How Much Worth Is Coordination of Mobile Robots for Exploration in Search and Rescue?

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Abstract. Exploration of unknown environments is an enabling task for several applications, including map building and search and rescue. It is widely recognized that several benefits can be derived from deploying multiple mobile robots in exploration, including increased robustness and efficiency. Two main issues of multirobot exploration are the exploration strategy employed to select the most convenient observation locations the robots should reach in a partially known environment and the coordination method employed to manage the interferences between the actions performed by robots. From the literature, it is difficult to assess the relative effects of these two issues on the system performance. In this paper, we contribute to filling this gap by studying a search and rescue setting in which different coordination methods and exploration strategies are implemented and their contributions to an efficient exploration of indoor environments are comparatively evaluated. Although preliminary, our experimental data lead to the following results: the role of exploration strategies dominates that of coordination methods in determining the performance of an exploring multirobot system in a highly structured indoor environment, while the situation is reversed in a less structured indoor environment.

Keywords: search and rescue, exploration, coordination, multirobot

1 Introduction

Robotic exploration of unknown environments is fundamental for several real-world applications, including map building and search and rescue. It is widely recognized that several benefits can be derived from deploying multiple mobile robots in exploration, ranging from an increased robustness of the whole system to a more efficient exploration [1–3]. Two important issues of multirobot exploration are exploration strategies and coordination methods. An exploration strategy is employed to select the most convenient observation locations the robots should reach in a partially known environment [4]; in short an exploration strategy is used to answer the question "where to go next?". A coordination method is employed to manage the interferences between the actions

performed by robots [5]; in the context of exploration, a coordination method is used to allocate tasks to robots and basically to answer the question "who goes where?". Prior work evaluates these two issues mostly in a separated way, making it difficult to assess their relative effects on exploration.

In this paper, we contribute to fill this gap by comparatively evaluating some coordination methods and exploration strategies in a search and rescue setting according to their contribution to an efficient exploration of indoor environments. We selected the search and rescue application because there is an international competition, namely the RoboCup Rescue Virtual Robots Competition¹, that provides a simulated common ground (e.g., metrics and software tools) for assessing the performance of exploring multirobot systems, enabling comparison and reproduction of results.

The general setting we consider is the following. A team of robots has to search an initially unknown environment for victims. Since no a priori knowledge about the possible locations of the victims is assumed to be available, we can reduce the problem of maximizing the number of victims found in a given time interval to the problem of maximizing the amount of area covered by robots' sensors in the same time interval. Broadly speaking, the robots operate according to the following steps: (a) they perceive the surrounding environment, (b) they integrate the perceived data within a map representing the environment known so far, (c) they decide where to go next and who goes where, and (d) they go to their destination locations and start again from (a). In our experiments, we employ a publicly available simulator [6] and controller [7]. In this way, we can focus on the exploration strategies and coordination methods (step (c)) exploiting an already tested framework for steps (a), (b), and (d).

The original contribution of this paper is not in proposing new exploration strategies or coordination methods, but in taking some initial steps in shedding light on their relative impact on the performance of multirobot systems employed in search and rescue applications. We contribute to answer the following question: With limited computing or time resources, should developers spend more efforts on developing an effective exploration strategy or coordination method?

2 Coordinated Multirobot Exploration

Robotic exploration can be defined as a process that discovers unknown features in environments by means of mobile robots. Coordinated multirobot exploration has been mainly studied for map building [8, 9] and for search and rescue [10]. Previous works on coordinated multirobot exploration have focused in a rather separated way on evaluation of either coordination methods or exploration strategies.

Exploration strategies are used to select locations that autonomous robots should reach in order to discover the physical structure of environments that are initially unknown. In the following, we survey a representative sample of the several exploration strategies that have been proposed in literature.

¹ http://www.robocuprescue.org/virtualsim.html

Unsurprisingly, most of the work on exploration strategies for discovering the physical structure of environments has been done for map building. The mainstream approach models exploration as an incremental Next Best View (NBV) process, i.e., a repeated greedy selection of the next best observation location. Usually, at each step, an NBV system considers a number of candidate locations on the frontier between the known free space and the unexplored part of the environment (in such a way they are reachable from the current position of the robot) and selects the best one [11]. The most important feature of an exploration strategy is the *utility function* it uses to evaluate candidate locations in order to select the best one.

