Tricomi Gleichung

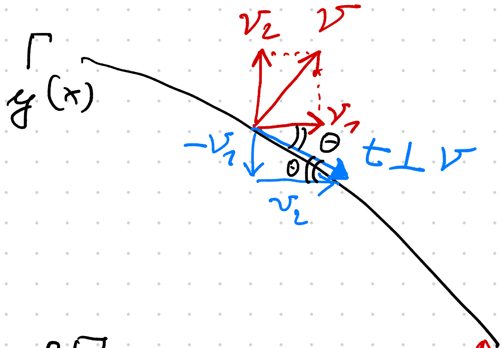
$$\underset{c=1}{u_{yy}} \underbrace{-\,y}_{=a}\, u_{xx} = 0 \quad ; \; x, y \in \mathbb{R} \qquad b = 0$$

[vgl. $a u_{xx} + 2b u_{xy} + c u_{yy} = f$]

$y > 0$: hyp., $\quad y < 0$: ell.

ges: Berechne alle char. Kurven im hyp. Bereich.

char. Richtungen



Cauchydaten: $u, \dfrac{\partial u}{\partial v}$ geg.

Normalrichtungen auf Γ:

$$a v_1^2 + 2b v_1 v_2 + c v_2^2 = 0 \;\Big|\; : v_2^2$$

... quadr. Gl. für $\dfrac{v_1}{v_2}$

$$\frac{v_1}{v_2} = \frac{-b \pm \sqrt{b^2 - ac}}{a} = \frac{\pm\sqrt{y}}{-y} = \mp \frac{1}{\sqrt{y}}$$

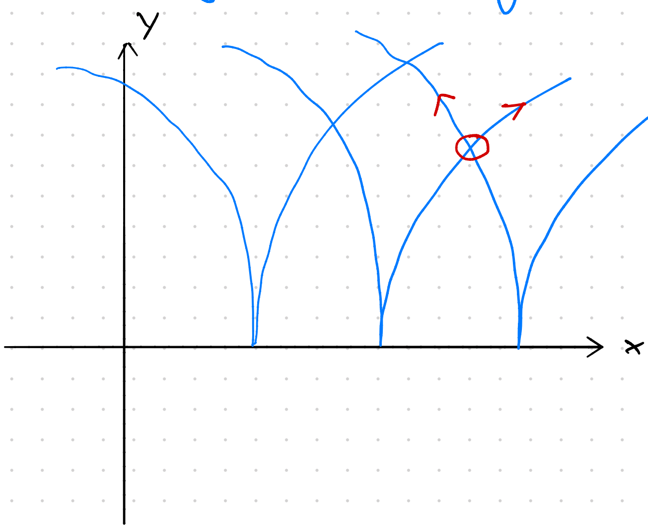
$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ ... orth. Richtung an Γ

$\begin{pmatrix} v_2 \\ -v_1 \end{pmatrix}$ ... tangent. Richtung an Γ

$$\Rightarrow \tan\theta = -\frac{v_1}{v_2} = \boxed{\frac{dy}{dx} = \pm\frac{1}{\sqrt{y}}}$$

$$\pm\sqrt{y}\,dy = dx$$

$$C \pm \frac{2}{3} y^{\frac{3}{2}} = x \quad , \quad y \geq 0$$



Neilsche Parabeln

$\psi : \mathbb{R} \to \mathbb{R}$, $\psi(x) =: \mathbb{1}_{(0,1)}(x)$; $\psi_\varepsilon(x) := \frac{1}{\varepsilon}\psi\left(\frac{x}{\varepsilon}\right)$

a) Frage: $\{\psi_\varepsilon^2\}_{\substack{\varepsilon>0 \\ \varepsilon\to 0}}$ konvergent in $\mathcal{D}'(\mathbb{R})$?

$\forall \varphi \in \mathcal{D}(\mathbb{R}) : \langle \psi_\varepsilon^2, \varphi \rangle = \int_{\mathbb{R}} \psi_\varepsilon^2 \varphi \, dx = \frac{1}{\varepsilon^2} \int_0^\varepsilon \varphi(x)\, dx =$

$$\overset{MWS}{=} \frac{1}{\varepsilon^2} \varepsilon \varphi(\underset{\in[0,\varepsilon]}{x_\varepsilon}) = \frac{1}{\varepsilon} \underbrace{\varphi(x_\varepsilon)}_{\to \varphi(0)} \Rightarrow \text{div.!}$$

b) Frage: $\{\psi_\varepsilon^2 - \frac{1}{\varepsilon}\delta_0\}_{\substack{\varepsilon>0 \\ \varepsilon\to 0}}$ konvergent in $\mathcal{D}'(\mathbb{R})$?

$\forall \varphi \in \mathcal{D}(\mathbb{R}) : \int_{\mathbb{R}} \psi_\varepsilon^2 \varphi \, dx - \frac{1}{\varepsilon}\varphi(0) = \frac{1}{\varepsilon^2}\int_0^\varepsilon \varphi(x)\,dx - \frac{1}{\varepsilon}\varphi(0) =$

$$= \underbrace{\frac{1}{\varepsilon^2}\int_0^\varepsilon [\varphi(x) - \varphi(0)]\,dx}_{} = [\text{l'Hôpital}]$$

$$\lim_{\varepsilon\to 0}\left[\frac{1}{\varepsilon^2}\int_0^\varepsilon [\varphi(x) - \varphi(0)]\,dx\right] = \lim_{\varepsilon \to 0} \frac{\varphi(\varepsilon) - \varphi(0)}{2\varepsilon} \overset{\text{l'Hôpital}}{=}$$

$$= \lim_{\varepsilon\to 0} \frac{\varphi'(\varepsilon)}{2} = \frac{\varphi'(0)}{2} \Rightarrow \text{konvergent}$$

$$\frac{d}{d\varepsilon}\int_{a(\varepsilon)}^{b(\varepsilon)} f(x,\varepsilon)\,dx = b'(\varepsilon) f(b(\varepsilon)) - a'(\varepsilon) f(a(\varepsilon)) + \int_{a(\varepsilon)}^{b(\varepsilon)} \frac{\partial f}{\partial \varepsilon}(x,\varepsilon)\,dx$$