

Structural Analysis.

Tutorial - II

Statically Indeterminate Structures.

Statical Indeterminacy $\rightarrow (m + r) - 2j$. (for truss)

Other formula $\rightarrow (\alpha m + r) - (\beta j + r) = SI$.

Degree of freedom = $n + 2$.

$$r = n - 1$$

FOR BEAM,

$$\alpha = 3$$

$$\beta = 3$$

External Indeterminacy = $R - 3$.

Internal " " =

$$\text{Beam} = 0$$

$$\text{Frame} = M - 2j - 3 \text{ (3 x closed loop)}$$

$$\text{Truss} = M - 2j - 3$$

$r = n - 1$, if n is no. of member meeting at the internal hinge.

$$\alpha = 3 \Rightarrow [AFD, BMD, SFD]$$

For Truss.

$$\alpha = 1 \text{ (only Axial)}$$

$$\beta = 2 \text{ (}\sum F_x = 0, \sum F_y = 0\text{)}$$

For Frame.

$$\alpha = 3$$

$$\beta = 3$$

α = No. of unknown action in mem.

m = No. of member

r = No. of ext. support reac.

β = No. of equilibrium eqn available at each joint

j = No. of joint

r = No. of internal release

For space structure.

$$\alpha = 6 \quad r = 3(n - 1)$$

$$\beta = 6$$

Kinematic Indeterminacy :

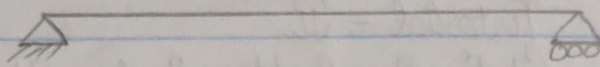
$$KI = (\beta j + r) - (\alpha m + r)$$

$\alpha = 0$, for extensible member condition

$\alpha = 1$, for inextensible member condition

Determine Static Indeterminacy of following :-

(1)



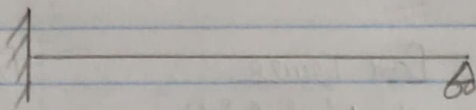
$$m=1$$

$$a=3$$

$$j=2$$

$$\begin{aligned} SI &= (m+a) - 2j \\ &= 1+3 - 2 \times 2 \\ &= \underline{\underline{0}} \end{aligned}$$

(2)



$$m=1$$

$$a=4$$

$$j=2$$

$$a=3$$

$$\beta=3$$

$$\gamma=0$$

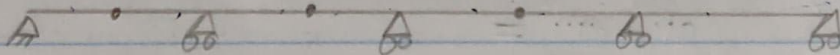
$$\begin{aligned} SI &= 1+4 - 2 \times 2 \\ &= \underline{\underline{1}} \end{aligned}$$

$$\begin{aligned} SI &= (3 \times 1 + 4) - 2 \times 3 \\ &= 7 - 6 = \underline{\underline{1}} \end{aligned}$$

$$EI = a - 3$$

$$= 4 - 3 = \underline{\underline{1}}$$

(3)



$$m=7$$

$$a=6$$

$$j=8$$

$$a=3$$

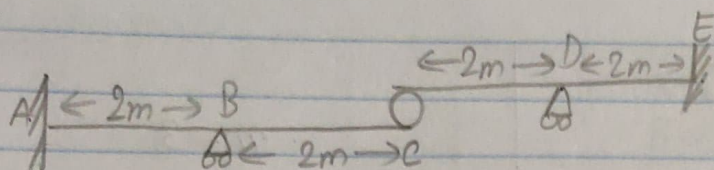
$$\beta=3$$

$$\gamma=3$$

$$\begin{aligned} SI &= (m+a) - 2j \\ &= 7+6 - 2 \times 8 \\ &= \underline{\underline{-3}} \end{aligned}$$

$$\begin{aligned} SI &= (3 \times 7 + 6) - (3 \times 8 + 3) \\ &= 21 + 6 - (24 + 3) \\ &= \underline{\underline{0}} \end{aligned}$$

