Modeling and evaluation

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In the previous chapter, the coordination mechanisms developed in [5] have been presented, giving particular attention to the contribution provided by proactivity. Concerning buddy system and reserve, their proactive versions move the idle set at the barycenter of the positions of the active agents. This ensures good results when compared with the base method, in particular for proactive reserve. However, the location for the idle set is computed naively and might be exploiting only a part of the information available at the moment related to the structure of the environment. For example, the barycenter of the positions of the active robots may fall into a small room, from which a robot in the idle set would have to get off once turned into active, making the move into the room almost useless. On the contrary, we are more interested in moving the robots in the idle set towards positions providing a good starting point for them.

To include the structure of the environment directly into the coordination mechanism, graphs are considered. They provide a representation of the free space and computing some suitable metrics, called centrality metrics, it is possible to find out a subset of nodes more influential on connectivity between spaces. In this way, the proactivity location, i.e., the location where the idle set is proactively moved, is a central point of the environment. The concept of centrality varies accordingly to the centrality metric used to compute such subset of nodes. In the following, the graphs and the centrality metrics used in this work will be presented and described in-depth. The last section is dedicated to providing an overview of the criteria used to evaluate the different coordination mechanisms tested.

1 Graphs

The use of graphs allows including a topological component in the coordination, which is done employing an embedded graph. A graph G embedded to a surface Σ is a representation of G on Σ such that points of Σ and arcs in it are associated with vertices and edges of G respectively. This concept is applied by creating a graph Υ embedded on the map M and then through the measures of the centrality of a node, nodes are ranked based on their values for the two centrality measures used in this work.

The definition of an embedded graph is independent of the actual implementation of it and this leaves the freedom to adapt the correspondence points-nodes and arcs-edges on the needs. Based on this, two different types of graphs are defined in the next sections.

These graphs differ strongly on how they are defined and how they model the environment. The first one is more focused on the topological properties of the environment, nodes and edges are computed based on the structure of the known free space. The model it provides is not strictly affected by team-dependent factors like agents distribution or team size, being built only considering the connectivity properties of the map. On the contrary, the second kind of graph defined below is built according to the positions of the agents moving in the environment. In this way, it includes information related to the disposition of the agents and the path followed. Moreover, it is also indirectly affected by the structure of the environment, being the agents only capable of moving in the free space, the routes they travel outline the connectivity among the different spaces. In the following, how frontiers are introduced into the graph building process is also explained.

The following sections explain in detail how the two types of graphs are built during the exploration process, starting with the topological graph and then moving on to the visibility one.

1.1 Topological graph

From now on, as *topological graph* will be identified a graph isomorphic to the graph used by the simulator to compute the path followed by the agents during the exploration. As presented in [1], the construction of this one is based on the structure of the occupancy grid known up to that moment, in which the skeleton of the free space is found by performing thinning. A discretization is then applied to the skeleton to find the set of nodes composing the graph and nodes linked by the skeleton are also connected by an edge, weighted according to the distance between the nodes. This process is sufficient to provide a simple topological graph embedded on the map of the environment.

On one side, the use of this graph is backed up by the implementation of the navigation system, because it is computed just once for both the applications and in this way it does not introduce further costs in the building and the update. On the other side, it is pretty limited in its representation. It suffices to provide a topological view of the environment, but it is hard to extend with elements like frontier nodes or information about the location of the robots. Two aspects that characterize the second kind of graph tested, the visibility graph.

In Figure 1.a, the topological graph of the environment presented in the previous chapter is shown. This one has been produced by applying the building procedure on the whole map of the environment when it has been completely mapped. This has been considered to show more clearly the properties of this graph. The disposition of the nodes is almost uniform and this can be particularly noticed by looking at their distribution in the lateral rooms. Being these spaces almost equal in structure, nodes are placed with the same pattern. Moreover, the distance among nodes tends to be almost the same on the entire map, with a slight reduction at the conjunctions. This almost uniform distribution highlights strongly the difference in structure with the visibility graph, shown in Figure 1.b, where nodes are provided with varying concentrations among the various spaces.

