

**Exercise 4.18:** A function  $f: L_1 \rightarrow L_2$  where  $L_1$  and  $L_2$  are lattices is *distributive* when  $\forall x, y \in L_1: f(x) \sqcup f(y) = f(x \sqcup y)$ .

(a) Show that every distributive function is also monotone.

(b) Show that not every monotone function is also distributive.

**证明:**

(a) Show that every distributive function is also monotone:

设  $\forall x, y \in L_1$ , 当  $x \sqsubseteq y$  时, 由练习 4.2 结论得知, 当  $x \sqcup y$  存在, 有  $x \sqcup y = y$ 。因为  $f: L_1 \rightarrow L_2$  为 distributive 函数, 故  $f(x) \sqcup f(y) = f(x \sqcup y) = f(y)$ , 再由练习 4.2 结论得  $f(x) \sqsubseteq f(y)$ 。所以设  $\forall x, y \in L_1: x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$ , 故该函数为 monotone, 条件 a 成立。

(b) Show that not every monotone function is also distributive:

假设  $f: L_1 \rightarrow L_2$  为单调函数, 则取  $\forall x, y \in L_1: x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$ 。

(1) 若  $f(x) \sqcup f(y)$  存在, 则  $f(x) \sqcup f(y) = f(y)$ , 为使得  $f(x) \sqcup f(y) = f(y) = f(x \sqcup y)$ , 还须  $x \sqcup y$  存在!

(2) 若  $x \sqcup y$  存在, 则  $f(x \sqcup y) = f(y)$ , 为使得  $f(x) \sqcup f(y) = f(y) = f(x \sqcup y)$ , 还须  $f(x) \sqcup f(y)$  存在!