# Artificial Neural Networks (ANN)

Nikola Popović nipopovic@vision.ee.ethz.ch





#### 1st wave of NN – The birth of the idea

- [1943] First artificial neuron model W. McCulloch, W. Pitts
- [1949] Hebb's learning law D. Hebb
- [1958] Perceptron F. Rosenblatt
- [1962] Delta Learning Rule B. Widrow, T. Hoff

#### 2nd wave – Excitement again

- [1986] Bacpropagation popularized D. Rumelhart, G. E. Hinton, R. Williams
- [1997] LSTM's H. Sepp, J. Schmidhuber
- [1998] CNN's- Y. LeCun, L. Bottu, Y. Bengio, P. Haffner

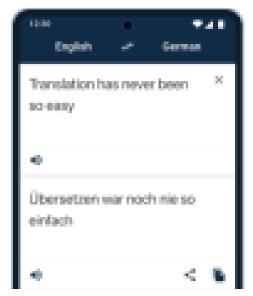
#### 3rd wave – ANN seem to work

- Much better hardware and software
- Much bigger data sets
- Some influential works
  - [2006] Deep Belief Networks G. E. Hinton, S. Osindero, Y.-W. Teh
  - [2009] Speech processing G. E. Hinton, L. Deng
  - [2012] Image classification A. Krizhevsky, I. Sutskever, G. E. Hinton

Many astonishing results followed

#### Deep Learning applications

Machine translation



https://www.deepl.com/translator

Autonomous



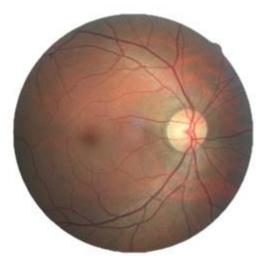
https://www.ptolemus.com/topics/autonomous-vehicles/

Speech recognition



https://play.google.com/store/apps/details?id =com.prometheusinteractive.voice\_launcher

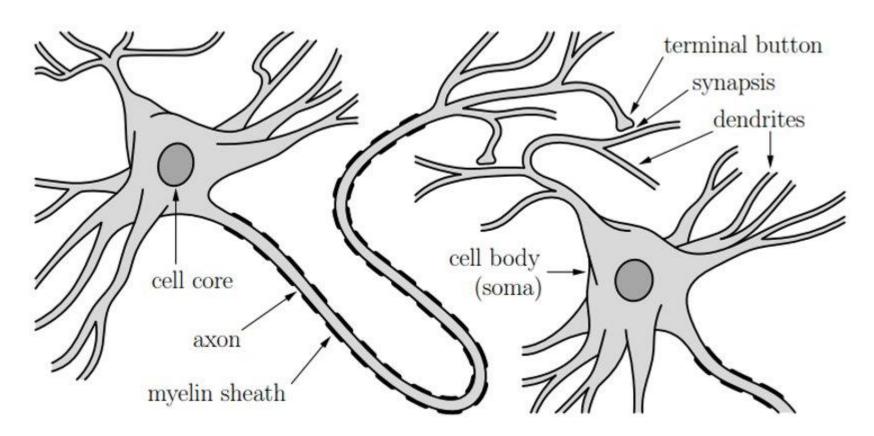
Diabetic retinopathy detection



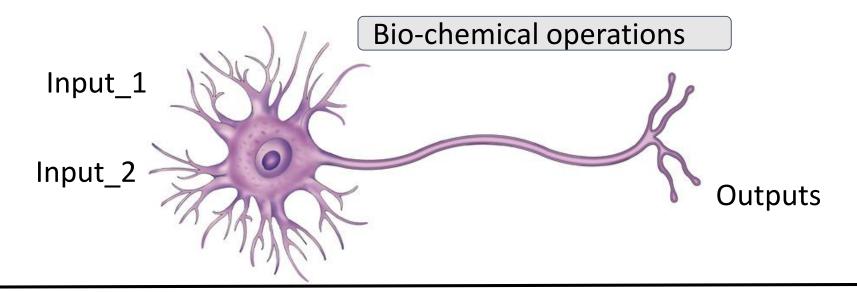
https://www.kaggle.com/c/diabetic-retinopathy-detection

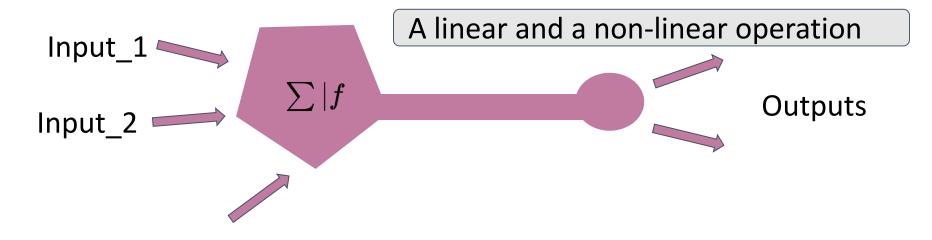
## Biological and Artificial Neurons

ANN were inspired by human nervous system



#### Biological and Artificial Neurons





#### One Artificial Neuron

• Input 
$$\mathbf{x} = [x_1, \dots, x_m]^T$$

• Weights 
$$\mathbf{w} = [w_1, ..., w_m]$$

- Bias b
- Pre-activation  $z = \sum_i x_i w_i + b = wx + b$
- Activation fn.  $f(\cdot)$
- Neuron output  $y = f(z) = f(\sum_i x_i w_i + b)$

#### ANN Single layer

- Input vector  $\mathbf{x} = [x_1, ..., x_m]^T$
- Different weights for each of k neurons

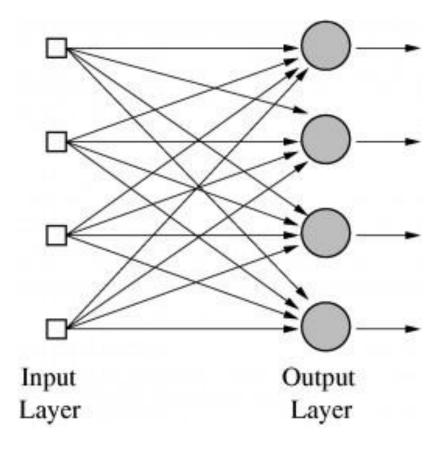
$$\mathbf{W} = \begin{bmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & \ddots & \vdots \\ w_{k1} & \cdots & w_{km} \end{bmatrix} = \begin{bmatrix} \mathbf{w_1} \\ \cdots \\ \mathbf{w_k} \end{bmatrix}$$

Pre-activation vector

$$\mathbf{z} = [z_1, \dots, z_k]^T = \mathbf{W}\mathbf{x} + \mathbf{b}$$

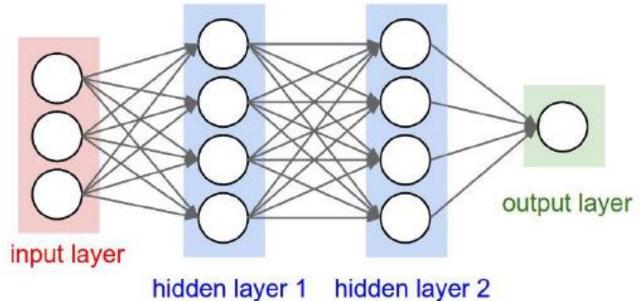
Output vector

$$y = [y_1, ..., y_k]^T = f(Wx + b)$$



## Multi Layer ANN

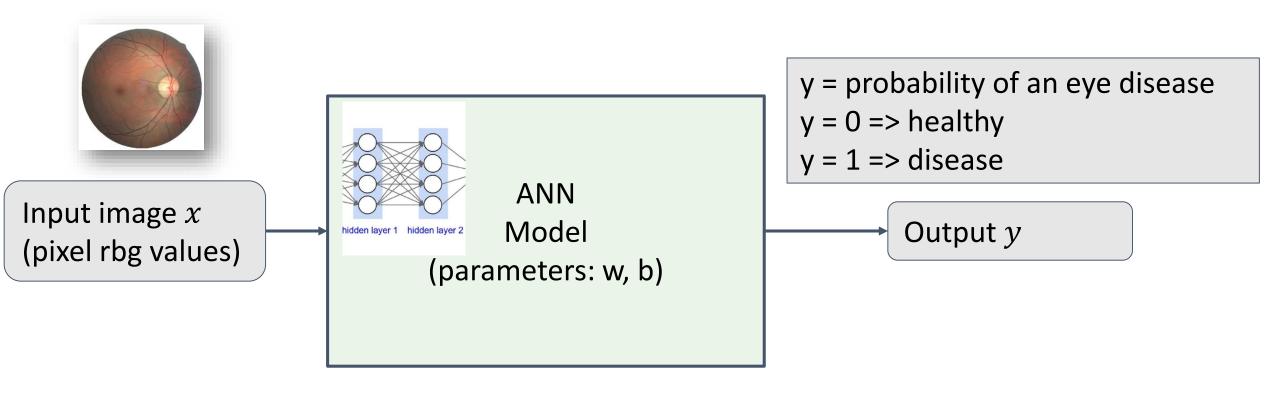
- Input layer
  - The input vector
- Hidden layer(s)
  - Intermediate computation
  - Extract features
- Output layer
  - Final computation and decision making
- Nowadays called multi layer perceptron (MLP)



