

# Computational Natural Language Processing

## Self-Attention and Transformers

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# Lecture Plan

1. From recurrence (RNN) to attention-based NLP models
2. The Transformer model
3. Great results with Transformers

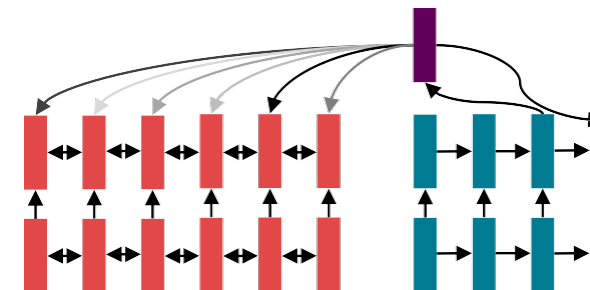
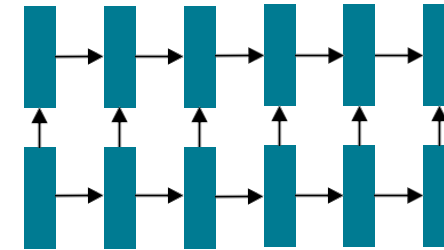
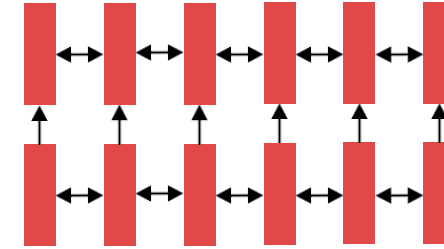
## Reminders:

Assignment 1 is due today!

Assignment 2 will be out today!

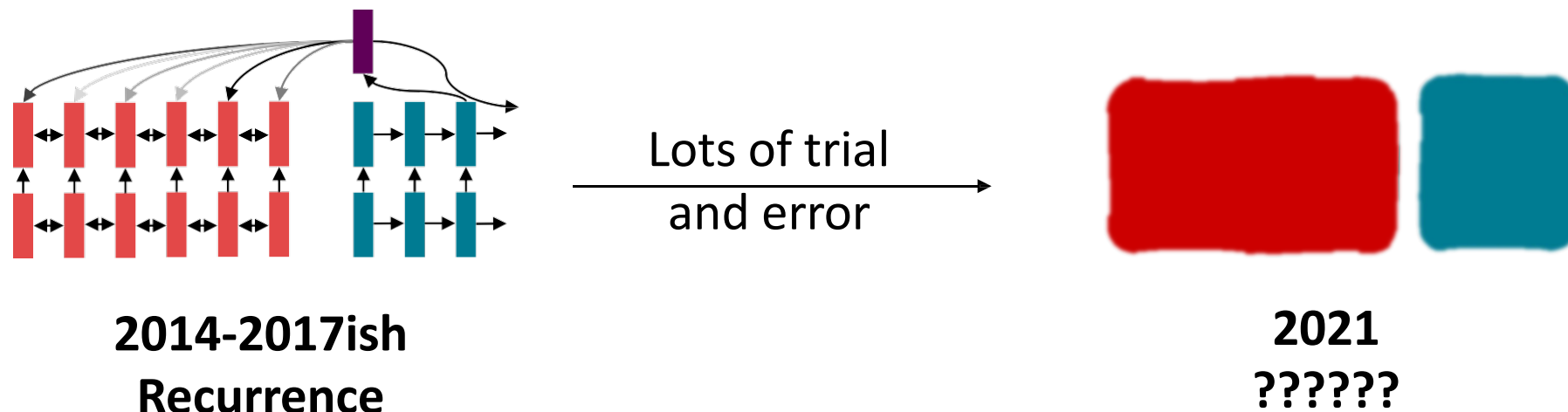
# As of last lecture: recurrent models for (most) NLP!

- Circa 2016, the de facto strategy in NLP is to **encode** sentences with a bidirectional LSTM: (for example, the source sentence in a translation)
- Define your output (parse, sentence, summary) as a sequence, and use an LSTM to generate it.
- Use attention to allow flexible access to memory



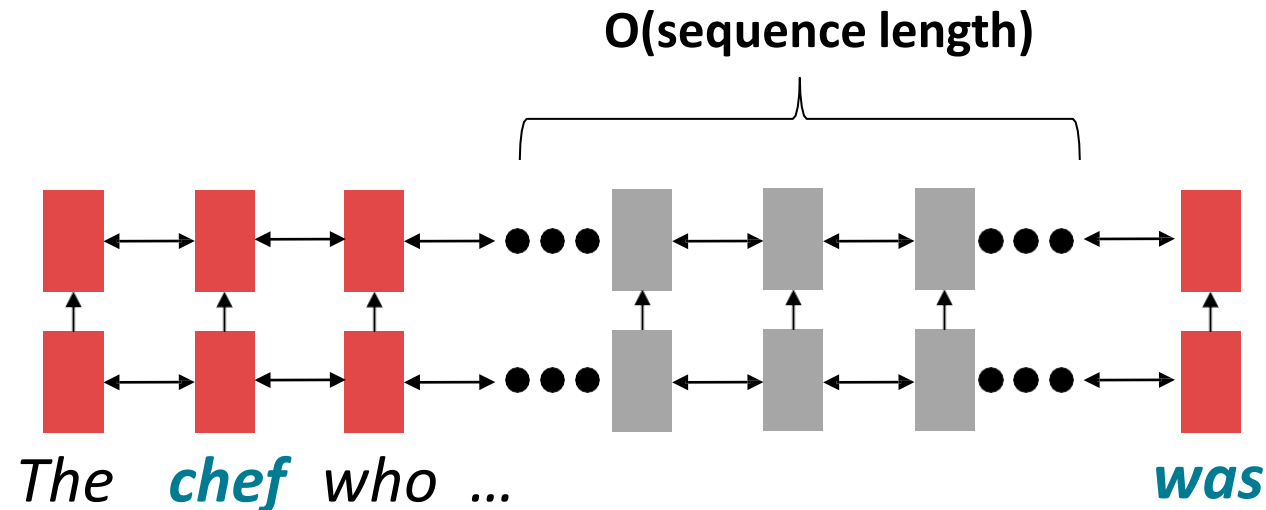
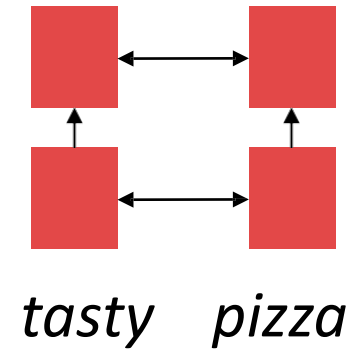
# Today: Same goals, different building blocks

- So far, we learned about sequence-to-sequence problems and encoder-decoder models.
- Today, we're **not** trying to motivate entirely new ways of looking at problems (like Machine Translation)
- Instead, we're trying to find the best **building blocks** to plug into our models and enable broad progress.



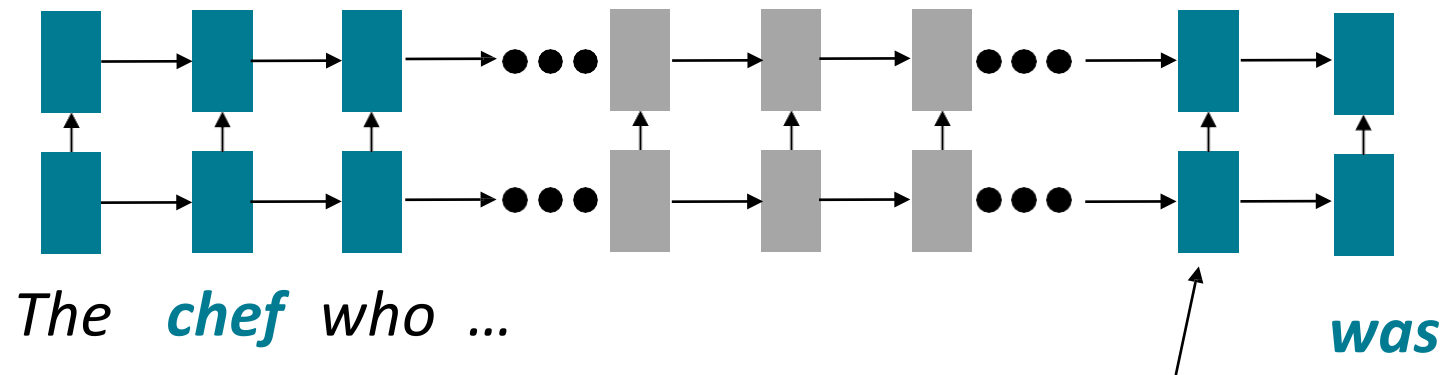
# Issues with recurrent models: Linear interaction distance

- RNNs are unrolled “left-to-right”.
- This encodes linear locality: a useful heuristic!
  - Nearby words often affect each other’s meanings
- **Problem:** RNNs take  $O(\text{sequence length})$  steps for distant word pairs to interact.



