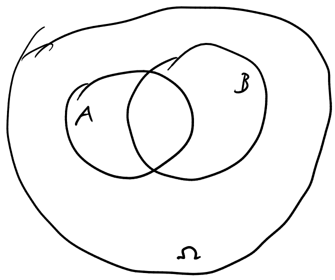
(3) **Independence**
Let A and B be two independent events.

(a) Prove that A^c and B^c are also independent.
(b) If we additionally know that $P(A|B) = 0.6$ and $P(B|A) = 0.3$, compute the probabilities of the following two events
 (i) at most one of A or B
 (ii) either A or B but not both.



a) $$\mathbb{P}(A^c \cap B^c) = \mathbb{P}((A \cup B)^c) = \mathbb{P}(\Omega \setminus (A \cup B)) = \mathbb{P}(\Omega) - \mathbb{P}(A \cup B)$$
$$= 1 - \mathbb{P}(A) - \mathbb{P}(B) + \mathbb{P}(A \cap B) = 1 - \mathbb{P}(A) - \mathbb{P}(B) + \mathbb{P}(A)\mathbb{P}(B)$$
$$= 1 - \mathbb{P}(B) + \mathbb{P}(A)(\mathbb{P}(B) - 1) = (1 - \mathbb{P}(B))(1 - \mathbb{P}(A)) = \mathbb{P}(A^c)\,\mathbb{P}(B^c)$$

b) (i) $$\mathbb{P}(\text{at most one of } A \text{ or } B) = 1 - \mathbb{P}(A \cap B) = 1 - \mathbb{P}(A)\mathbb{P}(B) = 1 - \mathbb{P}(A|B)\,\mathbb{P}(B|A) = 1 - \frac{6}{10}\frac{3}{10} = 1 - \frac{18}{100}$$
$$= 1 - \frac{9}{50} = \frac{41}{50}$$

(ii) $$\mathbb{P}(\text{either } A \text{ or } B \text{ but not both}) = \mathbb{P}(A) + \mathbb{P}(B) - 2\,\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - 2\,\mathbb{P}(A)\mathbb{P}(B)$$
$$= \frac{3}{10} + \frac{6}{10} - 2\,\frac{3}{10}\frac{6}{10} = \frac{9}{10} - \frac{36}{100}$$
$$= \frac{9}{10} - \frac{18}{50} = \frac{45-18}{50} = \frac{27}{50}$$