

## 实验一 手算各类型信号的 DFT

### 一. 实验目的:

手算 DFT, 计算八点不同数字频率的实信号和复信号的 DFT, 观察指定点数的向量图。

### 二. 实验原理

本次实验原理主要是 DFT 的计算公式, 如下所示:

设  $x(n)$  是一个长度为  $M$  的有限长序列, 则定义  $x(n)$  的  $N$  点离散傅里叶变换为

$$X(k) = \text{DFT}[x(n)] = \sum_{n=0}^{N-1} x(n)W_N^{kn} \quad k = 0, 1, \dots, N-1$$

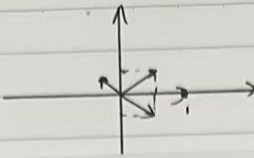
$X(k)$  的离散傅里叶逆变换 (Inverse Discrete Fourier Transform, IDFT) 为

$$x(n) = \text{IDFT}[X(k)] = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W_N^{-kn} \quad n = 0, 1, \dots, N-1$$

式中,  $W_N = e^{-j\frac{2\pi}{N}}$ ,  $N$  称为 DFT 变换区间长度,  $N \geq M$ 。

### 三. 实验内容

下图是手算 DFT 的过程以及对应的向量图:



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1. 求  $\text{Re}\{X(n)\}$

$$X(n) = \cos(2\pi n/4) \cdot \cos(\pi n/4) \quad e^{j\pi n/4} = \cos(\pi n/4) + j\sin(\pi n/4)$$

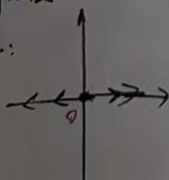
第一个点:  $X(0) = \cos(0) + \cos(\pi/4) + \cos(\pi/2) + \cos(3\pi/4) + \cos(\pi) + \cos(5\pi/4) + \cos(3\pi/2) + \cos(7\pi/4)$   
 $= 1 + \frac{\sqrt{2}}{2} + 0 + (-\frac{\sqrt{2}}{2}) + 1 + \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} = 0$

第二个点:  $X(1) = \cos(\pi/4) + e^{j\pi/4} \cos(\pi/4) + e^{j\pi/2} \cos(\pi/2) + e^{j3\pi/4} \cos(3\pi/4) + e^{j\pi} \cos(\pi) + e^{j5\pi/4} \cos(5\pi/4) + e^{j3\pi/2} \cos(3\pi/2) + e^{j7\pi/4} \cos(7\pi/4)$   
 $= 1 + e^{j\pi/4} \cos(\pi/4) + (-1) \cos(\pi/2) + (-\frac{\sqrt{2}}{2}) \cos(3\pi/4) + (-1) \cos(\pi) + (-\frac{\sqrt{2}}{2}) \cos(5\pi/4) + (-1) \cos(3\pi/2) + (-\frac{\sqrt{2}}{2}) \cos(7\pi/4)$   
 $= 1 + \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} = 4$

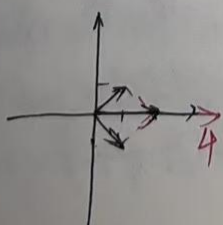
第三个点:  $X(2) = 1 + e^{j\pi/2} \cos(\pi/2) + e^{j\pi} \cos(\pi) + e^{j3\pi/2} \cos(3\pi/2) + e^{j2\pi} \cos(2\pi) + e^{j5\pi/2} \cos(5\pi/2) + e^{j3\pi} \cos(3\pi) + e^{j7\pi/2} \cos(7\pi/2)$   
 $= 1 + (-1) \cos(\pi/2) + (-1) \cos(\pi) + (-1) \cos(3\pi/2) + 1 \cos(2\pi) + 1 \cos(5\pi/2) + 1 \cos(3\pi) + 1 \cos(7\pi/2)$   
 $= 1 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + 1 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

复平面图:

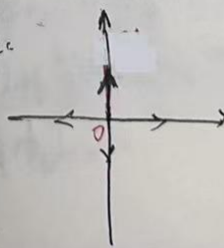
第一:



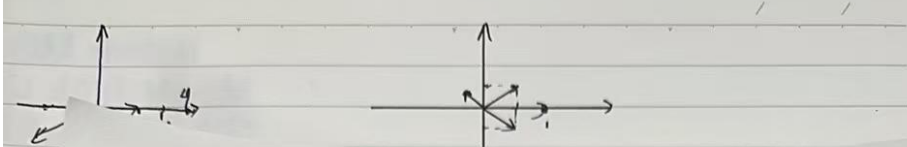
第二:



第三:



联系方式: \_\_\_\_\_



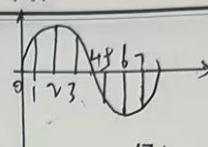
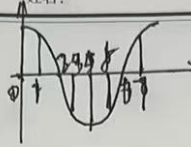
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2.  $N=8$  points,  $M=8$

$$x(n) = \cos(\frac{9\pi n}{8} + \frac{\pi}{4}), \cos(\frac{\pi n}{4})$$



第1点

四题新

$$\begin{aligned} x(n) &= \cos\frac{\pi}{4} + e^{j\frac{\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{2\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{3\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{4\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{5\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{6\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{7\pi}{4}}\cos\frac{3\pi}{4} \\ &= \frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(0+j) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + (-1)(\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(0+j) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) \\ &= \frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2}j) + (-\frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2} + (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2}j + (-\frac{\sqrt{2}}{2}) \\ &= -j \end{aligned}$$

$-j$

第2点

四题新

$$\begin{aligned} x(n) &= \cos\frac{\pi}{4} + e^{j\frac{\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{2\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{3\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{4\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{5\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{6\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{7\pi}{4}}\cos\frac{3\pi}{4} \\ &= \frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2})(0-j) + (-\frac{\sqrt{2}}{2})(-1) + (-\frac{\sqrt{2}}{2})(0+j) + (-\frac{\sqrt{2}}{2})(1) + (-\frac{\sqrt{2}}{2})(0-j) + (-\frac{\sqrt{2}}{2})(-1) + (-\frac{\sqrt{2}}{2})(0+j) \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} \\ &= 2\sqrt{2} + 2\sqrt{2}j \end{aligned}$$

第3点

四题新

$$\begin{aligned} x(n) &= \cos\frac{\pi}{4} + e^{j\frac{\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{2\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{3\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{4\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{5\pi}{4}}\cos\frac{3\pi}{4} + e^{j\frac{6\pi}{4}}\cos\frac{\pi}{4} + e^{j\frac{7\pi}{4}}\cos\frac{3\pi}{4} \\ &= \frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(0+j) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(-1) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{2}}{2})(0-j) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) \\ &= \frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}) - \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2} = 0 \end{aligned}$$

联系方式: \_\_\_\_\_

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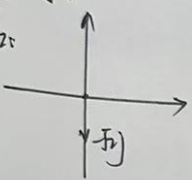
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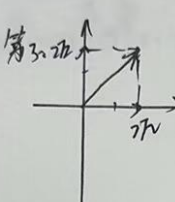
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复平面图

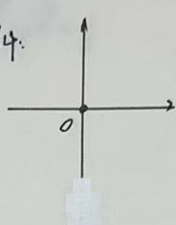
第2:



第3:



第4:



3 复信号 DFT,  $N=8$  Pre=3

$$x[n] = \exp(j \cdot 3 \cdot 2 \pi n / 8) = e^{j \frac{3n}{2}}$$

第1点:

$$X[1] = 1 + e^{j \frac{\pi}{4}} + e^{j \frac{\pi}{2}} + e^{j \frac{3\pi}{4}} + e^{j \pi} + e^{j \frac{5\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{7\pi}{4}}$$

先

$$= 1 + e^{j \frac{\pi}{4}} + e^{j \frac{\pi}{2}} + e^{j \frac{3\pi}{4}} + e^{j \pi} + e^{j \frac{5\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{7\pi}{4}}$$

$$= 1 + (0+j) + (1+j) + (0+j) + (1+j) + (0-j) + (1-j) + (0-j)$$

7 0

第3点:

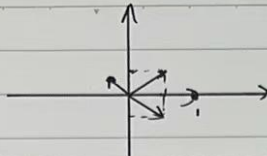
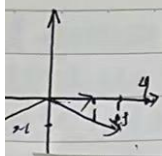
$$X[3] = 1 + e^{j \frac{3\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{9\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{9\pi}{4}} + e^{j \frac{9\pi}{4}} + e^{j \frac{21\pi}{4}}$$

$$= 1 + e^{j \frac{3\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{9\pi}{4}} + e^{j \frac{3\pi}{2}} + e^{j \frac{9\pi}{4}} + e^{j \frac{9\pi}{4}} + e^{j \frac{21\pi}{4}}$$

$$= 1 + (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) + (0-j) + (\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}) + (0-j) + (\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}) + (0-j) + (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2})$$

$$= (1 - \frac{\sqrt{2}}{2})$$

联系方式: \_\_\_\_\_



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第4个式:

$$x(2) = 1 + e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}} + e^{j\frac{\pi}{4}} e^{j\frac{\pi}{4}} + e^{j\frac{\pi}{4}} e^{j\frac{\pi}{4}} + \dots + e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}} = 8$$

联系方式: \_\_\_\_\_

#### 四. 实验心得体会：

通过本次实验：手算 DFT 的过程，一部分提高了我对于 DFT 本身公式的应用，对于矩阵的计算有复习了一边；同时，对于相位校正这一概念有了初步的体会和认识。