"A PRODUCTION-STORAGE AND DISTRIBUTION PROBLEM"

Assume that n number of manufacturing plants ($i = 1, \dots, n$) of a large company are selling their product (one kit only) to r number of customers ($k = 1, \dots, r$) directly and/or through m number of distributor stores ($j = 1, \dots, m$) located in different parts of the country. Distributorships are offered when needed to the dealers who Work independently. This production distribution system can be represented by a network as shown in figure 1

Let us define the following:

x_{ij} = quantity of the product (one kind of product only) sent from the plant i to the distributor , with a unit distribution cost c_{ij}.

z_{ik} = quantity of the product sent from the plant i to the customer k, with a unit distribution cost c_{ik}.

y_{jk} = quantity of the product sent from the distributor j to the customer k, with a unit distribution cost c_{jk}.

Plants	Distributors	Customers
$i = 1, \dots, n$	$(j = 1, \dots, m)$	$(k = 1, \dots, r)$

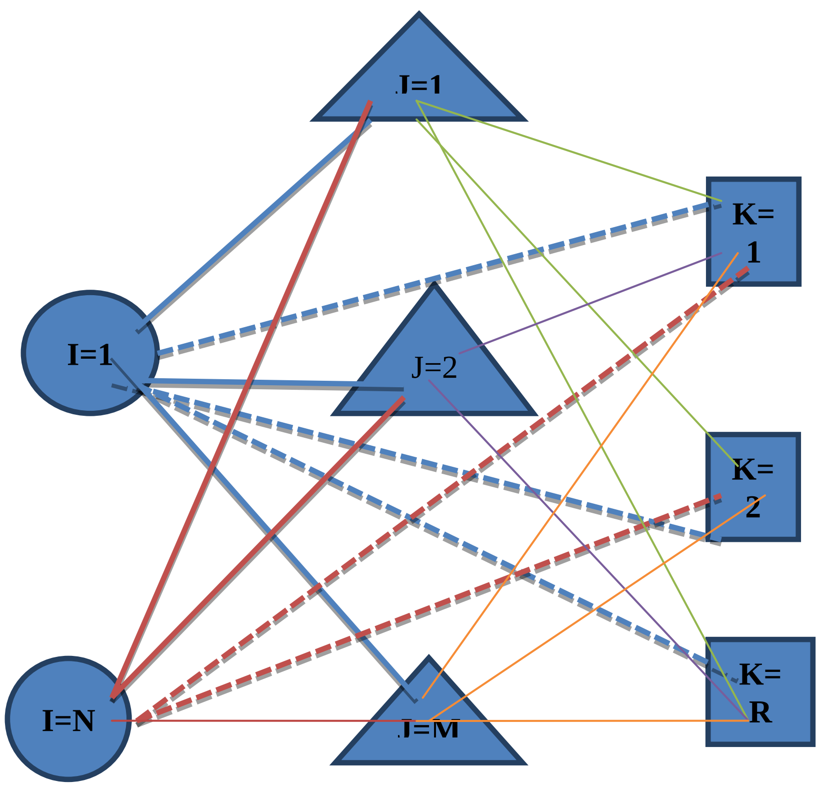

FIGURE 1 Production distribution system network.

Each plant has a monthly capacity of producing N_i, units of products. Each distributor has a monthly capacity to store and distribute M_j units of products, while each customer has a monthly demand of R_k, unist of products.

The problem deals with the question of how to distribute the products to the customers such that the total costs of distribution are minimized.
The model of the problem can be formulated as follows.

