

Cell Resolved Matlab® OOP Model of a Lithium Iron Phosphate Battery Pack

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Accessible properties of the dischargeFit class

Acronyms

BMS battery management system

EOL end of life

MEX Matlab® executable

OO object oriented

OOP object oriented programming

SP strings of parallel elements

PS parallel strings

List of Symbols

Symbol	Unit	Description
A_{c}	_	battery's age
A_{cal}	-	calendar ageing
$A_{c,eol}$	-	age at which the end of life is reached
A_{cyc}	-	cycle ageing
$A_{\sf tot}$	-	total ageing stress
C	Ah	battery capacity
C_{bu}	Ah	usable capacity
C_{dis}	Ah	discharge capacity
cDoC	%	cycle-depth-of-cycle
C_{n}	Ah	nominal capacity
DoD	-	depth of discharge
F	As/mol	Faraday constant
I	Α	current
$I_{\sf max}$	А	battery's maximum current
L_{cal}	а	calendar life
N_{f}	-	number of cycles to failure
P	W	power
$P_{\sf sd}$	W	self-discharge
p_{z}	-	impedance proportionality factor
SoC	%	state of charge
$SoC_{\sf max}$	%	$maximum\ SoC$
$SoC_{max,l}$	%	local maximum in an SoC profile
SoC_{min}	%	$minimum\ SoC$
SoH	%	state of health
R	$J/(mol \cdot K)$	universal gas constant
rmse	various	root mean squared error
T	K or $^{\circ}$ C	temperature
$t_{\sf s}$	S	simulation time step
V	V	voltage
V_{n}	V	nominal voltage
Z_{i}	Ω	internal impedance
z_{Li}	-	ionic charge number of lithium
$\Delta t_{\sf s}$	S	simulation time step size



1 Discharge curves

Many battery data sheets provide measured discharge curves, on which the charging and discharging behaviour of this model is based. Rather than determining the curves according to the internal impedance, a common approach [1], this model determins the curves directly by means of digitizing the images and creating a curve fit. The classes used for fitting and modelling the discharge curves are described in the following subsections.

1.1 Single discharge curve

For modelling a single discharge curve, the class dischargeFit is used, which implements the interface curveFitInterface. The curve is fitted according to [2], using a function that is loosely based on the Nernst equation with two exonential functions superimposed as a correction for the voltage drops at the beginning and end of the curve.

$$V(SoC) = x_1 - \frac{R \cdot T}{z_{\text{Li}} \cdot F} \cdot ln\left(\frac{SoC}{1 - SoC}\right) + x_2 \cdot SoC + x_3$$
$$+ (x_4 + (x_5 + x_4 \cdot x_6) \cdot SoC) \cdot exp(-x_6 \cdot SoC)$$
$$+ x_7 \cdot exp(-x_8 \cdot SoC)$$
(1)

where $x_1, ..., x_8$ are the fit parameters, $R=8.3144598 \text{ J/(mol} \cdot \text{K)}$ is the universal gas constant, $z_{\text{Li}}=1$ is the ionic charge number of lithium, F=96485.3328959 As/mol is the Faraday constant, SoC is the state of charge, V is the voltage in V and T is the temperature in K at which the curve was recorded. The curves are fitted using the levenberg-marquardt algorithm and either the <code>lsqcurvefit</code> method, the <code>fminsearch</code> method or a combination of both, depending on the user's preference.

1.1.1 Creation of a dischargeFit object

A dischargeFit object is created with the digitized raw data - the voltage V in V, the discharge capacity C_{dis} in Ah, the current I in A at which the curve was recorded and the temperature T in K at which the curve was recorded.

```
1 d = dischargeFit(V, C_dis, I, T);
```

V and C_dis are vectors containing the digitized raw data from the data sheet. Further options, such as initial values for the fit parameters $x_1, ..., x_8$ and the fit method can be passed to the constructor using Matlab's name-value pair syntax:

```
1 d = dischargeFit(V, C_dis, I, T, 'OptionName', OptionValue);
```

By default, the initial fit parameters are set to zero and the curve is fit by first using lsqcurvefit, followed by fminsearch. The initial fit parameters are stored in a vector x0

Section describing interface, etc.



of length 8, which can be passed via the option name 'x0', for example using the following syntax:

```
1 x0 = ones(8, 1);
2 d = dischargeFit(V, C_dis, I, T, 'x0', x0);
```

The method used for the curve fitting can be passed to the constructor using the option name 'mode'. The corresponding value must be one of the following three strings:

- 'lsq' for lsqcurvefit
- 'fmin' for fminsearch
- 'both' for lsqcurvefit followed by another fit using fminsearch

e.g.

```
1 d = dischargeFit(V, C_dis, I, T, 'mode', 'fmin');
2 d.plotResults
```

Depending on the curve and on the technology, one of the methods may return a better result.

1.1.2 Visual validation

A visual validation can be performed by calling the class's plotResults method (see above). In Figure 1, the results of two dischargeFit objects using the same raw data are compared.

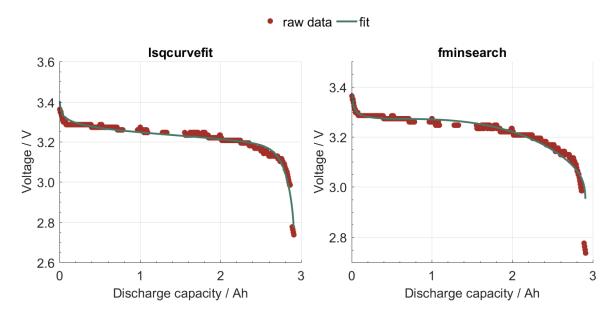


Figure 1: Fit results of the dischargeFit class using the fit methods lsqcurvefit and fminsearch, respectively. The raw data was extracted from [3].





Figure 2: Fit results of the dischargeFit class using the fit mode 'both' with the default parameter initialization (left) and with a custom parameter initialization (right). The raw data was extracted from [3].