# Issues with recurrent models: Linear interaction distance

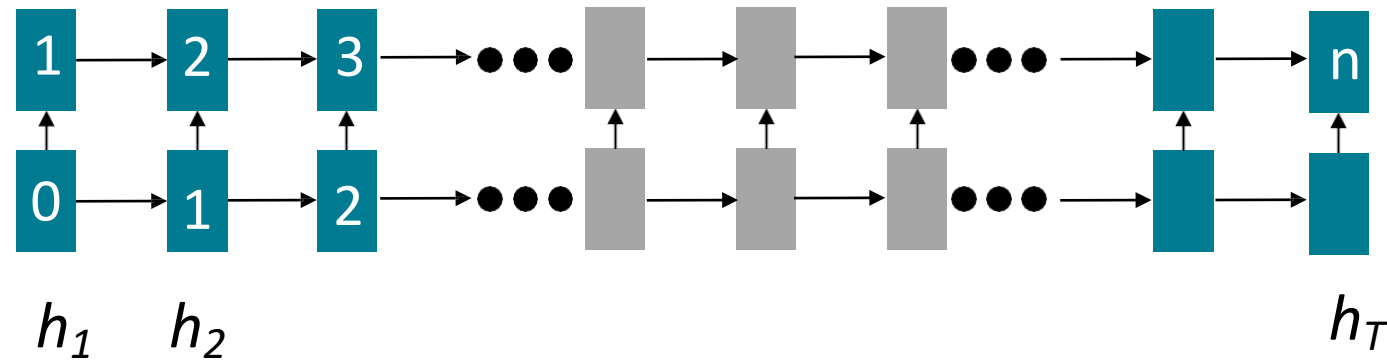
- **$O(\text{sequence length})$**  steps for distant word pairs to interact means:
  - Hard to learn long-distance dependencies (because gradient problems!)
  - Linear order of words is “baked in”; we already know linear order isn’t the right way to think about sentences...



Info of *chef* has gone through  $O(\text{sequence length})$  many layers!

# Issues with recurrent models: Lack of parallelizability

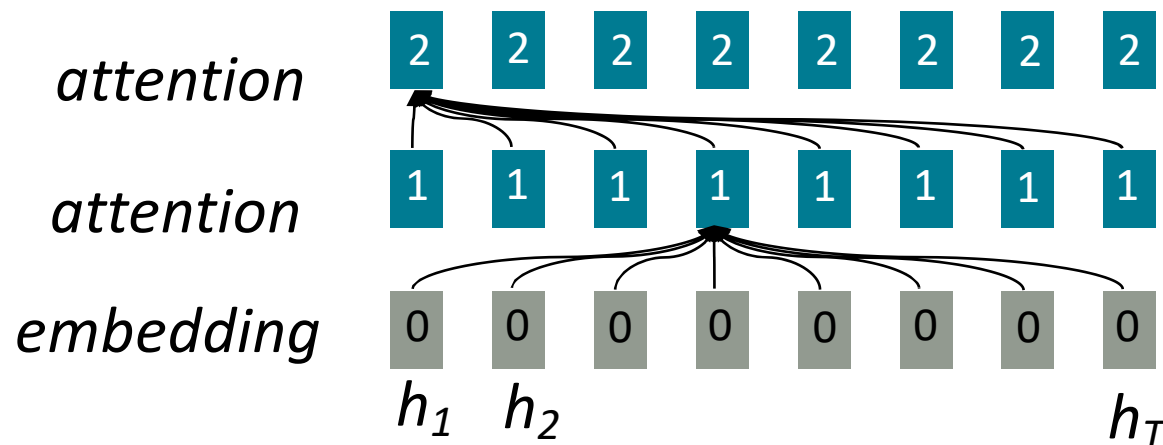
- Forward and backward passes have  **$O(\text{sequence length})$**  unparallelizable operations
  - GPUs can perform a bunch of independent computations at once!
  - But future RNN hidden states can't be computed in full before past RNN hidden states have been computed
  - Inhibits training on very large datasets!



Numbers indicate min # of steps before a state can be computed

# If not recurrence, then what? How about attention?

- **Attention** treats each word's representation as a **query** to access and incorporate information from **a set of values**.
  - We saw attention from the **decoder** to the **encoder**; today we'll think about attention **within a single sentence**.
- Number of unparallelizable operations does not increase with sequence length.
- Maximum interaction distance:  $O(1)$ , since all words interact at every layer!



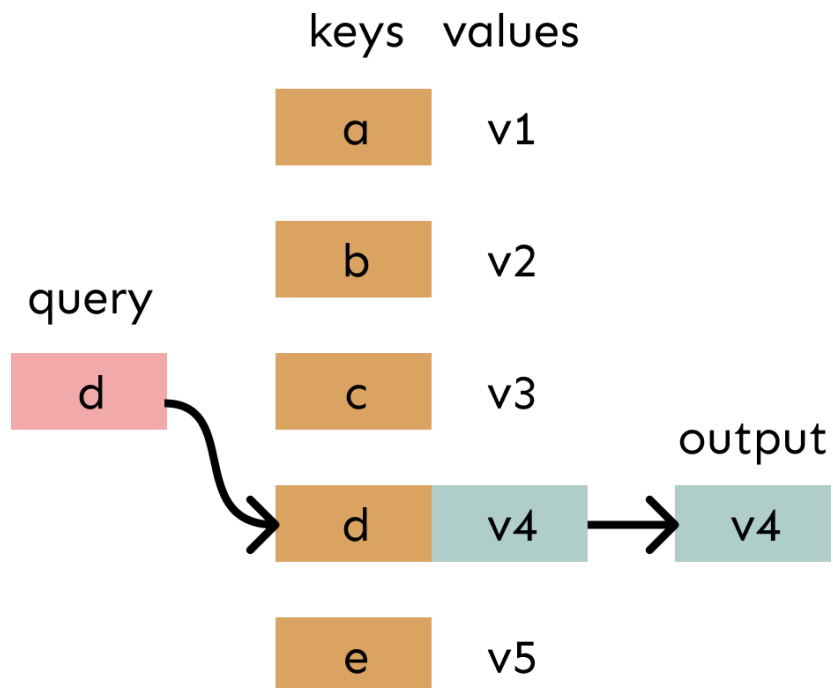
All words attend to all words in previous layer; most arrows here are omitted



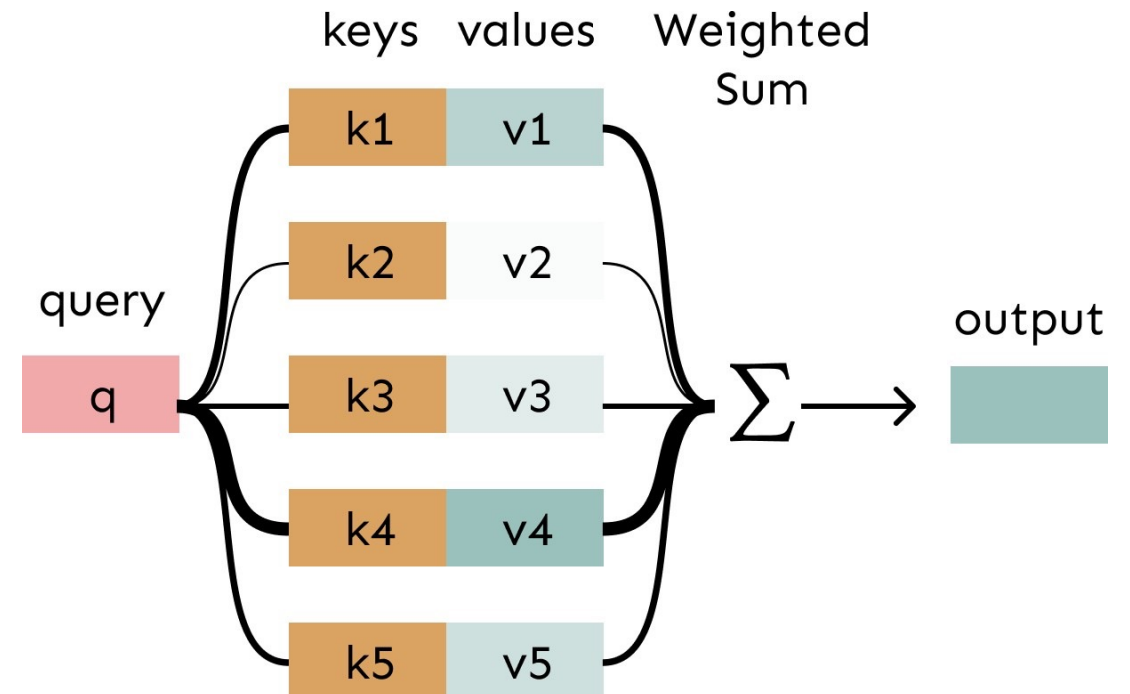
# Attention as a soft, averaging lookup table

