Application of Centrality Measures of Complex Network Framework in Power Grid

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***Abstract*—The classical definitions of various centrality measures are modified to incorporate electrical parameters of the power grid. In this paper three distinct measures of centrality are presented and they are described with suitable examples. The usefulness of these measures are described. Various standard test systems are simulated to find critical nodes of the system. Complex network is a new area of research in power system. Simulation of several systems suggests that the definitions proposed in this paper can be used as a standard.**

***Index Terms*—degree centrality, closeness centrality, betweenness centrality.**

I. Introduction

Typically a system like power grid consists of power plants, transformers, transmission lines, distribution lines and loads. The trouble with systems like this is that individual behavior of its components is reasonably well understood. It is designed to behave collectively in an orderly fashion but sometimes it shows chaotic, confusing attitude and sometimes behave destructively like when blackout occurs.

Power systems play an indispensable role in modern society. Prevention of large scale outage is attributed to security assessment and monitoring system. Recent series of blackouts occurring all over the world shows that the system designated for prevention of blackouts is not working well, which stimulates researchers to seek solutions from alternative means. Recently advances of research in complex network field have attracted the interest of researchers of power grid to model and analyze the century old power grid under complex network framework.

If the network structure is known several measures or matrices could be developed which can identify particular features of the network. Social scientists have used several centrality measures [1]–[4] to explain a person's influence within a network. Among these centralities most widely used measures are degree centrality, betweenness centrality, and closeness centrality. To analyze the vulnerability of the power grid or to measure which nodes are more important within a power network these centrality approaches were used by researchers [5]–[8]. Some considered the power grid as an abstract network and neglected concrete engineering features of the grid, whereas some considered various features like impedance or admittance of various lines within a network.

Three distinct centrality concepts were redefined and three measures were adopted for each concept to clarify the concept of centrality in social networks [1]. Centrality indices were used to detect community boundaries [2]. A new betweenness centrality measure was proposed to find out nodes with high centrality that do not fall in the shortest path set of the network between various node sets or could not be found using maximum flow minimum cut set [3]. A fast algorithm to calculate betweenness centrality in large-scale networks was proposed [4].

A centrality measure for electric power grid was proposed which considers electrical topology rather than physical topology [5]. Topological and electrical centralities to rank various substations of the power grid were proposed [6]. Based on graph edge betweenness a method was proposed to carry out contingency analysis in power grids [7]. Based on admittance and impedance matrix various centrality measures were proposed to rank relative importance of nodes and edges in an electrical network [8].

But the static bus admittance and impedance matrix of the power grid cannot always capture the true scenario in large interconnected dynamical power grid. In this paper various centrality measures are proposed based on power flow in the network found by solving non-linear algebraic equations of the network [9]–[11]. IEEE 30 bus, 57 bus and 118 bus [12] systems are simulated to find out various important nodes in these systems based on degree, closeness, and betweenness centralities.

The rest of the paper is organized as follows. Section II describes a model for analyzing power system within the context of complex networks. Section III -V gives various centrality measures as applied to power system. Section VI deals with simulation of standard IEEE test systems to find out various centrality measures in those systems. Some concluding remarks are given in Section VII.

II. System Model

In order to demonstrate the application of centrality measures of complex network framework in power grid, representation of the power grid as a graph is the first step [13].

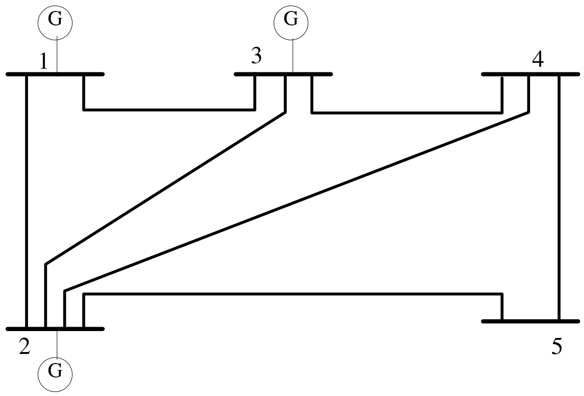

Fig. 1. Simple 5 bus system.