In this example, 'lsq' appears to return better results for the voltage drop at the end of the curve, while 'fmin' results in a more precise fit for the voltage drop at the beginning of the curve. Further differences can be seen in the fits' curvatures. The 'lsq' option results in a slightly flatter curve than the 'fmin' mode. The results of a dischargeFit object using the 'both' option are presented in Figure 2. Using the default fit parameter initialization of zeros (left) appears to improve the curvature and voltage drops slightly, compared to the other modes. Further improvements can be made by passing custom initial fit parameters to the constructor via the option 'x0' (see Figure 2, right).

1.1.3 Object properties

Further fit quality analysis can be performed via the mean difference in voltage between the raw data and the curve fit at the respective positions of the raw data $\overline{\Delta V}$ in V and the maximum difference between the raw data and the curve fit at the respective positions $\Delta V_{\rm max}$ in V. Additionally, every curve fit class (i.e. dischargeCurves, woehlerFit, etc.) in this package implements the curveFitInterface, which contains the root mean square error rmse as a property. The rmse for a curve fit with the raw data $y_{\rm raw}$ and the fitted data $y_{\rm fit}$ at the same respective x coordinates is defined as

$$rmse = \sqrt{\frac{\sum_{i=1}^{n} (|y_{\mathsf{raw},i} - y_{\mathsf{fit},i}|)^2}{n}} \tag{2}$$

where i is the index of the measurement and n is the number of measurements. In the case of a dischargeFit object, $y_{\text{raw},i}$ is the measured voltage at the discharge capacity $C_{\text{dis},i}$ and $y_{\text{fit},i}$ is the fitted voltage at $C_{\text{dis},i}$. Often used for forecasting models, the rmse provides



	Table 1:	: Accessible	properties	of the	discharc	geFit. cla :	SS.
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Name	Description	Unit	Set access
Х	8x1 vector of fit parameters	-	public
dV_mean	Mean voltage difference between raw data and fit	V	read only
dV_max	Max voltage difference between raw data and fit	V	read only
T	Temperature at which the curve was recorded	K	immutable
Z	Current of the curve	Α	immutable
mode	Method used for fitting ('fmin', 'lsq' or 'both')	-	public
rmse	Root mean square error	V	read only

a good measure of accuracy when comparing two models of the same data set [4]. In the previous examples, the curve fit using the 'lsq' method (Figure 1, left) has an rmse of 0.0244 V. Using the 'fmin' mode (Figure 1, right) improves the rmse to a value of 0.0162 V and using the fit mode 'both' (Figure 2, left) further improves it to 0.0157 V. The lowest rmse (0.0106 V) is achieved with the custom fit parameter initialization (Figure 2, right). A list of the class's accessible properties is provided in Table 1. The z property is inherited from the curveFitInterface. Setting the x or mode properties will cause the object to re-run the fitting algorithm, thus likely resulting in different values for x than were set by the user.

1.1.4 Usage of a dischargeFit object

In order to calculate a voltage for a given discharge capacity, the object can be treated like a function handle, by using subsref indexing.

```
1 d = dischargeFit(V, C_dis, I, T, 'mode', 'fmin');
2 Cd = 1.5; % Discharge capacity in Ah
3 V = d(Cd); % Voltage in V
4 Cd_vect = linspace(0, 3, 1000); % Vector of discharge capacities in Ah
5 V_vect = d(Cd_vect); % Corresponding vector of voltages in V
```

A dischargeFit object is not accessed directly by the battery model, but rather stored in a dischargeCurves object. After creating a dischargeFit, it can be added to a dischargeCurves collection by using the add() method (see section 1.2). Alternatively, it can be added directly to a subclass of the batteryInterface (see section) using it's addcurves() method.

section ref



1.2 Collection of discharge curves

A single discharge curve can be used to model the behaviour of a battery for a given current. However, in reality, a battery will often be charged or discharged with different currents. In many cases, the current may change from one simulation time step to another. In order to be able to determine the voltage as a function of $C_{\rm dis}$ and I, multiple dischargeFit objects are wrapped by a dischargeCurves object, which is described in the following sections.

1.2.1 Creation of a dischargeCurves object

There are two ways to initialize a dischargeCurves object. The first option is to create an empty object and using the class's dischargeFit() method to add curve fits. The dischargeFit() method has the same syntax as the dischargeFit class's constructor.

```
1 dC = dischargeCurves;
2 I = [0.6; 1; 3; 5; 10; 20]; % Vector of currents in A
3 T = 293; % Temperature in K
4 for i = 1:6
5     dC.dischargeFit(raw(i).V, raw(i).Cd, I(i), T)
6 end
7 % raw is a struct array containing the measured curve data
8 % from the data sheet.
```

This option has the advantage of reducing clutter in the workspace. However, changing the parameters and analysing the accuracy of the individual curve fits is more complicated. Alternatively, the dischargeFit objects can be created, modified and then passed to the dischargeCurves constructor.

If a dischargeFit is passed to a dischargeCurves object that already holds a reference to a dischargeFit with the same current, the stored reference is replaced by the new one. Similarly, if two or more dischargeFit objects with the same current are passed to a dischargeCurves constructor, the first one is ignored.



1.2.2 Interpolation between curves

The calculation of the voltage for any given current and discharge capacity is done via Matlab's built-in griddedInterpolant class, which is called from within the interp() method. The syntax for a dischargeCurves object dC is as follows:

```
1 V = dC.interp(I, Cd);
2 V = interp(dC, I, Cd); % equivalent
```

Where V is the voltage in V, I is the charging or discharging current in A and Cd is the discharge capacity after charging or discharging in Ah. If I is equal to one of the stored dischargeFit objects' currents, Cd is simply passed on to the respective object, which returns the voltage. If I does not match any of the stored objects and either of the input arguments is not found in the object's cache, Cd is passed on to each of the stored dischargeFit references, creating a vector of voltages for the different currents. Finally, a griddedInterpolant is created using the sample points, I is passed to it and the interpolated voltage is returned and cached. The interpolation method (the default is 'spline') can be changed by setting the property interpMethod.