We can think of **attention** as performing fuzzy lookup in a key-value store.

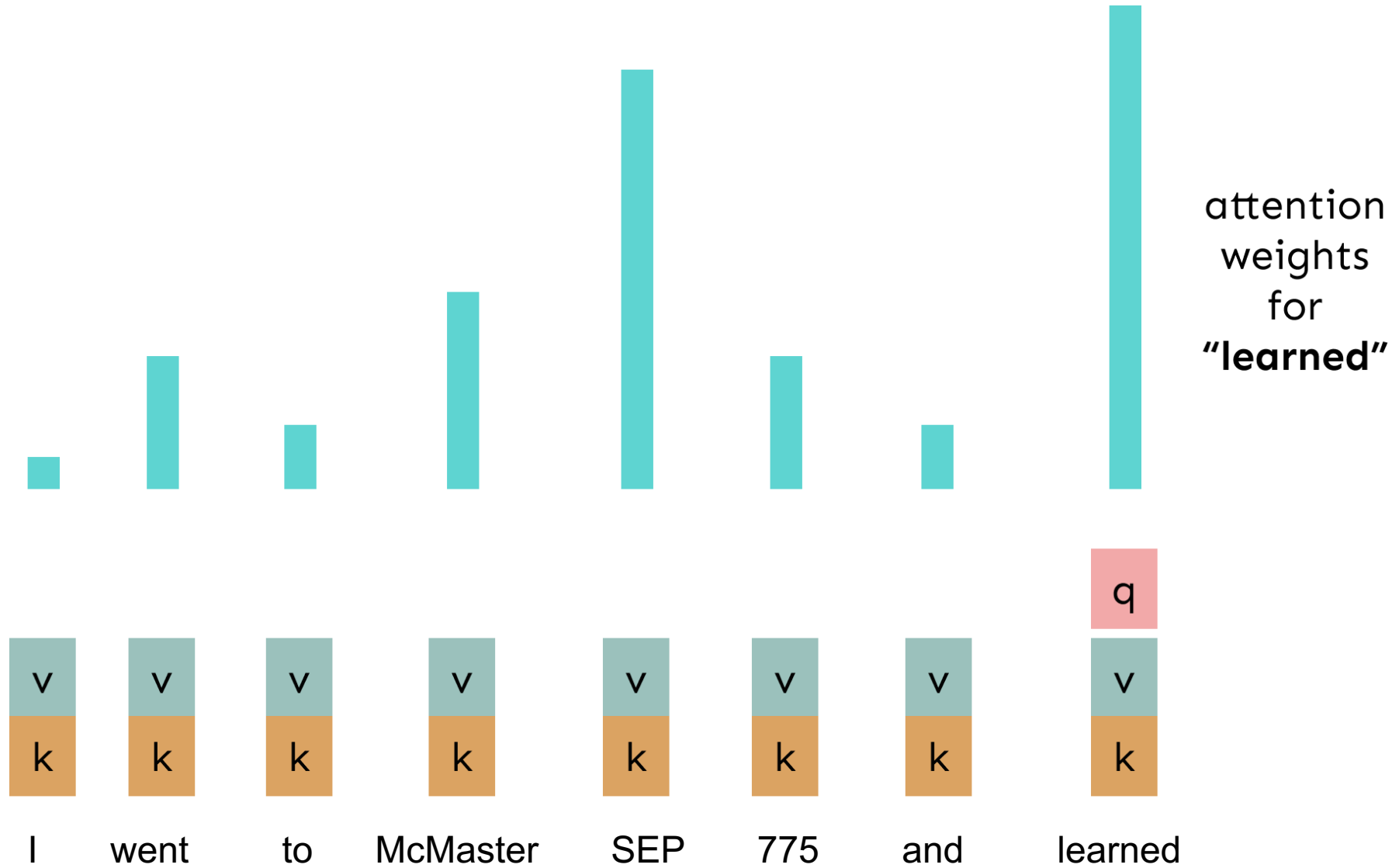
In a **lookup table**, we have a table of **keys** that map to **values**. The **query** matches one of the keys, returning its value.



In **attention**, the **query** matches all **keys** *softly*, to a weight between 0 and 1. The keys' **values** are multiplied by the weights and summed.



# Self-Attention Hypothetical Example



# Self-Attention: keys, queries, values from the same sequence

Let  $\mathbf{w}_{1:n}$  be a sequence of words in vocabulary  $V$ , like *Zuko made his uncle tea*.

For each  $\mathbf{w}_i$ , let  $\mathbf{x}_i = E\mathbf{w}_i$ , where  $E \in \mathbb{R}^{d \times |V|}$  is an embedding matrix.

1. Transform each word embedding with weight matrices  $Q, K, V$ , each in  $\mathbb{R}^{d \times d}$

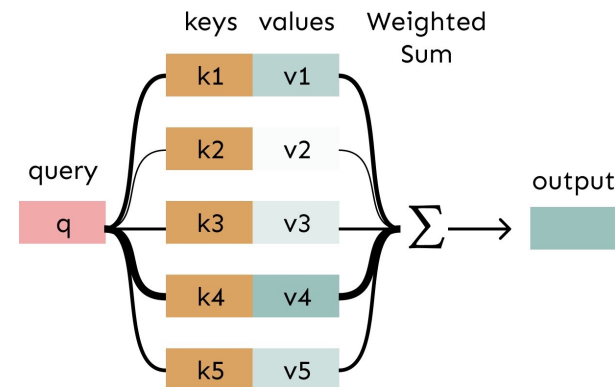
$$\mathbf{q}_i = Q\mathbf{x}_i \text{ (queries)} \quad \mathbf{k}_i = K\mathbf{x}_i \text{ (keys)} \quad \mathbf{v}_i = V\mathbf{x}_i \text{ (values)}$$

2. Compute pairwise similarities between keys and queries; normalize with softmax

$$\mathbf{e}_{ij} = \mathbf{q}_i^\top \mathbf{k}_j \quad \alpha_{ij} = \frac{\exp(\mathbf{e}_{ij})}{\sum_{j'} \exp(\mathbf{e}_{ij'})}$$

3. Compute output for each word as weighted sum of values

$$\mathbf{o}_i = \sum_j \alpha_{ij} \mathbf{v}_j$$



# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!