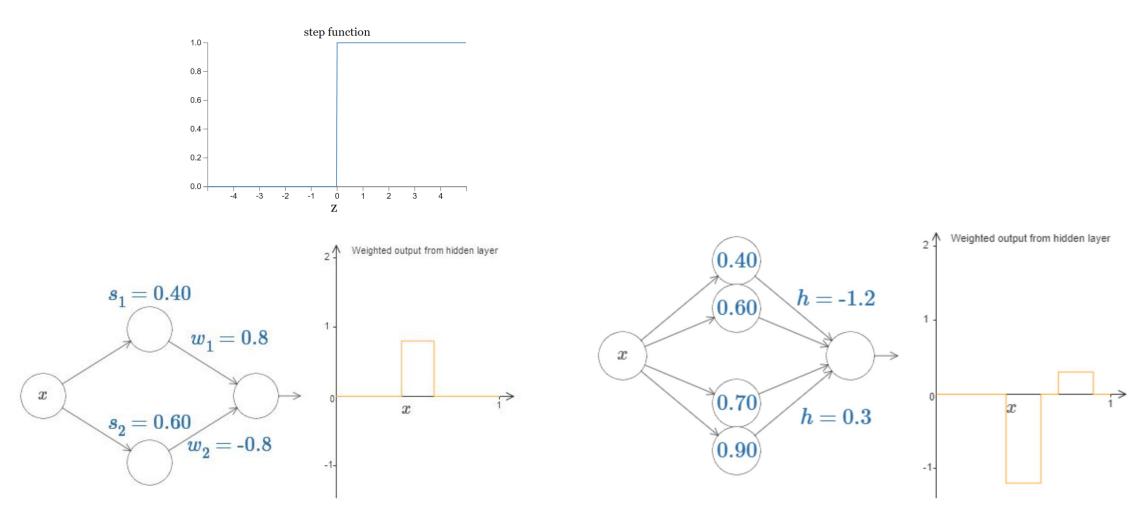
Stanford, CS231n course, Lecture 4 presentation

# High-level view

Binary classification example



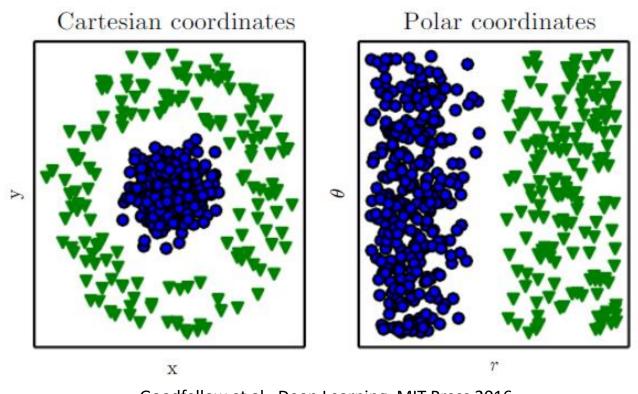
# Intuition behind the power of ANN



#### Motivation on need for good features

 The left plot cannot be seperated with a linear classifier

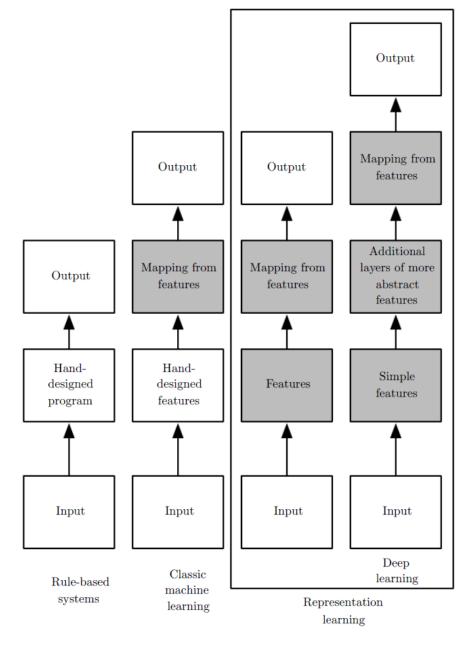
 With good features (right plot) we can seperate these two classes with a line



Goodfellow et al., Deep Learning, MIT Press 2016

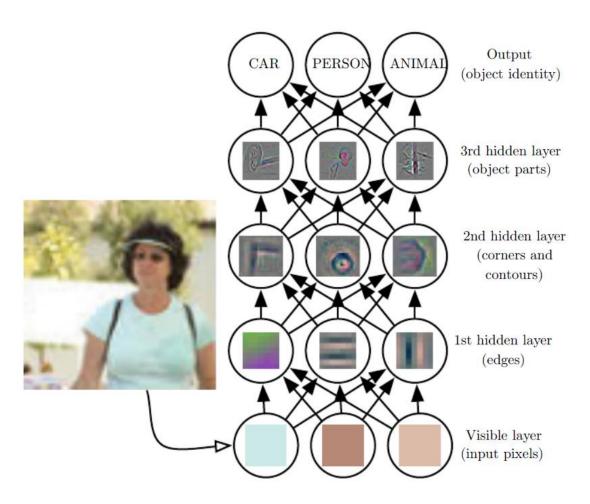
## What is Deep Learning?

- Before multiple stages
  - Designing features
  - Prediction algorithm (takes features as input)
- Now Deep Learning
  - Raw data as input
  - Prediction as output



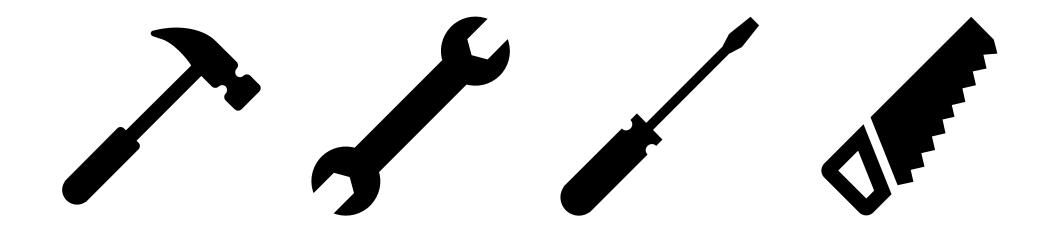
Goodfellow et al., Deep Learning, MIT Press 2016

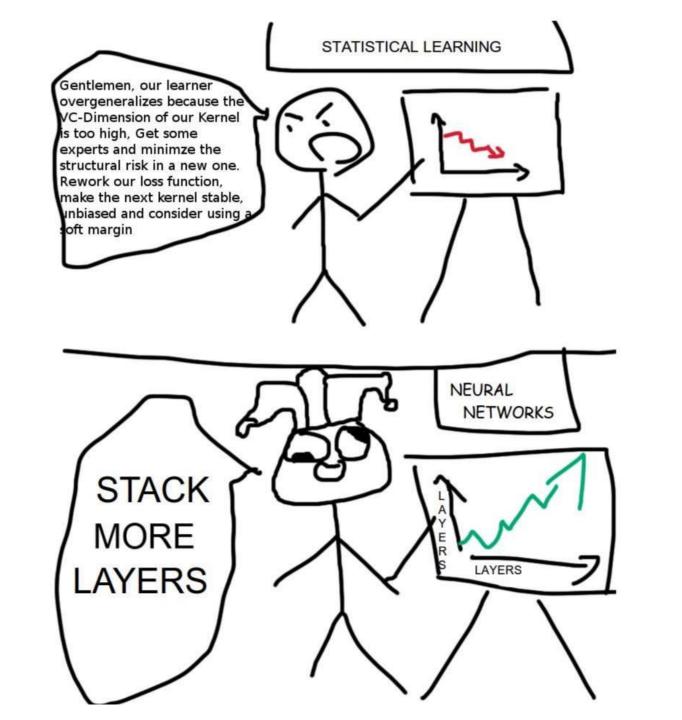
## Intuition behind Deep ANN



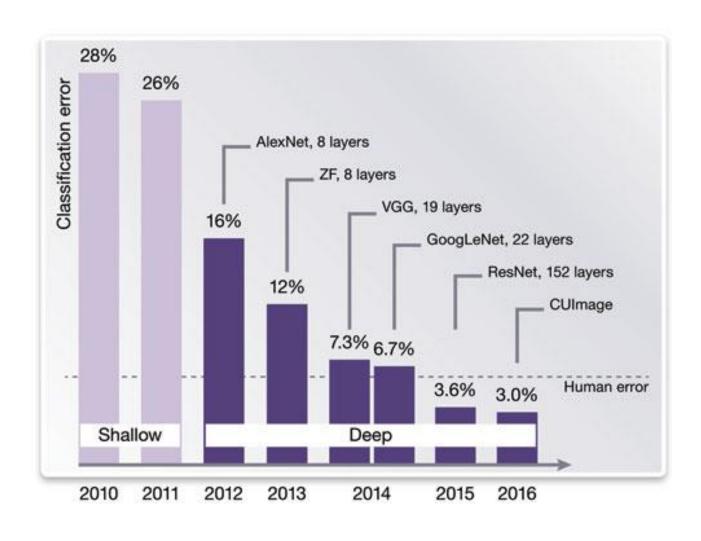
Goodfellow et al., Deep Learning, MIT Press 2016

