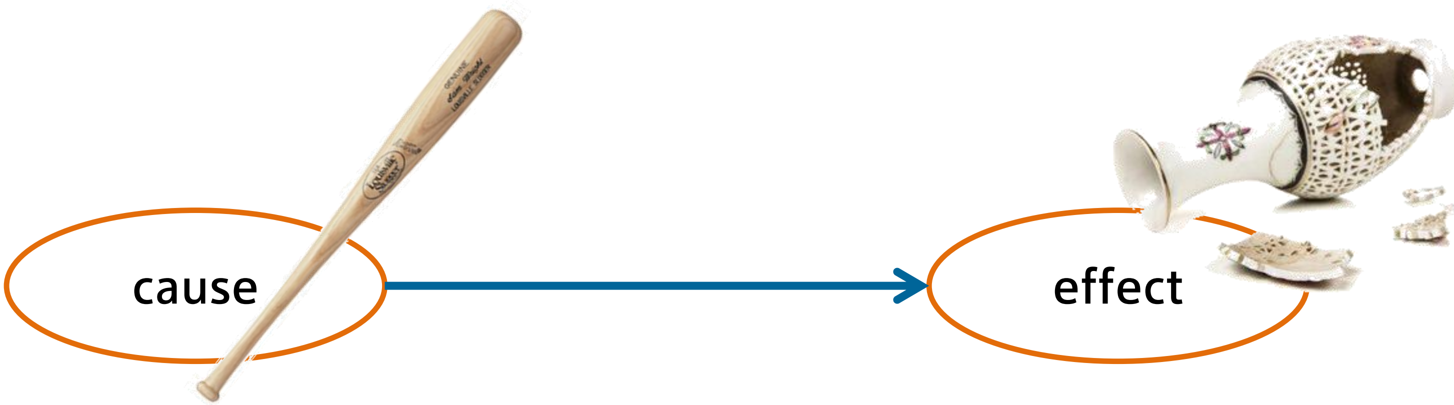
Perfect starting-point for learning about Modelling and Simulation

Key to develop SD Models

Causal thinking is the key to organizing ideas in a system dynamics study

(Roberts et al. 1983)



How to build a SD Model?

1. Identify system variables and system boundaries
2. Capture **links** of variables in a **Causal Loop Diagram (CLD)**
3. Build a **Stock and Flow Diagram (SFD)**

- Implement the model in a simulator

1. System Variables and Boundaries

Mathematical Modelling and Simulation

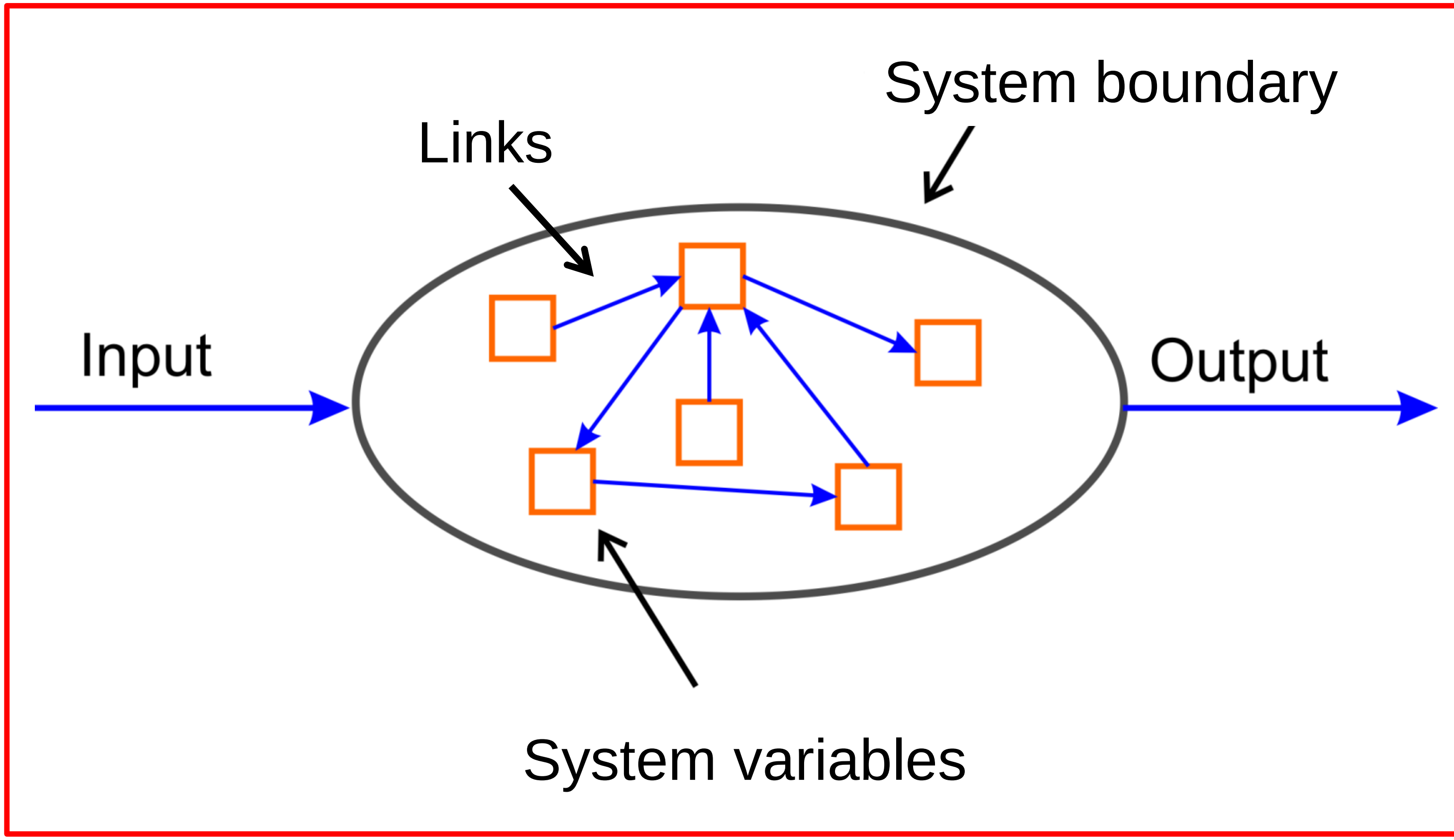
a. Analysis of the problem - Determining the purpose and the use of the model and defining a target for the simulation.

b. Start collecting information and data. Start developing hypothesis about the parts of the system.

c. Determine the elements of the system.

d. Determine causal relationships between the elements.

1. System Variables and Boundaries

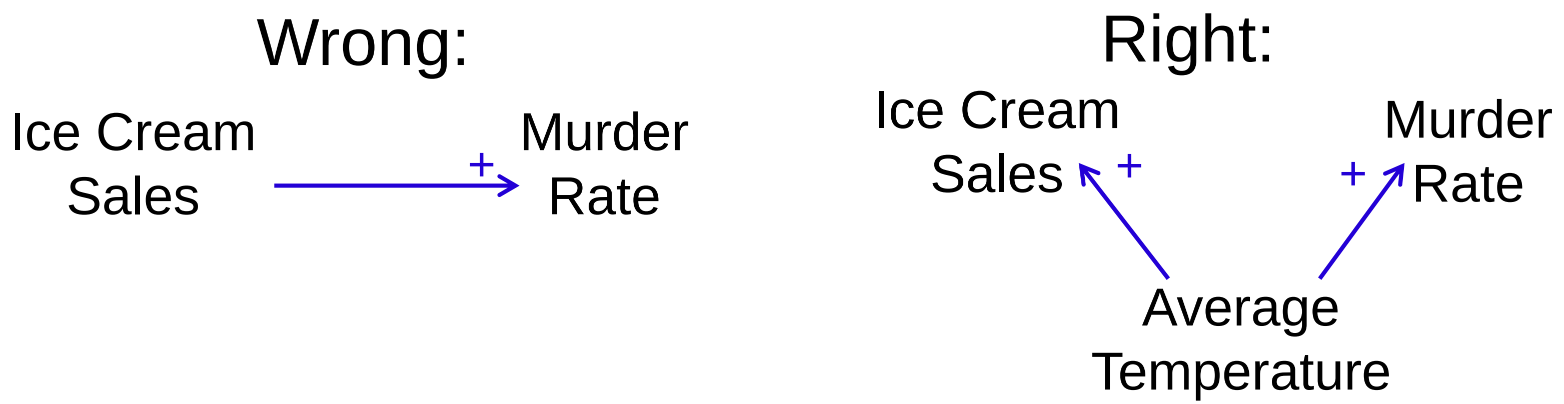


Causality vs Correlation

Mathematical Modelling and Simulation

Causation vs. Correlation

- ***Correlation*** represents past behavior and not the structure of the system
- ***Causation*** represents the causal links of the structure



Causality vs Correlation

TU WIEN Mathematical Modelling and Simulation

<u>Causation vs. Correlation</u>

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Correlation	Wrong Causal Implication	Lesson?
Smoking, Lung Cancer (+)	People suffering from lung cancer are more likely to start smoking	??