## Solutions

# Fixing the first self-attention problem: **sequence order**

- Since self-attention doesn't build in order information, we need to encode the order of the sentence in our keys, queries, and values.
- Consider representing each **sequence index** as a **vector**

$\mathbf{p}_i \in \mathbb{R}^d$ , for  $i \in \{1, 2, \dots, n\}$  are position vectors

- Don't worry about what the  $\mathbf{p}_i$  are made of yet!
- Easy to incorporate this info into our self-attention block: just add the  $\mathbf{p}_i$  to our inputs!
- Recall that  $\mathbf{x}_i$  is the embedding of the word at index  $i$ . The positioned embedding is:

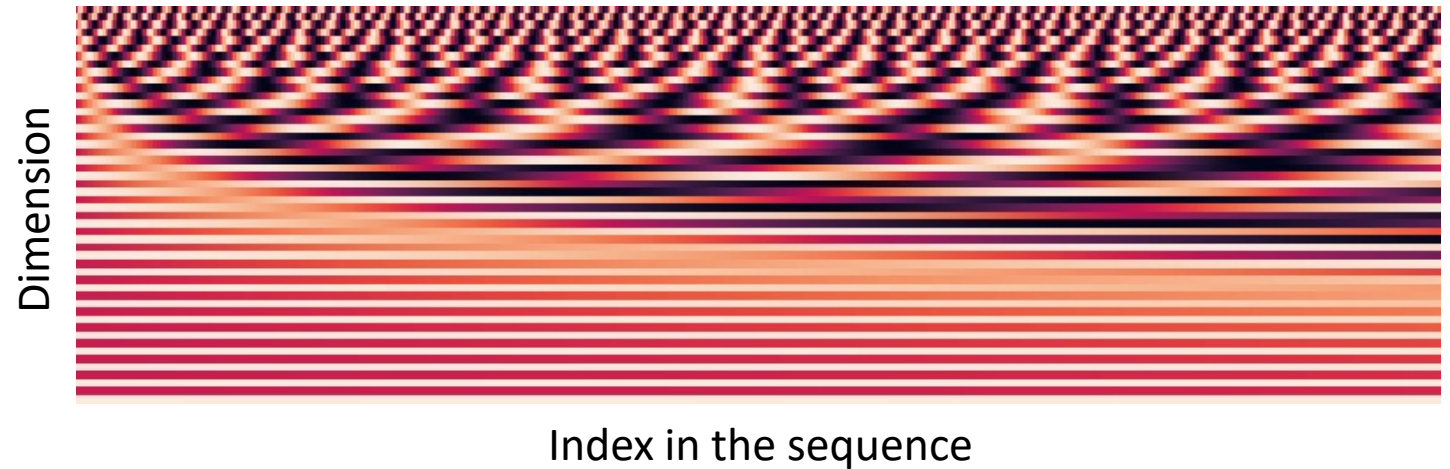
$$\tilde{\mathbf{x}}_i = \mathbf{x}_i + \mathbf{p}_i$$

In deep self-attention networks, we do this at the first layer! You could concatenate them as well, but people mostly just add...

# Position representation vectors through sinusoids

- **Sinusoidal position representations:** concatenate sinusoidal functions of varying periods:

$$\mathbf{p}_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$



- Pros:
  - Periodicity indicates that maybe “absolute position” isn’t as important
  - Maybe can extrapolate to longer sequences as periods restart!
- Cons:
  - Not learnable; also the extrapolation doesn’t really work!

# Position representation vectors learned from scratch

- **Learned absolute position representations:** Let all  $p_i$  be learnable parameters!  
Learn a matrix  $\mathbf{p} \in \mathbb{R}^{d \times n}$ , and let each  $\mathbf{p}_i$  be a column of that matrix!
- Pros:
  - Flexibility: each position gets to be learned to fit the data
- Cons:
  - Definitely can't extrapolate to indices outside  $1, \dots, n$ .
- Most systems use this!
- Sometimes people try more flexible representations of position:
  - Relative linear position attention [\[Shaw et al., 2018\]](#)
  - Dependency syntax-based position [\[Wang et al., 2019\]](#)

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning! It's all just weighted averages



## Solutions

- Add position representations to the inputs

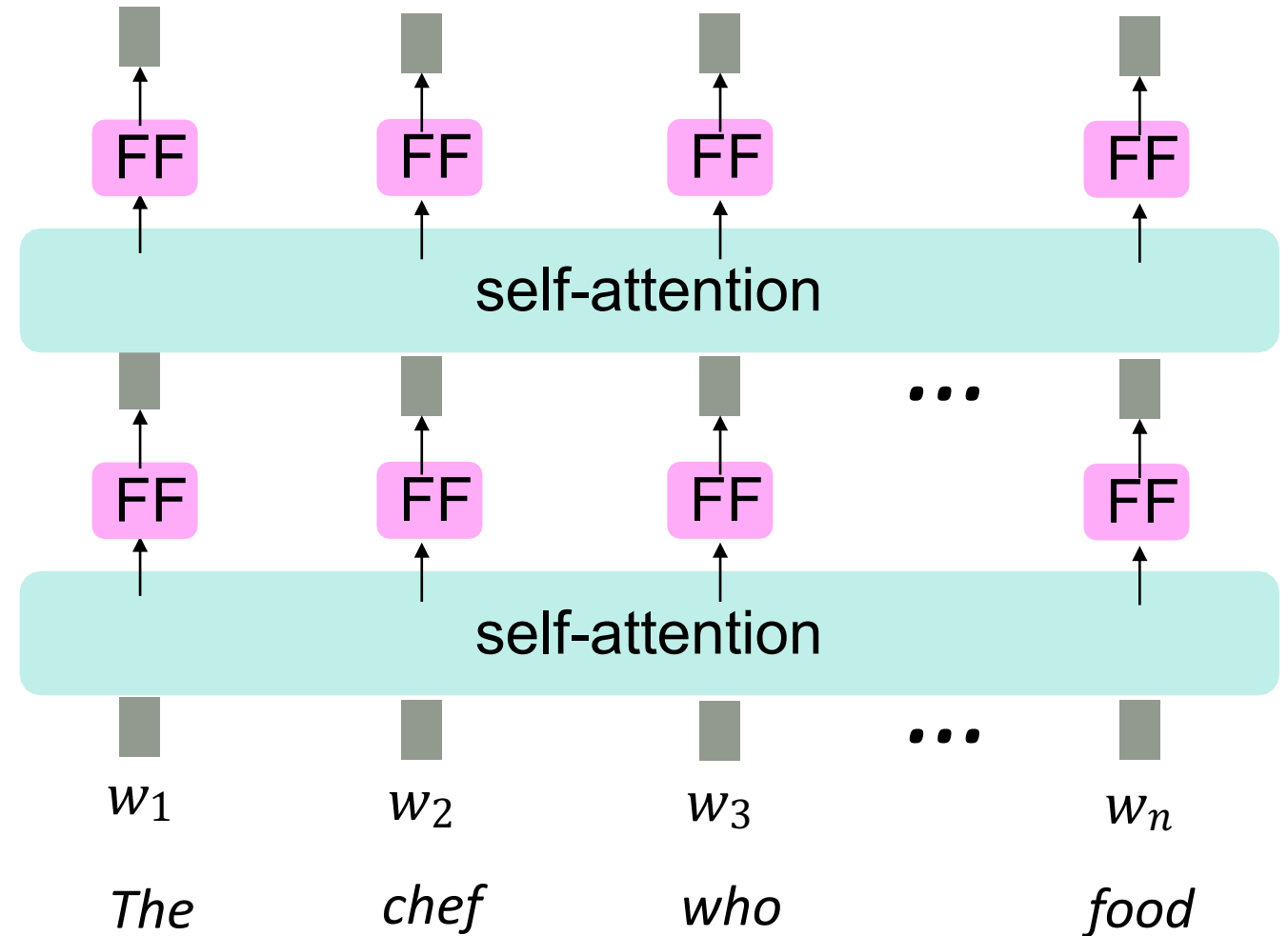




# Adding nonlinearities in self-attention

- Note that there are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages **value** vectors (Why?)
- Easy fix: add a **feed-forward network** to post-process each output vector.

$$\begin{aligned} m_i &= MLP(\text{output}_i) \\ &= W_2 * \text{ReLU}(W_1 \text{output}_i + b_1) + b_2 \end{aligned}$$



Intuition: the FF network processes the result of attention

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
  - Like in machine translation
  - Or language modeling



## Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.

