

These problems are to be presented on **June 22, 2021**. They should be ticked and solution paths uploaded by **23:59 on June 21, 2021**.

(1) **Comparing Two Populations 1**

A study of the differences in cognitive function between normal individuals and patients diagnosed with schizophrenia was published in the American Journal of Psychiatry (Apr. 2010). The total time (in minutes) a subject spent on the Trail Making Test (a standard psychological test) was used as a measure of cognitive function. The researchers theorize that the mean time on the Trail Making Test for schizophrenics will be larger than the corresponding mean for normal subjects. The data for independent random samples of 41 schizophrenics and 49 normal individuals yielded the following results:

	Schizophrenia	Normal
Sample size	41	49
Mean time	104.23	62.24
Standard deviation	62.24	16.34

- (a) Define the parameter of interest to the researchers.
- (b) Set up the null and alternative hypothesis for testing the researchers' theory.
- (c) The researchers conducted the test, part (b), and reported a p -value of .001. What conclusions can you draw from this result? (Use $\alpha = 0.01$)
- (d) Find a 99% confidence interval for the target parameter. Interpret the result. Does your conclusion agree with that of the previous part?

(2) **Comparing Two Populations 2**

Suppose you wish to compare a new method of teaching reading to slow learners with the current standard method. You decide to base your comparison on the results of a reading test given at the end of a learning period of six months. Of a random sample of 22 slow learners, 10 are taught by the new method and 12 are taught by the standard method. All 22 children are taught by qualified instructors under similar conditions for the designated six-month period. The results of the reading test at the end of this period are given below.

New Method: 80, 76, 70, 80, 66, 85, 79, 71, 81, 76.

Standard Method: 79, 73, 72, 62, 76, 68, 70, 86, 75, 68, 73, 66.

- (a) Use the data in the table to estimate the true mean difference between the test scores for the new method and the standard method. Use a 95% confidence interval.
- (b) Interpret the interval you found in the previous part.
- (c) What assumptions must be made in order that the estimate be valid? Are they reasonably satisfied?

(3) **Missing Information**

An investigation of ethnic differences in reports of pain perception was presented at the annual meeting of the American Psychosomatic Society (Mar. 2001). A sample of 55 blacks and 159 whites participated in the study. Subjects rated (on a 13-point scale) the intensity and unpleasantness of pain felt when a bag of ice was placed on their foreheads for two minutes. (Higher ratings correspond to higher pain intensity.) A summary of the results is provided in the following table.

	Blacks	Whites
Sample Size	55	159
Mean pain intensity	8.2	6.9

- (a) Why is it dangerous to draw a statistical inference from the summarized data? Explain.
- (b) What values of the missing sample standard deviations would lead you to conclude (at $\alpha = 0.05$) that blacks, on average, have a higher pain intensity rating than whites?
- (c) What values of the missing sample standard deviations would lead you to an inconclusive decision (at $\alpha = 0.05$) regarding whether blacks or whites have a higher mean intensity rating?

(4) **χ^2 -test for independence**

100 students from major mathematics of three Viennese universities were randomly chosen and asked which lecture, either a: calculus, b: algebra, or c: probability, they enjoyed most. The frequencies are given in the following table:

	Uni A	Uni B	Uni C
calculus	10	5	5
algebra	10	20	10
probability	20	5	

Perform a χ^2 -test to test whether the preference for a lecture is independent from the university, on a 5% significance level.

- (a) Only use the following table which gives the 95%-quantile q of the χ^2 -distribution with df degrees of freedom.

df	1	2	3	4	5	6	7	8	9
q	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92

- (b) Solve the previous exercise using R.

(5) **Hypnosis**

Some researchers claim that susceptibility to hypnosis can be acquired or improved through training. To investigate this claim six subjects were rated on a scale of 1-20 according to their initial susceptibility to hypnosis and then given 4 weeks of training. Each subject was rated again after the training period. In the ratings below, higher numbers represent greater susceptibility to hypnosis.

Subject	Before	After
1	10	18
2	16	19
3	7	11
4	4	3
5	7	5
6	2	3

Specify and perform the appropriate hypothesis test. What potential issues exist with this analysis?