#### ANN are tools





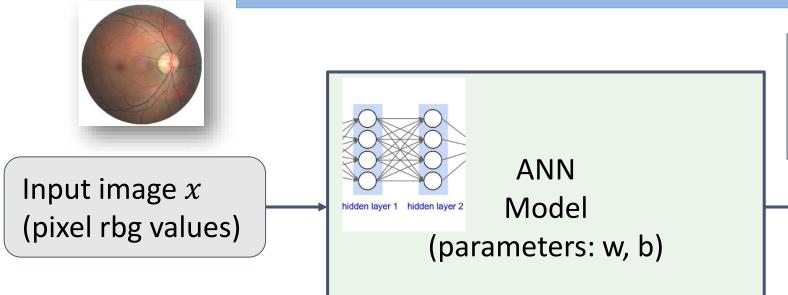
### ImageNet Challenge



#### Learning

How do we get the model parameters (w,b)?

By designing loss functions and optimizing them.



y = probability of an eye disease y = 0 => healthy

y = 1 => disease

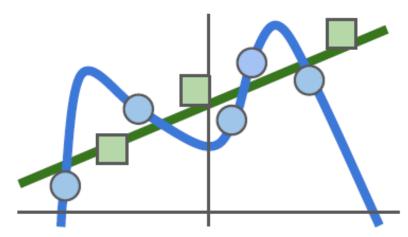
Output *y* 

## Supervized learning objective

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(f(x_i, W), y_i)$$

#### **Loss function**

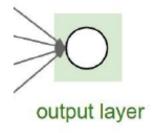
Model predictions should match training data



Stanford, CS231n course, Lecture 3 presentation

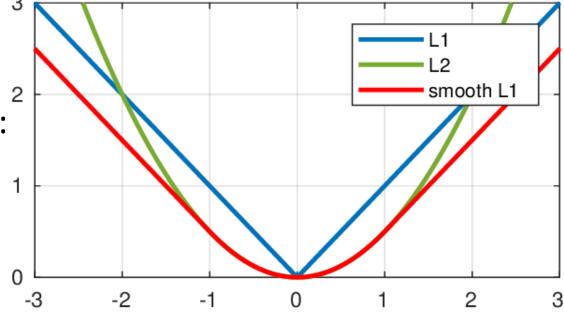
#### Regression

• In case of one output:  $\hat{y}^{(i)} = f(x^{(i)},W)$ 



- $L_2$  loss(MSE):  $L_i = (\hat{y}^{(i)} y^{(i)})^2$
- $L_1$  loss:  $L_i = |\hat{y}^{(i)} y^{(i)}|$
- Smooth  $L_1$  loss (less sensitive to outliers):

$$L_{i} = \begin{cases} L_{2} \log s & |\hat{y}^{(i)} - y^{(i)}| \le a \\ L_{1} \log s & |\hat{y}^{(i)} - y^{(i)}| > a \end{cases}$$

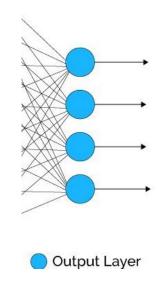


https://www.researchgate.net/figure/Plots-of-the-L1-L2-and-smooth-L1-loss-functions fig4 321180616

#### Regression

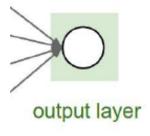
• For multiple outputs, summ the losses for each output

$$L_i = \sum_j L_{ij}$$



# Sigmoid output layer (Classification)

Binary classification problem with one output

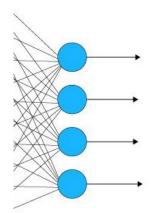


- Network output:  $\hat{p} = P(Y = 1 | X = x^{(i)}) = \frac{1}{1 + e^{-net^{(i)}}}$ 
  - $net^{(i)} = f(x^{(i)},W)$  (score for class 1)
- Cross-entropy loss

$$L_i = -\log P(Y = y^{(i)} | X = x^{(i)})$$
  
=  $-y^{(i)} \log(\hat{p}) - (1 - y^{(i)}) \log(1 - \hat{p})$ 

## Softmax output layer (Classification)

• Multi-class classes classification problem (m classes)



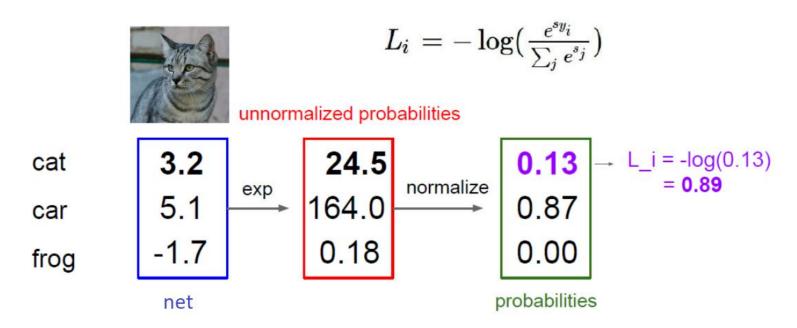
Output Layer

- Network outputs:  $\hat{p}_k = P(Y = k | X = x^{(i)}) = \frac{e^{net_k^{(i)}}}{\sum_{i=1}^m e^{net_j^{(i)}}}$ 
  - $net_k^{(i)} = f(x^{(i)}, W)$  score for class k
- Negative log likelihood

$$L_{i} = -\log P(Y = y^{(i)} | X = x^{(i)})$$

$$= -\log \left(\frac{e^{net}_{y^{(i)}}}{\sum_{j=1}^{m} e^{net_{j}^{(i)}}}\right)$$

## Softmax output layer (Classification)



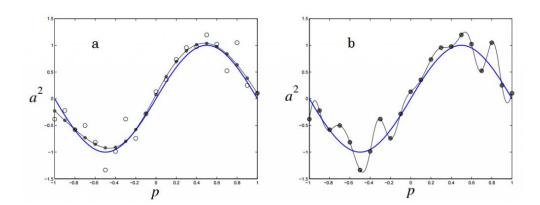
• Worst prediction  $L_i = -\log(0) \to +\infty$ 

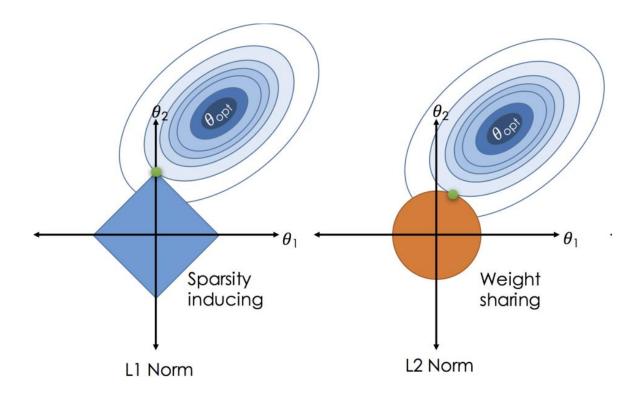
• Best prediction  $L_i = -\log(1) = 0$ 

Slide taken from - Stanford, CS231n course, Lecture 3 presentation (http://cs231n.stanford.edu/slides/2017/cs231n\_2017\_lecture3.pdf)

#### Regularization

- $\frac{1}{N}\sum_{i=1}^{N}L_i(f(x_i,W),y_i) + \lambda R(W)$ 
  - $\lambda$  regularization strength (hyperparameter)
- Commonly used
  - $L_2 R(W) = \sum W^2$  (Weight decay)
  - $L_1 R(W) = \sum |W|$





# Gradient Descent Optimization



Stanford, CS231n course, Lecture 3 presentation

# Gradient descent (GD)

• Technique for solving numerical optimization problems

#### • Algorithm:

- 1) Initialize parameters (W weights, b biases)
- 2) Repeat
  - Compute error gradient for each parameter respectively  $\frac{\partial L}{\partial w_{ij}}$

$$i = 1..m, j = 1..k$$

- Update parameters using gradients  $w_{ij}(n+1) = w_{ij}(n) \alpha \frac{\partial L}{\partial w_{ij}}$
- Parameter  $\alpha$  is a **learning rate** and it should be chosen carefully!