Minimice: $\boldsymbol{Z = \sum_{ij} c_{ij} x_{ij} + \sum_{ik} c_{ik} z_{ik} + \sum_{jk} c_{kj} y_{jk}}$

Subject to:

*Constraint of the total manufacturing capacity

$$\sum_{j=1}^{m} x_{ij+} \sum_{k=1}^{r} z_{ik} \leq N_i \; for \; i = 1, \ldots., n$$

* Constraint of the storage capacity for each distributor

$$\sum_{i=1}^{n} x_{ij} \leq M_j \; for \; j = 1, \ldots., m$$

* Constraint of the balance of input and output to and from distributor

$$\sum_{i=1}^{n} x_{ij} \geq \sum_{k=1}^{r} y_{jk} \; for \; j = 1, \ldots., m$$

* Constraint of the demand by each customer

$$\sum_{i=1}^{n} z_{ik+} \sum_{j=1}^{m} y_{jk} \geq R_k \; for \; k = 1, \ldots. r$$

*Nonnegativity and integrality constraint

$$x_{ij}, z_{ik}, y_{jk} \geq 0, integer$$

Example

Comfortable Slacks Company produces slacks in two plants located in Dallas and San Antonio , The company has a restricted marked in the St. Louis, Oklahoma City, Houston, and Santa Fe arenas. In addition to its direct sales to customers, the company also maintains distributor depots in Austin, Texas and