1.2 Visibility graph

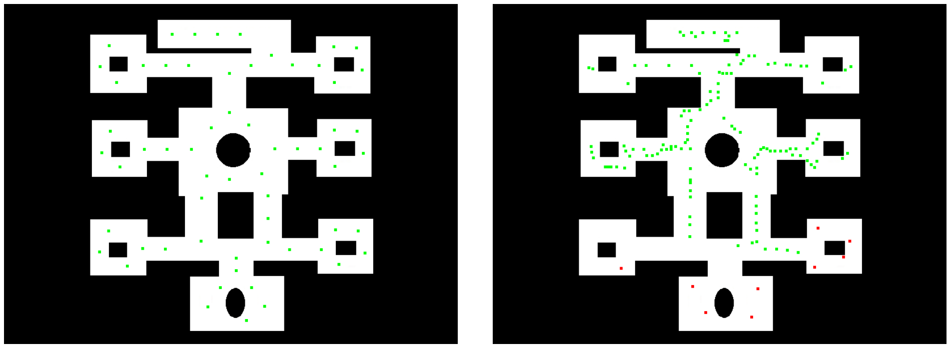
The *visibility graph* as defined in this work consists of a graph built on the notion of visibility, rather than the navigability from node to node. It is composed of two different types of nodes, the *pose nodes,* and the *frontier nodes*.

The first kind is the one composing the vast majority of the graph and each node has an historical meaning, being a pose assumed by an active robot during the exploration. As active agents proceed in their mapping task, their locations are stored as nodes every time a re-plan for one of them or a new location for the robots in the idle set is needed. These two events happen quite frequently in the initial part of the exploration, and thus they trigger the graph building function enough times. However, the frequency with which this is done is variable, for this reason, the distribution of the nodes might be loose in some spaces and more tight in others. This allows keeping the number of nodes in the graph reduced with respect to the case in which every time an active agent moves, its pose is used to set up a pose node. Nevertheless, as long as connectivity is ensured, the effect on the information obtainable by the graph is almost the same and by keeping the size of the graph reduced, the complexity of computing centrality measures remains manageable. To enforce this aspect, the location of an active agent is added as a pose node as long as there are no other pose nodes within a certain radius. Once a pose node is added to the graph, it is fixed and never modified as the exploration goes on. The motion of agents in the idle set is excluded to avoid an excessive concentration of nodes in some crucial areas, which would negatively affect the computation of centrality measures.

The second kind of node is frontier nodes. As the name suggests, they allow to include the frontiers computed by the exploration strategy into the graph. This is a fundamental characterization of this graph in the distinction from the topological one. At each step in which the list of frontiers is updated, the same is done for the list of frontier nodes: the old ones which have been explored are removed and the new ones are added, making this type of nodes variable in time differently from pose nodes which are fixed once they are put in the graph.

An edge models the notion of visibility: two nodes n_1 and n_2 are linked by an edge if and only if $n_1$is within the sensing range of a robot placed in n_2 and there are no obstacles along the straight line connecting them. This relation holds in both the directions, thus the visibility graph is an undirected graph by construction, same as the topological one. Edges are also characterized by a weight equal to the distance between the nodes.

Figure 1.b shows the visibility graph built by a team of robots during the exploration. Accordingly to what stated previously, the distribution of nodes depends strongly on the path followed by the robots and on the frequency with which the graph is updated. The structural differences with the topological graph



(a) Topological graph. Green dots represent the nodes

(b) Visibility graph. Green dots represent pose nodes, red ones are the frontier nodes

Figure 1: Examples of graphs at the end of the exploration of the environment presented in the previous chapter. Edges are omitted for clarity

in Figure 1.a are clean just by looking at the two figures and all relate to the distribution of nodes, being widely less uniform. Frontier nodes, here painted in red, are also a discriminating factor between the two.