Causality vs Correlation

Mathematical Modelling and Simulation

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Darkness, Electricity Consumption (-)	If it was darker, we could reduce our energy problems	??

Causality vs Correlation

Mathematical Modelling and Simulation

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Darkness, Electricity Consumption (-)	If it was darker, we could reduce our energy problems	Always look for direct causalities! Don't foget that people sleep when its dark...
Murder Rate, Ice Cream Sales (+)	Ice cream makes people potential murderes	??

Causality vs Correlation

Mathematical Modelling and Simulation

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Darkness, Electricity Consumption (-)	If it was darker, we could reduce our energy problems	Always look for direct causalities! Don‘t foget that people sleep when its dark…
Murder Rate, Ice Cream Sales (+)	Ice cream makes people potential murderes	Always look for confounding factors! E.g. the average Temperature?

Causality vs Correlation

Mathematical Modelling and Simulation

Causation vs. Correlation

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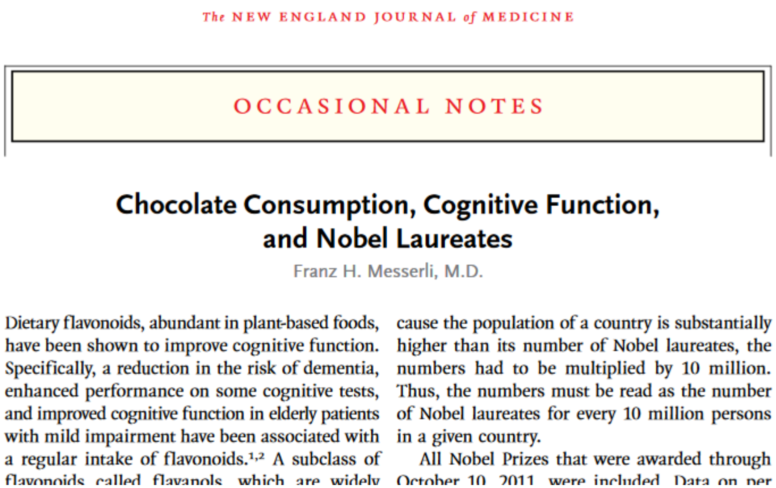
Famous example (1):



The NEW ENGLAND JOURNAL of MEDICINE

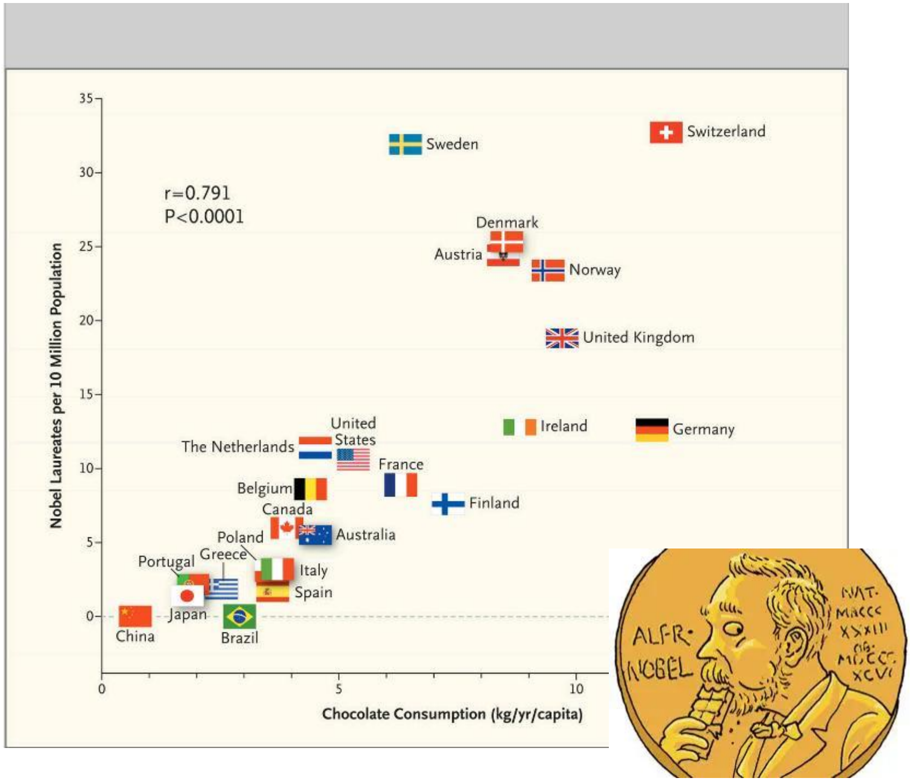
OCCASIONAL NOTES

Chocolate Consumption, Cognitive Function, and Nobel Laureates

Franz H. Messerli, M.D.

Dietary flavonoids, abundant in plant-based foods, have been shown to improve cognitive function. Specifically, a reduction in the risk of dementia, enhanced performance on some cognitive tests, and improved cognitive function in elderly patients with mild impairment have been associated with a regular intake of flavonoids.[1,2] A subclass of flavonoids called flavanols which are widely cause the population of a country is substantially higher than its number of Nobel laureates, the numbers had to be multiplied by 10 million. Thus, the numbers must be read as the number of Nobel laureates for every 10 million persons in a given country.

All Nobel Prizes that were awarded through October 10, 2011 were included. Data on per



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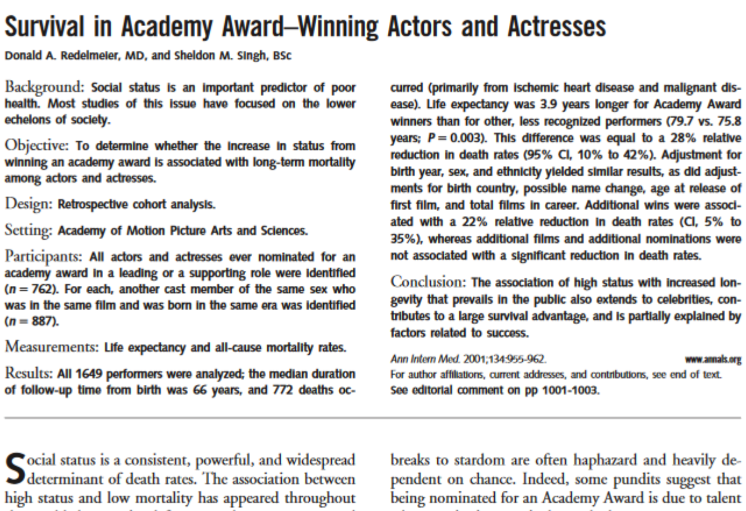
Famous example (2):



Survival in Academy Award–Winning Actors and Actresses

Donald A. Redelmeier, MD, and Sheldon M. Singh, BSc

Background: Social status is an important predictor of poor health. Most studies of this issue have focused on the lower echelons of society.

Objective: To determine whether the increase in status from winning an academy award is associated with long-term mortality among actors and actresses.

Design: Retrospective cohort analysis.

Setting: Academy of Motion Picture Arts and Sciences.

Participants: All actors and actresses ever nominated for an academy award in a leading or a supporting role were identified ($n = 762$). For each, another cast member of the same sex who was in the same film and was born in the same era was identified ($n = 887$).

Measurements: Life expectancy and all-cause mortality rates.

Results: All 1649 performers were analyzed; the median duration of follow-up time from birth was 66 years, and 772 deaths occurred (primarily from ischemic heart disease and malignant disease). Life expectancy was 3.9 years longer for Academy Award winners than for other, less recognized performers (79.7 vs. 75.8 years; $P = 0.003$). This difference was equal to a 28% relative reduction in death rates (95% CI, 10% to 42%). Adjustment for birth year, sex, and ethnicity yielded similar results, as did adjustments for birth country, possible name change, age at release of first film, and total films in career. Additional wins were associated with a 22% relative reduction in death rates (CI, 5% to 35%), whereas additional films and additional nominations were not associated with a significant reduction in death rates.