A visual validation of the interpolation using the 'linear' and 'spline' methods, respectively, is depicted in Figure 3. A collection of dischargeFit objects for six currents was created and the fit results were plotted. Then, all fits except for the one at 10 A were added to a dischargeCurves object. Finally, the interp() method was called for a current of 10 A and a range of discharge capacities, in an attempt to replicate the dischargeFit results using interpolation. The linearly interpolated curve (Figure 3, left) is almost identical to the fit until the beginning of the voltage drop at the end. However, the spline interpolation results



Figure 3: Comparison of the dischargeCurves results using linear interpolation and spline interpolation, respectively. The raw data was extracted from [3].





Figure 4: Result of the interp() method for a current below the lowest measured current without output limitation (left) and with output limitation (right). The raw data was extracted from [3].

in an overall more precise replication of the fit if the entire curve is regarded. This indicates that the most suitable interpolation method may depend on the maximum depth of discharge DoD of the modelled battery. As can be seen in Figure 3, the interpolation bends slightly at the end of the curve (close to a discharge capacity of 3 Ah). This is due to the fact that the voltage returned by a dischargeFit object is limited to the minimum and maximum of the raw data, respectively. If it were not limited, it could return -Inf or Inf, causing the interpolation to fail. Since most lithium ion batteries' DoD are limited to 0.8 or 0.9, this bend should rarely cause any issues.

As demonstrated Figure 4 (left), the interp method using spline interpolation does not provide a good extrapolation of currents. In order to correct this, the voltage output is limited by the curve fit with the lowest current I_{\min} and by the curve fit with the highest current I_{\max} , respectively. As a result, the dischargeFit recorded at I_{\min} is called for any current below I_{\min} (see Figure 4, left) and the dischargeFit recorded at I_{\max} is called for any current above I_{\max} . In this model, the battery's maximum discharging current is limited by the dischargeCurves object's I_{\max} (see section).

ref section

1.2.3 Usage of a dischargeCurves object

Similarly to a dischargeFit, the results of a dischargeCurves object can be visually validated using the plotResults() method. Individual curve fit references removed using the remove() method and the respective currents.



```
1 % d = dischargeFit object
2 % I = current
3 dC.add(d) % add d to dischargeCurves dC
4 dC.remove(I) % remove the dischargeFit object with current I from dC
```

In order to access the dischargeFit references stored within a dischargeCurves object, the createIterator() method can be used. This creates an iterator object, a Matlab[®] implementation of the java.util.iterator interface [5]. The object can be used to iterate through the wrapped dischargeFit objects using a similar syntax to that of a JAVA[™] iterator.

```
1 it = dC.createIterator; % returns an scIterator object
2 while it.hasNext % returns true if there is another object to
3 % iterate through
4 d = it.next; % returns a dischargeFit object
5 % more code here
6 end
7 it.reset % resets the scIterator
```

For usage in a battery model, a dischargeCurves object is passed to an implementation of the batteryInterface (see section) using it's addCurves() method.

section



2 Age model

The age model is implemented using the Observer design pattern via Matlab's "Events and Listenersⁱ" [6]. This way, various age models (predefined or custom) can be dynamically added to a battery model at run time or even left out completely. Ageing can be simulated on the battery pack level (by treating all cells as one entity) or on the cell level (by observing each cell separately)ⁱⁱ. The event oriented age model provided in this package is based solely on cycle counting, for which a mathematical approach developed by [7] is implemented. Descriptions of the counting algorithm and the classes used to implement the age model are provided in the following sections.

ref section in footnote

2.1 Overview

Cycle counting algorithms are designed to count cycles from a set of measured data. A challenge for a running simulation or a battery management system (BMS) that relies on cycle counting is to decide when to count the cycles of an accumulated data set. Counting could be done at fixed time intervals or it could be triggered by a certain event. The latter is the approach implemented by the cycleCounter interface, which acts both as an observer of a battery cell or pack as well as a subject for the eoAgeModel class.

The observation of charge cycles is handled by the abstract cycleCounter interface, in which all methods except for the count () method are predefined. An object that implements the interface is regularly updated with the observed battery's SoC, which is stored within the object's memory. The cycle counting occurs every time the SoC reaches an upper threshold, i.e. the observed battery's maximum SoC. After counting, the NewCycle event is triggered, causing all of the object's observers (i.e. an eoAgeModel object) to be notified that new data is available for simulation. The age model then uses the data to determine the battery's new state of health SoH and passes it on to the battery.

An Observer Pattern class diagram of the age model is depicted in Figure 5. The observation is handled by the respective abstract interfaces, while the actual simulation is handled by the implementations. This makes the model highly flexible. For example, a lightweight implementation could be to observe a batteryPack using a single cycleCounter and a single batteryAgeModel. Another option could be to use multiple cycleCounter and ageModel objects in order to simulate the ageing of each batteryCell within a pack individually. The cycle counting can be implemented by various algorithms (two are provided in this package). And advanced users could even replace the default age model implementation (eoAgeModel) with a custom class that takes other factors into account, e.g. calendar ageing or thermal influences. In large simulations, it may be of interest to neglect the battery ageing in order to save simulation time. This can either be done by simply not linking up the components

in Matlab[®], an observer is often referred to as a "listener". However, "observer" is the more common term in OOP design pattern terminology and will be used throughout this documentation.

iisee section





Figure 5: Overview of the Observer implementation of the age model with communication flows and inheritance links.

at runtime or by including a dummyCycleCounter and a dummyAgeModel. These classes implement the cycleCounter and batteryAgeModel interfaces, respectively. However, calling their methods does nothing. The former option is faster, due to reduced method overhead, but the latter may be more robust in some cases.