2 Centrality measures

Centrality measures have been exploited widely in the literature, especially in fields related to social networks [3], power grids [7], disease [8] and computer virus spreading [9]. This is possible since centrality measures are applicable as long as the system is modeled by means of a graph and they provide a ranking of the nodes according to the metric applied. In fact, different metrics may provide different rankings, depending on the network topology, because of the mismatching concept of central node. An example of this is provided by the kite graph [2]. It is a simple graph composed of 10 nodes and 18 edges and it is depicted in Figure 2. The particularity of it is to be the smallest possible graph for which the nodes having the highest values of the three most basic centrality measures, namely degree, closeness, and betweenness, are all different.

To the author's knowledge, there are no previous works trying to apply the use of centrality measures to enhance the coordination of a team of robots and due to the variability depending on the centrality measure used, in this work both closeness and betweenness have been tested. In the following sections, they are formally introduced with the reasons for their use.

2.1 Closeness

The closeness centrality of a node is defined as the reciprocal of the sum of the length of the shortest paths between the node and all the other nodes in the graph [3]. This definition is strongly dependent on the number of nodes N in

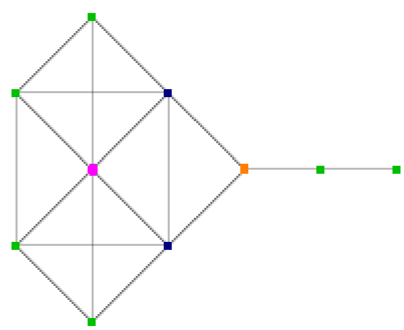


Figure 2: Kite graph. The pink node has the highest degree, the orange node has the highest betweenness and the blue nodes have the highest closeness. Green nodes are the remaining nodes.

the graph, thus closeness is usually normalized by dividing for $N-1$. In this way, closeness can be defined as the reciprocal of the average distance between the node and all the other nodes and allows to compare its value for graphs of different sizes.

Formally, let x and y be nodes of the graph, and let d be a real-valued function which provides the length of the shortest path connecting two nodes, then the normalized closeness value $C(x)$ is defined as

$$C(x) = \frac{N-1}{\sum_{y \neq x} d(x, y)}$$

with N the total number of nodes in the graph, as defined above.

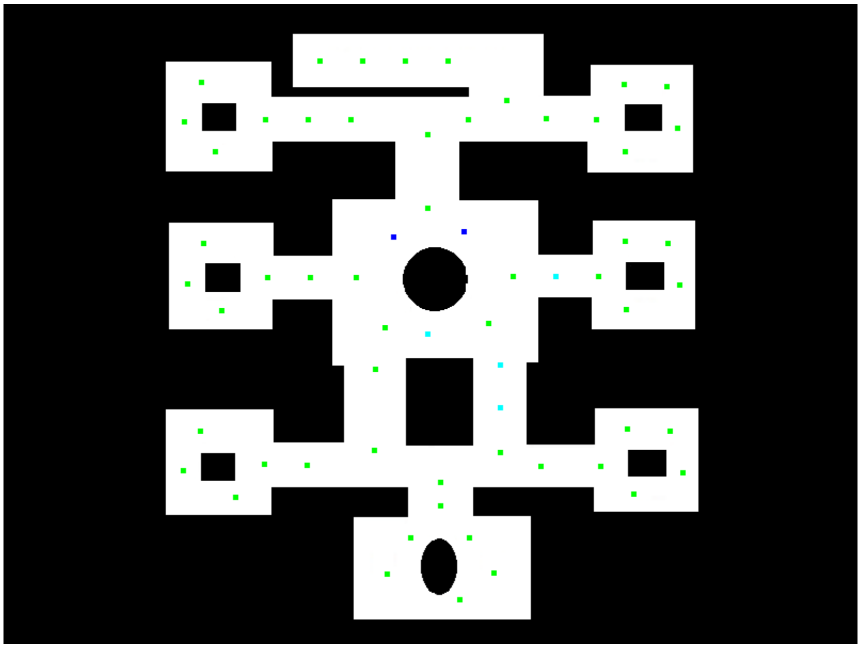
The original formulation of closeness centrality considers only unweighted graphs, but it has been extended to be applied also to weighted ones. The formal definition is the same assuming an appropriate modification in the implementation of the distance function d. Indeed, in an unweighted graph, it simply has to count the number of edges along the shortest path linking the two nodes in input, while on a weighted graph the cost of traversing an edge is proportional to its weight and thus it has to be included accordingly [4]. In the latter case, the distance function d is $d(x, y) = \sum_{e \in E} w(e)$, where x and y are nodes of the graph, E is the set of edges of the graph composing the shortest path from x to y, and w is a real-valued function returning the weight of the edge.