Conclusion: The association of high status with increased longevity that prevails in the public also extends to celebrities, contributes to a large survival advantage, and is partially explained by factors related to success.

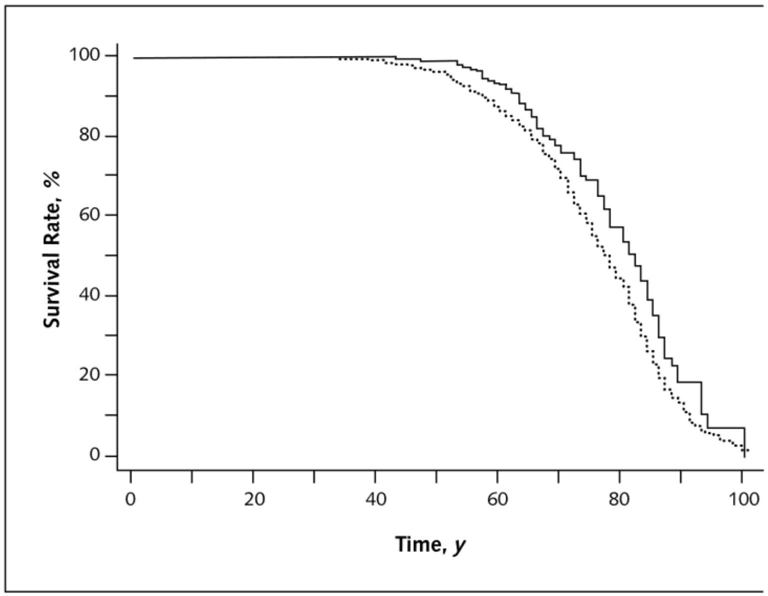
Ann Intern Med. 2001;134:955-962. www.annals.org

For author affiliations, current addresses, and contributions, see end of text.

See editorial comment on pp 1001-1003.

Social status is a consistent, powerful, and widespread determinant of death rates. The association between high status and low mortality has appeared throughout the world, has persisted for more than a century, and

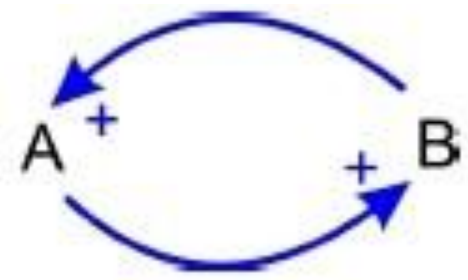
breaks to stardom are often haphazard and heavily dependent on chance. Indeed, some pundits suggest that being nominated for an Academy Award is due to talent whereas winning one is due to luck.



Analysis is based on log-rank test comparing 235 winners (99 deaths) with 887 controls (452 deaths). The total numbers of performers available for analysis were 1122 at 0 years, 1056 at 40 years, 762 at 60 years, and 240 at 80 years. $P = 0.003$ for winners vs. controls.

2. Causal Loop Diagram

Capture the **behavior** and **links** of and within the system by interlinking system variables that are related to each other



Behavior of system due to:

- Feedback Loops
- System memory (stocks)
- Delays in material and information delays

2. Causal Loop Diagram

<u>Main components of CLDs:</u>

- **System variables:** names of elements
- **Link - positive:**

Represented by a plus-sign

Increase in variable *Eating* results in an increase in variable *Weight*

<u>Main components of CLDs:</u>

- **Link – negative:**

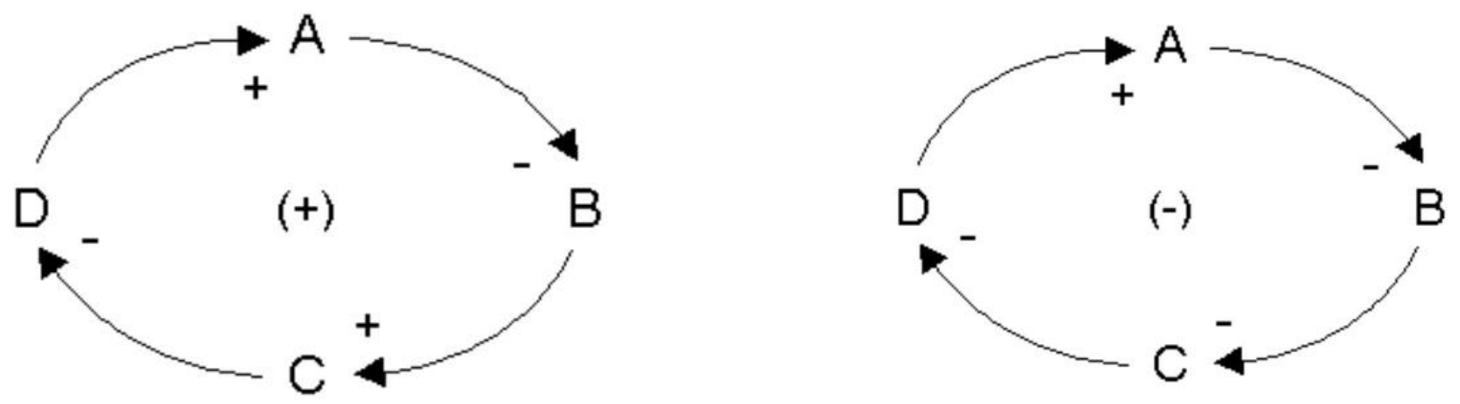
Represented by minus-sign.

Increase in variable *Diet* results in a decrease in variable *Weight*

2. Causal Loop Diagram

Main components of CLDs:

- **Feedback Loops:** are closed loops of arrows, represented by a:
"(+)" (or "(R)" for **reinforcing**) or
"(-)" (or "(B)" for **balancing**) sign in the middle.



2. Causal Loop Diagram

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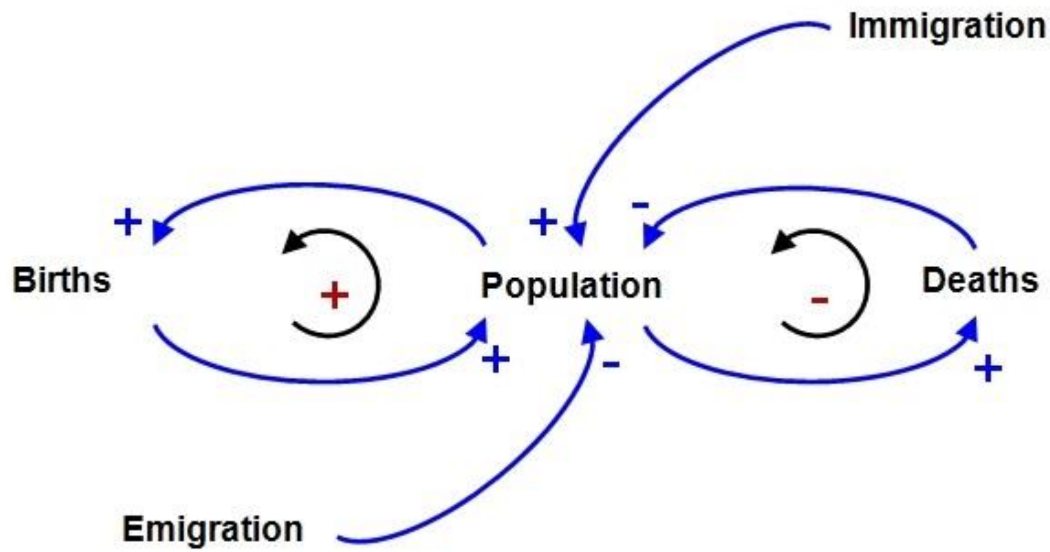