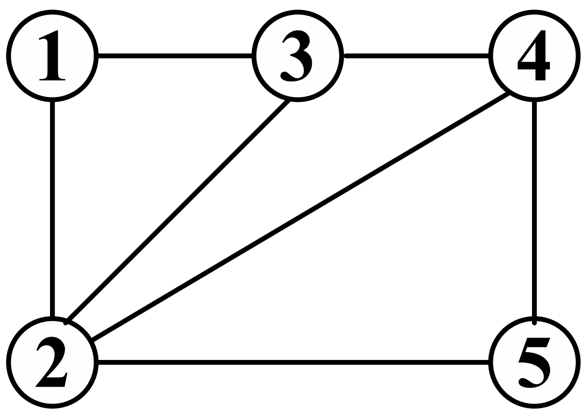

Fig. 2. Physical topology graph simple test system.

From the perspective of network theory, a graph is an abstract representation of a set of objects, called nodes or vertices, where some pairs of the objects are connected via links or edges.

To portray the assemblage of various components of power system, engineers use single-line or one-line diagram which provides significant information about the system in a concise form [11]. Power is supplied form the generator nodes to the load nodes via transmission and/or distribution lines. The principle of mapping is described as follows [14]:

a) all impedances between any bus and neutral are neglected,

b) all transmission and/or distribution lines are modeled except for the local lines in the plants and substations,

c) all transmission lines and transformers are modeled as weighted lines, the weight is equal to the admittance between the buses, and

d) parallel lines between buses are modeled as an equivalent single line.

Any power system network can be represented by a graph $G = (V, E, W)$ comprising of a set V, whose elements are called vertices or nodes, a set E of ordered pairs of vertices, called edges or lines . An element $e = (x, y)$ of the edge set E, is considered to be directed from x to y. y is called the head and x is called the tail of the edge. A set W, whose elements

TABLE I
SYSTEM DATA FOR NETWORK IN FIG. 1

From Bus	To Bus	R in pu	$\frac{1}{2}B$ in pu	X in pu	Power Flow
1	2	0.02	0.12	0.030	58.7
1	3	0.08	0.18	0.025	27.6
2	3	0.06	0.12	0.020	16.3
2	4	0.02	0.05	0.020	85.5
2	5	0.04	0.75	0.015	08.8
3	4	0.01	0.26	0.010	20.6
4	5	0.08	0.09	0.025	54.4

are weights of edge set elements. There exists a one-to-one correspondence between set E and set W.

To illustrate mapping of a single-line diagram to a directed graph, a simple example of 5 bus system [9] is used here. Fig. 1 depicts the system with 5 bus bars, and 7 links connecting them. Fig. 2 is the corresponding mapped graph. It contains 5 nodes/vertices which correspond to the slack, voltage-controlled, and load bus bars of the original system. The transmission lines are represented by the 7 links/edges which connects various nodes. The system data is given in Table I.

III. MEASURE OF CONNECTIVITY-DEGREE CENTRALITY

Degree centrality is the simplest form of centrality measures for networks. Although it is very simple, it has a great significance. It represents the connectivity of a node to the network [15]. Individuals who have more links with other persons are more connected to the network in the sense that they have more resource, access of information than others. A non-social network example is the use of citation counts in the evaluation of scientific papers. The number of citations of a paper can be regarded as its impact on research [16].

For example, node 2 in Fig. 2 is adjacent to four other nodes, it's degree is four. In a 5 node graph any node can be adjacent to only remaining four nodes. So, this node has got highest connectivity. In literatures degree centrality is defined as:

$$C_D(k) = \frac{deg(k)}{n-1} \tag{1}$$

where, $deg(k)$ is the degree of node k.