In evaluating candidate locations, different criteria can be used. A simple one is the distance from the current position of the robot [12], according to which the best observation location is the nearest one. Most works combine different criteria in more complex utility functions. For example, in [13] the cost of reaching a candidate location is linearly combined with its benefit. Another example of combination of different criteria is [14], in which the distance of a candidate location from the robot and the expected information gain of the candidate location are combined in an exponential function. In [15], a technique based on relative entropy is used to combine traveling cost and expected information gain. In [16], several criteria (such as uncertainty in landmark recognition and number of visible features) are combined in a multiplicative function.

The above strategies aggregate different criteria in utility functions that are defined *ad hoc* and are strongly dependent on the criteria they combine. In [17], the authors proposed a more theoretically-grounded approach based on multiobjective optimization, in which the best candidate location is selected on the Pareto frontier. Following the same theoretically-grounded approach, decision theoretical tools have been applied to the definition of exploration strategies [18]. More details on this approach will be illustrated in Section 3.2.

Compared with exploration strategies for map building, relatively few exploration strategies for autonomous search and rescue have been proposed. A work that explicitly addressed this problem is [7], which proposes to combine some criteria in an $ad\ hoc$ utility function that will be described in Section 3.2. In [10], traveling cost to reach a location is used as the main criterion for evaluating candidate locations, while the utility of the locations (calculated according to the proximity of other robots) is used as a tie-breaker. The exploration strategy for search and rescue of [19] uses a formalism based on Petri nets for exploiting $a\ priori$ information about the victims' distribution to improve the search.

In this paper, we evaluate, relatively to some coordination methods, the exploration strategies proposed in [18] and [7], as representative samples of theoretically-grounded and *ad hoc* exploration strategies, respectively.

Coordination methods are used to manage the interactions between multiple robots. Here we are interested in coordination methods that are used to allocate locations to the robots during exploration. One of the earliest works in the field of multirobot exploration is by Yamauchi [12], in which robots navigate, in an

uncoordinated way, to the closest accessible unvisited frontiers and integrate their local maps in a global map of the environment.

A series of works [1,2] (and, partially, [20]) propose an interesting approach in which the coordination method is embedded within the exploration strategy. In particular, the utility value of a candidate location is reduced according to the number of robots that can view it. In this way, robots are pushed to select different locations to reach. Experimental results show that this coordinated behavior has better performance than uncoordinated behavior (in which different robots can select the same location to reach) and slightly worse performance than a method that finds the optimal allocation of candidate locations to robots.

Coordination methods based on market mechanisms have been extensively studied. For example, in [21] coordination of mobile robots is performed by a central executive that, beyond collecting local maps and combining them into a single global map, manages an auction by asking bids to the robots and assigning tasks (i.e., locations to reach) according to the received bids. Bids contain information about expected utility for pairs robot-location; utility are calculated as the expected information gain at the location minus the cost for reaching it. A similar coordination method is presented in [22] in connection with three techniques for generating the locations that the robots should reach (a random technique, a closest-point greedy technique, and a quadtree-based technique). These points are evaluated using an utility function similar to that used in [21]. Experimental results show that the auction-based coordination method performs better with a random and a quadtree-based generation of locations, while (as expected) outperforming the uncoordinated methods. Qualitatively similar findings are reported also in [23], which proposes an auction-based coordination method not only for task assignment, but also for coalition formation.

In this paper, we evaluate, relatively to some exploration strategies, some variants of the coordination method employed in [7], which produces the same allocation of the market-based coordination method of [21]. Our results complement those of [22], by considering more complex ways for generating the locations allocated to robots.

3 The Search and Rescue Setting

In this section, we describe the search and rescue setting in which we investigated the relative impact of exploration strategies and coordination methods on performance of exploring multirobot systems. In our setting, the goal is to explore an initially unknown indoor environment for finding the largest number of human victims within a given time. Assuming no a priori knowledge about the possible locations of the victims, the problem of maximizing the number of victims found in a given time interval is equivalent to the problem of maximizing the amount of area covered by robots' sensors in the same interval. We consider a time interval of 15 minutes. We first describe the adopted simulation environment and robot controller. Then, we describe the exploration strategies and the coordination methods we consider.