2.2 Cycle counting

In this pack, two classes have been created to implement the cycleCounter's count() method. A cycleCounter subclass can be constructed in one of the following ways:

```
1 c = cycleCounter; % sets the initial SoC to 0.2 and the max. SoC to 1
2 c = cycleCounter(init_soc); % sets the initial SoC
3 c = cycleCounter(init_soc, soc_max); % sets the initial SoC and the
4 % max. SoC
```



where cycleCounter must be replaced with the name of the respective class that is being constructed (e.g. dambrowskiCounter or rainflowCounter). To register the object as an observer of a battery object batⁱ, the initAgeModel() method can be used.

ref section

```
1 % Extract intitial SoC and max. SoC from battery
2 init_soc = bat.SoC;
3 soc_max = bat.socMax;
4 % Replace "cycleCounter" with the respective subclass
5 c = cycleCounter(init_soc, soc_max);
6 % Intitialize event oriented age model with cycle counter c
7 bat.initAgeModel('ageModel', 'EO', 'cycleCounter', c)
```

Note that the 'ageModel' option must be specified, otherwise bat will internally replace c with a dummyCycleCounter object to prevent runtime errors. If this happens, a warning message is printed to the command window.

2.2.1 The rainflowCounter class

The state of the art algorithm for cycle counting, "rainflow", was originally developed for mechanical stress modelling [8] and has recently become popular in the field of battery charge cycle counting [9]. The rainflowCounter class was added to this package for the purpose of demonstrating the flexibility of the age model implementation. It acts as an adapter for the popular FileExchange contribution, "Rainflow Counting Algorithm" by Adam Nieslony [10]. In order for the class to work, the MEX functions must be downloaded from [10] and placed within Matlab's search path. They are not included in this package and attempting to construct a rainflowCounter object will fail if they are not found. To register a rainflowCounter with a battery bat, the above syntax must be used, whereby cycleCounter (init_soc, ... soc_max) is replaced by rainflowCounter (init_soc, soc_max).

2.2.2 The dambrowskiCounter class

In 2012, J. Dambrowski, S. Pichlmaier and A. Jossen developed a mathematical definition of a battery's charge cycles along with an algorithm for counting them [7]. Since the counting algorithm was not named, the dambrowskiCounter class that implements it in this package was named after one of the authors. In their approach, so-called pre-cycles are counted and compared with each other. This is visualized in Figure 6. The twice depicted SoC curve (grey) has two local maxima $SoC_{\max,l,i}$, since the last value is counted as a local maximum. Starting from an $SoC_{\max,l,i}$, a pre-cycle of "prior equality" is defined as the SoC within an interval between the respective $SoC_{\max,l}$ and the last point at which the SoC was equal to $SoC_{\max,l}$. Two such pre-cycles are depicted in Figure 6 (left) and coloured in red and blue, respectively. A pre-cycle of "subsequent equality" (Figure 6, right, coloured in blue) is defined as the SoC

bat can be an object of any class that implements the batteryInterface.





Figure 6: Qualitative visualization of pre-cycle counting according to [7]: Two pre-cycles of prior equality (left) and a pre-cycle of subsequent equality (right).

within an interval between the respective $SoC_{\max,l}$ and the subsequent point at which the SoC is equal to $SoC_{\max,l}$. Finally, a pre-cycle is counted as a cycle if there is no larger pre-cycle that encompasses the same interval and shares the same local minimum. This is not the case for the small cycle (coloured red) in Figure 6 (left); so the depicted curve contains two cycles.

In this package, dambrowskiCounter is the default cycle counter if an age model is specified. Thus, it does not have to be passed as an argument in a battery's initAgeMode() method.

```
1 bat.initAgeModel('ageModel', 'EO')
```

- 2 % Automatically initializes a dabrowskiCounter object with init_soc
- 3 % and soc_max set according to the battery's properties and links.

2.2.3 Comparison of the cycle counters

Each cycleCounter object's count () method converts the saved SoC profile into a cycle-Depth-of-Cycle cDoC curve - a vector containing the depths of discharge DoD of all the counted cycles. The simulation results of two batteries using a dambrowskiCounter and a rainflowCounter, respectively, are compared in Figure 7. The cycles' DoDs are each sorted into 50 BINs and compared in a histogram. Overall, the histograms appear very similar, thus proving that both classes produce good results. However, more cycles are counted using the dambrowskiCounter class, possibly causing the simulated battery to age slightly faster. While both classes use different methods for determining the extremaⁱ, the amount local maxima found is the same. Thus, the determination of extrema can be ruled out as a cause

idambrowskiCounter uses cycleCounter's iMaxima() method and rainflowCounter uses the sig2ext() function [10].



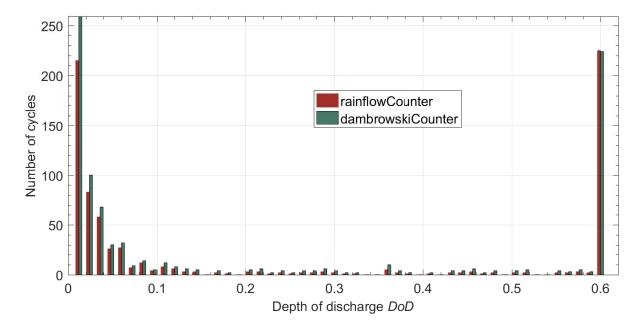


Figure 7: Comparison of the counted cycles and their DoD between two simulations using the rainflowCounter and the dambrowskiCounter using the same SoC profile.

and the root of the discrepancy must lie within the different counting approaches.

2.3 Event oriented ageing model

The event oriented ageing model - a very simple and lightweight model - is implemented by the eoAgeModel class, which subclasses the abstract batteryAgeModel interface. Cycle ageing is calculated based on a curve fit of the battery's number of cycles to failure $N_{\rm f}$ vs DoD curve.