#### **Gradient Descent**

Gradient Descent

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^N \nabla_W L_i(f(x_i, W), y_i) + \lambda \nabla_W R(W)$$

• (Mini-)Batch GD

$$\nabla_{W} L(W) = \frac{1}{N_{mb}} \sum_{i=1}^{N_{mb}} \nabla_{W} L_{i}(f(x_{i}, W), y_{i}) + \lambda \nabla_{W} R(W)$$

Stochastic (incremental) GD (SGD)

$$\nabla_W L(W) = \nabla_W L_i(f(x_i, W), y_i) + \lambda \nabla_W R(W)$$

#### Gradient Descent Implementations

#### Gradient Descent

- Too slow for large datasets
- We need to keep intermediate results for each datapoint forward pass
  - Memory problems
- May be quickly stuck in local minimum

#### Mini-batch GD

- This is used in Deep Learning practice
- Gradient approximation
- May skip some local minima
- $N_{mb}$  is usually 32/64/128/256 (depends on data/model/working memory size)

#### Chain rule

• Backpropagation is based on the chain rule:

$$h = f_1(z)$$

$$g = f_2(h)$$

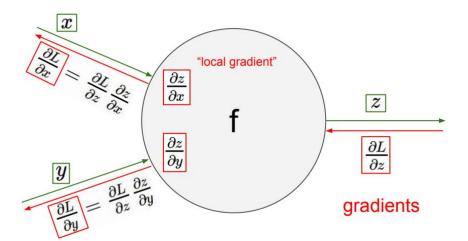
$$y = f_3(g)$$

$$y = f_3(f_2(f_1(z)))$$

$$\frac{\partial y}{\partial z} = \frac{\partial y}{\partial g} \frac{\partial g}{\partial h} \frac{\partial h}{\partial z}$$

## Backpropagation

#### 



#### Learning representations by back-propagating errors

David E. Rumelhart\*, Geoffrey E. Hinton† ©1986 Nature Publishing Group & Ronald J. Williams\*

$$E = \frac{1}{2} \sum_{c} \sum_{j} (y_{j,c} - d_{j,c})^{2}$$
 (3)

The backward pass starts by computing  $\partial E/\partial y$  for each of the output units. Differentiating equation (3) for a particular case, c, and suppressing the index c gives

$$\partial E/\partial y_j = y_j - d_j \tag{4}$$

We can then apply the chain rule to compute  $\partial E/\partial x_i$ 

$$\partial E/\partial x_j = \partial E/\partial y_j \cdot dy_j/dx_j$$

Differentiating equation (2) to get the value of  $dy_j/dx_j$  and substituting gives

$$\partial E/\partial x_i = \partial E/\partial y_i \cdot y_i (1 - y_i) \tag{5}$$

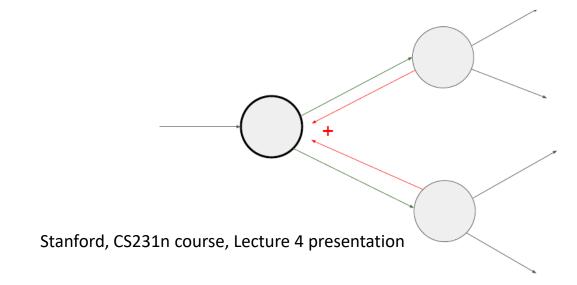
This means that we know how a change in the total input x to an output unit will affect the error. But this total input is just a linear function of the states of the lower level units and it is also a linear function of the weights on the connections, so it is easy to compute how the error will be affected by changing these states and weights. For a weight  $w_{ji}$ , from i to j the derivative is

$$\partial E/\partial w_{ji} = \partial E/\partial x_j \cdot \partial x_j/\partial w_{ji}$$

$$= \partial E/\partial x_i \cdot y_i$$
(6)

## Computational graphs

Neuron with multiple outputs – gradients add

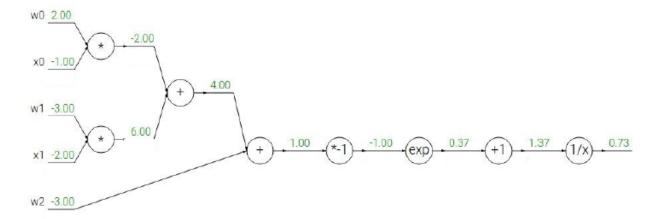


• Forward and backward passes in these graphs can be vectorized (Jacobian matrix instead of gradient)

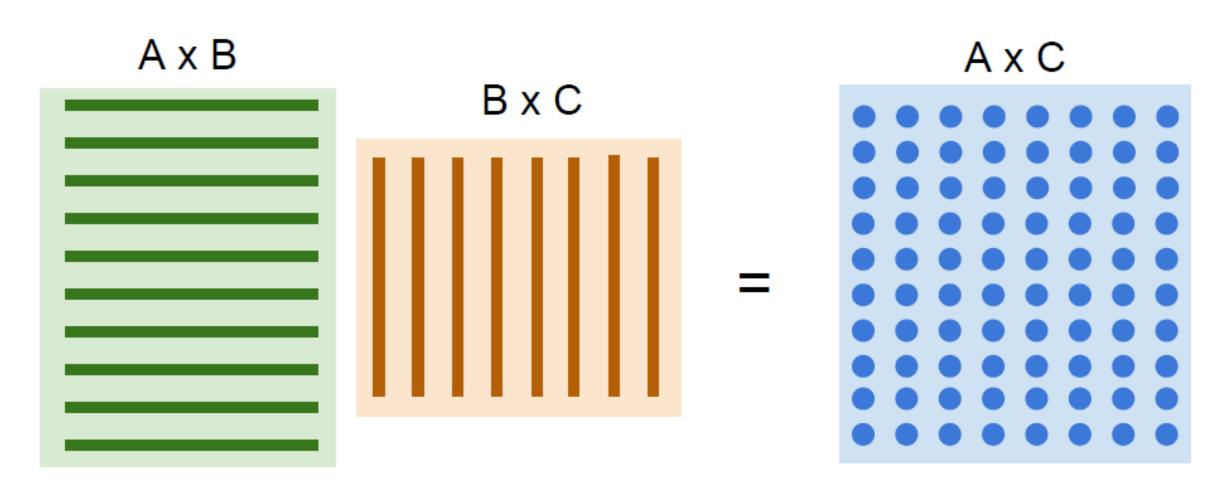
## Computational graphs

- ANN will be very large: impractical to calculate all gradient formulas by hand
- Implementation of DL algorithms comes down to implementing computational graphs and calculating their partial derivatives
  - This is called backpropagation

$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



## Matrix multiplications can be parallelized ⇒ GPUs



#### Common practices

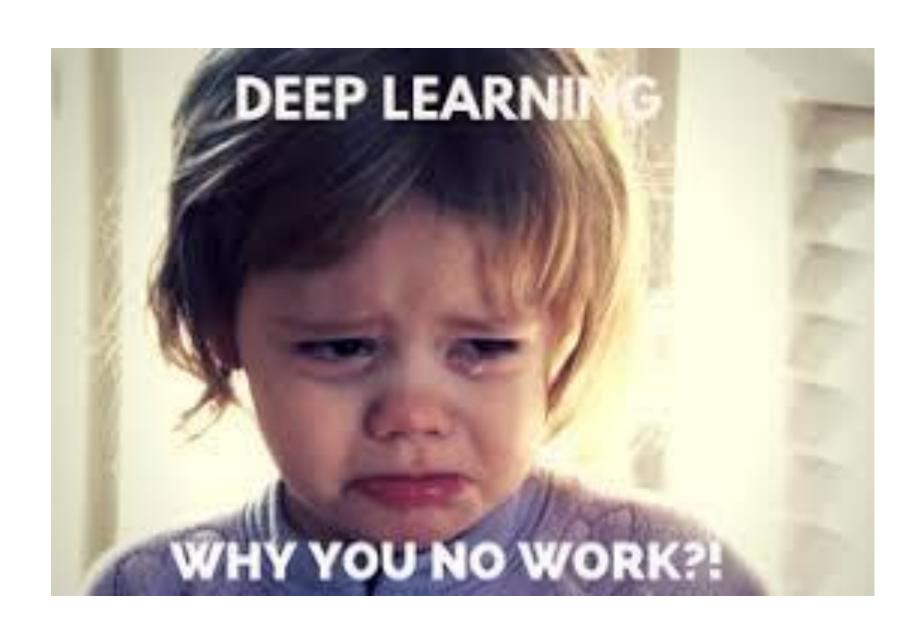
- Usually we start with certain amount of labeled samples
  - $(x_i, y_i)$ ; i = 1, ..., n
- The dataset is divided into three partitions
  - Training set used in training
  - Validation set used to tune hyperparameters
  - Test set used to measure quality of the network
- Neural network are "data" hungry
- Randomization
  - Present samples in random order

