(5) Cumulative distribution function

Let a cumulative distribution function (cdf) F of a continuous random variable Y be given by

$$F(y) = \begin{cases} 0, & y \leq 0 \\ \frac{2}{5}y, & 0 < y \leq 1 \\ ay - b, & 1 < y \leq 2 \\ 1, & y > 2 \end{cases},$$
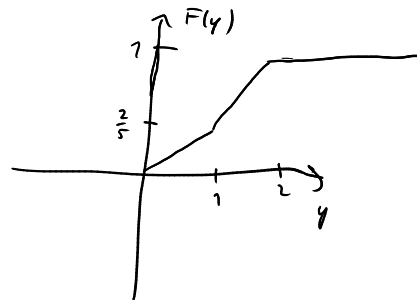
where a and b are real constants.

(a) Find out the values of a and b.

(b) Write down the probability density function (pdf) of Y.

(c) What is the probability that an observed random variable Y is greater than 1.8, given that it is greater than 1?

a) $\quad \frac{2}{5} = F(1) \overset{!}{=} \lim\limits_{y \to 1+} F(y) = \lim\limits_{y \to 1+} ay - b = a - b \;\; \Rightarrow \;\; a = b + \frac{2}{5}$

$2a - b = F(2) \overset{!}{=} \lim\limits_{y \to 2+} F(y) = \lim\limits_{y \to 2+} 1 = 1$

$\Rightarrow \; 2\left(b + \frac{2}{5}\right) - b = 1 \Leftrightarrow \; b + \frac{4}{5} = 1 \Leftrightarrow b = \frac{1}{5} \;\; \Rightarrow \;\; a = \frac{3}{5}$

b)
$$f(y) = \begin{cases} 0, & \text{if } y \leq 0 \vee y > 2 \\ \frac{2}{5}, & \text{if } 0 < y \leq 1 \\ \frac{3}{5}, & \text{if } 1 < y \leq 2 \end{cases}$$

c) $\mathbb{P}\left(Y > \frac{18}{10} \mid Y > 1\right) = \dfrac{\mathbb{P}\left(Y > \frac{18}{10} \wedge Y > 1\right)}{\mathbb{P}(Y > 1)} = \dfrac{1 - F\left(\frac{18}{10}\right)}{1 - F(1)} = \dfrac{1 - \frac{3}{5}\frac{18}{10} + \frac{1}{5}}{1 - \frac{2}{5}}$

$= \dfrac{1 - \frac{27}{25} + \frac{1}{5}}{\frac{3}{5}} = \dfrac{5 - \frac{27}{5} + 1}{3} = \dfrac{\frac{30 - 27}{5}}{3}$

$= \dfrac{\frac{3}{5}}{3} = \dfrac{1}{5}$