(4)



$$M = 4$$

$$u = 8$$

$$\alpha = 3$$

$$\gamma = 2$$

$$\beta = 3$$

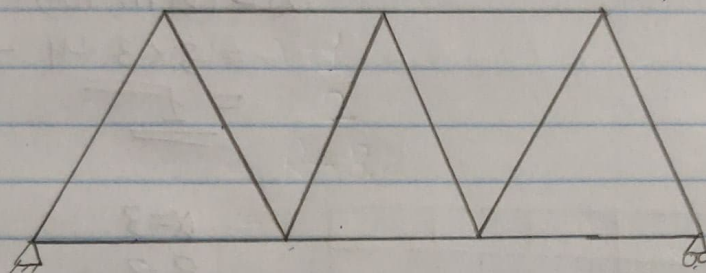
$$j = 5$$

$$SI = (4 \times 3 + 8) - (3 \times 5 + 2)$$

$$= 20 - 17$$

$$= \underline{\underline{3}}$$

(5)



$$SI = m + u - 2j$$

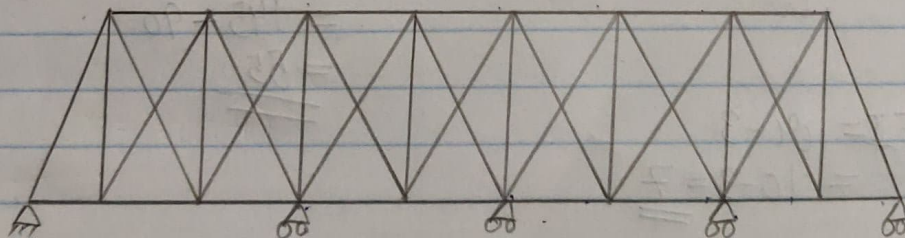
$$m = 11$$

$$u = 3$$

$$j = 7$$

$$SI = 11 + 3 - 14 = \underline{\underline{0}}$$

(6)



$$SI = m + u - 2j$$

$$m = 40$$

$$u = 6$$

$$j = 18$$

$$SI = 40 + 6 - 2 \times 18 = 4 + 6 = \underline{\underline{20}}$$

$$EI = 6 - 3 = 3$$

$$IS = 40 - 2 \times 18 + 3$$

$$= \underline{\underline{7}}$$

radius

$$\alpha = 3 \quad \gamma = 0$$

$$\beta = 3$$

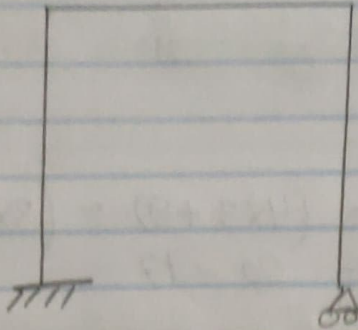
$$3 \times m + \alpha - 3 \times j$$

$$m = 3$$

$$\alpha = 4$$

$$j = 4$$

7



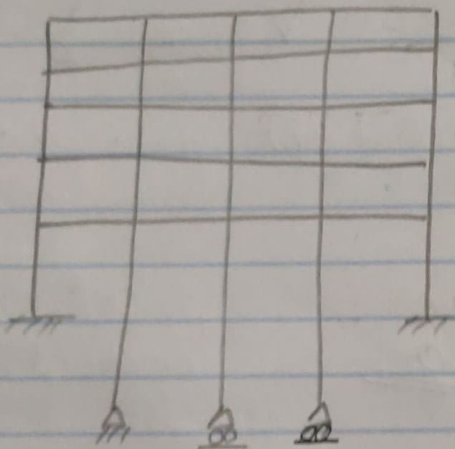
$$\begin{aligned} SI &= (3m + \alpha) - 3j \\ &= 3 \times 3 + 4 - 3 \times 4 \\ &= \underline{\underline{1}} \end{aligned}$$

$$\alpha = 3, \beta = 3, \gamma = 0$$

~~Ans~~

$$\begin{aligned} SI &= (\alpha m + \alpha) - (\beta j + \gamma) \\ &= 3 \times 3 + 4 - 3 \times 4 \\ &= \underline{\underline{1}} \end{aligned}$$

8



$$\alpha = 3$$

$$\beta = 3$$

$$\gamma = 0$$

$$m = 45$$

$$\alpha = 10$$

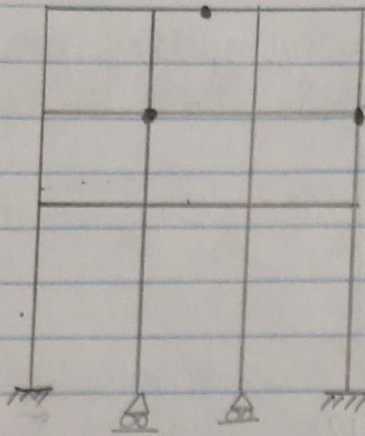
$$j = 30$$

$$\begin{aligned} SI &= (3 \times 45 + 10) - (3 \times 30 + 0) \\ &= 145 - 90 \\ &= \underline{\underline{55}} \end{aligned}$$

$$\begin{aligned} EI &= \alpha - 3 \\ &= 10 - 3 = \underline{\underline{7}} \end{aligned}$$

$$\begin{aligned} II &= 3 \times 45 - 3 \times 30 + 3 \\ &= \underline{\underline{48}} \end{aligned}$$