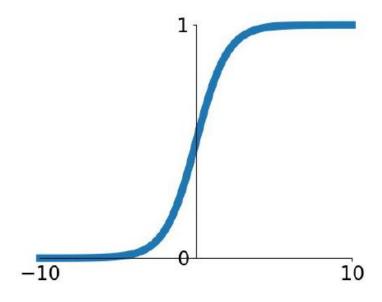
## Some terminology

- Forward pass: Calculate the network outputs  $y_i$  based on the inputs  $x_i$
- Backward pass: Calculate gradients  $\frac{\partial L}{\partial w_{ij}}$ , by using the chain rule and intermediate calculations from the forward pass
- Backpropagation: Update network weights  $w_{ij}$  by using the calculated gradients  $\frac{\partial L}{\partial w_{ij}}$
- Mini-batch: examples whose gradients are averaged before backpropagation
- Epoch: The ENTIRE dataset is passed ONCE forwards and backwards
  - A complete training goes through multiple epoch

#### You can train an ANN now



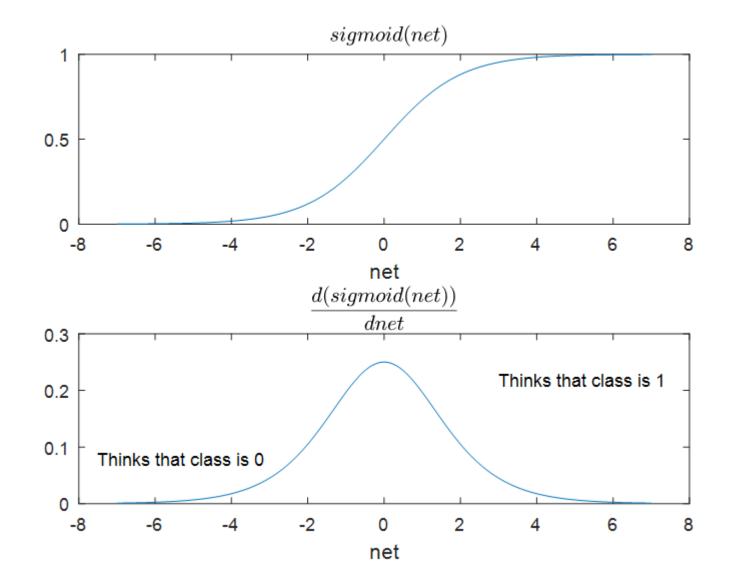


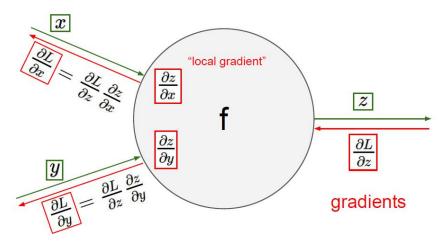


Saturation "kills" the gradients X

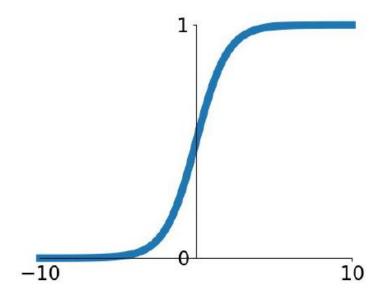
Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \frac{1}{1 + e^{-net}}$$





Stanford, CS231n course, Lecture 4 presentation



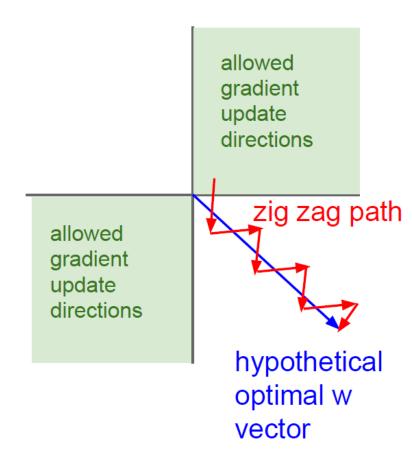
- Saturation "kills" the gradients X
- Sigmoid always outputs a positive number X

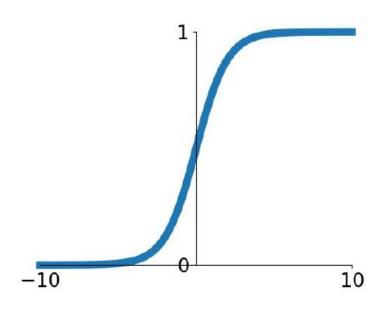
Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \frac{1}{1 + e^{-net}}$$

• 
$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial w_i}$$
  
•  $z = \sum_i x_i w_i + b$  (pre-activation)

- 1. When inputs  $x_i$  are always positive
  - $\Rightarrow \frac{\partial z}{\partial w_i} = x_i$  is always positive
- 2. Each  $w_i$  uses the same gradient  $\frac{\partial L}{\partial z}$
- $\Rightarrow$  Gradient update rule is positive/negative for all  $w_i$



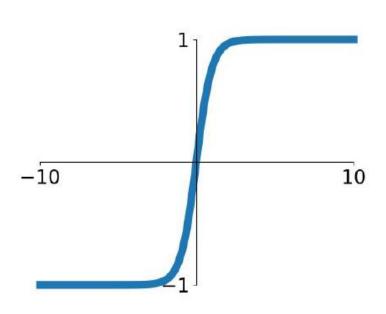


Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \frac{1}{1 + e^{-net}}$$

- Saturation "kills" the gradients X
- Sigmoid always outputs a positive number X
- exp() is a bit computationally expensive X
  - Not a big problem

# Activation Functions – Hyperbolic Tangent



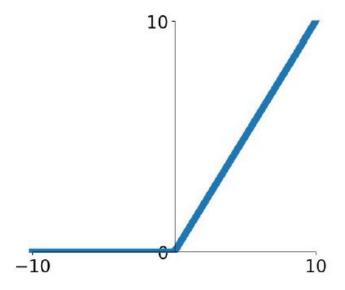
Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \tanh(net)$$

Zero centered √

- Saturation "kills" the gradients X
- A bit computationally expensive X
   (not a big problem)

#### Activation Functions — ReLU



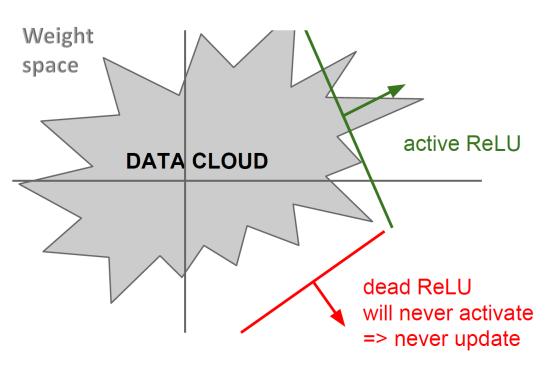
Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \max(0, net)$$

- Doesn't saturate in + region √
- Computationally efficient √
- Converges much faster than sigmoid/tanh in practice √

- Output not zero-centered X
- Gradient is 0 in region X
   (saturation)

#### Activation Functions — ReLU



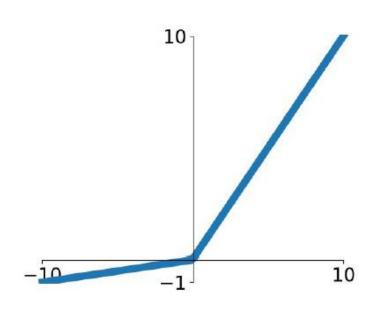
Stanford, CS231n course, Lecture 6 presentation

- If datapoints don't activate the ReLU, it won't update weights in a BP step
  - ReLUs with bad initialized weights will stay dead