2. Causal Loop Diagram

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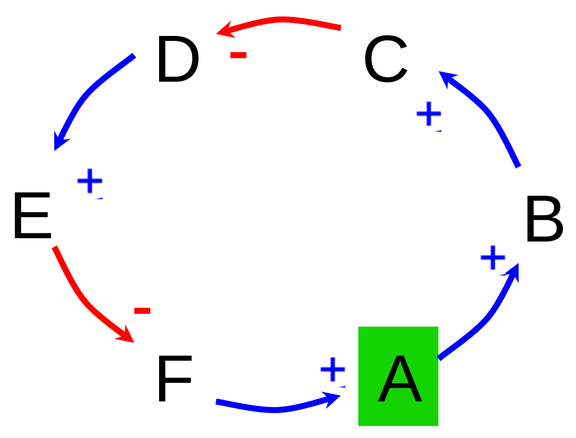
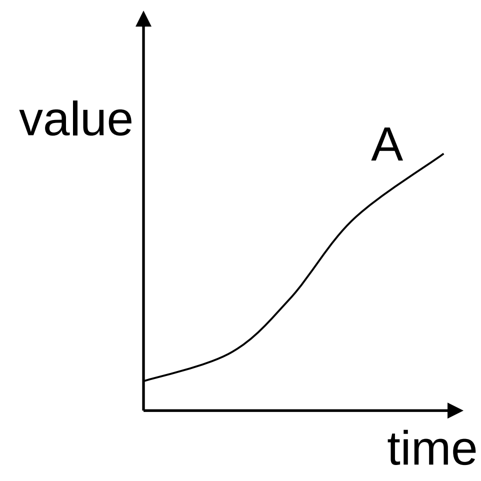
Feedback Loops:

– **Reinforcing**: A system variable effects itself (via other system variable(s) of the loop), *resulting in a reinforcing of the original state* of the system variable

Even number of negative links

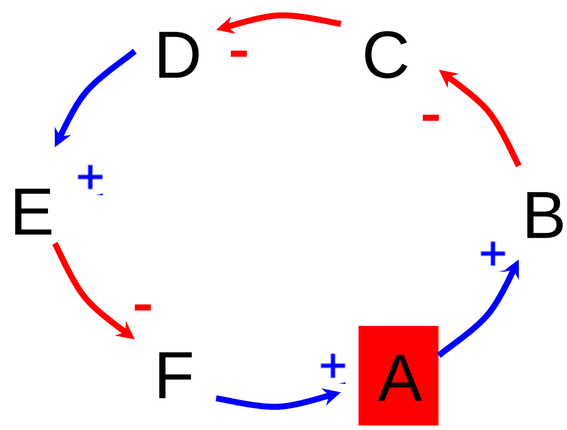


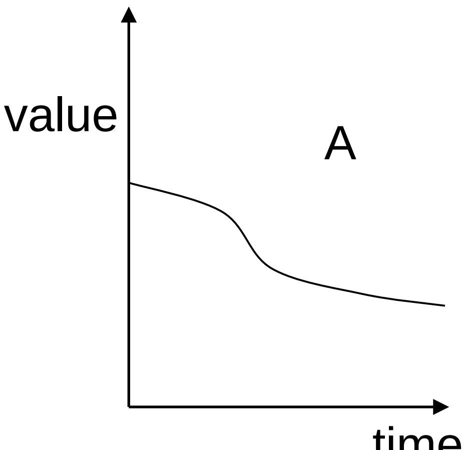


2. Causal Loop Diagram

__Feedback Loops:__

– **Balancing:** A system variable effects itself (via other system variable(s) of the loop), resulting in a balancing of the original state of the system variable
Uneven number of negative links





2. Causal Loop Diagram

Feedback Loops

- Search to identify closed, causal feedback loops is one key element of System Dynamics
- The most important causal influences will be exactly those that are enclosed within feedback loops

2. Causal Loop Diagram

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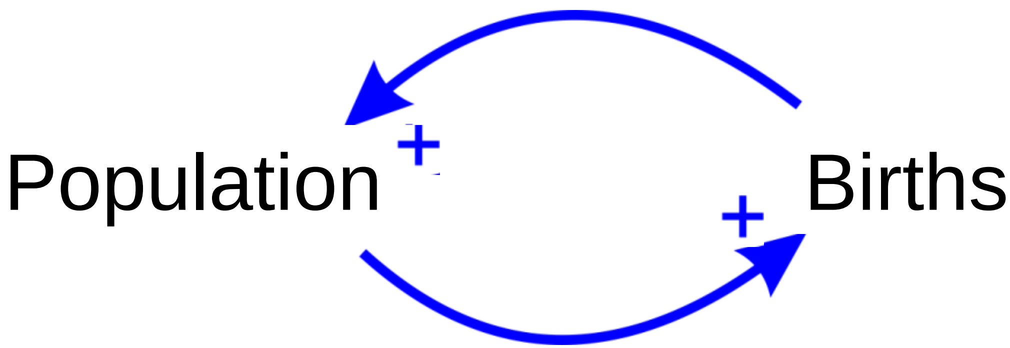
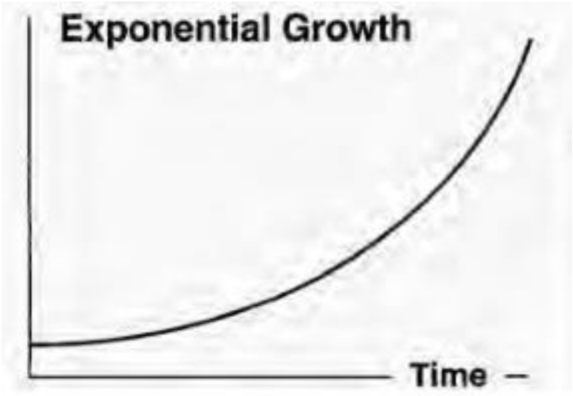
Types of behavior due to loops:

- **Exponential Growth**: arises from **positive (reinforcing) feedback** loop.

Example:





2. Causal Loop Diagram

Mathematical Modelling and Simulation

Types of behavior due to loops:

- **S-shaped Growth**: arises from a **combination of positive and negative feedback loops (nonlinear interactions)**

Important here:

– ***Carrying capacity***: *Number of organisms a habitat can support and it is determined by the resources available in the environment and the resource requirements of the population. When the population reaches its carrying capacity the net increase rate slows down until it is zero and the population reaches its equilibrium (limit of growth)*

2. Causal Loop Diagram

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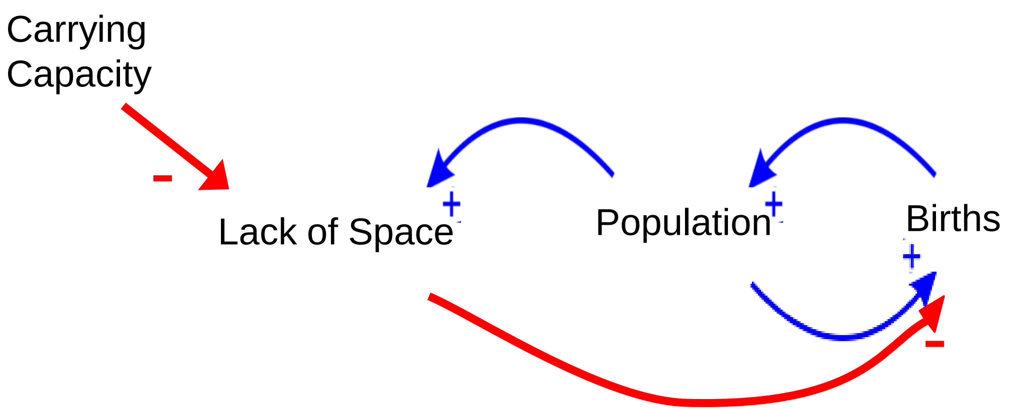
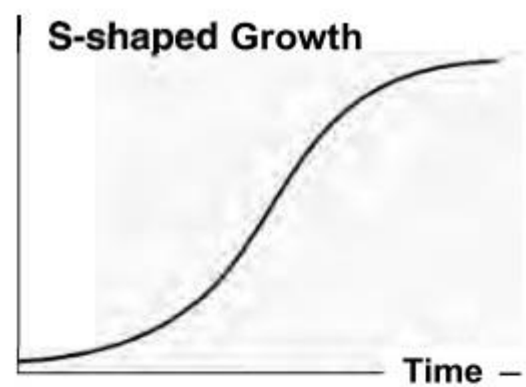




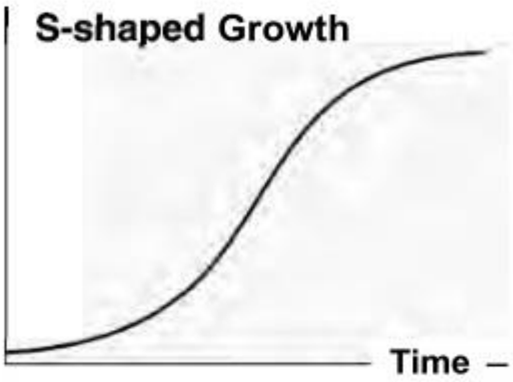
Types of behavior due to loops:

- **S-shaped Growth**: arises from a **combination of positive and negative feedback loops (nonlinear interactions)**

Necessary requirements:

– Negative feedback loops must no include any significant delays

– Carrying capacity must be fixed



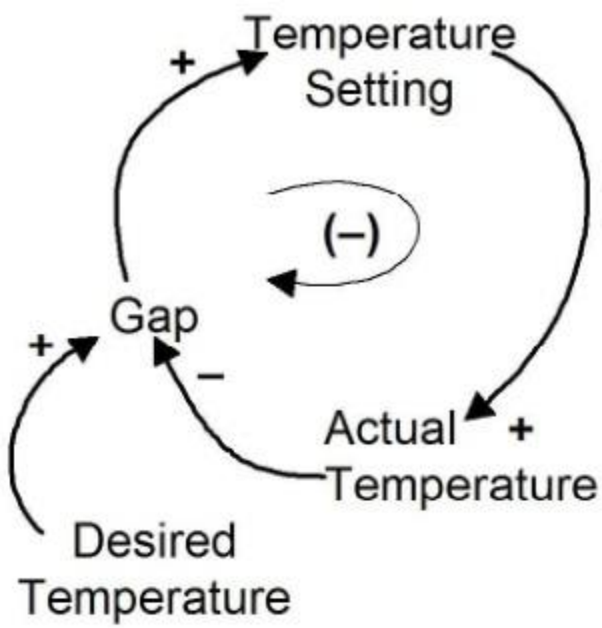
2. Causal Loop Diagram

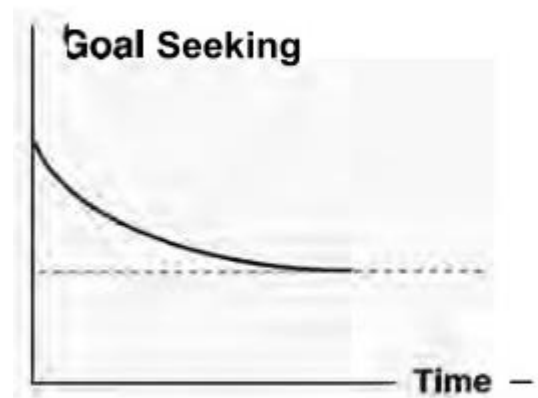
<u>Types of behavior due to loops:</u>

- **Goal Seeking Behavior:** arises from **negative (balancing) feedback** loop.

Example:

Corrective action





2. Causal Loop Diagram

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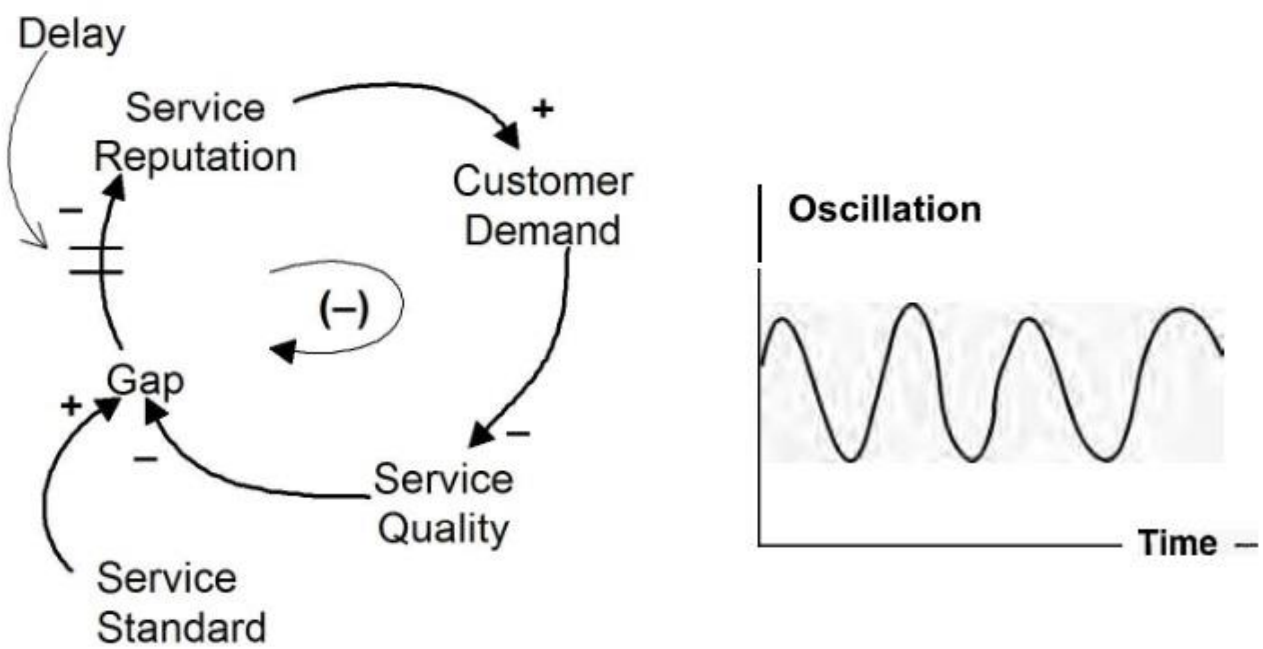
Types of behavior due to loops:

- **Oscillation**: arises from **negative feedback with delays.**

Example:



2. Causal Loop Diagram

Types of behavior due to loops:

- **Oscillation**: arises from **negative feedback with delays.**

The state of the system is compared to the desired state of the system and corrective actions are taken. The goal is constantly overshot, then corrects / reverses and then undershoots the system and so on.

2. Causal Loop Diagram

Mathematical Modelling and Simulation

Types of behavior due to loops:

- **Oscillation**: arises from **negative feedback with delays.**

Special oscillations are:

- **Damped oscillation**: e.g. pendulum
- Chaotic oscillations

Types of behavior due to loops:

- **Oscillation**: arises from **negative feedback with delays.**

Special oscillations are:

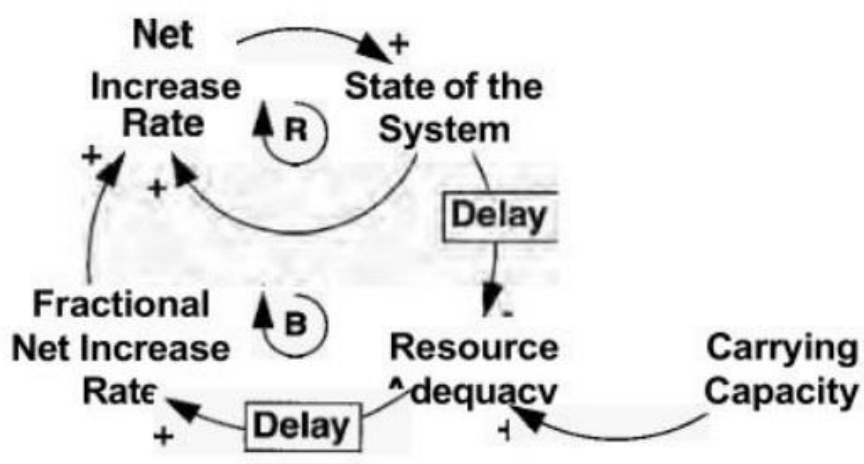
- **Expanding oscillation and limit cycles**: If an oscillatory system is given a nudge off its equilibrium, its swings grow larger and larger until they are constrained by various nonlinearities this oscillation is called limit cycles. Predator prey populations are cycles.

2. Causal Loop Diagram

<u>Types of behavior due to loops:</u>

- **Growth with overshoot and oscillation**: is basically s-shaped growth with additional delay in the negative feedback loop.

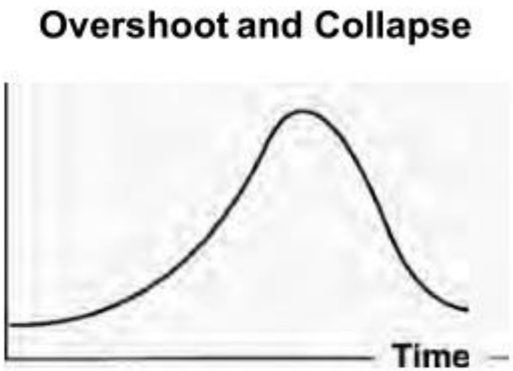
Example:



Types of behavior due to loops:

- **Overshoot and collapse**: is basically s-shaped growth but with a not fixed carrying capacity

Example: A population in a forest that grows so large, that they overbrowse the vegetation, leading to starvation and a decline in the population. If there is no regeneration of the carrying capacity, the equilibrium of the system is extinction.