2.3.1 Cycle life curve fits

Due to the fact that the cycle ageing can vary strongly between technologies, it can be difficult to find a good fit for the $N_{\rm f}$ vs DoD curve. In order to provide some flexibility, three different classes, each implementing the curveFitInterface, are provided for fitting such curves in this package: woehlerFit, nrelcFit and deFit. They only differ in their class names, the number of fit parameters and in the functions used for fitting. The function used in woehlerFit is based on a metal fatigue curve (also known as a "Wöhler curve") [11]

$$N_{\mathsf{f}}(DoD) = x_1 \cdot DoD^{-x_2} \tag{3}$$

with the fit parameters x_1 and x_2 . The nrelcFit class bases it's fit method on an older model [12].

$$N_{f}(DoD) = x_{1} \cdot \frac{1}{DoD} \cdot exp\left(x_{2} \cdot \left(1 - \frac{1}{DoD}\right)\right) \tag{4}$$

Finally, the deFit class uses a double exponential function that was originally developed for lead-acid batteries [13]. However, it also seems to provide decent results for lithium-ion



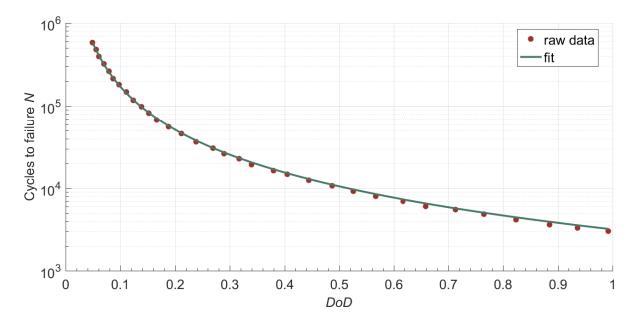


Figure 8: Example for a lithium ion battery's N_f vs DoD cycle life curve fitted using the woehlerFit class.

batteries in some cases.

$$N_{\mathsf{f}}(DoD) = x_1 + x_2 \cdot exp(-x_3 \cdot DoD) + x_4 \cdot exp(-x_5 \cdot DoD) \tag{5}$$

An example for the fit results of a woehlerFit object is depicted in Figure 8. Due to a lithium-ion battery's extremely large amount of cycles to failure at low DoDs, large relative errors may occur, especially at DoDs close to 1. In order to reduce the rmse, as many raw data points as possible should be provided for fitting.

A cycle life curve fit object is initialized with the raw data and the optional name-value pairs that the dischargeFit and every other subclass of the curveFitInterface accepts (see section 1.1.1). It can then be passed on to an age model via it's constructor or by setting it's wFit property.

```
1 % DoD = depth of discharge, N = number of cycles to failure
2 fit1 = woehlerFit(DoD, N);
3 % Plot results to new figure window
4 fit1.plotResults;
5 % Example using lsqcurvefit
6 fit2 = deFit(DoD, N, 'mode', 'lsq');
7 % Example using fminsearch and with custom initial params
8 fit3 = nrelcFit(DoD, N, 'mode', 'fmin', 'x0', [0.5; 1]);
9 % Pass cycle counter cy and fit1 to age model via it's constructor
10 am = eoAgeModel(cy, fit1);
11 % Replace fit1 with fit2 in age model
12 am.wFit = fit2;
```



Alternatively, a cycle life curve fit can be passed directly to a battery object bat via it's addcurves() method.

```
1 fit = woehlerFit(DoD, N);
2 bat.initAgeModel('ageModel', 'EO')
3 bat.addcurves(fit, 'cycleLife')
```

If the age model has not been initialized when the curve fit is added, it is stored for later use.

```
1 bat.addcurves(fit, 'cycleLife')
2 bat.initAgeModel('ageModel', 'EO') % fit is automatically passed
3 % to the age model
```

As well as curve fit objects, function handles to functions of one variable are accepted. However, anonymous functions are not recommended, due to their significant performance penalty.

```
1 fit = @(x)(3000 * x.^(-1.73));
2 bat.addcurves(fit, 'cycleLife')
```

2.3.2 Cycle ageing

A battery's age A_c is the opposite of the SoH, which is the usable capacity C_{bu} divided by the nominal capacity C_n .

$$A_{c} = 1 - SoH = 1 - \frac{C_{bu}}{C_{p}}$$
 (6)

In the eoAgeModel class, the ageing due to cycling stress A_{cyc} with n cycles and their respective DoD is determined from the curve fit $N_{\text{f}}(DoD)$.

$$A_{\text{cyc}} = \sum_{i=1}^{n} \frac{1}{N_{\text{f}}(DoD_i)} \tag{7}$$

Every time a set of cycles is counted, A_{cvc} is determined and added to A_{c} .

2.3.3 Age model initialization

The default constructor syntax of a batteryAgeModel is as follows:

```
1 am = batteryAgeModel; % Replace batteryAgeModel with the subclass name
2 % eoAgeModel requires at least one input (a cycle counter cy)
3 am = eoAgeModel(cy);
4 am = eoAgeModel(cy, cfit); % adds a cycle life curve fit
5 % Specify the SoH at which the end of life is reached (default: 0.2)
6 am = eoAgeModel(cy, cfit, soh_eol);
7 am = eoAgeModel(cy, cfit, soh_eol, soc_ini); % Set the initial SoC
```



To initialize the age model directly from a battery bat, use it's initAgeModel() method.

```
1 bat.initAgeModel('ageModel', 'EO') % default eoAgeModel
```