# Masking the future in self-attention

- To use self-attention in **decoders**, we need to ensure we can't peek at the future.
- At every timestep, we could change the set of **keys and queries** to include only past words. (Inefficient!)
- To enable parallelization, we **mask out attention** to future words by setting attention scores to  $-\infty$ .

$$e_{ij} = \begin{cases} q_i^\top k_j, & j \leq i \\ -\infty, & j > i \end{cases}$$

For encoding these words

We can look at these (not greyed out) words

	[START]	The	chef	who
[START]		$-\infty$	$-\infty$	$-\infty$
The			$-\infty$	$-\infty$
chef				$-\infty$
who				

# Barriers and solutions for Self-Attention as a building block

## Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
  - Like in machine translation
  - Or language modeling

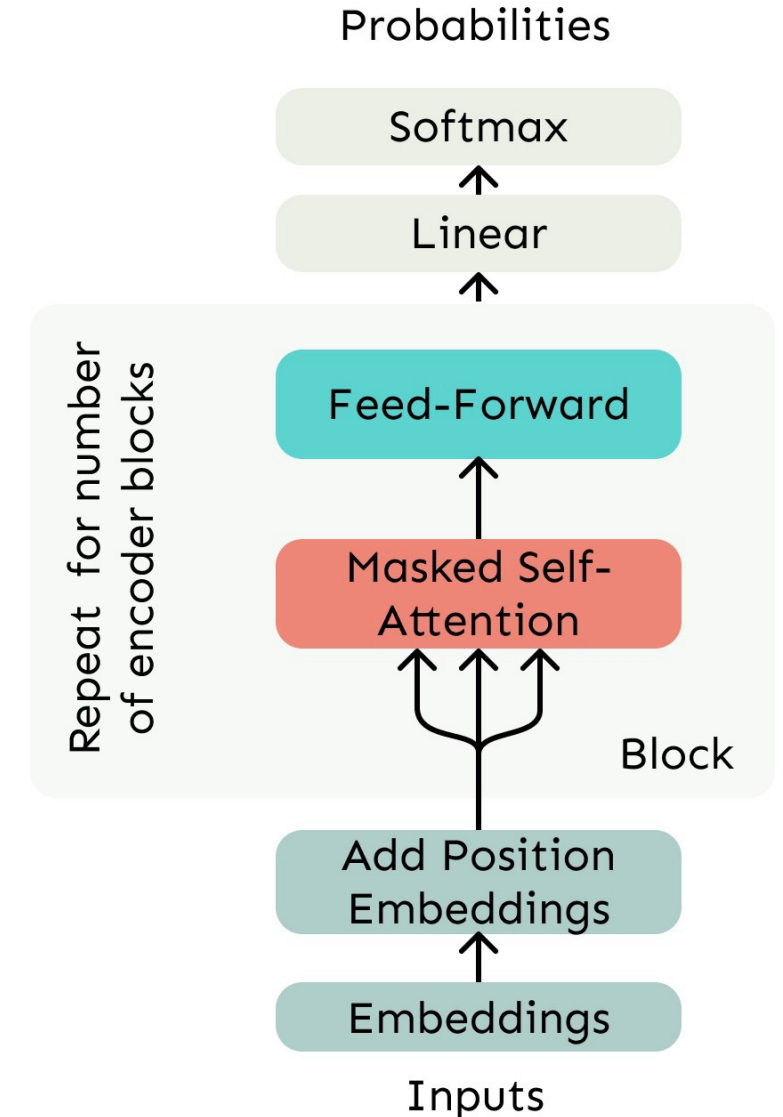


## Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.
- Mask out the future by artificially setting attention weights to 0!

# Necessities for a self-attention building block:

- **Self-attention:**
  - the basis of the method.
- **Position representations:**
  - Specify the sequence order, since self-attention is an unordered function of its inputs.
- **Nonlinearities:**
  - At the output of the self-attention block
  - Frequently implemented as a simple feed-forward network.
- **Masking:**
  - In order to parallelize operations while not looking at the future.
  - Keeps information about the future from “leaking” to the past.