Dominating Loop

- There are systems which have more than one feedback loop within them
- A particular loop in a system of more than one loop is most responsible for the overall behavior of that system
- The dominating loop might shift over time
- When a feedback loop is within another, one loop must dominate
- Stable conditions will exist when negative loops dominate positive loops

2. Causal Loop Diagram

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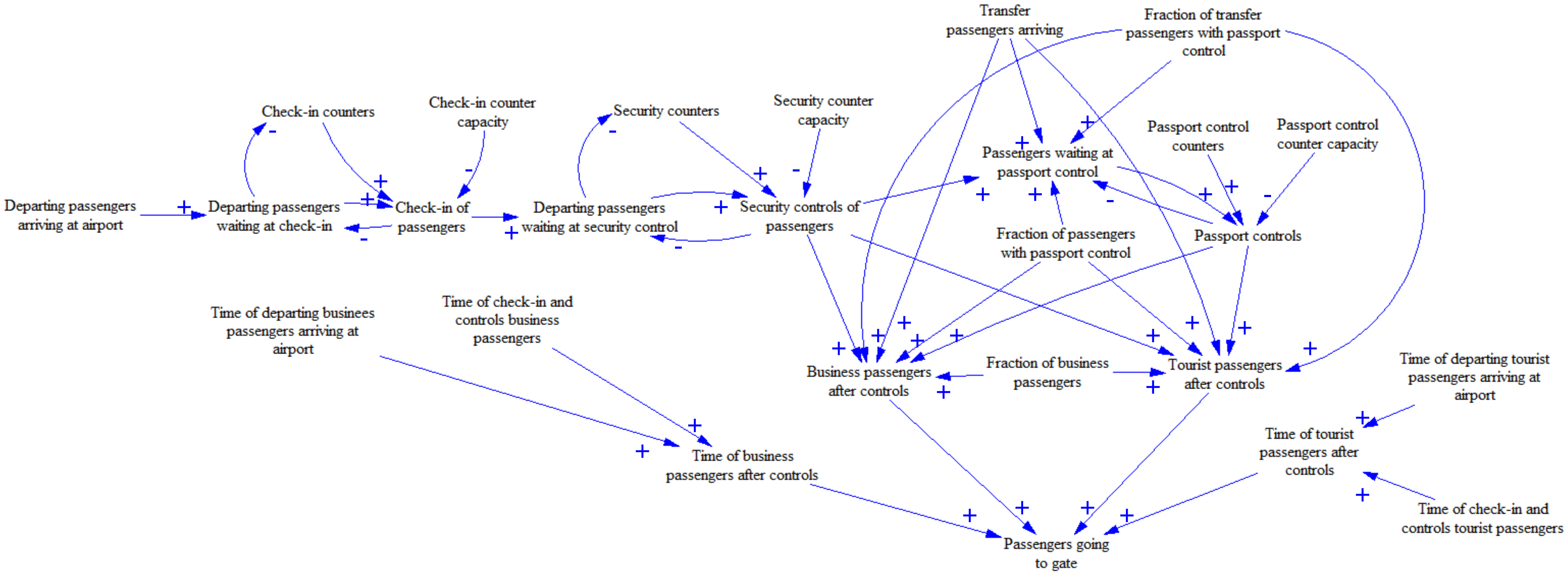
Example:



3. Stock and Flow Diagram

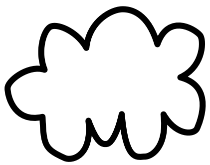
Mathematical Modelling and Simulation

Problem: Not all system elements are system variables!

Solution: distinguish between

- Sources/Sinks
- Levels/Stocks
- Flows
- Auxiliaries
- Paramters
- Links

Sources/Sinks:



Source represents systems of levels and rates outside the boundary of the model

Sink is where flows terminate outside the system

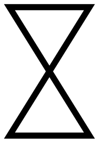
E.g.: Raw Material (Source for „Construction“ Flow), Graveyard (Sink for „Dying“ Flow)

Levels/Stocks/System variables:

A quantity that accumulates over time and changes its value continuously.

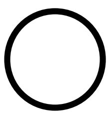
E.g.: Size of a population, Number of people waiting in a queue, Number of goods waiting to be transported, etc.

Flow/Rate/Activity/Movement:



Changes the values of levels. Every level has at least to be connected to one flow in order to change its value.

E.g.: Birth (Changes the value of the stock „population“), Eating (Changes the value of the stock „amount of food“), etc.



Auxiliary:

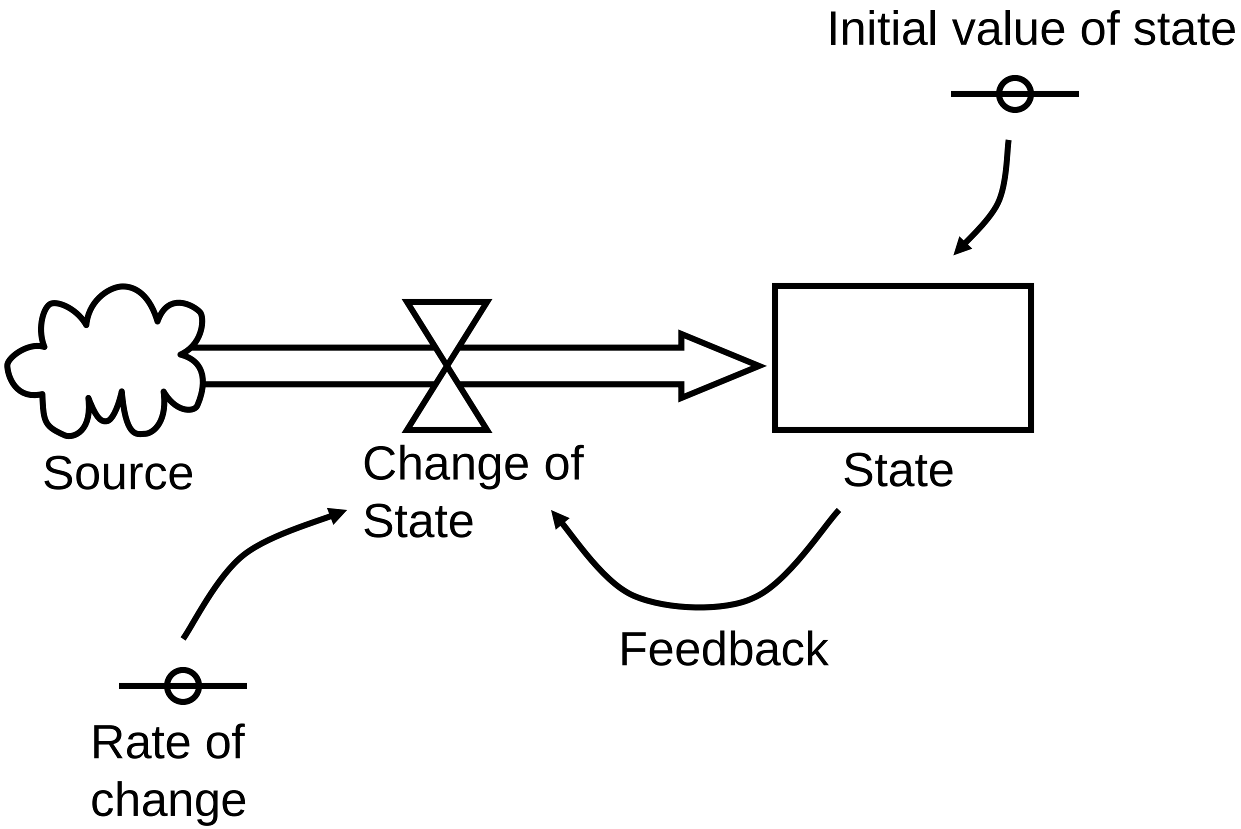
Everything that can directly/analytically be calculated out of stocks and constants. Often useful, to avoid confusing models.