- Learning rate too high
  - ReLU could fall into the dead ReLU region

 Initializing ReLU biases with small positive numbers often helps

## Activation Functions — Leaky ReLU



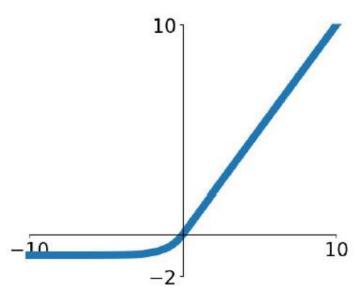
Stanford, CS231n course, Lecture 6 presentation

$$\sigma(net) = \max(0.01net, net)$$

- Doesn't saturate in + region √
- Computationally efficient √
- Converges much faster than sigmoid/tanh in practice √
- Doesn't saturate in region √
   (will not die)

 Slope in the – region could be parametrized (learn it with backprop)

#### Activation Functions — ELU



Stanford, CS231n course, Lecture 6 presentation

- Similar to leaky ReLU √
- Saturation in the region adds some robustness to noise compared to leaky ReLU √

exp() is a bit computationally expensive X

$$\sigma(net) = \begin{cases} net & net > 0 \\ \alpha(e^{net} - 1) & net \le 0 \end{cases}$$

#### Activation Functions – Maxout

$$\sigma(net) = (w_1^T x + b_1, w_2^T x + b_2)$$

- Generelizes ReLU and Leaky ReLU √
- Doesn't saturate √
- Doesn't die √

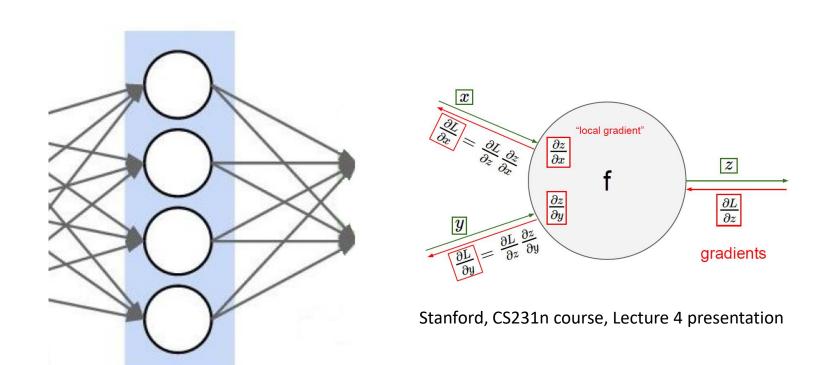
Twice as many parameters as ReLU X

#### Activation Functions — Practical advice

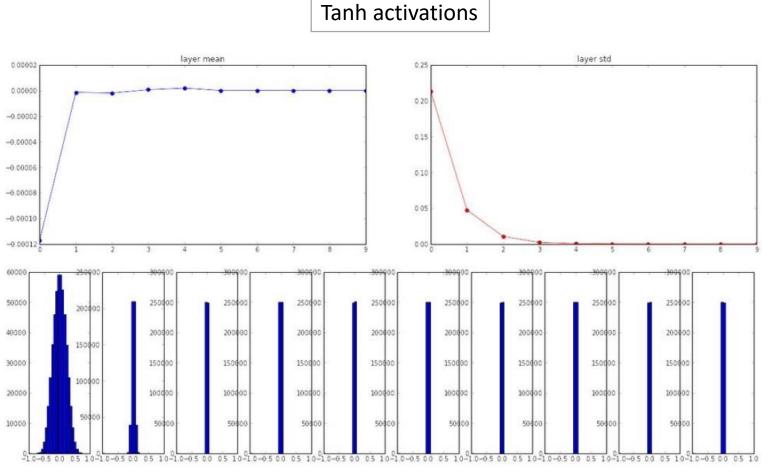
- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU
- Try out tanh but don't expect much
- Don't use sigmoid

#### Weight Initialization – W=0

What will happen if we initialize all weights to 0?



#### Weight Initialization – Small Random Values

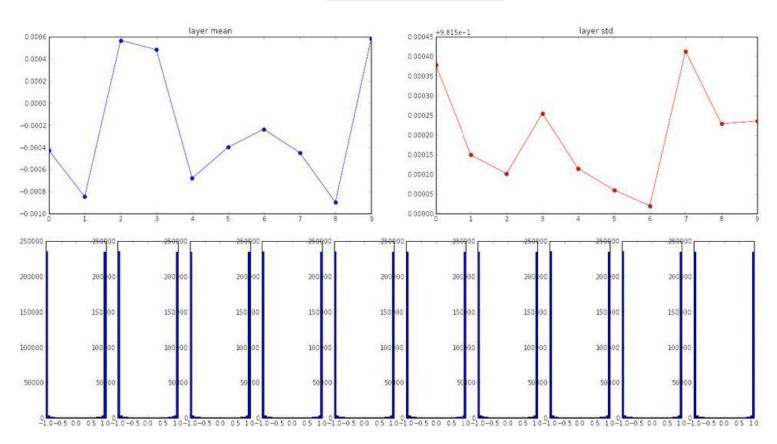


 Deeper into the network activations become zero

- During backpropagation each layer is multiplied by w
  - Slow learning

#### Weight Initialization – Big Random Values

#### Tanh activations

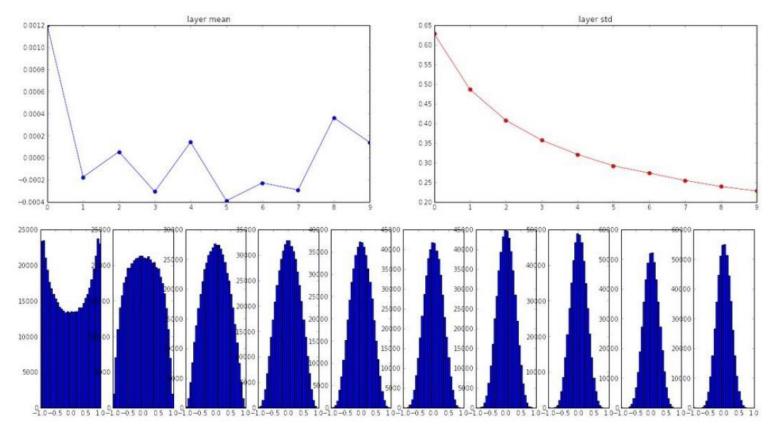


 Most of the activations go into saturation.

- Gradients will be small because of the saturations
  - Slow learning

#### Weight Initialization — Xavier





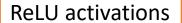
- Glorot et al., 2010
  - Wants the same variance at input & output of a layer
- Layer initialization:
  - Typically Gaussian or Uniform

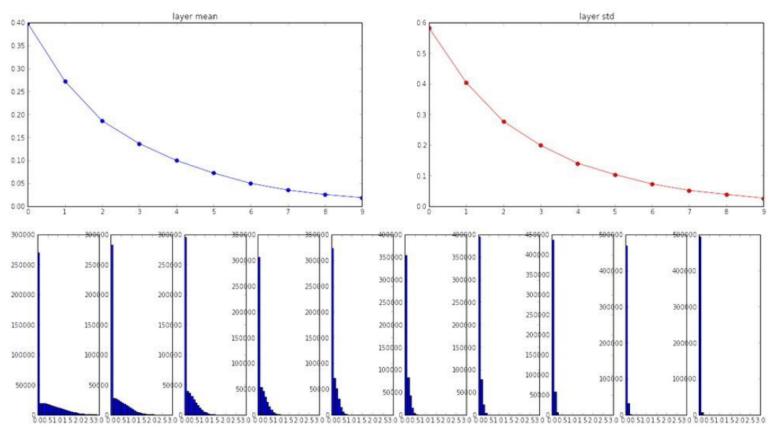
$$\mu(W) = 0$$

$$\sigma(W) = \frac{2}{n_{in} + n_{out}}$$

- $n_{in}$  number of units in the previous layer
- $n_{out}$  number of units in the next layer

#### Weight Initialization – Xavier



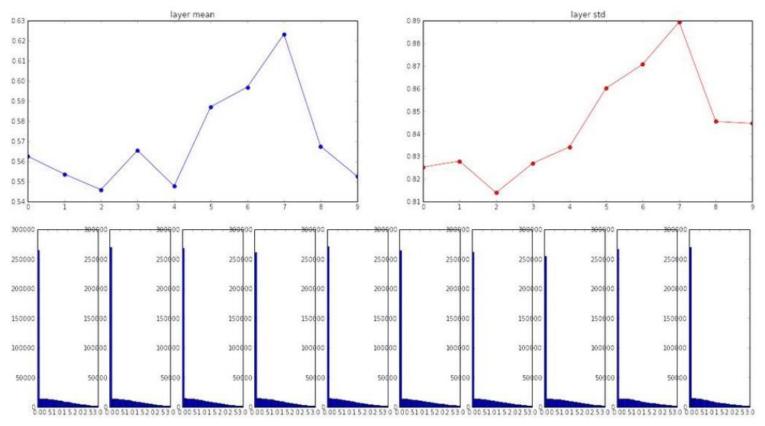


 Doesn't work well on ReLU

Stanford, CS231n course, Lecture 6 presentation

#### Weight Initialization – Recent Recommendation





- He et al., 2015
- Layer initialization:
  - Typically Gaussian or Uniform  $\mu(W)=0$   $\sigma(W)=\frac{2}{2}$
  - $n_{in}$  number of units in the previous layer