In case of electrical network, the power flowing in the adjacent links of the node in concern can be regarded as a degree of the node and the definition of the electrical degree centrality can be given as:

$$C_D^E(k) = \frac{\sum\limits_{k \sim t} P_{kt}}{n-1} \tag{2}$$

where, $k \sim t$ indicates that node k are t are connected. P_{kt} indicates power flowing in line connected in between nodes k and t.

Table II shows the degree centrality of simple 5 bus system in Fig. 1 using classical and proposed approach.

TABLE II
DEGREE CENTRALITY FOR NETWORK IN FIG. 1

Bus	$C_D(k)$	$C_D^E(k)$
1	0.50	21.58
2	1.00	42.33
3	0.75	16.13
4	0.75	40.13
5	0.50	15.80

TABLE III
CLOSENESS CENTRALITY FOR NETWORK IN FIG. 1

Bus	$C_C(k)$	$C_C^E(k)$
1	0.17	1.28
2	0.25	2.13
3	0.20	1.23
4	0.20	1.92
5	0.17	1.14

IV. MEASURE OF INDEPENDENCE-CLOSENESS CENTRALITY

This approach of centrality measure is based upon the degree to which a node is close to all other nodes in the network [1]. Fig. 3 shows closeness in classical sense and to illustrate the idea of electrical closeness centrality Fig. 4 is drawn to show the closeness of various nodes of the simple 5 bus system in Fig. 1 in terms of electrical distance found in Table I. It is clear from Fig. 4 that node 2 is adjacent to three other nodes (nodes 1, 3, and 4) in terms of electrical distance, while nodes 1, 3, and 4 being adjacent to two nodes. Node 5 is adjacent to one node only. So node 2 is the closest to other nodes than the rest of the nodes in the network.

In the social network theory, closeness is a sophisticated measure of centrality. It is defined as the mean geodesic distance (i.e., the shortest path) between a vertex k and all other vertices reachable from it [17]. In mathematical form, the closeness centrality of a vertex k, $C_C(k)$ in a network of n vertices is given by:

$$C_C(k) = \frac{\sum_{t \in V \backslash k} d(k,t)}{n-1} \tag{3}$$

where, $d(k,t)$ being the shortest path length between vertices k and t. This definition of closeness centrality gives a measure of distance of particular vertex from other vertices. So, some researchers have used the reciprocal of the shortest path to quantify closeness centrality as follows:

$$C_C(k) = \frac{1}{\sum_{t \in V \backslash k} d(k,t)} \tag{4}$$

The electrical closeness centrality was defined as [8]:

$$C_{Cz}(k) = \frac{n-1}{\sum_{t \in V \backslash k} d_z(k,t)} \tag{5}$$

where, $d_z(k,t)$ is taken as the shortest electrical distance between nodes k and t. Resistance was neglected since they considered only transmission systems. But in order to generalize the concept in both transmission and distribution system we cannot neglect resistance of the network lines which is a significant portion of the line impedance in case of distribution lines. The numerator was taken as $n-1$. This was adopted in (3) to average the distance. But when it comes in the numerator it just scales the parameter. So, in this paper we propose our electrical closeness centrality as:

$$C_C^E(k) = \frac{1}{\sum_{t \in V \backslash k} d(k,t)} \tag{6}$$

where, $d(k,t)$ is the weight of the shortest electrical path from node k to all other nodes t reachable from k.

Table III shows the closeness centrality of simple 5 bus system in Fig. 1 in classical as in (4) and proposed approach.

The independence of a node is determined by the closeness centrality of the node [1]. In Fig. 2 node 2 is in direct contact with nodes 1, 3, and 4. He must depend upon node 4 to communicate with node 5. So, node 5 needs only one relayer to communicate with all other nodes of the network. On the other hand, node 1 needs node 2 to communicate with node 4 and both needs 2 and 4 to communicate with node 5. So we can say that node 2 is more independent than node 1. So closeness centrality can be used to quantify independence of various nodes within a electrical power grid.