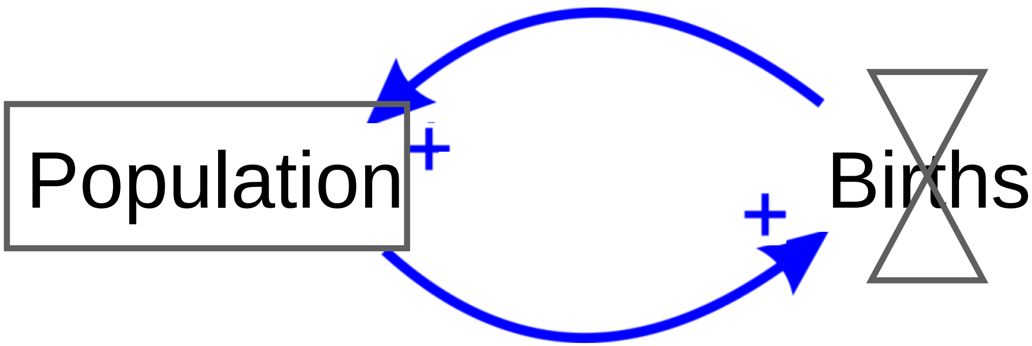
E.g.: Density (can directly be calculated by the stocks/constants „mass“ and „volume“), Quelength (calculated by stock „people in queue“ and constant „average size of one person“), etc.

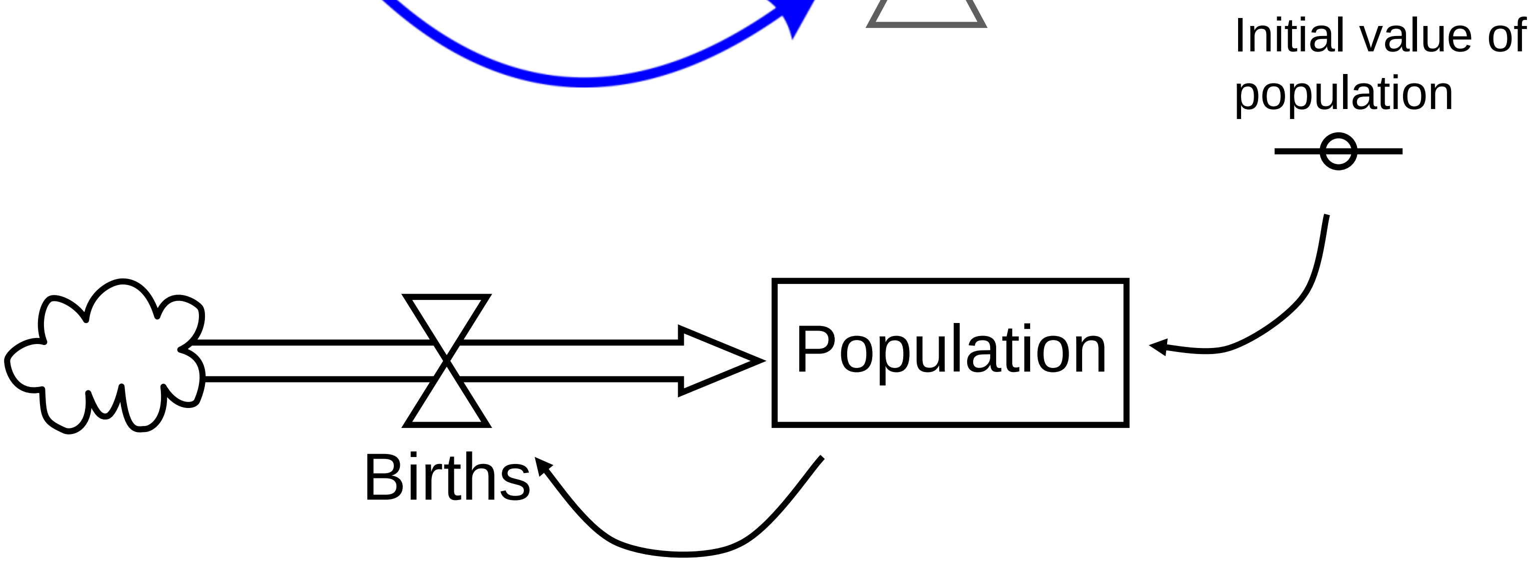
Parameter /Constant

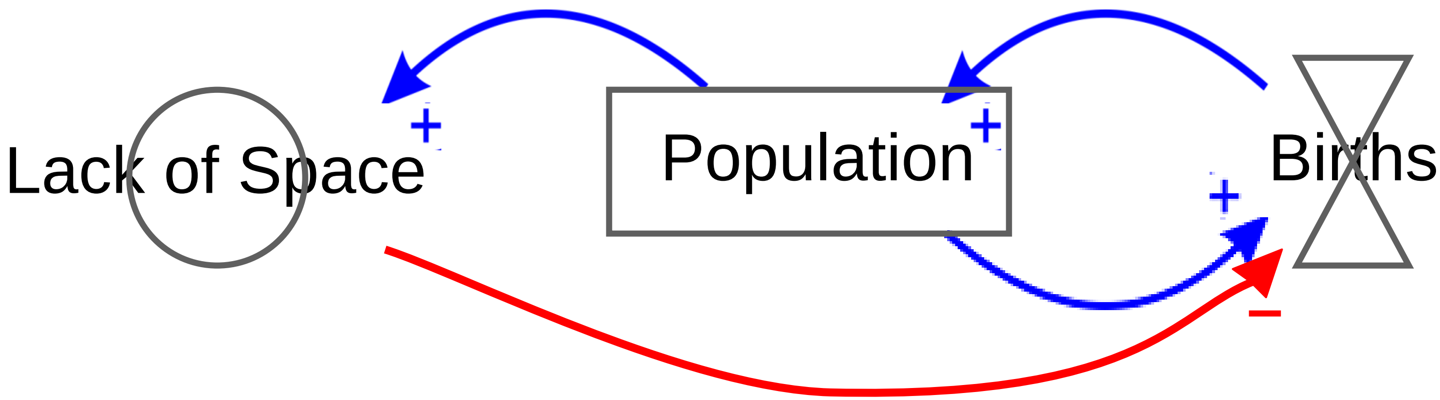
Everything that is predefined for the whole simulation – usually it is a constant but can be a function too.

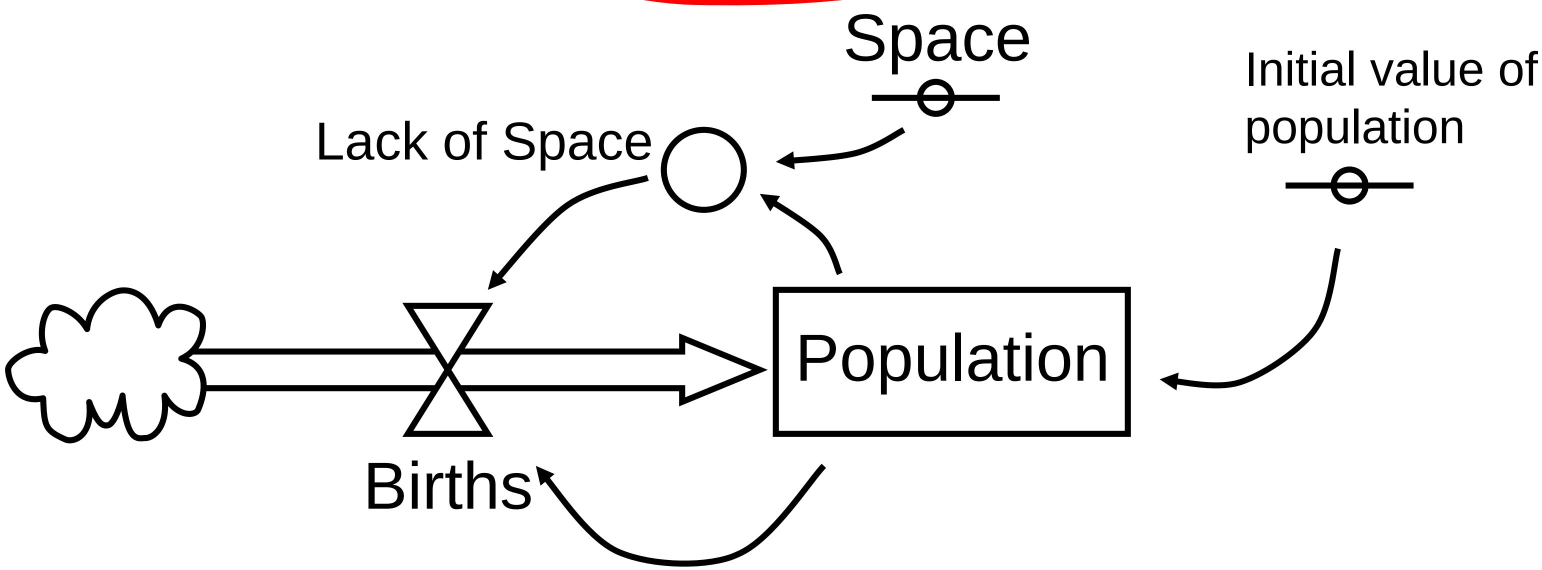
E.g.: Average Temperature, Number of Cash Desks (In a supermarket), Birth Rate, Maximum capacity of a Room, etc.