Stanford, CS231n course, Lecture 6 presentation

## Weight Initialization

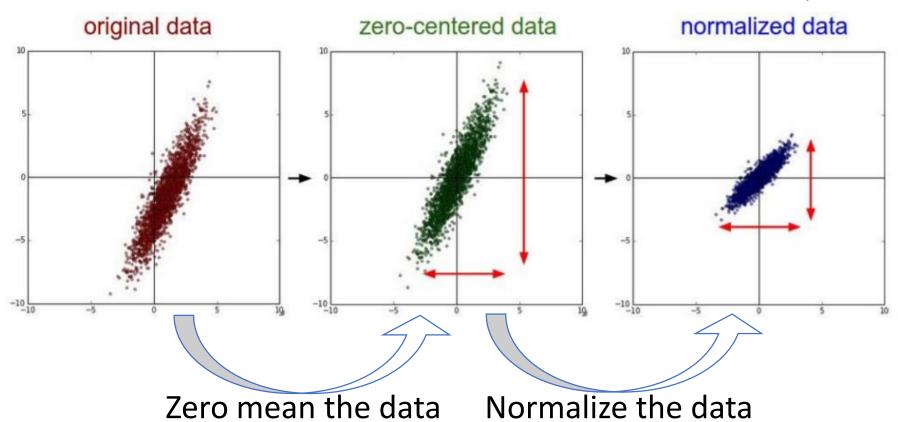
• Having a good initialization scheme is important in practice

Some networks couldn't be trained at all without a good initialization

Still an open area of research

#### Data preprocessing

Stanford, CS231n course, Lecture 6 presentation



All features are in the same range ⇒ they all contribute equally

#### In practice

#### Training phase:

- Subtract the mean for each image
- Normalize with standard deviation

#### Testing phase:

Subtract empirical mean (obtained from training data)

Only helps for the first layer!

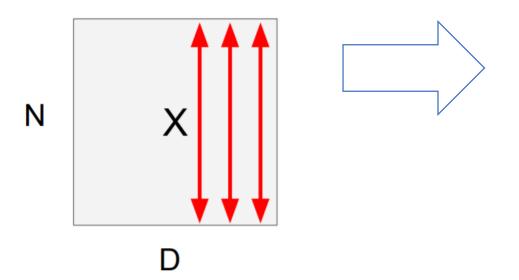
#### Batch normalization

$$\hat{x}^{(k)} = \frac{x^{(k)} - E\{x^{(k)}\}}{\sqrt{var\{x^{(k)}\}}}$$
$$y^{(k)} = \gamma^{(k)} x^{(k)} + \beta^{(k)}$$

- $\beta \& \gamma$  are hyperparameters that can change during training
- Network can learn to undo the normalization of not suitable

$$\gamma^{(k)} = \operatorname{var}\{x^{(k)}\}\$$
$$\beta^{(k)} = \operatorname{E}\{x^{(k)}\}\$$

#### Batch normalization



 $x^{(k)}$  – current batch

N – training examples in the batch

D – dimension of each batch

Compute empirical mean and variance independently for each dimension

$$\mathbf{x} : \mathbf{N} \times \mathbf{D}$$
Normalize
$$\boldsymbol{\mu}, \boldsymbol{\sigma} : \mathbf{1} \times \mathbf{D}$$

$$\mathbf{y}, \boldsymbol{\beta} : \mathbf{1} \times \mathbf{D}$$

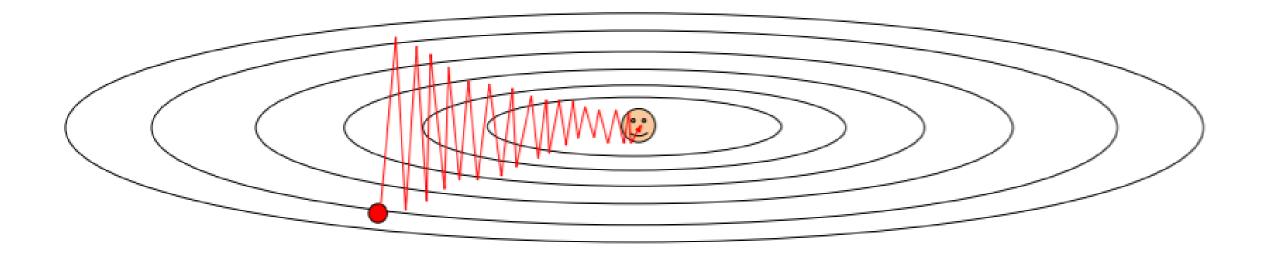
$$\mathbf{y} = \mathbf{y}(\mathbf{x} - \boldsymbol{\mu}) / \boldsymbol{\sigma} + \boldsymbol{\beta}$$

#### Batch normalization

```
Input: Values of x over a mini-batch: \mathcal{B} = \{x_{1...m}\};
          Parameters to be learned: \gamma, \beta
Output: \{y_i = BN_{\gamma,\beta}(x_i)\}
                                                    // mini-batch mean
                                               // mini-batch variance
                                                             // normalize
    y_i \leftarrow \gamma \hat{x}_i + \beta \equiv BN_{\gamma,\beta}(x_i)
                                                       // scale and shift
```

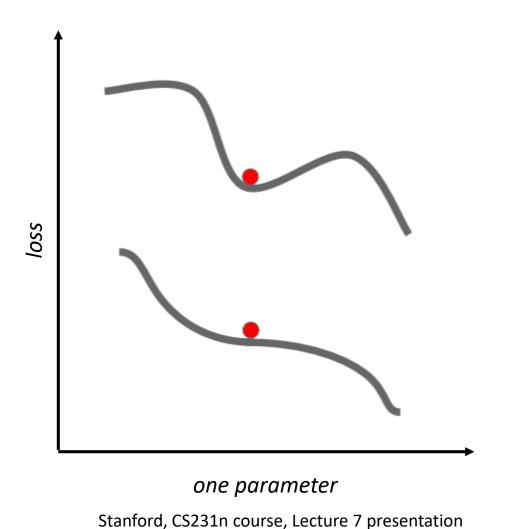
- More robust to:
  - Higher learning rates
  - Different kinds of weight init.
- Improves gradient flow
  - Easier to train
- Some kind of regularization
- It is not the same during training and testing

# 1° problem with SGD



- Different sensitivity to different dimensions
  - Slow progress along shallow dimension
  - Zigzag behavior along steep dimension

# 2° problem with SGD

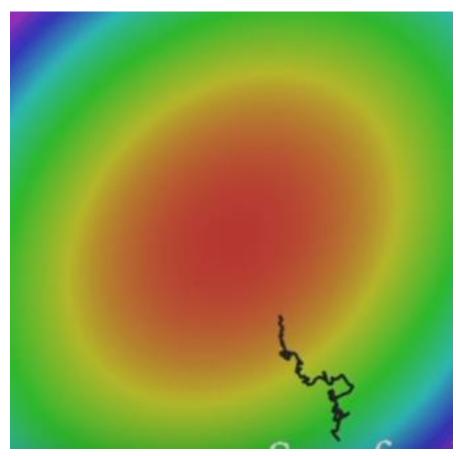


Can get stuck in local minima

 Local minima is not a big problem for high dimensions

- Saddle point is more common problem
  - Gradient is very small

#### 3° Problem with SGD



Stanford, CS231n course, Lecture 7 presentation

- We usually use minibatches
  - We are only getting the noisy estimation of the gradient in the current minibatch

#### Momentum

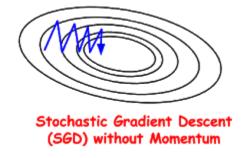
SGD

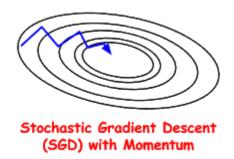
$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