V. MEASURE OF CONTROL OF COMMUNICATION-BETWEENNESS CENTRALITY

This type of centrality is based upon the frequency with which a node falls between pairs of other nodes on the shortest or geodesic paths connecting them [1]. This idea is illustrated by ten possible shortest paths in the network of Fig. 1 as shown in Fig. 5. Node 2 comes four times between other points in the six geodesics. Node 4 comes three times. So node 2 is more central in terms of betweenness.

The betweenness centrality $C_B(k)$ for vertex k is computed as follows [17]:

1. Find the shortest path set of the network,
2. Find out the fraction of shortest path containing node k for each pair of vertices,
3. Sum this fraction over all pairs.

Mathematically,

$$C_B(k) = \sum_{s=1}^{n} \sum_{t=1}^{n} \frac{\sigma_{st}(k)}{\sigma_{st}}, s \neq t \neq k \in V \tag{7}$$

where, σ_{st} is the number of shortest paths from s to t, and $\sigma_{st}(k)$ is the number of shortest paths from s to t that pass through a vertex k.

As in closeness centrality the shortest paths for a electrical network can be calculated from the line impedance. And the power flowing in the line is taken as a measure of betweenness

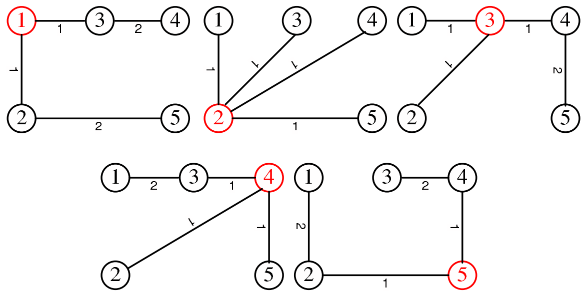

Fig. 3. Classical closeness of various nodes of the simple 5 bus system in Fig. 1.

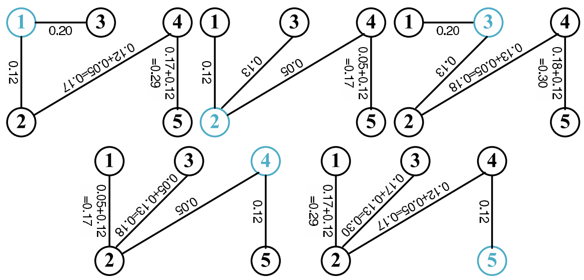

Fig. 4. Electrical closeness based on line impedance of various nodes of simple 5 bus system.

TABLE IV
BETWEENNESS CENTRALITY FOR NETWORK IN FIG. 1

Bus	$C_B(k)$	$C_B^E(k)$
1	$\frac{0}{10}$	$\frac{0}{670}$
2	$\frac{1}{10}$	$\frac{192}{670}$
3	$\frac{1}{10}$	$\frac{0}{670}$
4	$\frac{1}{10}$	$\frac{93.3}{670}$
5	$\frac{0}{10}$	$\frac{0}{670}$

[14]. The electrical betweenness centrality of a node k in a network of n nodes is defined as:

$$C_B^E(k) = \sum_{s=1}^{n}\sum_{t=1}^{n} \frac{P_{st}(k)}{P_{st}}, s \neq t \neq k \in V \tag{8}$$

where, P_{st} is the maximum power flowing in the shortest electrical path between buses s and t , and $P_{st}(k)$ is the maximum of inflow and outflow at bus k within the shortest electrical path between buses s and t. Fig. 6 illustrates the concept of electrical shortest path and shows ten possible geodesics in the simple 5 bus test system.

Table IV shows the betweenness centrality of simple 5 bus system in Fig. 1 using classical and proposed approach.

VI. SIMULATION OF VARIOUS STANDARD IEEE TEST SYSTEMS

There are some critical nodes in every networks which when removed from the system can make the system very vulnerable to attack. Previously researchers used complex network theory to explain blackouts or cascading effects in power system. Very few works were done to identify critical nodes of the system. System reliability can be improved a lot if these critical nodes can be identified beforehand by monitoring them regularly and servicing them when subjected to deterioration. Critical nodes can be found from calculating various centrality measures as outlined in previous sections.