Closeness tends to consider central nodes the ones in which distance from all the other nodes is lower on average. Therefore, going back to an exploration context, placing an agent in the location corresponding to the highest closeness node makes it possible to reach an assigned position, not known before, in an expected time lower than any other starting location with a lower closeness. This reasoning has been applied to the idle set, which placed in the node with the highest closeness is likely to already be in a good spot when turned into active.

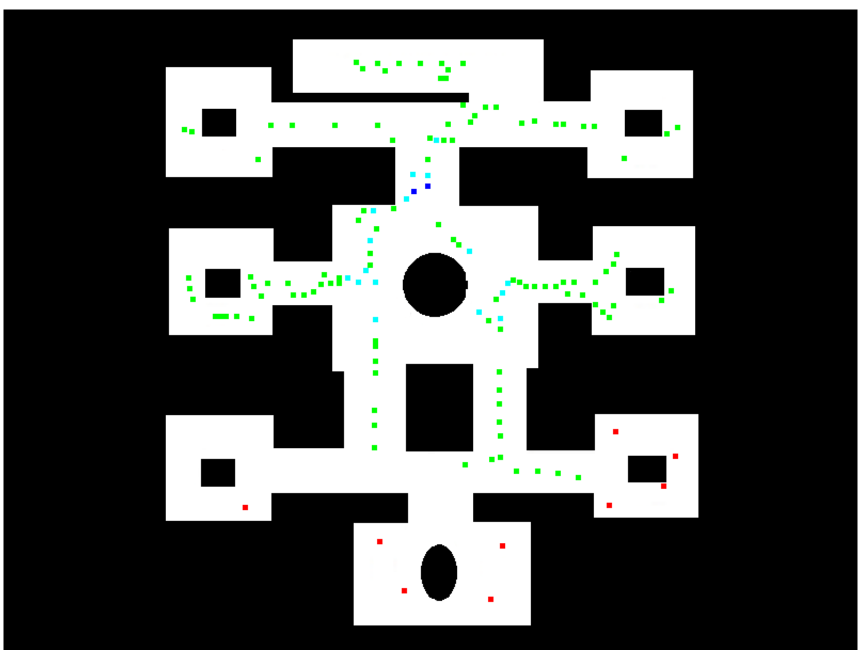
In Figure 3 an example of the distribution of high closeness nodes is shown for each kind of graph. For the sake of the example, as high closeness nodes are considered nodes with a value of closeness higher than the 75% of the max value. It is interesting to point out how these nodes are distributed mostly on one corner of the central space for the topological graph, while they wrap the central obstacle in the visibility case. Despite the marked differences in their structures and the number and disposition of high closeness nodes, it is remarkable that the distance among the highest closeness nodes is really low.

2.2 Betweenness

Betweenness centrality for a certain node of the graph measures how much of the total number of shortest paths between other nodes passes through that node [3]. Let s, v, and t be three nodes in a connected graph, σ_{st} be the total number of shortest paths connecting s and t and $\sigma_{st}(v)$ be the number of those paths



(a) Topological graph. Green dots are the nodes with a value of closeness lower than the 75% of the max value.



(b) Visibility graph. Green dots are the remaining pose nodes and red ones are the frontier nodes.