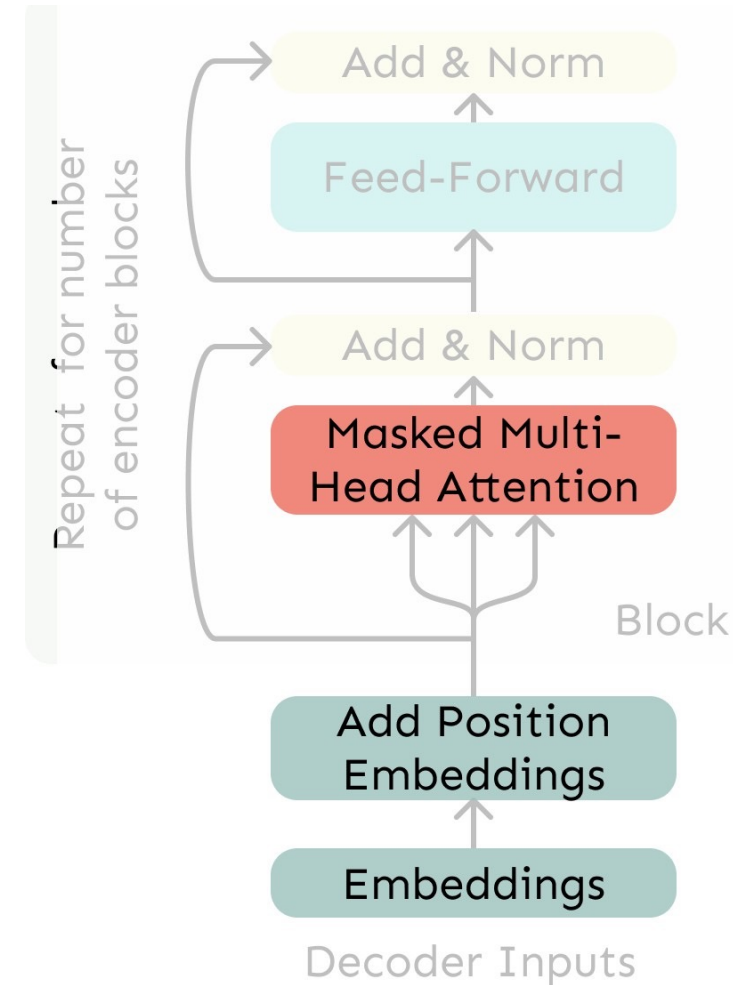


# Outline

1. From recurrence (RNN) to attention-based NLP models
2. The Transformer model
3. Great results with Transformers

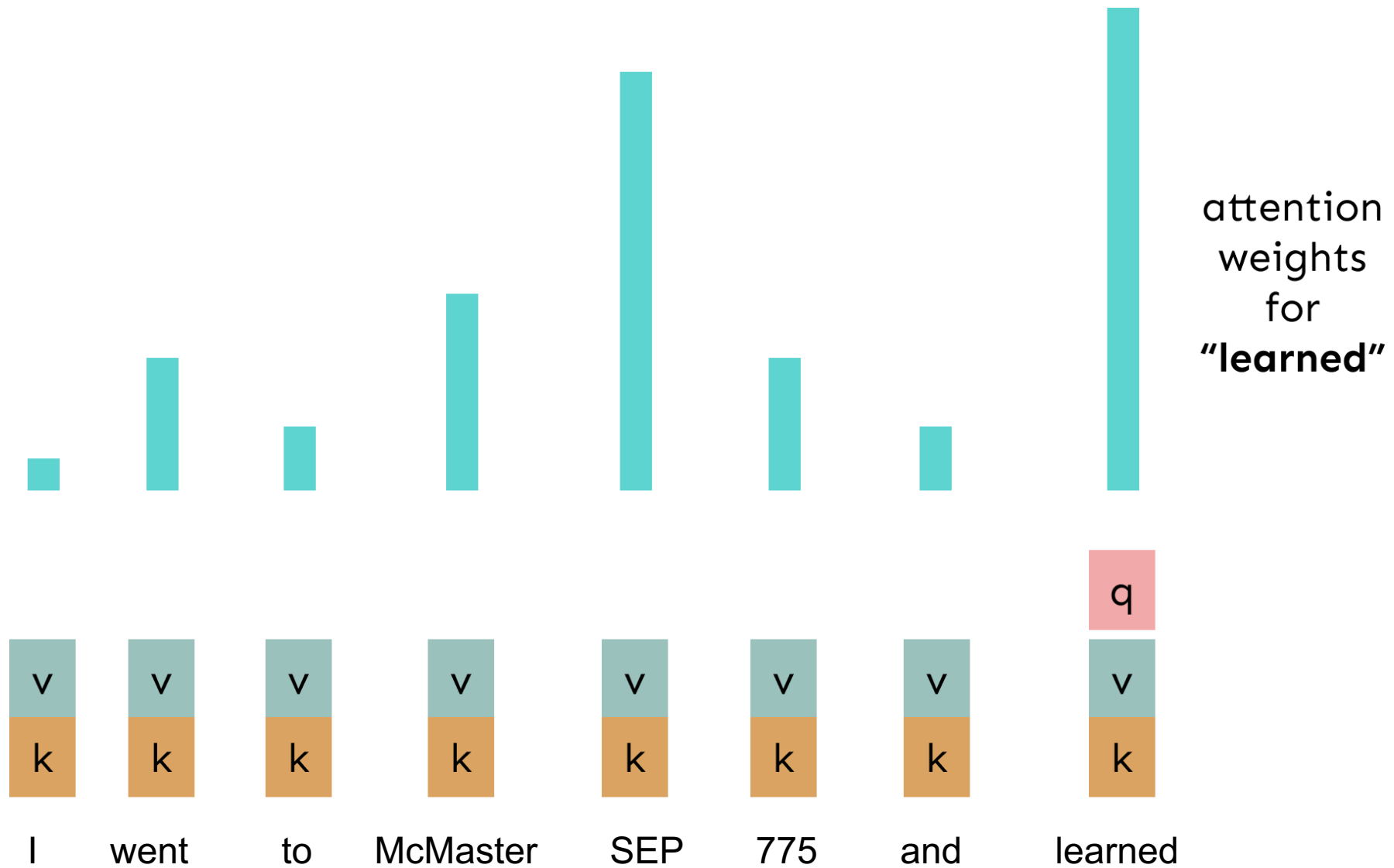
# The Transformer Decoder

- A Transformer decoder is how we'll build systems like **language models**.
- It's a lot like our minimal self-attention architecture, but with a few more components.
- The embeddings and position embeddings are identical.
- We'll next replace our self-attention with **multi-head self-attention**.



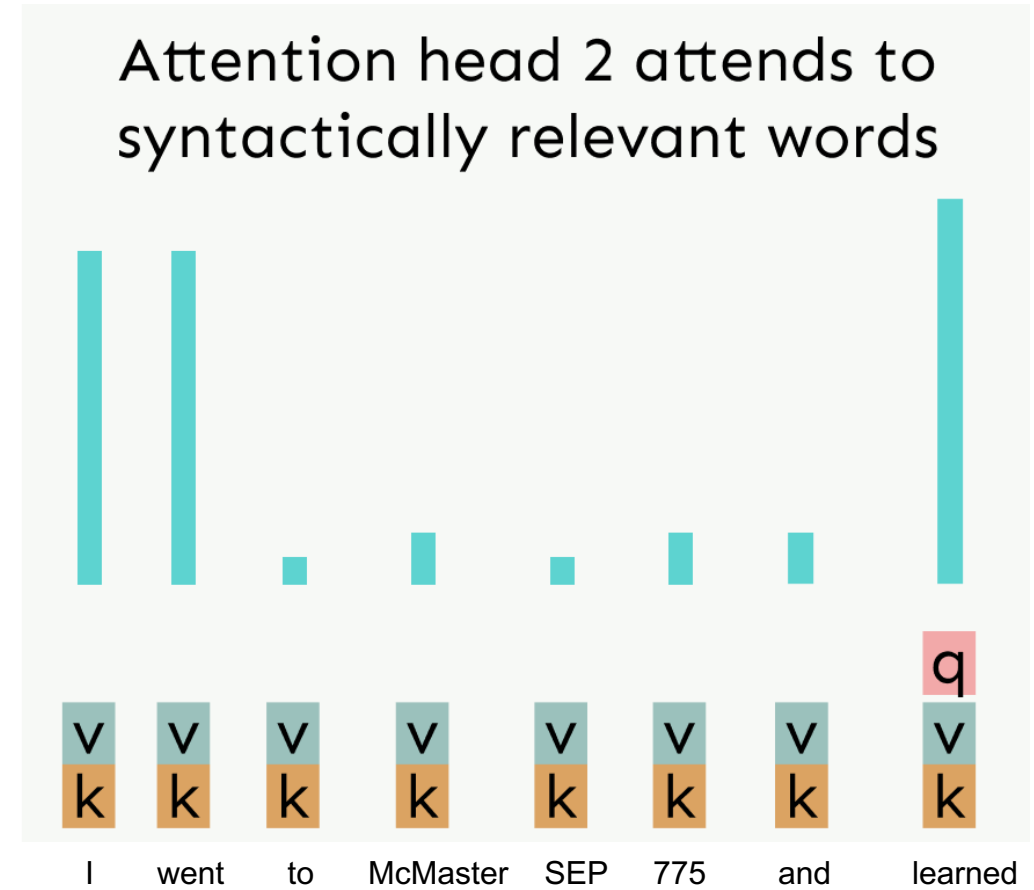
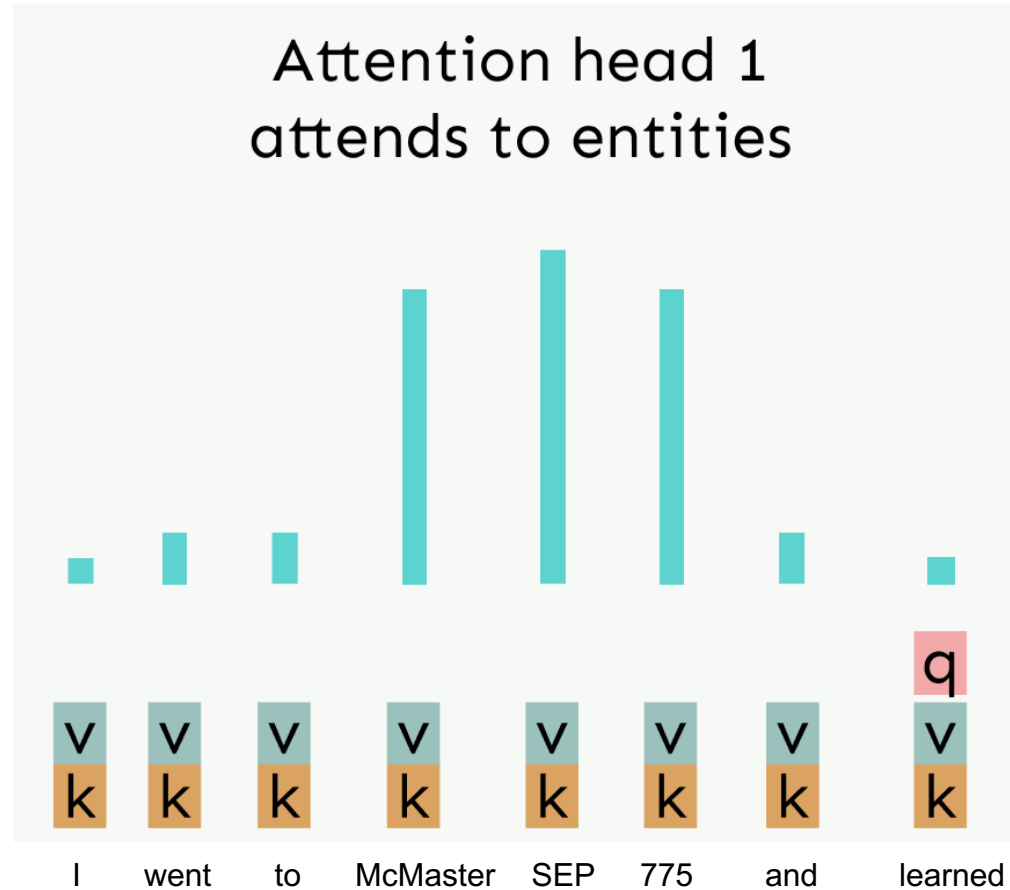
Transformer Decoder

# Recall the Self-Attention Hypothetical Example





# Hypothetical Example of Multi-Head Attention



# Sequence-Stacked form of Attention

- Let's look at how key-query-value attention is computed, in matrices.
  - Let  $X = [x_1; \dots; x_n] \in \mathbb{R}^{n \times d}$  be the concatenation of input vectors.
  - First, note that  $XK \in \mathbb{R}^{n \times d}$ ,  $XQ \in \mathbb{R}^{n \times d}$ ,  $XV \in \mathbb{R}^{n \times d}$ .
  - The output is defined as  $\text{output} = \text{softmax}(XQ(XK)^T)XV \in \mathbb{R}^{n \times d}$ .