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$$\alpha = 3$$

$$\beta = 3$$

$$\gamma = \sum (n-1)$$

$$= (2-1) + (3-1) + (4-1)$$

$$= 1 + 2 + 3 = 6$$

$$m = 22$$

$$u = 8$$

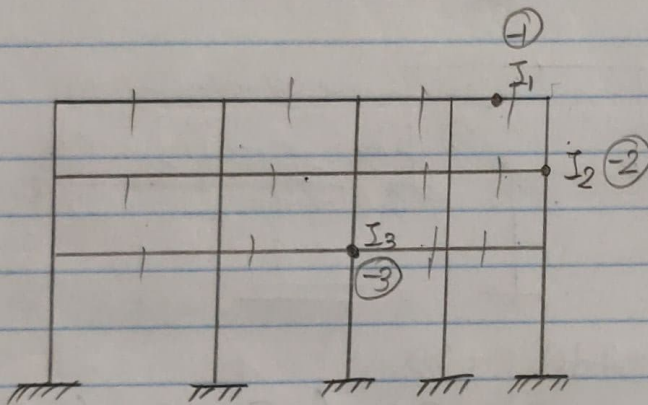
$$j = 17$$

$$SI = (3 \times 22 + 8) - (3 \times 17 + 6)$$

$$= 66 + 8 - 51 - 6$$

$$= 74 - 57 = 17$$

20



$$\alpha = 3$$

$$\beta = 3$$

$$\gamma = \sum (n-1) = (2-1) + (3-1) + (4-1) = 6$$

$$m = 28$$

$$u = 15$$

$$j = 21$$

$$SI = (3 \times 28 + 15) - (3 \times 21 + 6)$$

$$= 99 - 69$$

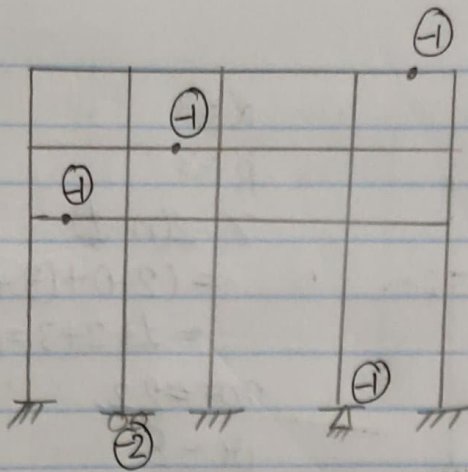
$$= \underline{\underline{30}}$$

Using tree method,
No. of internal release =
 $21 + 41 + 31 = 6$
No. of cut = 12.

$$SI = 3 \times 12 - 6$$

$$= 36 - 6 = \underline{\underline{30}}$$

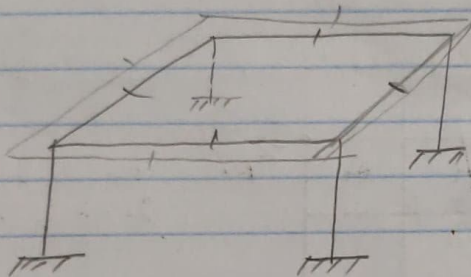
11.



No. of cut = $3 \times 4 = 12$.
 No. of internal releases = $1 + 1 + 1 + 2 + 1 = 6$.

$SI = 3 \times 12 - 6 = 36 - 6 = \underline{\underline{30}}$

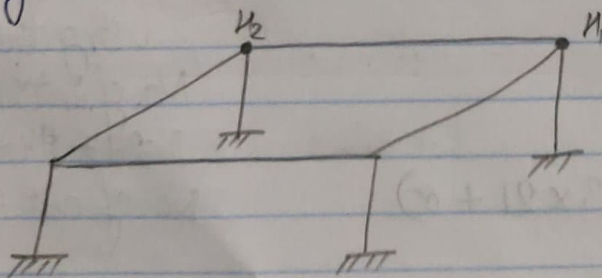
12.



$\alpha = 6$.
 $\beta = 6$.
 $\gamma = 3(n-1) = 0$.
 $m = 8$.
 $\alpha = 12$.
 $\beta = 8$.

$SI = (6 \times 8 + 12) - (6 \times 8 + 0)$
 $= \underline{\underline{12}}$

13.