2.3.4 Creating a user-defined age model

The eoAgeModel class does not take into account calendar ageing, which may be needed in some cases. It was left out of the default class because in many cases, a weighted degradation factor may be sufficient. The following source code snippet provides an example of how the eoAgeModel could be subclassed to extend it with a simple calendar ageing model. A property which holds the battery's calendar life is added and initialized in the constructor, taking into account the end of life age (typically 0.2). Finally, an addCalAge() method is added, that calculates calendar ageing as a linear function of the time step size. This method can be called from within the main simulation. To create a completely different age model (i.e. one that takes thermal influences into account), subclass the batteryAgeModel class instead.

```
classdef myCalendarAgeModel < lfpBattery.eoAgeModel</pre>
  %MYCUSTOMAGEMODEL: An example for a user-defined age model.
  %Combines the event oriented age model with a linear calendar age model.
      properties
           L_cal; % calendar life in s
      end
      methods
           function obj = myCalendarAgeModel(1, varargin)
10
               % l = calendar life in years
11
               % varargin = input args of eoAgeModel constructor
12
               %% call superclass constructor
13
               obj = obj@lfpBattery.eoAgeModel(varargin{:});
               obj.L_cal = 1 * 525600 / obj.eolAc; % set L_cal in
15
               % seconds taking end of life age into account
16
17
           function addCalAge(obj, dt)
18
               % Adds to the battery's age using the simulation time
19
               % step size.
20
               % dt = simulation time step size in s
21
               % obj.Ac = 1 - obj.SoH
               obj.Ac = obj.Ac + dt / obj.L_cal; % increment age
23
           end
       end
  end
26
```



A user-defined age model can be added to a battery bat using it's initAgeModel() method.

```
1 L_cal = 20; % calendar life in years
2 am = myCalendarAgeModel(L_cal, cy); % user-defined age model
3 bat.initAgeModel('ageModel', am)
```

The above is meant as a rough example for how a user-defined age model could be defined. However, it adds calendar ageing on top of the cycle stress every time the addCalAge method is called. This could lead to an unwanted acceleration of the simulated ageing process. A better solution is included in this package.

2.3.5 Calendar ageing

The eoCalAgeModel class is an extension of eoAgemodel that adds the possibility of calendar ageing. It is similar to the example in section 2.3.4. However, calendar ageing is not simply added on top of cycle stress, but set against it. An eoCalAgeModel object is created with the same input arguments as an eoAgemodel object, with the addition of the battery's calendar life L_{cal} in years as the first argument.

```
1 am = eoCalAgeModel(L_cal, cy, __);
```

As within the example in section 2.3.4, the calendar ageing $A_{\rm cal}$ is modelled as a linear function of the simulation time step size $\Delta t_{\rm s}$

$$A_{\mathsf{cal}} = \frac{\Delta t_{\mathsf{s}} \cdot A_{\mathsf{c},\mathsf{eol}}}{L_{\mathsf{cal}}} \tag{8}$$

where $A_{\rm c,eol}$ is the age at which the end of life EOL is reached. The addCalAge() method must be called manually from within the main simulation at the end of each time step (after cycling the battery). The total stress for each simulation time step $t_{\rm s}$ is the maximum of cycle and calendar ageing.

$$A_{\mathsf{tot}}(t_{\mathsf{s}}) = \max(A_{\mathsf{cvc}}(t_{\mathsf{s}}), A_{\mathsf{cal}}(t_{\mathsf{s}})) \tag{9}$$



3 Battery Composition

The battery pack is modelled using a variation of the Composite design pattern with multiple composite classesⁱ. This way, cells can be combined flexibly in various different topologies.

3.1 Overview

The batteryInterface is the abstract component that defines the interface for all objects in the composition. It is subclassed by all other battery elements. The batteryCell objects are the "leaves" and a composite can be one of the following classes:

- parallelElement: A set of components in parallel.
- seriesElementAE: A set of components in series with active equalization.
- seriesElementPE: A set of components in series with passive equalization.

Each component can either be another composite object or a leaf. Figure 9 provides a visual overview of the topologies that are possible using different compositions. Using this variation

ⁱThe basic Composite design pattern has one component interface, one composite class and one leaf class.

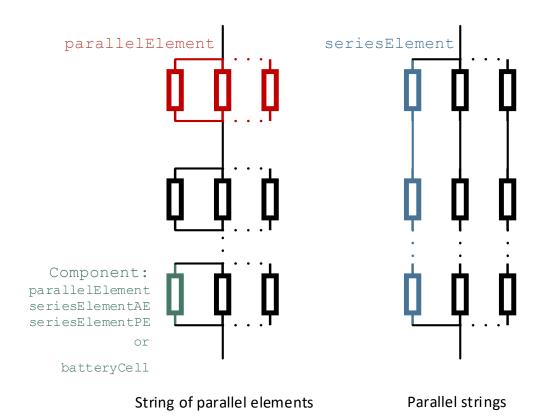


Figure 9: Visualization of the possible battery topology compositions.



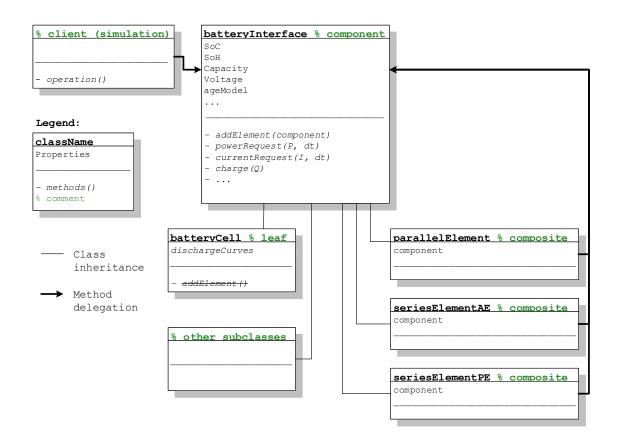


Figure 10: Class diagram of the battery composition with communication flows and inheritance links.

of the Composite design pattern, the components can be combined in any possible way at runtime. The most common battery topologies are strings of parallel elements (SP) and parallel strings of cells (PS) [14]. In Figure 9, these would be the case if the composition's leaf nodes (cells) were all in the second layer (marked green). However, since every component can be either a cell or another composite, more complicated topologies are made possible in this package.