First, take the query-key dot products in one matrix multiplication:  $XQ(XK)^T$

$$XQ \quad K^T X^T = XQK^T X^T \in \mathbb{R}_{n \times n}^{n \times n}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$$\text{softmax} \left( XQK^T X^T \right) XV = \text{output} \in \mathbb{R}^{n \times d}$$

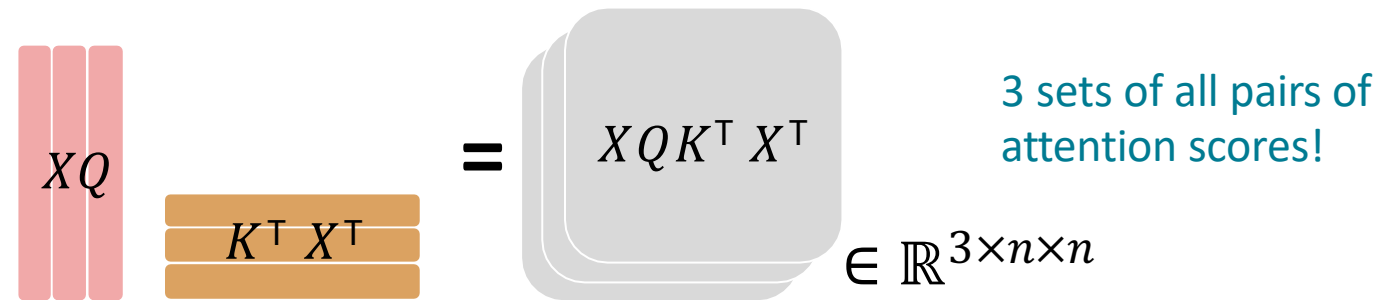
# Multi-headed attention

- What if we want to look in multiple places in the sentence at once?
  - For word  $i$ , self-attention “looks” where  $x_i^\top Q^\top K x_j$  is high, but maybe we want to focus on different  $j$  for different reasons?
- We’ll define **multiple attention “heads”** through multiple Q,K,V matrices
- Let,  $Q_\ell, K_\ell, V_\ell \in \mathbb{R}^{d \times \frac{d}{h}}$ , where  $h$  is the number of attention heads, and  $\ell$  ranges from 1 to  $h$ .
- Each attention head performs attention independently:
  - $\text{output}_\ell = \text{softmax}(X Q_\ell K_\ell^\top X^\top) * X V_\ell$ , where  $\text{output}_\ell \in \mathbb{R}^{d/h}$
- Then the outputs of all the heads are combined!
  - $\text{output} = [\text{output}_1; \dots; \text{output}_h] Y$ , where  $Y \in \mathbb{R}^{d \times d}$
- Each head gets to “look” at different things, and construct value vectors differently.

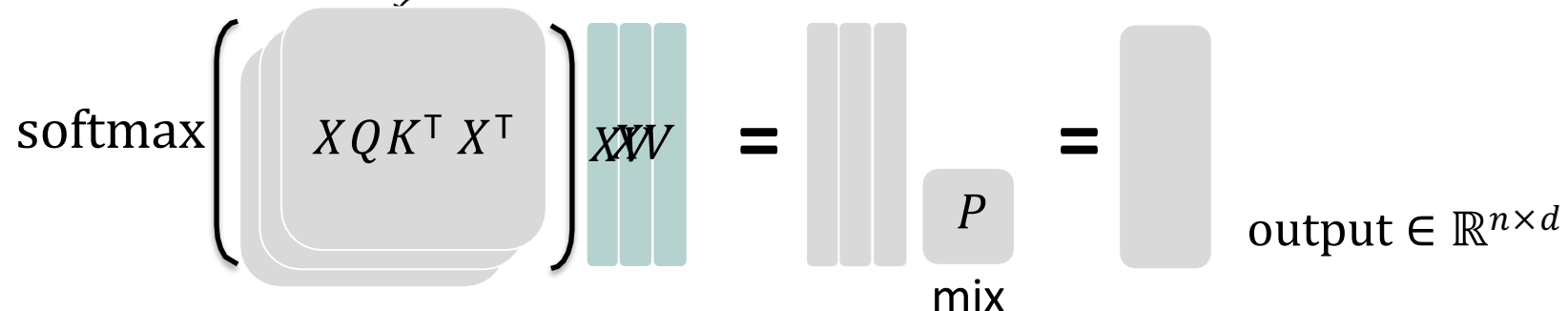
# Multi-head self-attention is computationally efficient

- Even though we compute  $h$  many attention heads, it's not really more costly.
  - We compute  $XQ \in \mathbb{R}^{n \times d}$ , and then reshape to  $\mathbb{R}^{n \times h \times d/h}$ . (Likewise for  $XK$ ,  $XV$ .)
  - Then we transpose to  $\mathbb{R}^{h \times n \times d/h}$ ; now the head axis is like a batch axis.
  - Almost everything else is identical, and the **matrices are the same sizes**.

First, take the query-key dot products in one matrix multiplication:  $XQ(XK)^\top$

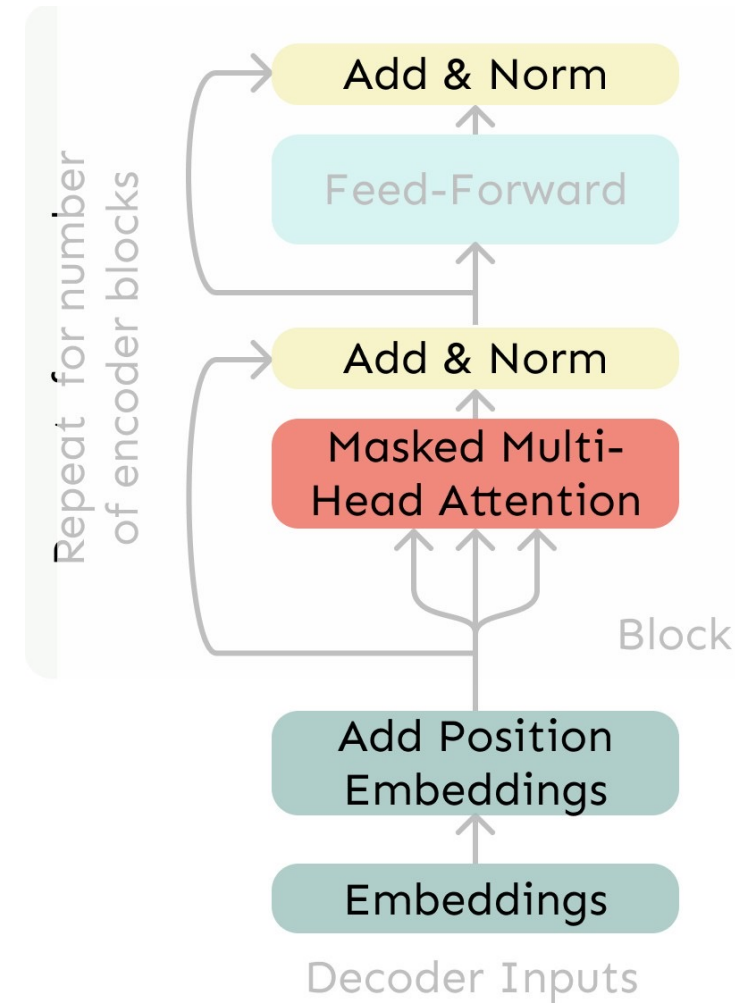


Next, softmax, and compute the weighted average with another matrix multiplication.



# The Transformer Decoder

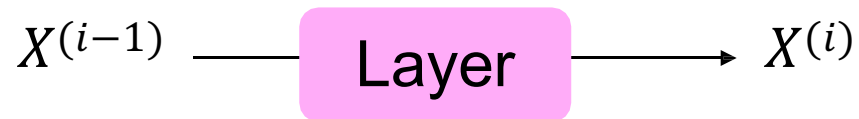
- Now that we've replaced self-attention with multi-head self-attention, we'll go through two **optimization tricks** that end up being :
  - **Residual Connections**
  - **Layer Normalization**
- In most Transformer diagrams, these are often written together as “Add & Norm”



Transformer Decoder

# The Transformer Encoder: **Residual connections** [[He et al., 2016](#)]

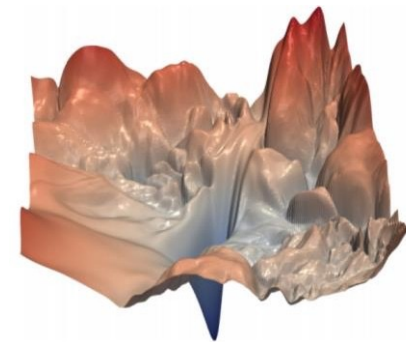
- **Residual connections** are a trick to help models train better.
  - Instead of  $X^{(i)} = \text{Layer}(X^{(i-1)})$  (where  $i$  represents the layer)



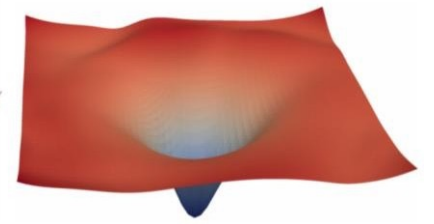
- We let  $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$  (so we only have to learn “the residual” from the previous layer)



- Gradient is **great** through the residual connection; it's 1!
- Bias towards the identity function!