TABLE V
TOP TEN CRITICAL NODES ACCORDING TO DEGREE CENTRALITY OF VARIOUS STANDARD IEEE TEST SYSTEMS.

30 Bus	$C_B^E(k)$	57 Bus	$C_B^E(k)$	118 Bus	$C_B^E(k)$
2	12.5841	1	7.9668	12	12.2114
6	9.2330	4	5.3512	69	5.6372
1	9.0528	2	4.9898	70	5.1916
4	7.0180	3	4.4910	80	4.8233
3	5.7375	15	3.9604	7	4.7227
5	3.7382	6	3.5725	11	4.7200
10	2.4665	17	3.4561	32	4.4156
9	2.3013	24	2.9787	46	4.1532
7	2.2122	23	2.0769	75	3.8160
12	2.1811	13	2.0628	34	3.2639

TABLE VI
TOP TEN CRITICAL NODES ACCORDING TO CLOSENESS CENTRALITY OF VARIOUS STANDARD IEEE TEST SYSTEMS.

30 Bus	$C_C^E(k)$	57 Bus	$C_C^E(k)$	118 Bus	$C_C^E(k)$
6	2.2366	14	1.7785	65	3.0553
4	2.1676	13	1.7596	68	3.0249
28	2.0587	46	1.7399	116	2.9884
8	2.0438	47	1.7120	81	2.9366
3	2.0108	48	1.7042	38	2.8773
9	2.0029	15	1.6993	64	2.8353
10	1.9662	38	1.6775	69	2.8298
7	1.9069	11	1.6663	80	2.8158
12	1.8110	3	1.6167	66	2.8119
21	1.7877	12	1.6149	30	2.7189

IEEE 30, 57, and 118 bus systems were used to simulate various centrality measures and results are given in Tables V–VII. Results in Tables V–VII show that different approaches

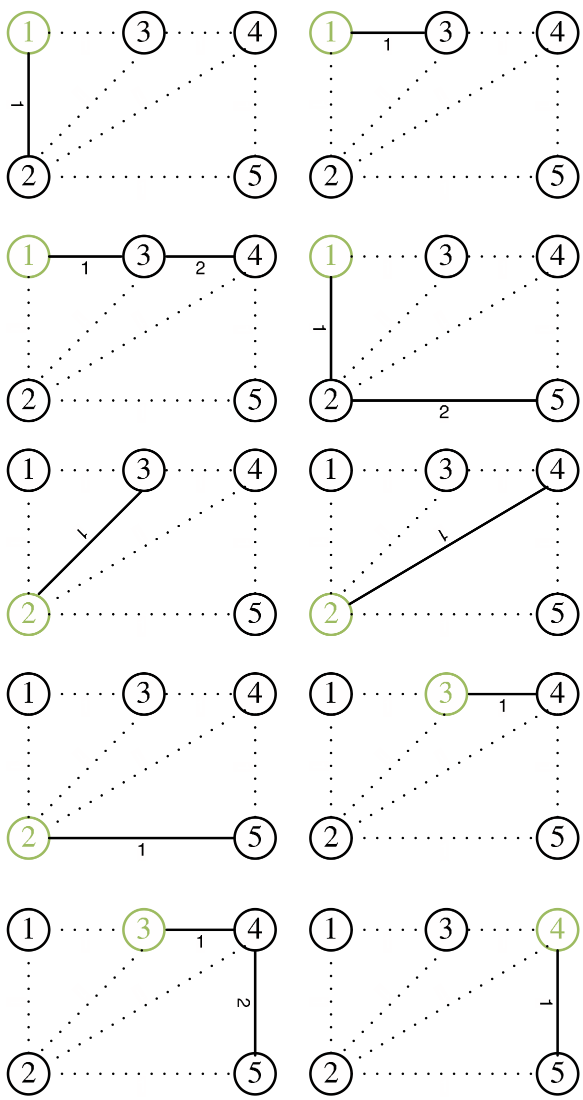

Fig. 5. Illustration of betweenness in 10 possible shortest path set of the test system.