SGD + Momentum

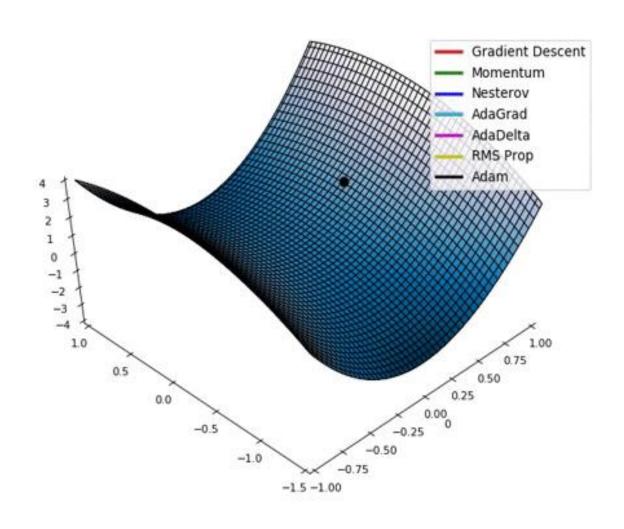
$$v_{t+1} = \rho v_t + \alpha \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

- SGD always stepping in the direction of the gradient
- SGD + Momentum stepping in the direction of velocity



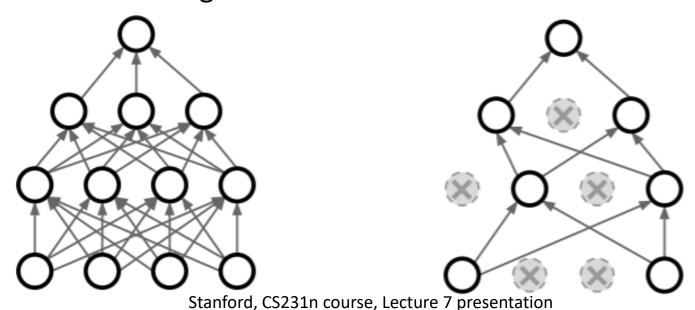


# More sophisticated options



#### Regularization - dropout

- In each pass through the NN we randomly set some of the neurons to zero, with probability p one layer at the time
- Dropping probability p is a hyperparameter
- Each dropout gives us a different subset of the NN
  - Dropout is like learning whole ensemble of networks at the same time



# Dropout – why is it good?

- Helps to prevent co-adaptation of features
- The algorithm doesn't depend to much on one feature

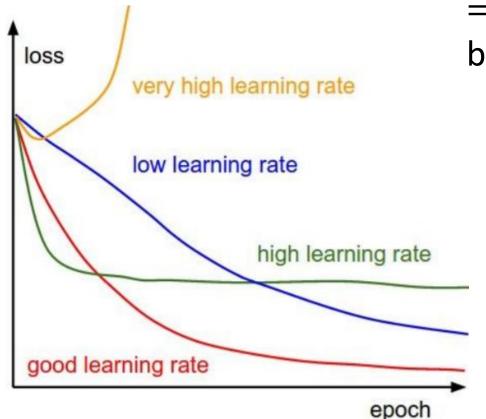


Can help in overfitting!

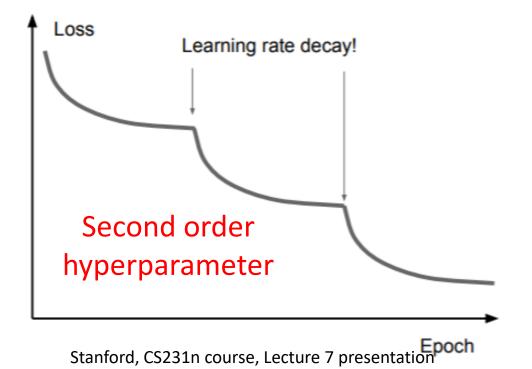


# Learning rate adjusting

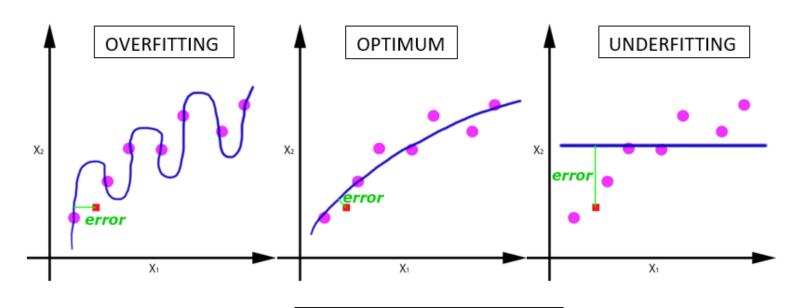
• SGD, SGD + Momentum, AdaGrad, RMSProp and Adam have learning rate as hyperparameter



⇒ Learning rate decay over time: start with bigger value, and decrease it over time



# What is Overfitting?





Fighting overfitting

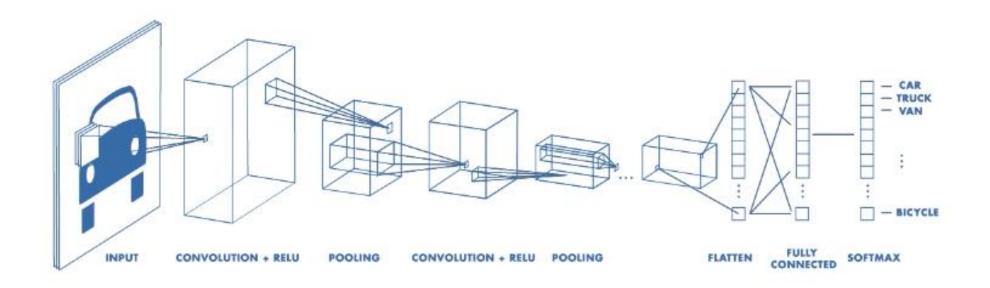
- Smaller network
- Weight decay
- Dropout

- Early stopping
- More data
  - Data augmentation

Source: <a href="https://towardsdatascience.com/">https://towardsdatascience.com/</a>

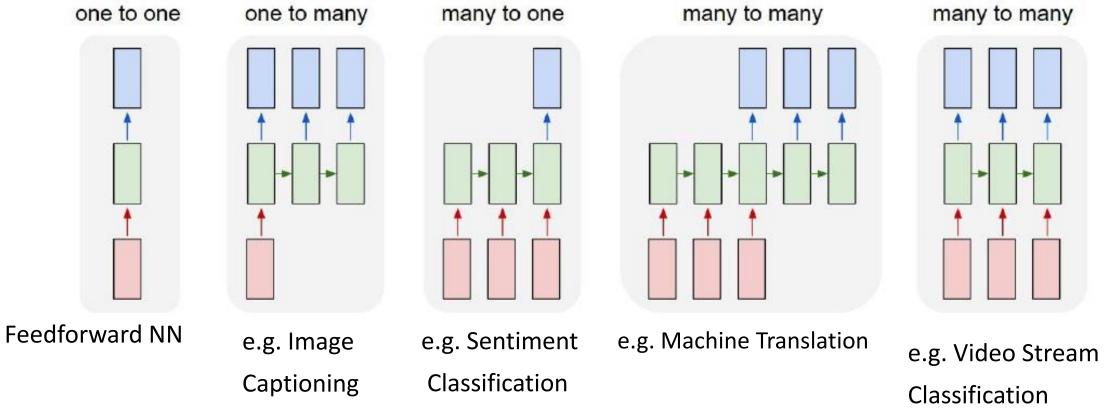
#### Popular Deep Learning models

• Convolutional Neural Networks (CNN) – process Image-like data



## Popular Deep Learning models

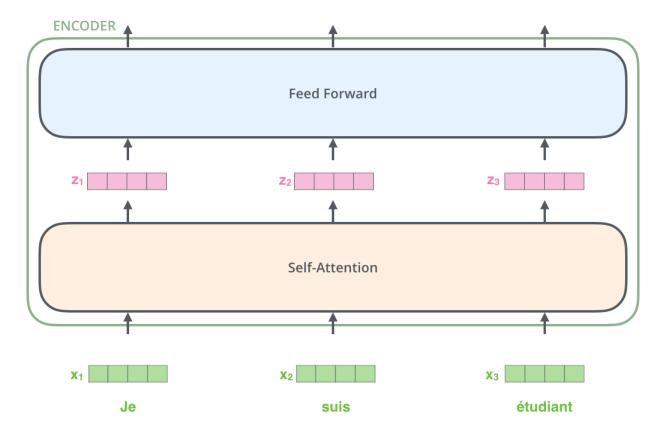
• Recurrent Neural Networks (RNN) –process sequences



Slide taken from - Stanford, CS231n course, Lecture 10 presentation http://cs231n.stanford.edu/slides/2018/cs231n 2018 lecture10.pdf

#### Popular Deep Learning models

Transformers – process sets and sequences



https://jalammar.github.io/illustrated-transformer/

# Some terminology

- Optimizer
- Loss
- Weights
- Backprop
- Learning rate
- Batch size
- Weight decay
- Dropout
- Hyperparameters
- Batch Norm
- Layer
- Activations
- Augmentation
- Softmax
- 0

# Thank you!

