## (1) Card game

A deck of 52 cards has 13 ranks ($2, 3, 4, 5, 6, 7, 8, 9, 10,$ J, Q, K, A) and 4 suits ($\heartsuit, \spadesuit, \diamondsuit, \clubsuit$).
Three cards are drawn randomly without replacement from a deck of 52 cards.

(a) What is the probability that all three cards are in the same suit?

(b) What is the probability that all three cards have the same rank?

(c) What is the probability that the three cards contain exactly one pair (a pair means two cards with the same rank from two different suits)?

a) $\mathbb{P}(\text{all three cards are in the same suit}) = 4 \cdot \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = 4 \cdot \frac{1}{4} \cdot \frac{4}{17} \cdot \frac{11}{50} = \frac{2}{17} \cdot \frac{11}{25}$

b) $\mathbb{P}(\text{all three cards have the same rank}) = 13 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = 13 \cdot \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25} = \frac{1}{17} \cdot \frac{1}{25}$

c) $\mathbb{P}(\text{the three cards contain exactly one pair}) = \frac{48}{50} \cdot \frac{3}{51} + 2 \cdot \frac{48}{51} \cdot \frac{3}{50} = 3 \cdot \frac{1}{17} \cdot \frac{24}{25} = \frac{72}{425}$



draw first card

$\frac{3}{51}$     $\frac{48}{51}$

second card forms pair with first     second card does not form pair with first

$\frac{2}{50}$   $\frac{48}{50}$     $\frac{3}{50}$   $\frac{3}{50}$

third card forms pair with second   first card does not form pair with second   third card forms pair with first   third card forms pair with second