3.1 The Simulation Environment and the Robot Controller

In order to perform repeated tests under controlled conditions, we use a robot simulator. We selected USARSim [6] because it is a high fidelity 3D robot simulator and it is employed in the RoboCup Rescue Virtual Robots Competition.

From an analysis based on availability of code and performance obtained in the RoboCup Rescue Virtual Robots Competition, we selected the controller developed by the Amsterdam and Oxford Universities (Amsterdam Oxford Joint Rescue Forces, AOJRF²) for the 2009 competition [24]. The main reason for using an existing controller is that we can focus only on the exploration strategies and on the coordination methods, exploiting existing and tested methods for navigation, localization, and mapping. The controller manages a team of robots. The robotic platform used is a Pioneer P2AT equipped with range scanner sensors and sensors able to detect human victims. The map of the environment is maintained by a base station, whose position is fixed in the environment, and to which robots periodically send data. The map is two-dimensional and represented by three occupancy grids. The first one is obtained with a small-range (3 meters) scanner and constitutes the safe area, i.e., the area where the robots can safely move. The second one is obtained from maximum-range scans (20 meters) and constitutes the free area, i.e., the area which is believed to be free but not yet safe. Moreover, a representation of the clear area is maintained as a subset of the safe area that has been checked for the presence of victims (this task is accomplished with simulated sensors for victim detection). Given a map represented as above, a set of (connected) boundaries between safe and free regions are extracted. The center point of the free area beyond each boundary is considered as a candidate location to reach. The utility u(p,r) of a candidate location p for a robot r is evaluated as discussed in the next section.

3.2 Exploration Strategies

As discussed in Section 2, exploration strategies differ in the utility functions they use to evaluate and select the candidate locations. The following criteria are combined in our utility functions:

- A(p) is the amount of free area beyond the frontier of p computed according to the free area occupancy grid;
- -P(p) is the probability that a robot, once reached p, will be able to transmit information (e.g., the perceived data or the locations of victims) to the base station (whose position in the environment is known), this criterion depends on the distance between p and the base station;
- -d(p,r) is the distance between p and current position of robot r, this criterion can be calculated with two different methods: $d_{EU}()$, using an approximate method that calculates the Euclidean distance, and $d_{PP}()$, using a path planner procedure that returns the exact value of the distance (if no safe path completely contained in the explored area can be found, then $d_{PP}() = \infty$); obviously, calculating $d_{PP}()$ requires more time than calculating $d_{EU}()$;

² http://www.jointrescueforces.eu/

-b(r) is the battery level of robot r (from 0, full, to 1, empty); the larger its value, the smaller the amount of residual energy in the battery.

Given these criteria, we define two exploration strategies. The first one is a slight variation of the strategy proposed in [7] and is called *AOJRF strategy*. It integrates the above criteria in an *ad hoc* utility function:

$$u(p,r) = \frac{A(p)P(p)}{d(p,r)^{b(r)}}. (1)$$

The second exploration strategy is called MCDM strategy and combines the criteria of the set $N = \{A, P, d, b\}$ using the Multi-Criteria Decision Making (MCDM) approach. Refer to [18] for a complete description; here we just sketch how the approach works. We call $u_j(p,r)$, with $j \in N$, the utility value for candidate location p and robot r according to criterion j. To apply MCDM, utilities have to be normalized to a common scale I = [0,1]. We use a linear relative normalization. For example, given a robot r, the utility of a candidate p related to the distance q (is normalized using q (in q) and q (in q) where q is the set of candidate locations. Note that the larger q (q, q), the better the pair q and q.

Basically, the MCDM strategy replaces function (1) with the following function:

$$u(p,r) = \sum_{i=1}^{4} (u_{(j)}(p,r) - u_{(j-1)}(p,r))\mu(A_{(j)}), \tag{2}$$

where $\mu: \mathcal{P}(N) \to [0,1]$ ($\mathcal{P}(N)$ is the power set of set N) is such that $\mu(\{\emptyset\}) = 0$, $\mu(N) = 1$, and, if $A \subset B \subset N$, then $\mu(A) \leq \mu(B)$. That is, μ is a normalized fuzzy measure on the set of criteria N that will be used to associate a weight to each group of criteria. $u_{(j)}$, with $(j) \in N$, indicates the j-th criterion according to an increasing ordering with respect to utilities, i.e., after that criteria have been ordered to have, for candidate p and robot r, $u_{(1)}(p,r) \leq \ldots \leq u_{(n)}(p,r) \leq 1$. It is assumed that $u_{(0)}(p,r) = 0$. Finally, the set $A_{(j)}$ is defined as $A_{(j)} = \{i \in N | u_{(j)}(p,r) \leq u_i(p,r) \leq u_{(n)}(p,r) \}$.