Figure 3: Distribution of nodes with a closeness higher than the 75% of the max value for the topological and the visibility graphs. Blue dots are the highest closeness nodes, cyan dots are the ones with a high value of closeness but not the maximum, and edges are omitted for clarity.

which go through v, then the betweenness $B(v)$ for the node v is defined as

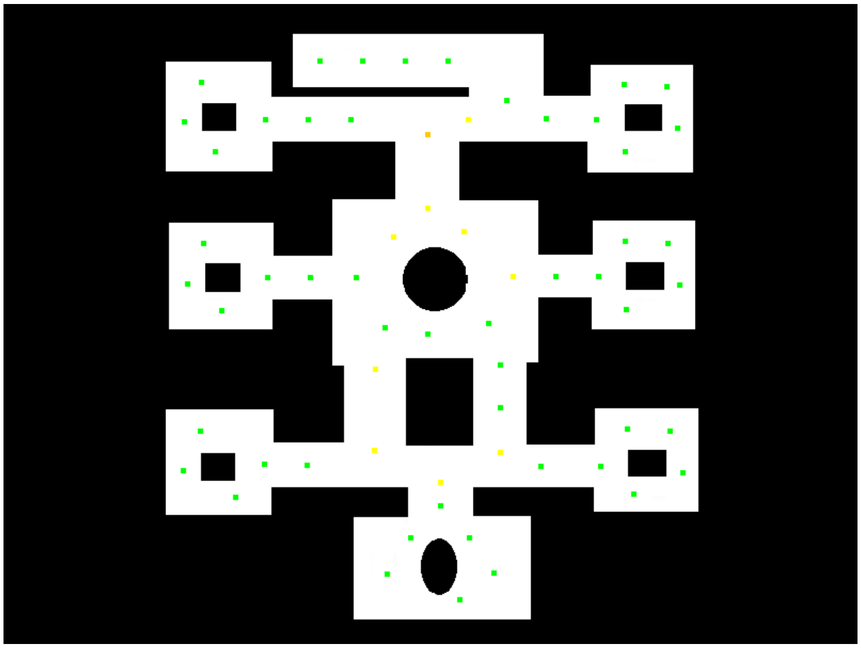
$$B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where the sum is performed over each pair of nodes in the graph. The graph needs to be connected, otherwise, each σ_{st} where s or t is a disconnected node would result in a division by zero. However, this is always granted in the context of this work because of the way in which graphs are built.

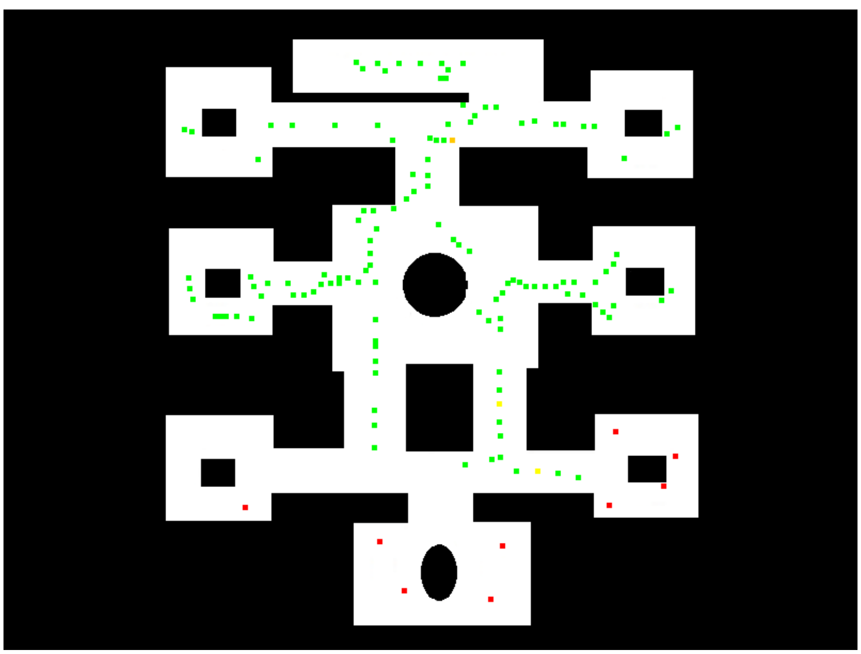
Betweenness followed an evolution similar to the one of closeness, being at first defined on unweighted graphs [3] and then extended to the case of weighted ones [4]. In the case of weighted graphs, weights impact how shortest paths are computed, making necessary the use of algorithms like Dijkstra's or Breadth-First Search to deal with them. Once the shortest paths are provided, the algorithm to compute the betweenness is the same. Similarly to closeness, where the introduction of weights on the edges only affects the computation of the distances between nodes.