3.2 Method delegation

A pattern diagram of the classes used for the topology composition is depicted in Figure 10. Every composite element holds a reference to a component and delegates the methods called on it to said component. The delegated methods are wrapped with the rules of the respective topology in a similar fashion as is done with the Decorator design pattern. An example of the method delegation for a PS configuration - a parallelElement that holds a set of seriesElement objects, each in turn holding a set of batteryCell objects - is visualized in Figure 11. In this example, a current I and the simulation time step size is passed to the parallelElement via a getVoltage() method. The parallelElement determines



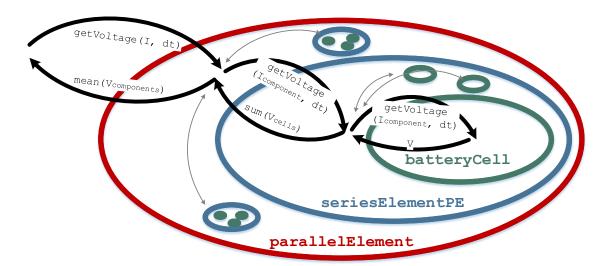


Figure 11: Example of a method being delegated across a battery pack composition.

which portion of I to send to each of it's components and delegates the method. Each seriesElement does the same and delegates the method to it's batteryCell objects. These return their voltages back to the seriesElement objects, which sum up the results received from their cells and pass the sum back to the parallelElement. Finally, the parallelElement calculates the mean of all the summed up voltages it received and passes the end-result back to the client. The following operations are delegated by an object that implements the batteryInterface:

- Determination of the new voltage after charging or discharging a with a certain current and time step size. This is delegated to each <code>batteryCell</code> object's <code>dischargeCurves</code> reference.
- Charging or discharging the battery.
- Determining the state of the battery if it were to be charged or discharged.
- Determining the maximum charging or discharging current.
- Calculating the pack's SoH.
- Getters and setters for the component's voltage and capacity properties.
- Getter for the component's internal impedance.

With the number of subcomponents n, a component's voltage is determined as

$$V_{\mathsf{component}} = \begin{cases} \frac{\sum_{i=1}^{n} V_{\mathsf{subcomponent},i}}{n} & \text{for a parallel element} \\ \sum_{i=1}^{n} V_{\mathsf{subcomponent},i} & \text{for a series element} \end{cases}$$
 (10)



And a component's capacity is

$$C_{\mathsf{component}} = \begin{cases} \sum_{i=1}^{n} C_{\mathsf{subcomponent},i} & \text{for a parallel element} \\ \min_{i=1}^{n} C_{\mathsf{subcomponent},i} & \text{for a series element with passive equalization} \\ \frac{\sum_{i=1}^{n} C_{\mathsf{subcomponent},i}}{n} & \text{for a series element with active equalization} \end{cases} \tag{11}$$

Since the SoH is derived directly from the capacity (see Equation 6), a component's SoH can be determined in the same fashion.

$$SoH_{\mathsf{component}} = \begin{cases} \sum_{i=1}^{n} SoH_{\mathsf{subcomponent},i} & \text{for a parallel element} \\ \min_{i=1}^{n} SoH_{\mathsf{subcomponent},i} & \text{for a series element with passive equalization} \\ \frac{\sum_{i=1}^{n} SoH_{\mathsf{subcomponent},i}}{n} & \text{for a series element with active equalization} \end{cases}$$
 (12)

Due to the fact that the model is based on curve fits, the internal impedance Z_i property is not used for modelling the charging behaviour directly. It does however, determine how the voltages and currents are distributed across the subcomponents when charging or discharging. The impedance proportionality factor p_z of a subcomponent with index j is the component's Z_i divided by the sum of all subcomponents' Z_i .

$$p_{z,j} = \frac{Z_{i,j}}{\sum_{i=1}^{n} Z_{i,i}}$$
 (13)

When charging, a series element with active equalization will distribute it's voltage equally across all of it's subcomponents to account for balancing, while a series element with passive equalization will distribute it's voltage according to $p_{z,j}$. For a parallel element, the current is distributed in such a way that the subcomponent j with the lowest Z_i receives the highest current.

$$I_{\text{subcomponent},j} = \frac{\frac{1}{p_{z,j}}}{\sum_{i=1}^{n} \frac{1}{p_{z,i}}} \cdot I_{\text{component}}$$
(14)

3.3 Battery Interface

The battery interface is described in the following subsections. Every component implements the batteryInterface, so the methods described in this section can be called on batteryCell objects and on the composites.

3.3.1 Battery object initialization

To initialize a battery object at runtime, the nominal capacity C_n in Ah and the nominal voltage V_n in V must be passed to a batteryCell constructor. A composite can be initialized as an "empty" circuit element and the cell (or other composites) can be added to it via it's addElements () methodⁱ.

 $^{^{}i}$ Here, "empty" is referred to in the sense of not holding any cells, not in the sense of an empty Matlab $^{\textcircled{R}}$ variable.



```
1 % Initialize an "empty" parallel element
2 bat = parallelElement;
3 Cn = 3; % Nominal cell capacity in Ah
4 Vn = 3.2; % Nominal cell voltage in V
5 % Initialize 3 battery cells and add them to bat
6 for i = 1:3
7     b = batteryCell(Cn, Vn);
8    bat.addElements(b);
9 end
```

The addElements() method also accepts component arrays...

```
1 for i = 1:3
2    b(i) = batteryCell(Cn, Vn);
3 end
4 bat.addElements(b);
```

...and multiple inputs:

```
1 b1 = batteryCell(Cn, Vn);
2 b2 = batteryCell(Cn, Vn);
3 b3 = batteryCell(Cn, Vn);
4 bat.addElements(b1, b2, b3);
```

To create a composition like the example in Figure 11 (see also Figure 9, right), the following syntax could be used:

```
1 % Initialize "empty" parallel element
2 bat = parallelElement;
3 % Initialize 3 "empty" series elements each holding 3 cells
4 for i = 1:3
5     se = seriesElementPE; % passive equalization
6     for j = 1:3
7         se.addElements(batteryCell(Cn, Vn))
8     end
9     % Add series elements to bat
10     bat.addElements(se)
11 end
12 % Further initialization operations, e.g. bat.addcurves() here...
```