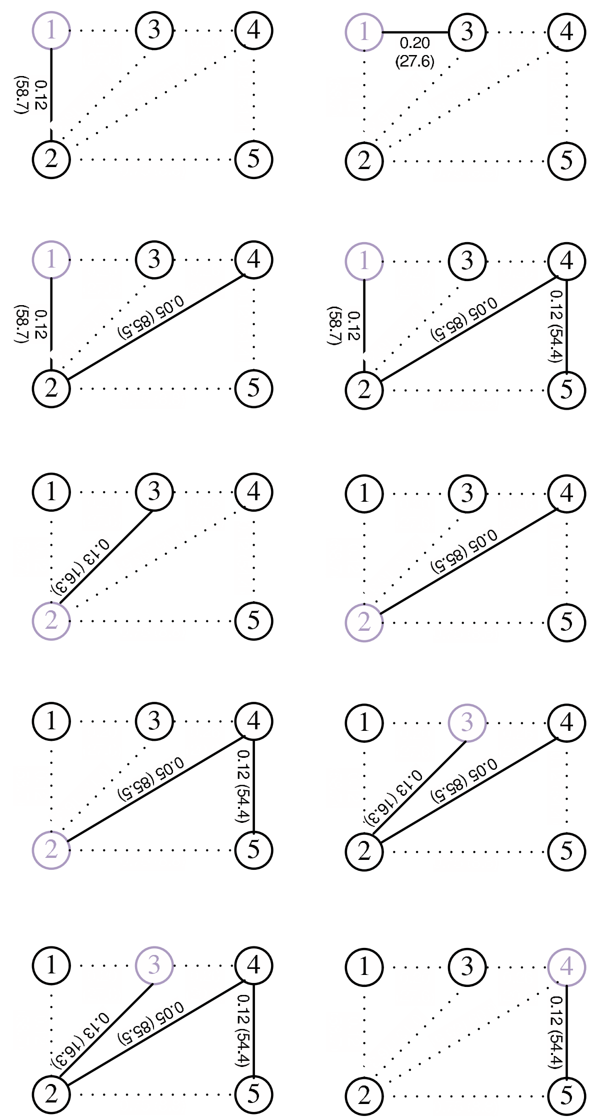

Fig. 6. Ten possible shortest path set in terms of electrical distance in simple 5 bus system.

give different nodes as critical in order of priority. This is expected because these three centrality measures are based on three different approaches. So if we are dealing with connectivity, degree based centrality is the one to consider. If some cases require the criticality measure based on independence on the node, closeness centrality would be the best option to look at. However, the last option – control of communication can be measured using betweenness centrality. Since the future smart grid will rely on control of communication along with power transfer, the third measure of centrality could be very useful for future control room engineers and planners to take necessary action in critical events.

VII. Conclusion

Various centrality measures based on electrical parameters is proposed. These centrality measures could be used to identify critical nodes of the system. New definitions are proposed based on network impedance and power flow. Usefulness of various centrality measures are demonstrated via example. Simulation of various standard IEEE test systems using the proposed definitions could be used to find several critical nodes in the. Various simulation suggests that the definitions used in the paper could be used as standard for further study of power grid using complex network theory.

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TABLE VII
TOP TEN CRITICAL NODES ACCORDING TO BETWEENNESS CENTRALITY OF VARIOUS STANDARD IEEE TEST SYSTEMS.

30 Bus	$C_B^E(k)$	57 Bus	$C_B^E(k)$	118 Bus	$C_B^E(k)$
2	0.6117	1	0.6117	12	0.7030
1	0.5546	2	0.4862	7	0.4331
6	0.3114	17	0.4142	11	0.4310
4	0.3103	3	0.3380	2	0.3431
3	0.2972	15	0.2271	3	0.0780
5	0.1668	16	0.1427	6	0.0629
7	0.0547	4	0.1398	14	0.0350
8	0.0490	6	0.0566	117	0.0340
9	0.0420	14	0.0544	13	0.0286
10	0.0406	5	0.0498	4	0.0219

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