The idea behind this metric is to assign higher importance to the nodes which are along more shortest paths linking couples of other nodes. In different works about social networks analysis [3], betweenness is used to find out which are the nodes having more control over the information flow. Nodes with a high value of betweenness are along more shortest paths, thus more information goes through them and an eventual disconnection may cause loss of information or a separation of the graph into two sub-graphs. However, a node of this kind is likely to be fundamental for what concerns the connectivity of the graph, differently from a node with low betweenness. According to this, the approach based on betweenness has been conceived. Being interested in an effective positioning of the idle set, a location with a high value of betweenness is an ideal candidate because it guarantees to be a crucial point for the connectivity of the environment and when the idle set is turned into active, it is likely to be in a good position to navigate towards the assigned frontier.

In Figure 4, the nodes with high betweenness are plotted over the representation of the graphs provided before. The first thing that catches the eye is the difference in the number of this kind of nodes between the topological and the visibility graphs, being a lot more in the first one. This can be justified by considering the reduced size of the graph in that case, which makes every node on more shortest paths, thus with a higher value of betweenness. Also for this metric holds what stated previously about the highest closeness node. Despite the different structures of the two graphs and the even more marked difference in the distribution of high betweenness nodes, the highest values of the metric are measured in two extremely near locations. This enforces the idea that these metrics can characterize the environment mapped and by an analysis of them, it might be even possible to discriminate among different types of it.



(a) Topological graph. Green dots are the nodes with a value of betweenness lower than the 50% of the max value.



(b) Visibility graph. Green dots are the remaining pose nodes and red ones are the frontier nodes.

Figure 4: Graphs with nodes having a high value of betweenness highlighted. Orange nodes are the ones with the highest betweenness, while in yellow nodes with a betweenness comprised between the 50% and the max value are drawn. Edges are omitted for clarity.

3 Comparative metrics

The different methods proposed are compared based both on practical measures, as the time taken to fulfill the termination criterion and the distance traveled by the robots, and on theory-based measures, namely interference among robots and their availability.

The use of time and distance traveled as comparison metrics is intuitive if looking at some of the application contexts. Teams of robots are often used in search and rescue scenarios, where the time needed to complete the exploration is a fundamental aspect to take into account. Thus, a faster approach is overall preferable to a slower one in such a scenario. The distance traveled is a less important factor in discriminating among different mechanisms, but it may provide an interesting and more complete overview of an approach compared to others.

In the simulations run in this work, the termination criterion used is the exploration of the 95% of the environment. At the end of each run, the number of discrete time steps taken and the average distance traveled by the robots are stored and then compared during the analysis of the results. The decision of considering the number of time steps rather than the time taken by the exploration has been carried on to exclude machine-dependent aspects from the results. Some of the mechanisms implemented are way more computationally intensive than the ones based on the barycenter computation, and comparing their results on the effective time taken would have been faked by the computing capability of the machine running simulations.

These two metrics have been considered sufficient to characterize from a practical point of view each mechanism analyzed. Nevertheless, two further measures are used to examine the mechanisms, which are named interference and availability. They are at first introduced in [6] and formalized in [5].

3.1 Interference

Interference quantifies the average distance held by agents during the exploration and the higher the distance, the lower the value. A high value of interference is desirable because as the average distance among robots increases, it reduces the possibility of incidents and the complexity in the management of the system. Moreover, it is also an indirect measure of how parallel the exploration is being carried on because it increases as the robots are spread on the environment. According to this, the value of interference for a particular coordination mechanism can provide useful information about the amount of parallelizability exploited as compared to other mechanisms.

