AEM 591 Project 5

Optimal Vehicle State Estimation Part 2

This project is intended to introduce the setup and use of the Unscented Particle Filter (UKF) and basic Particle Filter (PF) using SciPy to estimate the position of a ground vehicle using two sensors with comparison to the Extended Kalman Filter (EKF)

Thus, one should use the same dynamics and problem setup as for project 4.

Project Assignment and Deliverables

For this project, perform the following tasks in Python:

1. set up functions in python to perform:
   - the initialization of the EKF;
   - the prediction step of the EKF; and
   - the correction step of the EKF.
2. use the provided dubins.py to setup a nominal Dubins path for $R = 5$:
   - set the nominal trajectory from $(0, -15, -90°)$ to $(-5, 20, -180°)$; and
   - sample the Dubins path with 5000-10000 grid points for the optimal positions and heading.
3. simulate 1 ground vehicle trajectory, i.e. (x_k, y_k) for $k = 1, \ldots,$ with a randomly sampled velocity along the Dubins path:
   - assume the speed is nominally $s[k] = 1$ for all k, but has been corrupted by zero-mean additive white Gaussian process noise, w_1, with covariance $\sigma_s^2 = 0.1^2$;
   - numerically integrate the position of the car along the Dubins path every $\Delta t = 0.5$ seconds by interpolating between the computed optimal heading for θ_{k-1} and using the generated speed, $s + w_1[k-1]$ to obtain a "true" trajectory of the ground vehicle along the Dubins path;
   - terminate the trajectory when one is "close" to the final point.
4. generate 1 sets of bearing measurements from these "true" positions to 1 radar with zero-mean additive white Gaussian measurement noise, v, with covariance, $\sigma_\beta^2 = 9$ degrees^2, positioned at $(-15, -10)$ and $(-15, 5)$
   - generate 1 set of heading measurements corrupted with zero-mean additive Gaussian white measurement noise with covariance $\sigma_\theta^2 = 5$ degree^2;
   - Assume all measurements are uncorrelated with each other.
5. estimate the position of the ground vehicle using an EKF, a UKF, and a PF, for the correction step where the following two equations form the state equation and the measurement equation:

$$
\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta t(s + w_1)\cos\theta_{k-1} \\ y_{k-1} + \Delta t(s + w_1)\sin\theta_{k-1} \\ \theta_{k-1} + w_2 \end{bmatrix}
$$
$$
\begin{bmatrix} \beta_{1,k} \\ \beta_{2,k} \\ \theta_k \end{bmatrix} = \begin{bmatrix} \arctan\left(\frac{y_k - y_1}{x_k - x_1}\right) + v_1 \\ \arctan\left(\frac{y_k - y_2}{x_k - x_2}\right) + v_2 \\ \theta_k + v_3 \end{bmatrix} \tag{1}
$$

- Note that w_2 has been added to the model so that the dynamics model allows θ to change. Set $\sigma_{w_2}^2 = \frac{1}{R}^2 \Delta t^2$ to model the possible maximum turning radius for following the Dubins path.

6. plot the true trajectory and the estimated trajectories for the EKF, UKF, and PF.
7. plot the *a posteriori* state **errors** for the EKF, UKF, and PF, i.e. $\hat{x}_{k|k}$, $\hat{y}_{k|k}$, and $\hat{\theta}_{k|k}$ relative to their true values, as well as $2\times$ the standard deviations of the estimate, i.e. the square root of the diagonal entries of the *a posteriori* covariance, $P_{k|k}$ for all three filters.
8. comment on the plots and the differences between the filters.