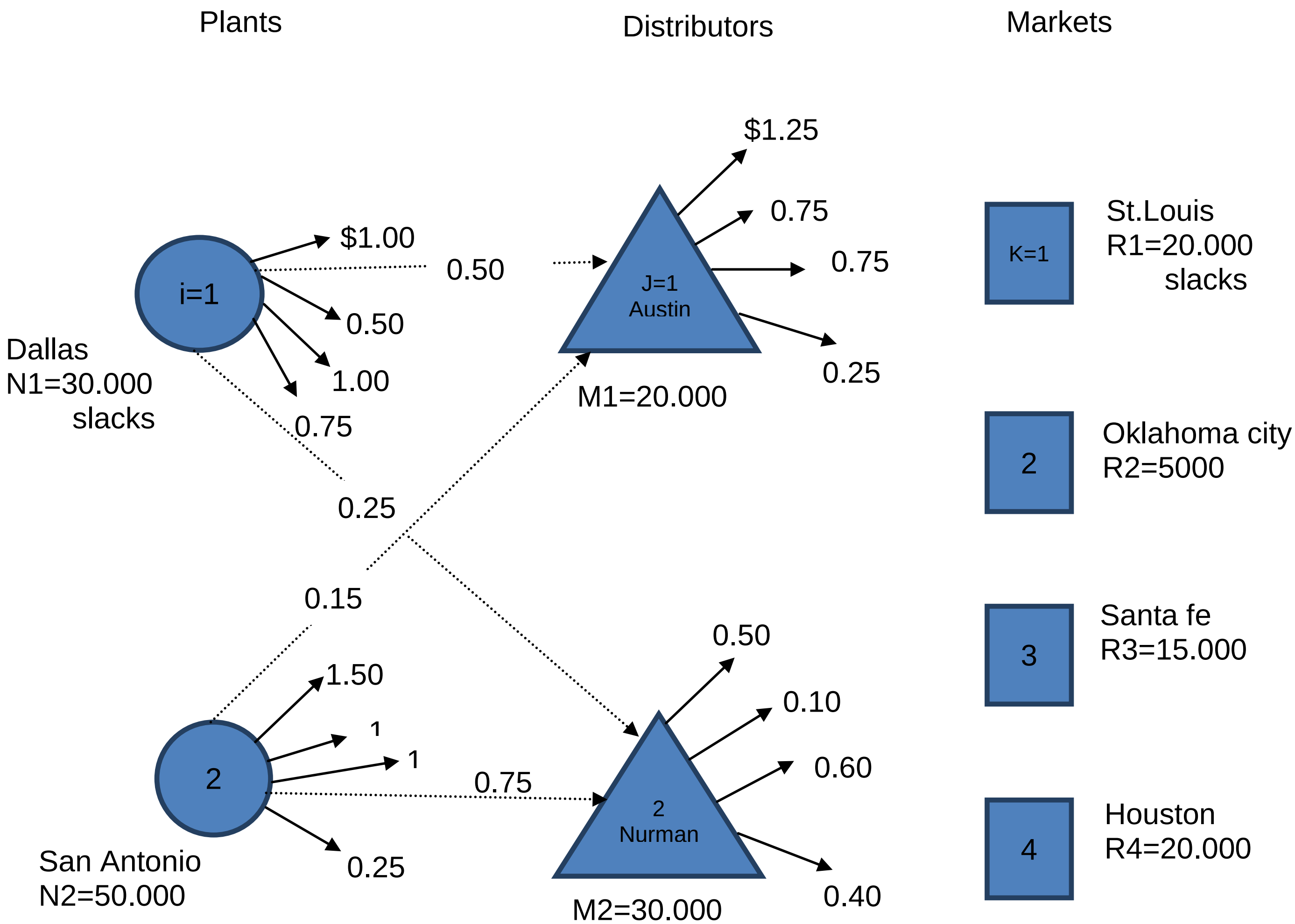

FIGURE 2 Schematic representation of the distribution system,

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Norman, Oklahoma that can supply any market upon request. The monthly capacity, demand, and unit distribution cost data shown in fig. 8.3.
How should the company distribute is slacks so that the total monthly distribution cost is minimized? The model of the problem is shown below:

Minimize: $z = 0.5x_{11} + 0.25x_{12} + 0.15x_{21} + 0.75x_{22}$
$+ z_{11} + 0.50z_{12} + z_{13} + 0.75z_{14}$
$+ 1.5z_{21} + z_{22} + z_{23} + 0.25z_{24}$
$+ 1.25y_{11} + 0.75y_{12} + 0.75y_{13} + 0.25y_{14}$
$+ 0.5y_{21} + 0.1\,y_{22} + 0.60\,y_{23} + 0.40y_{24}$

Subject to:

- Total manufacturing capacity constraint
  $x_{11} + x_{12} + z_{11} + z_{12} + z_{13} + z_{14} \leq 30{,}000$
  $x_{21} + x_{22} + z_{21} + z_{22} + z_{23} + z_{24} \leq 50{,}000$

- Distributor storage capacity constraint

$x_{11} + x_{21} \leq 20{,}000$

$x_{12} + x_{22} \leq 30{,}000$

Examples for Integer Programming Model Formulation

- Input-output balance constraint for distributors

$$y_{11} + y_{12} + y_{13} + y_{14} \leq x_{11} + x_{21}$$
$$y_{21} + y_{22} + y_{23} + y_{24} \leq x_{12} + x_{22}$$

- Customer demand constraint

$$z_{11} + z_{21} + y_{11} + y_{21} \geq 20{,}000$$
$$z_{12} + z_{22} + y_{12} + y_{22} \geq 5{,}000$$
$$z_{13} + z_{23} + y_{13} + y_{23} \geq 15{,}000$$
$$z_{14} + z_{24} + y_{14} + y_{24} \geq 20{,}000$$
$$x_{ij}\,, z_{ik}\,, y_{jk} \geq 0$$

Solution of Example: The solution of the problem is given in Table 1

TABLE 1

Monthly Production and Distribution Plan

From	**To**	**Solution**	**Solution Value**
Dallas	Norman	x_{12}	30,000 slacks
San Antonio	Austin	x_{21}	20,000 slacks
San Antonio	Santa Fe	z_{23}	10,000 slacks
Norman	St.Louis	y_{21}	20,000 slacks
Norman	Oklahoma City	y_{22}	5,000 slacks
Norman	Santa Fe	y_{23}	5,000 slacks
Austin	Houston	y_{14}	20,000 slacks

Minimized Monthly Distribution Cost $Z = \$34{,}000$

Tabla 2 Interpretación de los resultados obtenidos.

DESDE	HASTA	SOLUCIÓN	VALOR DE LA SOL.
Dallas	Norman	x_{12}	30000 slacks
San Antonio	Austin	x_{21}	10000 slacks
San Antonio	Houston	z_{24}	20000 slacks
Austin	St. Louis	y_{11}	5000 slacks
Austin	Oklahoma City	y_{12}	5000 slacks
Norman	St. Louis	y_{21}	15000 slacks
Norman	Santa Fe	y_{23}	15000 salacks

Costo de Distribución Mensual Minimizado Z_0=$31500