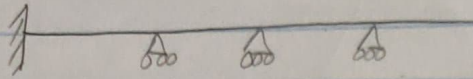


$\alpha = 6$.
 $\beta = 6$.
 $\gamma = 3(2) \times 2 = 12$.
 $m = 8$.

$\alpha = 12$.
 $\beta = 8$.
 $SI = (6 \times 8 + 12) - (6 \times 8 + 12)$
 $= \underline{\underline{0}}$

For the following diagrams, calculate Kinematic Indeterminacy as well as static indeterminacy:

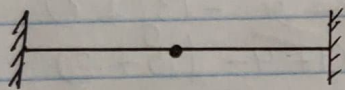
(1)



$$\begin{aligned} SI &= 3m + r - (3j + r) \\ &= 3 \times 4 + 6 - 3 \times 5 \\ &= 12 + 6 - 15 = \underline{\underline{3}} \end{aligned}$$

$$\begin{aligned} KI &= 3j + r - r \\ &= 3 \times 5 - 6 = 15 - 6 = \underline{\underline{9}} \end{aligned}$$

(2)



$$m=2$$

$$r=6$$

$$j=3$$

$$r=1$$

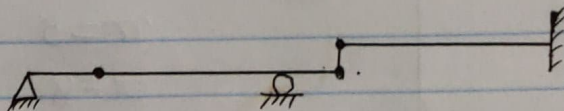
Static Indeterminacy \Rightarrow

$$\begin{aligned} SI &= 3m + r - (3j + r) \\ &= 6 + 6 - (9 + 1) \\ &= 2 \end{aligned}$$

Kinematic Indeterminacy

$$\begin{aligned} KI &= 3j + r - r \\ &= 3 \times 3 + 1 - 6 = \underline{\underline{4}} \end{aligned}$$

(3)



$$m=5$$

$$r=6$$

$$r=3$$

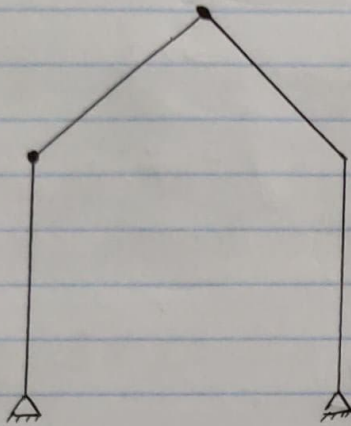
$$j=6$$

$$\begin{aligned} SI &= 3m + r - (3j + r) \\ &= 3 \times 5 + 6 - (3 \times 6 + 3) \\ &= 0 \end{aligned}$$

Hence, statically determinate beam.

$$\begin{aligned}
 KI &= 3j^o + r - a \\
 &= 3 \times 0 + 3 - 0 \\
 &= \underline{\underline{15}}
 \end{aligned}$$

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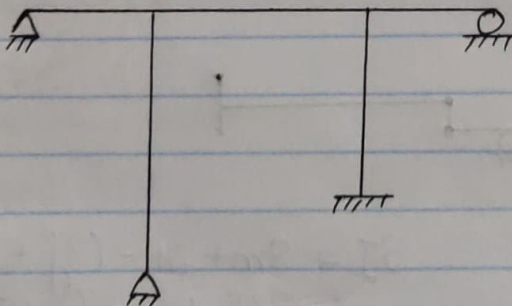
$$\begin{aligned}
 m &= 4 \\
 r &= 2 \\
 a &= 4 \\
 j &= 5
 \end{aligned}$$

$$\begin{aligned}
 SI &= am + r - (3j^o + a) \\
 &= 3m + r - (3j^o + a) \\
 &= 3 \times 4 + 4 - (15 + 2) \\
 &= \underline{\underline{-1 < 0}}
 \end{aligned}$$

Unstable frame.

$$\begin{aligned}
 KI &= (3j^o + r) - a \\
 &= 3 \times 0 + 2 - 4 \\
 &= \underline{\underline{13}}
 \end{aligned}$$

5.

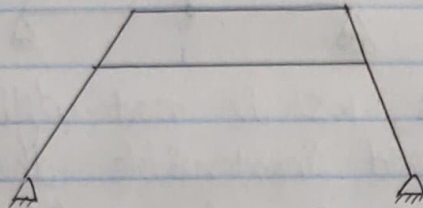


$$\begin{aligned}
 m &= 5 \\
 a &= 8 \\
 j &= 0 \\
 r &= 0
 \end{aligned}$$

$$\begin{aligned}
 SI &= 3m + r - (3j^o + a) \\
 &= 3m + r - (3j^o + a) \\
 &= 23 - 18 = \underline{\underline{5}}
 \end{aligned}$$

$$\begin{aligned}
 KI &= (3j^{\circ} + r) - \alpha \\
 &= 18 - 8 \\
 &= \underline{\underline{10}}
 \end{aligned}$$

6.

