(5) **Exponential family**
Show that a Poisson family of distributions $\mathcal{P}oi(\lambda)$, with unknown $\lambda > 0$ belongs to the exponential family.

$$h(x) := \begin{cases} \frac{1}{x!}, & \text{if } x \in \mathbb{N}_0 \\ 0, & \text{else} \end{cases} \qquad t_1(x) := x; \quad t_2(x) := -1$$

$$w_1(\lambda) := \log(\lambda); \quad w_2(\lambda) := \lambda$$

$$c(\lambda) := 1$$

$$\forall x \in \mathbb{N}_0:$$

$$h(x)\, c(\lambda)\, e^{(w_1(\lambda) t_1(x) + w_2(\lambda) t_2(x)} = \frac{1}{x!} e^{(x \log(\lambda) - \lambda)} = \frac{\lambda^x}{x!} e^{-\lambda} = f(x \mid \lambda)$$