The value of interference λ for an agent a is computed at each step t of the exploration by calculating the average distance between a and all the other agents a' in the team A, thus it can be formally written as

$$\lambda_t\left(a\right) = \frac{1}{N-1} \sum_{a' \in A | a' \neq a} d\left(P_t\left(a\right), P_t\left(a'\right)\right)$$

where N is the size of the team, P_t is a function providing the position of an agent at time t, while d is the function that computes the distance between two positions of the environment.

This definition only relates to one agent at a particular instant of the exploration. To completely characterize the value of interference for the whole exploration, it needs to be at first generalized over the set of agents composing the team and then over the entire time needed to complete the exploration. In this way, its value computed for a specific coordination mechanism is comparable with the ones provided by other mechanisms over the same exploration problem.

The first generalization consists in extending the definition of interference to all the robots in the team, rather than considering only a single robot, and this is simply done by averaging the values of interference of each agent:

$$\lambda_t = \frac{1}{N} \sum_{a \in A} \lambda_t \left(a \right)$$

At this point, it is possible to integrate this expression over the entire time taken by the exploration, which is considered to take values in $[0, \ldots, t_T]$ with t_T being the time step of the fulfillment of the termination criterion T. The definition of the interference for the coordination mechanism applied to a specific instance of the exploration problem considered is

$$\lambda = \frac{1}{t_T + 1} \sum_{t=0}^{t_T} \lambda_t$$

This states that the interference for the whole exploration can be computed by averaging over the total time taken the values of interference for the team. In this way, the value of interference obtained can be exploited to compare different coordination mechanisms, avoiding that differences in the duration of the exploration impact on this measure.

3.2 Availability

Availability is a measure of the distance between an agent and its assigned location. It can be formally defined at first for a single agent a in a certain time step t as

$$\alpha_t \left(a \right) = d \left(P_t \left(a \right), G_t \left(a \right) \right)$$

where α is the symbol used for the availability and G_t is a function returning the location of the frontier assigned to the agent in input. The two auxiliary functions d and P_t are the same presented in the previous section to define the interference.

Similarly to what done for the interference, this definition can be generalized to the whole team and the whole exploration. Following similar reasoning and with the same meaning of the symbols, the availability for the whole team at a certain instant t of the exploration is

$$\alpha_t = \frac{1}{N} \sum_{a \in A} \alpha_t \left(a \right)$$

recalling that A is the set of agents composing the team and N is its cardinality. Averaging this result over the whole exploration time t_T defines availability for a coordination mechanism applied to a specific instance of the exploration problem, thus it turns out to be

$$\alpha = \frac{1}{t_T + 1} \sum_{t=0}^{t_T} \alpha_t$$

This value of the availability is independent of the time taken to meet the termination criterion and therefore, allows a comparison among different coordination mechanisms, without being affected by their respective performance in terms of time. It is important to highlight how availability has the opposite trend of interference. Indeed, a mechanism with low availability assigns robots to locations near their positions, which makes the agents update the environment more frequently than a scenario in which robots have to travel a long distance before scanning unknown portions of it.

This metric has a two-fold interpretation. From one side, it shows whether a mechanism assigns robots to far or close targets. On the other side, it can provide insights on the effectiveness of the proactivity when comparing two mechanisms which differ only in this aspect. This, in particular, is the setting of this thesis. The differences of the mechanisms analyzed concerns only the proactive allocation of the idle set, and thus a mechanism that has a lower value of availability is likely to place this set of robots in a position nearer to the frontiers. Strong evidence of this relation between proactivity and availability is provided by [5] in the comparison between reserve and proactive reserve, where by moving the idle set towards the barycenter of locations of the active agents, the latter method ensures agents to travel a shorter distance once turned into active with respect to the former one.

It is worth to point out also that the absolute value of both this metric and the interference is highly affected by the particular configuration of the environment explored and by the team size, for this reason, comparisons among mechanisms based on them do make sense only if done on the same instance of exploration problem.

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