3.3.2 Battery charging and discharging

Battery charging ⁱ is handled by the methods powerRequest() and currentRequest(). Both functions are called in a similar manner. The syntax is as follows:

```
1 [P, V, I] = bat.powerRequest(P, dt);
2 [P, V, I] = powerRequest(bat, P, dt); % equivalent
3 [P, V, I] = bat.currentRequest(I, dt);
4 [P, V, I] = currentRequest(b, I, dt); % equivalent
```

Where P is the requested power P in W, I is the requested current I in A and \det is the simulation time step size $\Delta t_{\rm s}$ in s. The methods return the actual power throughput in W, the battery's voltage V at the end of the time step and the actual current throughput in A. The returned power and current is limited by the SoC or the cells' maximum currents, among other factors. Figure 12 contains a flow chart of the charging process. The client sends a request to the battery. If the requested power is not equal to zero and the battery's SoC is not already at it's upper or lower limit, a charge iteration is performed (the iteratePower() and

ⁱDischarging will also be referred to as charging (with a negative current) in this documentation.

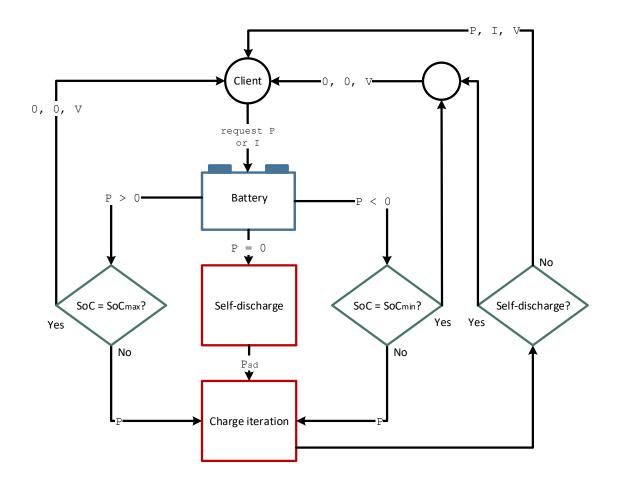


Figure 12: Flow chart of the powerRequest() and currentRequest() methods.



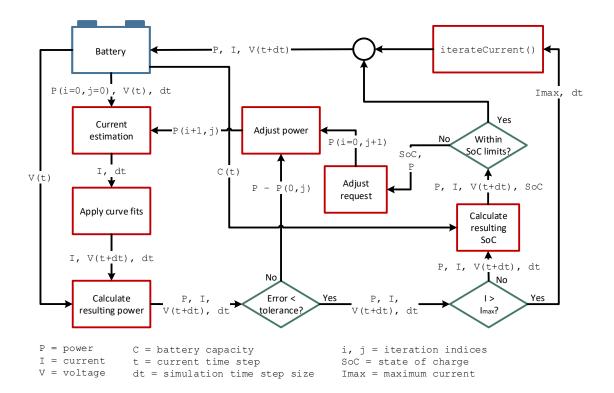


Figure 13: Flow chart of the iteratePower() method.

iterateCurrent () methods are called, respectively) and the resulting power, current and voltage are returned to the client. A positive input to the charge iteration indicates charging and a negative input specifies discharging. If the request is zero, signalling that the battery is in an idle state, a logical flag is set to true and the charge iteration is called with the battery's self-discharge $P_{\rm sd}$. The logical flag is checked after every call to the charge iteration methods in order to return a power and current of zero to the client if it was set to true. If the SoC is either at it's upper or lower limit, the battery simply returns it's voltage along with a power and current of zero.

A flow chart of the <code>iteratePower()</code> method is depicted in Figure 13. First, a current is estimated from the requested power and the battery's voltage. The current and the time step size are then delegated to the battery cells' <code>dischargeCurves</code> objects, in order to determine the resulting voltage. An approximation of the power is determined from the mean of the returned voltage and the battery's old voltage. This is repeated through recursion until the difference between the iterated power and the originally requested power meets a certain tolerance. If the resulting current is greater than the battery's maximum current I_{max} , the <code>iterateCurrent()</code> method is called using I_{max} as an input. It's output current, the resulting power and voltage are returned. Otherwise, the SoC is determined and compared the battery's upper and lower limit. If the SoC is within the interval $[SoC_{\min}, SoC_{\max}]$, the power, current and voltage are returned. Otherwise, the requested power is adjusted according to the difference between the SoC and the respective limit that was exceeded, thus starting the iteration again.



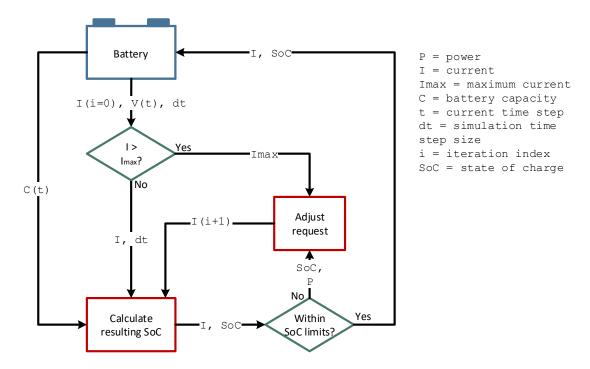


Figure 14: Flow chart of the iterateCurrent() method.

Figure 14 depicts a flow chart of the iterateCurrent () function. Using this method is a lot faster than using the iteratePower () function, due to it's comparative simplicity. However, the current may need to be determined separately in some cases. Before the iteration, the current is limited to $I_{\rm max}$. Finally, the another limitation is performed if the SoC is not within the interval $[SoC_{\rm min}, SoC_{\rm max}]$. Normally, one or two iterations should suffice for returning the current and SoC. The voltage and power are not calculated and must be determined by calling the getNewVoltage () method if required.

3.3.3 Age model level

ⁱFor example, this is done within the currentRequest() method, which does return the voltage and power.



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