Using (2) is a more principled way than (1) to compute utilities, because it allows to consider criteria's importance and their mutual dependency relations. Criteria belonging to a group $G \subseteq N$ are said to be redundant if $\mu(G) < \sum_{i \in G} \mu(i)$, synergic if $\mu(G) > \sum_{i \in G} \mu(i)$, and independent otherwise.

We use the weights reported in the following table, which have been manually set in order to obtain good performance (according to [18]).

criteria	A	d	P	b	A, d	A, P	A, b	d, P	d, b	P, b	A, d, P	A, d, b	A, P, b	d, P, b
$\mu()$	0.4	0.3	0.05	0.25	0.75	0.55	0.55	0.4	0.32	0.28	0.9	0.8	0.85	0.4

The two exploration strategies have been selected because they are representative of the two main classes of strategies that have been proposed for exploration of unknown environments (see Section 2). In particular, the AOJRF strategy represents ad hoc strategies, while the MDCM strategy represents more theoretically-grounded strategies.

3.3 Coordination Methods

While exploration strategies evaluate the goodness of a candidate location p for a robot r, coordination methods are used to assign candidate locations to robots. We define three coordination methods for allocating candidate locations to robots. They start from a set of candidate locations (generated as discussed in Section 3.1) and a set of robots, and their goal is to assign a location to each robot.

The first coordination method, which is executed by each robot independently, knowing (from the base station) the current map and the positions of the other robots is derived directly from [7]:

- 1. compute the global utility u(p,r) of allocating each candidate p to each robot r (using (1) or (2)) where d(p,r) is calculated using the Euclidean distance $d_{EU}()$ (namely using an underestimate of the real distance),
- 2. find the pair (p^*, r^*) such that the previously computed utility is maximum, $(p^*, r^*) = \arg \max_{p,r} u(p, r)$,
- 3. re-compute the distance between p^* and r^* using $d_{PP}()$ with the path planner (namely considering the real distance) and update the utility of (p^*, r^*) using such exact value instead of the Euclidean distance,
- 4. if (p^*, r^*) is still the best allocation, then allocate location p^* to robot r^* , otherwise go to Step 2,
- 5. eliminate robot r^* and candidate p^* and go to Step 2.

This first coordination method is called AOJRF original coordination. The reason behind the utility update of Step 3 is that computing $d_{PP}()$ requires a considerable amount of time. Calculating it for all the candidate locations and all robots would be not affordable in the rescue competition, since a maximum exploration time is enforced. Although in pathological cases all pairs (p, r) could be re-evaluated, in practice this is done only for few of them. Note that, being $d_{EU}()$ an underestimate of the real distance and being (1) and (2) monotonically decreasing with d(), the method is guaranteed to select the best pair (p^*, r^*) according to u() calculated with $d_{PP}()$.

The AOJRF original coordination method produces the same results of the market-based mechanism proposed in [21] and applied considering (1) or (2) to calculate utilities for bids. Both methods first select the pair (p^*, r^*) with the largest utility u(), then, among the pairs left after elimination of those involving p^* and r^* , they select the pair (p^{**}, r^{**}) with the largest utility, and so on.

The second coordination method, called *AOJRF simplified coordination*, is similar to the previous one, but does not re-compute the distance in the Step 3. It selects the best pair (p^*, r^*) only on the basis of the Euclidean distance.

In the third coordination method, called *no coordination*, each robot selfishly selects its best candidate location, without considering the presence of other robots. This means that Steps 1-4 are performed only for one robot r^* (the robot that is running the method) and that Step 5 is skipped. Note, however, that Step 3 is executed and distance re-computed.

