Exercise 4.30: Does the fixed-point theorem also hold without the assumption that the lattice has finite height? If yes, give a proof; if no, give a counterexample.

## 解答:

如果格高度无限长,则不动点定理不成立。

举例: 定义函数  $f: L \to L$ ,在格 $(L, \Box)$ 中,L 为整数集, $\Box$  为 $\leq$ , $\forall x \in L$ ,f(x) = x + 1。解释: f(x)与 g(x) = x, $x \in Z$  无交点,也即 $\forall k$ ,  $f^{k+1}(\bot) = f^k(\bot) + 1$ ,故  $f^k(\bot) \neq f^{k+1}(\bot)$ 。