Initial value of state
Source
Change of State
State
Feedback
Rate of change


Population
Births
+
+


Initial value of
population
Population
Births


Lack of Space
+
Population
+
Births
+
−


Space
Initial value of population
Lack of Space
Population
Births

Quantification?

$$lack\_of\_Space = Space - Population?$$
$$lack_{of_{Space}} = Space - 3 * Population?$$
$$lack\_of\_Space = \frac{Space}{Population}?$$
$$lack\_of\_Space = \frac{Population}{Space}?$$

$$Births = 3 * Population + lack\_of\_Space\ ?$$
$$Births = 10 * Population - lack\_of\_Space?$$
$$Births = 0.2 * Population + \frac{1}{lack\_of\_Space}?$$

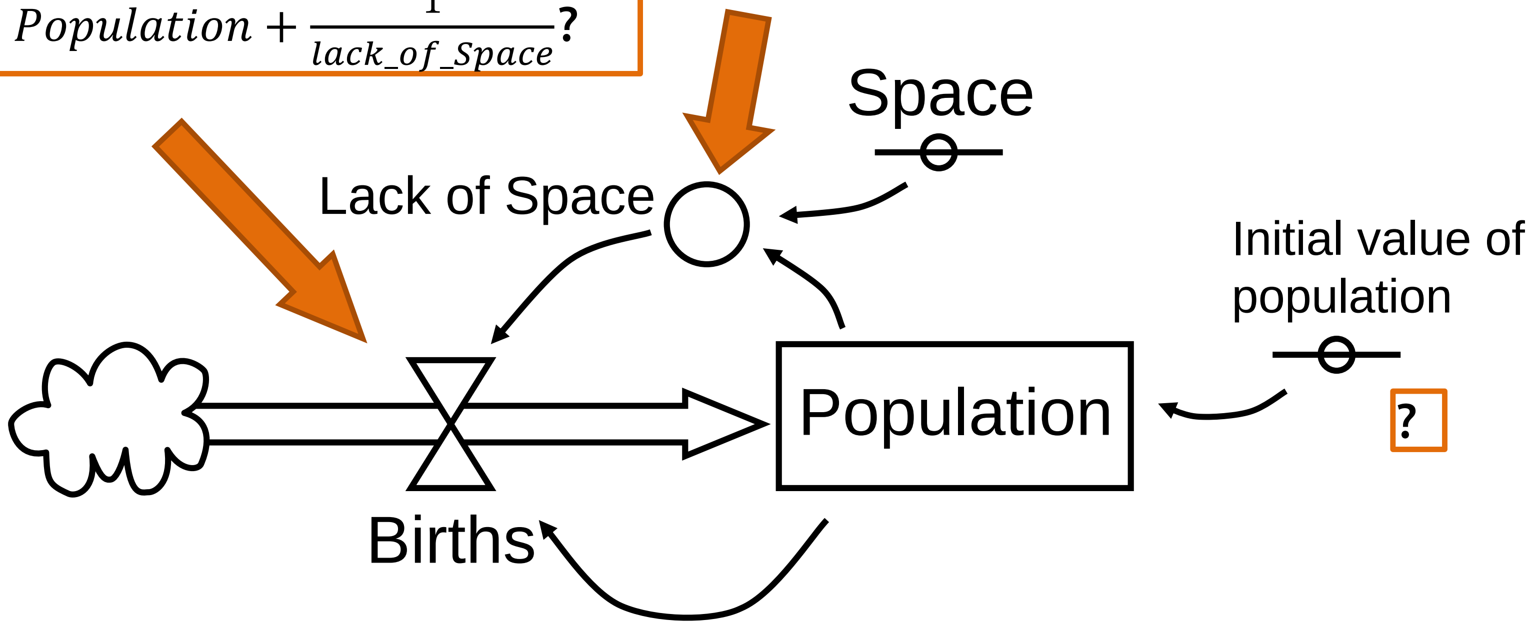


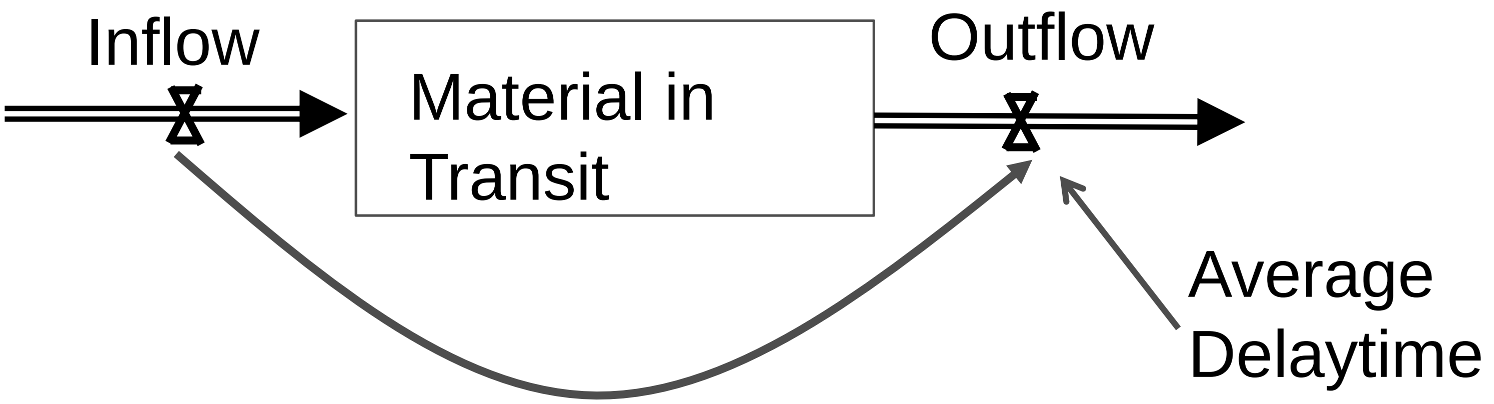
Table Function

- Responsible for nonlinar relationships
- Uses pairs of numbers
  - Interpolation inbetween:
    linear, step, spline, approximation
  - Out of range:
    error, repeat, extrapolate

Delays

The Value of the input will be time-delayed for the delay time:

Output = Material in Transit / Delaytime



Analysis of SD Models

- **Analytical**: Evaluation of equilibrium, behaviour and stability in an area (ordinary differential equations)
  But: For large systems this can be difficult and not useful for time variant values

- **Base Run**:
  The Model runs with the predefined set of parameters (which represent the best information available at this time).

Simple Structure

Stock and Flow with two flows

Differential Equation:

$$\dot{Stock}(t) = Inflow(t) - Outflow(t)$$

Integral equation*:*

$$Stock = \int_{t0}^{t} \left(Inflow(s) - Outflow(s)\right) ds + stock(t0)$$

Equilibrium

Static Equilibrium:
Inflow and *Outflow* are 0;
State of the system remains unchanged.

Dynamic Equilibrium:
Inflow and *Outflow* are the same;
State of the system remains unchanged

Analysis of SD Models

Mathematical Modelling and Simulation

- **Optimization / Calibration**:
  With specific algorithms some – unknown – parameter values can be calculated by macthing a objective function.
- **Parametervariation / Sensitivity Analysis**:
  Multiple simultion runs are simulated with different sets of parameter values, which are gained from
  - even distributed intervals or
  - stocastically from a probability function

Simulators

Mathematical Modelling and Simulation

- SD-simulators at least offer the most important elements (Flows, Levels, Auxiliaries, Table-functions, etc.) to be preimplemented.
- Additionally parametervariation and optimization is possible with most SD simulators.
- Examples: AnyLogic (does not only support SD), Vensim, Stella, PowerSim...

Conclusion

Mathematical Modelling and Simulation

- System-Dynamics is a top-down modelling approach. Its graphical representation is broadly standartized.
- Important Elements: causal relationships, causal loops, stock and flow diagrams
- It is equivalent to a DE model. Thus results can be analysed using the same methods.
- Simulators: AnyLogic, Vensim, Stella, PowerSim...

Thank you for your attention!

Questions?