AEM 591 Project 4

Optimal Vehicle State Estimation

This project is intended to introduce the setup and use of the Extended Kalman Filter (EKF) using SciPy to estimate the position of a ground vehicle using two sensors.

To that end, recall the Dubins path problem which used the Dubins car dynamics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\vec{x}, \vec{u}) = \begin{bmatrix} s\cos\theta \\ s\sin\theta \\ u \end{bmatrix} \tag{1}$$

where s is the constant speed of the car, (x, y) is the position of the car, θ is the attitude angle, and the control input is u, the instantaneous curvature which is constrained by

$$\frac{-1}{R} \leq u \leq \frac{1}{R} \tag{2}$$

where R is the maximum radius of curvature. Recall that it was shown that the optimal path is the concatenation of arcs of circles of radius R and line segments all parallel to some fixed direction ϕ.

Furthermore, consider the following discrete-time dynamics model of a ground vehicle *attempting* to follow the Dubins path.

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta t s\cos\theta_{k-1} \\ y_{k-1} + \Delta t s\sin\theta_{k-1} \\ \theta_{k-1} + u\Delta t \end{bmatrix} \tag{3}$$

where Δt is the time step length from $k-1$ to k and notably assumes a zero-order hold on u. Second, consider the following bearing measurement to the Dubins car

$$\vec{\beta}_k = \arctan\left(\frac{y_k - y_r}{x_k - x_r}\right) \tag{4}$$

where β is the bearing angle from a radar located at (x_r, y_r).

Project Assignment and Deliverables

For this project, perform the following tasks in Python:

1. set up functions in python to perform:
   - the initialization of the EKF;
   - the prediction step of the EKF; and
   - the correction step of the EKF.
2. use https://pypi.org/project/dubins/ to setup a nominal Dubins path for $R = 5$:

- set the nominal trajectory from $(0, -15, -90°)$ to $(-5, 20, -180°)$; and
- sample the trajectory with 5000-10000 grid points for the optimal positions and heading.

3. simulate 1 ground vehicle trajectory with a randomly sampled velocity along the Dubins path:
   - generate one set of speed profiles, $s[k] = 1$, corrupted by zero-mean additive white Gaussian process noise, w_1, with covariance $\sigma_s^2 = 0.05^2$;
   - numerically integrate the position of the car along the Dubins path every $\Delta t = 0.5$ seconds by interpolating between the computed optimal heading and using the generated velocity to obtain a "true" trajectory of the ground vehicle along the Dubins path;
4. generate 2 sets of bearing measurements from these "true" positions to 2 different radars with zero-mean additive white Gaussian measurement noise, v, with covariance, $\sigma_\beta^2 = 9$ degrees^2, positioned at two different locations
   (a) One set: $(-15, -10)$ and $(-15, 5)$
   (b) Second set: $(-100, -10)$ and $(-100, 5)$
   - generate 1 set of heading measurements corrupted with zero-mean additive Gaussian white measurement noise with covariance $\sigma_\theta^2 = 5$ degree^2;
   - Assume all measurements are uncorrelated with each other.
5. estimate the position of the ground vehicle using two different EKFs, one for each set of radars for the correction step where the following two equations form the state equation and the measurement equation:

$$
\begin{aligned}
\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} &= \begin{bmatrix} x_{k-1} + \Delta t(s + w_1)\cos\theta_{k-1} \\ y_{k-1} + \Delta t(s + w_1)\sin\theta_{k-1} \\ \theta_{k-1} + w_2 \end{bmatrix} \\
\begin{bmatrix} \beta_{1,k} \\ \beta_{2,k} \\ \theta_k \end{bmatrix} &= \begin{bmatrix} \arctan\left(\frac{y_k - y_1}{x_k - x_1}\right) + v_1 \\ \arctan\left(\frac{y_k - y_2}{x_k - x_2}\right) + v_2 \\ \theta_k + v_3 \end{bmatrix}
\end{aligned} \tag{5}
$$

   - Note that w_2 has been added to the model so that the dynamics model allows θ to change. Set $\sigma_{w_2}^2 = \frac{1}{R}^2 \Delta t^2$ to model the possible maximum turning radius for following the Dubins path.
6. plot the true trajectory and the estimated trajectories for both EKFs
7. plot the *a posteriori* states (i.e. $\hat{x}_{k|k}$, $\hat{y}_{k|k}$, and $\hat{\theta}_{k|k}$ relative to the true values) as well as $2\times$ the standard deviations of the estimate, i.e. the square root of the diagonal entries of the *a posteriori* covariance, $P_{k|k}$.
8. plot the innovations, $\tilde{y}_k$ as well as $2\times$ the standard deviations of the innovation, i.e. $2\times$ the square root of the diagonal entries of the innovation covariance, S_k.
9. comment on the plots and the differences between the sets of radars.

In SciPy, use the stats sub-package to generate the multivariate Gaussian samples for the simulation.