The three coordination methods are in decreasing order of "optimality" in allocating locations to the robots, with the AOJRF original coordination method producing the best allocation and the no coordination method the worst. The last two methods can end up with sub-optimal allocations in which a robot is assigned a location that is supposed to be close but is actually far (AOJRF simplified coordination method) or in which two robots are assigned the same location (no coordination method).

4 Experimental Results

We consider teams of two and three robots (plus the base station) deployed in the "DM-compWorldDay4b_250" and "DM-VMAC1" environments, called office and open environments, respectively (see Fig. 1). Both the environments are indoor with the office environment (about 800 m²) presenting an intricate cluttered structure and the open environment (about 1300 m²) presenting more open spaces. We define a configuration as an environment, a number of robots, an exploration strategy, and a coordination method. For each configuration, we execute 5 runs (with randomly selected starting locations for the mobile robots such that they are separated by about 20 meters) of 15 minutes each. We assess performance by measuring the amount of free, safe, and clear area every 30 seconds of the exploration. Due to space limitations, we report only data on safe area at the end of runs (free area is less significant and clear area is similar to the safe area). Of course, the larger the mapped safe area within 15 minutes, the better the performance. Under the assumption that victims are uniformly spread in the environment, this metric is basically equivalent to the metric that counts the number of victims found. Experiments have been run in real-time as in the competition, to realistically account for time spent in movements and in computation.



Fig. 1. The office environment (left) and the open environment (right)

To have a base line in comparing the results, we consider a *random coordination* method that randomly assigns robots to candidate locations, without evaluating them. We expect this random method to perform worse than other combinations of exploration strategies and coordination methods.

Tab. 1(a) shows results for the office environment. With all the three coordination methods, the MCDM strategy seems to behave better than the AOJRF strategy, although differences are not statistically significant, according to an ANOVA analysis with a threshold for significance p-value < 0.05 [25]. The difference between the safe area mapped at the end of the 15 minutes is more evident with the AOJRF original coordination method. Conversely, the difference between the two exploration strategies is less evident with the AOJRF simplified coordination method. These results can be explained by saying that MCDM better exploits the more precise information used with the AOJRF original coordination method (a precise distance value obtained with path planning procedures instead of an approximate Euclidean distance value). Multirobot exploration introduces some benefits, as shown by the configurations with three robots that consistently outperform those with two robots (consider that a single robot maps approximately 250 m² with the MCDM strategy and 230 m² with the AOJRF strategy). Finally, the random method has, as expected, the worst performance (we tested it only with two robots).

(a) office environment

	2 ro	bots	3 robots		
	AOJRF strategy	MCDM strategy	AOJRF strategy	MCDM strategy	
AOJRF original coordination	299.77(53.60)		()	387.41(66.67)	
AOJRF simplified coordination	257.53(54.65)	262.43(15.62)	320.40(63.71)	325.14(42.21)	
	()	()	332.58(42.03)	374.28(40.31)	
random	211.68(18.86)	211.68(18.86)			

(b) open environment

	2 ro		3 robots		
	AOJRF strategy	MCDM strategy	AOJRF strategy	MCDM strategy	
AOJRF original coordination	430.18(78.86)	498.45(51.12)	483.46(130.14)	511.83(118.35)	
AOJRF simplified coordination	()		673.48.77(85.61)		
		425.05(99.01)	458.55(80.30)	498.08(81.03)	
random	472.71(115.48)	472.71(115.48)			

Table 1. Average safe area (and standard deviation) mapped after 15 minutes (units are m²)

The performance of the AOJRF original coordination method and that of the method without coordination are very similar and better than that of the AOJRF simplified coordination method. The difference between the safe area mapped at 15 minutes with the AOJRF original coordination and with the AOJRF simplified coordination methods is statistically significant for the MCDM strategy (p-value= $2.05 \cdot 10^{-5}$ for two robots and p-value= 0.04321 for three robots), but not for the AOJRF strategy (p-value= 0.25 for two robots and p-value= 0.6972 for three robots). Similarly, the difference between the no coordination and the AOJRF simplified coordination methods is statistically significant for the MCDM strategy (p-value= 0.0147 for two robots and p-value= 0.0485 for

three robots), but not for the AOJRF strategy (p-value= 0.23797 for two robots and p-value= 0.7304 for three robots).