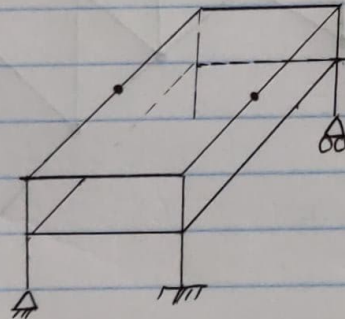


$$\begin{aligned}
 m &= 0 \\
 j &= 0 \\
 r &= 0 \\
 \alpha &= 3 \\
 \alpha &= 4
 \end{aligned}$$

$$\begin{aligned}
 SI &= 3m + \alpha - (3j^{\circ} + r) \\
 &= 3 \times 0 + 4 - 3 \times 0 \\
 &= \underline{\underline{4}}
 \end{aligned}$$

$$KI = 3j^{\circ} + r - \alpha = 18 + 0 - 4 = \underline{\underline{14}}$$

7



$$SI = \alpha m + \alpha - (3j^{\circ} + r)$$

$$\begin{aligned}
 SI &= 6 \times m + \alpha - (6j^{\circ} + r) \\
 &= 18 \times 6 + 16 - (6 \times 4 + 6) \\
 &= 124 - 30 = \underline{\underline{94}}
 \end{aligned}$$

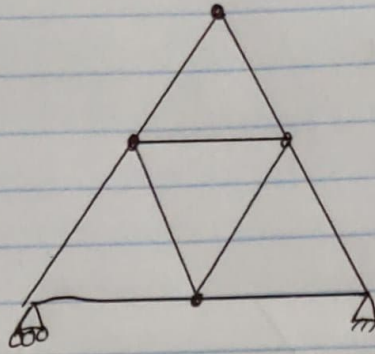
TREE METHOD.

$$\begin{aligned}
 SI &= 6 \times \text{No. of cut} - 3 \times \text{No. of hinge} - 5 \times \text{No. of roller} \\
 &\quad - 3 \times \text{No. of hinge} \\
 &= 6 \times 8 - 3 \times 2 - 5 - 3 = \underline{\underline{34}}
 \end{aligned}$$

$$\begin{aligned}
 \alpha &= 6 \\
 \beta &= 6 \\
 m &= 18 \\
 j &= 14 \\
 r &= 3 \times 2 = 6 \\
 \alpha &= 6 \times 2 + 1 + 3 = \underline{\underline{16}}
 \end{aligned}$$

$$\begin{aligned}
 KI &= (3j^{\circ} + r) - (\alpha m + \alpha) \\
 &= 6j^{\circ} + r - \alpha \quad (\alpha = 0) \\
 &= 6 \times 14 + 6 - 16 \\
 &= 84 + 6 - 16 = \underline{\underline{74}}
 \end{aligned}$$

100



In Trusses, KI is not defined because if mem. are considered inextensible then there is nothing to calculate KI as only axial forces are present at each members.

$SI =$

$m=6$

$j=5$

$r=3$

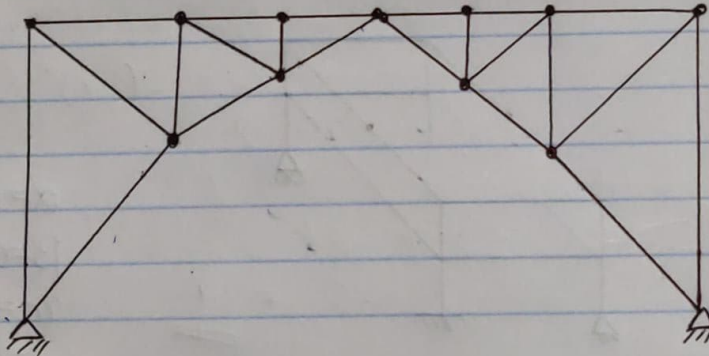
$$SI = m + r - 2j$$

$$= 6 + 3 - 10$$

$$= -1$$

Statically indeterminate.

101



$m = 22$

$j = 13$

$r = 4$

$$SI = m + r - 2j$$

$$= 22 + 4 - 26 = 0$$

Statically determinate.