Tab. 1(b) shows the results for the open environment. Also in this case, the MCDM strategy seems to behave better than the AOJRF strategy with all the three coordination methods, although differences are not statistically significant. The difference between the safe area mapped at the end of the 15 minutes is more evident with the no coordination method, suggesting that a theoretically-grounded exploration strategy like MCDM can be more effective in limiting the problems of uncoordinated robots in the open environment.

In the open environment, the AOJRF simplified coordination method outperforms the other methods. The difference between the safe area mapped at 15 minutes with the AOJRF simplified coordination and with the AOJRF original coordination methods is statistically significant both for the MCDM strategy (p-value= $5.00 \cdot 10^{-4}$ for two robots and p-value= 0.0123 for three robots) and for the AOJRF strategy (p-value= 0.0113 for two robots and p-value= 0.02594 for three robots). Similarly, the difference between the AOJRF simplified coordination and the no coordination methods is statistically significant both for the MCDM strategy (p-value= $9.00 \cdot 10^{-4}$ for two robots and p-value= 0.00131 for three robots) and for the AOJRF strategy (p-value= $8.00 \cdot 10^{-4}$ for two robots and p-value= 0.0035 for three robots).

The results for the office environment are rather surprising: coordinately allocating tasks to robots and allocating tasks without any coordination lead to the same performance. Although the initial separation of robots could help to decompose the problem, this observation can be explained by saying that what is predominantly important in exploring the highly structured office environment is the quality of the information used to evaluate the candidate locations (like the distance returned by path planning procedures instead of the Euclidean distance). This result does not contradict previous results that concluded that coordinated robots perform better than uncoordinated robots (see Section 2). It seems rather to complement previous works, which considered much simpler exploration strategies than those used in this paper. The use of exploration strategies, like MCDM and AOJRF strategies, that efficiently exploit good quality information to select observation locations effectively balances computational effort and accuracy of information. Indeed, although obtaining more accurate information (i.e., planning a path between the current location of the robot and the candidate location) requires more time and could represent a problem with the 15 minutes deadline, the resulting selection of a good observation location has a global benefit in highly structured environments.

The results for the open environment suggest that coordination becomes more important when the environment is less structured. This can be explained by noting that, in the office environment, robots can choose from many candidate locations and the intricate structure of the environment "pushes" robots to spread, while, in the open environment, the number of candidate locations is smaller and robots need to be coordinated to effectively spread across the environment and map it. Accordingly, in the open environment, the worst per-

formance is obtained with the no coordination method, which is outperformed also by the random method, suggesting that assigning candidate locations randomly to robots is more effective than letting robots independently choosing their best candidate locations. In the open environment, the quality of information seems not so important (AOJRF simplified coordination method using Euclidean distance outperforms AOJRF original coordination method using distance returned by path planning procedures), mainly because obtaining accurate information requires some efforts, thus leaving less time to exploration, which can be performed quickly in uncluttered open environments.

5 Conclusion

This paper offered a first contribution to assess the relative influence of exploration strategies and coordination methods on the performance of multirobot systems employed in search and rescue applications. One of our results is that the quality of information used to evaluate candidate locations seems more relevant than assigning locations to robots in a coordinated way for a highly structured indoor environment. We are not claiming that coordination is useless, but that, in some settings, its impact on the exploration performance is less important than that of exploration strategies. From the other hand, in a less structured environment, coordination methods have a stronger impact than exploration strategies on the amount of area discovered.

The above conclusions are not yet definitive and need more efforts to be further assessed. For example, larger multirobot systems and other environments, exploration strategies, coordination methods, and integrated approaches will be considered. Also, more realistic situations involving real physical robots (with issues like damaged robots and loss of communication) and human disaster response teams will be considered. Finally, generalization of the outcomes of this paper to other applications involving exploration (like map building, where the quality of the map is an issue) could be investigated.

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References

- Burgard, W., Moors, M., Schneider, F.: Coordinated multi-robot exploration. IEEE T ROBOT 21(3) (2005) 376–378
- Burgard, W., Fox, D., Moors, M., Simmons, R., Thrun, S.: Collaborative multirobot exploration. In: Proc. ICRA. (2000) 476–481
- 3. Sariel, S., Balch, T.: Real time auction based allocation of tasks for multi-robot exploration problem in dynamic environments. In: Proc. AAAI Workshop on Integrating Planning and Scheduling. (2005) 27–33

- Amigoni, F.: Experimental evaluation of some exploration strategies for mobile robots. In: Proc. ICRA. (2008) 2818–2823
- Gerkey, B., Mataric, M.: A formal analysis and taxonomy of task allocation in multi-robot systems. INT J ROBOT RES 23 (2004) 939–954
- Carpin, S., Lewis, M., Wang, J., Balakirsky, S., Scrapper, C.: USARSim: A robot simulator for research and education. In: Proc. ICRA. (2007) 1400–1405
- Visser, A., Slamet, B.: Including communication success in the estimation of information gain for multi-robot exploration. In: Proc. WiOPT. (2008) 680–687
- 8. Lopez-Sanchez, M., Esteva, F., Lopez de Mantaras, R., Sierra, C., Amat, J.: Map generation by cooperative low-cost robots in structured unknown environments. AUTON ROBOT 5 (1998) 53–61
- 9. Ko, J., Stewart, B., Fox, D., Konolige, K., Limketkai, B.: A practical, decision-theoretic approach to multi-robot mapping and exploration. In: Proc. IROS. (2003)
- Marjovi, A., Nunes, J., Marques, L., de Almeida, A.: Multi-robot exploration and fire searching. In: Proc. IROS. (2009) 1929–1934
- 11. Tovey, C., Koenig, S.: Improved analysis of greedy mapping. In: Proc. IROS. (2003) 3251–3257
- 12. Yamauchi, B.: Frontier-based exploration using multiple robots. In: Proc. Int'l Conf. Autonomous Agents. (1998) 47–53
- 13. Stachniss, C., Burgard, W.: Exploring unknown environments with mobile robots using coverage maps. In: Proc. IJCAI. (2003) 1127–1134
- 14. Gonzáles-Baños, H., Latombe, J.C.: Navigation strategies for exploring indoor environments. INT J ROBOT RES **21**(10-11) (2002) 829–848
- Amigoni, F., Caglioti, V.: An information-based exploration strategy for environment mapping with mobile robots. ROBOT AUTON SYST 5(58) (2010) 684–699
- Tovar, B., Munoz, L., Murrieta-Cid, R., Alencastre, M., Monroy, R., Hutchinson,
 S.: Planning exploration strategies for simultaneous localization and mapping.
 ROBOT AUTON SYST 54(4) (2006) 314–331
- Amigoni, F., Gallo, A.: A multi-objective exploration strategy for mobile robots.
 In: Proc. ICRA. (2005) 3861–3866
- 18. Basilico, N., Amigoni, F.: Exploration strategies based on multi-criteria decision making for searching environments in rescue operations. AUTON ROBOT $\bf 31(4)$ (2011) 401–417
- 19. Calisi, D., Farinelli, A., Iocchi, L., Nardi, D.: Multi-objective exploration and search for autonomous rescue robots. J FIELD ROBOT 24(8-9) (2007) 763–777
- 20. Fox, D., Ko, J., Konolige, K., Limketkai, B., Schulz, D., Stewart, B.: Distributed multirobot exploration and mapping. PROC IEEE **94**(7) (2006) 1325–1339
- 21. Simmons, R., Apfelbaum, D., Burgard, W., Fox, D., Moors, M., Thrun, S., Younes, H.: Coordination for multi-robot exploration and mapping. In: Proc. AAAI. (2000)
- 22. Zlot, R., Stentz, A., Dias, M.B., Thayer, S.: Multi-robot exploration controlled by a market economy. In: Proc. ICRA. (2002) 3016–3023
- 23. Hawley, J., Butler, Z.: Hierarchical distributed task allocation for multi-robot exploration. In: Proc. DARS. (2010)
- 24. Visser, A., de Buy Wenniger, G., Nijhuis, H., Alnajar, F., Huijten, B., van der Velden, M., Josemans, W., Terwijn, B., Sobolewski, R., Flynn, H., de Hoog, J.: Amsterdam Oxford joint rescue forces Team description paper RoboCup 2009. In: Proc. RoboCup. (2009)
- 25. Pestman, W.: Mathematical Statistics: an Introduction. de Gruyter (1998)