[no residuals]



[residuals]

[Loss landscape visualization,  
[Li et al., 2018](#), on a ResNet]

# The Transformer Encoder: **Layer normalization** [[Ba et al., 2016](#)]

- **Layer normalization** is a trick to help models train faster.
- Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean and standard deviation **within each layer**.
  - LayerNorm's success may be due to its normalizing gradients [[Xu et al., 2019](#)]
- Let  $x \in \mathbb{R}^d$  be an individual (word) vector in the model.
- Let  $\mu = \frac{1}{d} \sum_{j=1}^d x_j$ ; this is the mean;  $\mu \in \mathbb{R}$ .
- Let  $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$ ; this is the standard deviation;  $\sigma \in \mathbb{R}$ .
- Let  $\gamma \in \mathbb{R}^d$  and  $\beta \in \mathbb{R}^d$  be learned “gain” and “bias” parameters. (Can omit!)
- Then layer normalization computes:

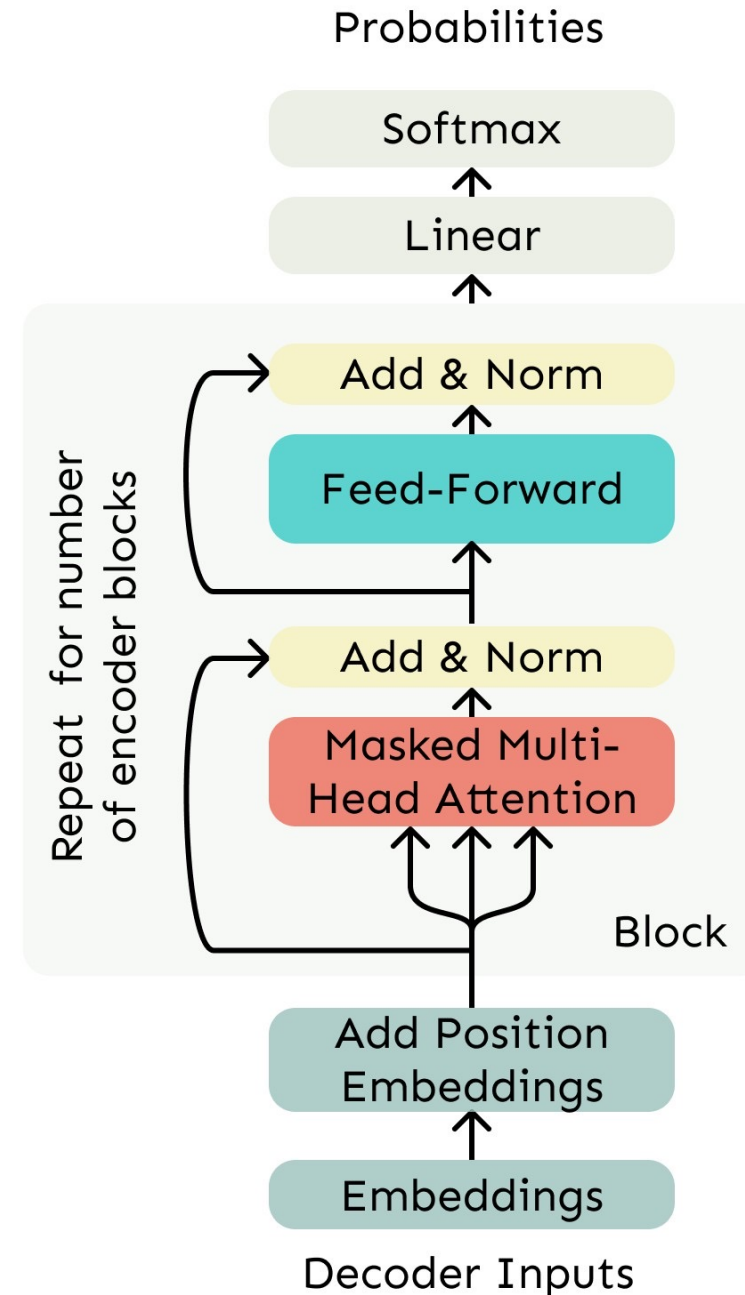
$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

Modulate by learned elementwise gain and bias

# The Transformer Decoder

- The Transformer Decoder is a stack of Transformer Decoder **Blocks**.
- Each Block consists of:
  - Self-attention
  - Add & Norm
  - Feed-Forward
  - Add & Norm
- That's it! We've gone through the Transformer Decoder.

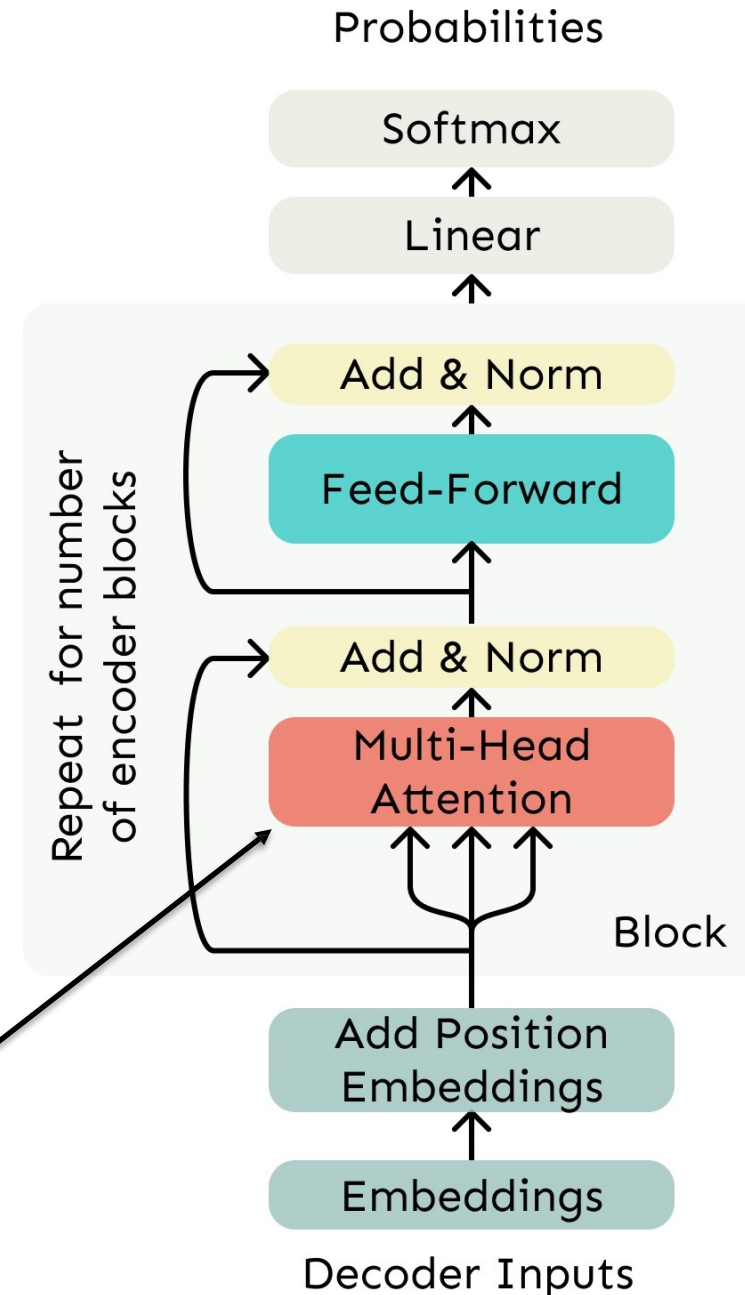




# The Transformer Encoder

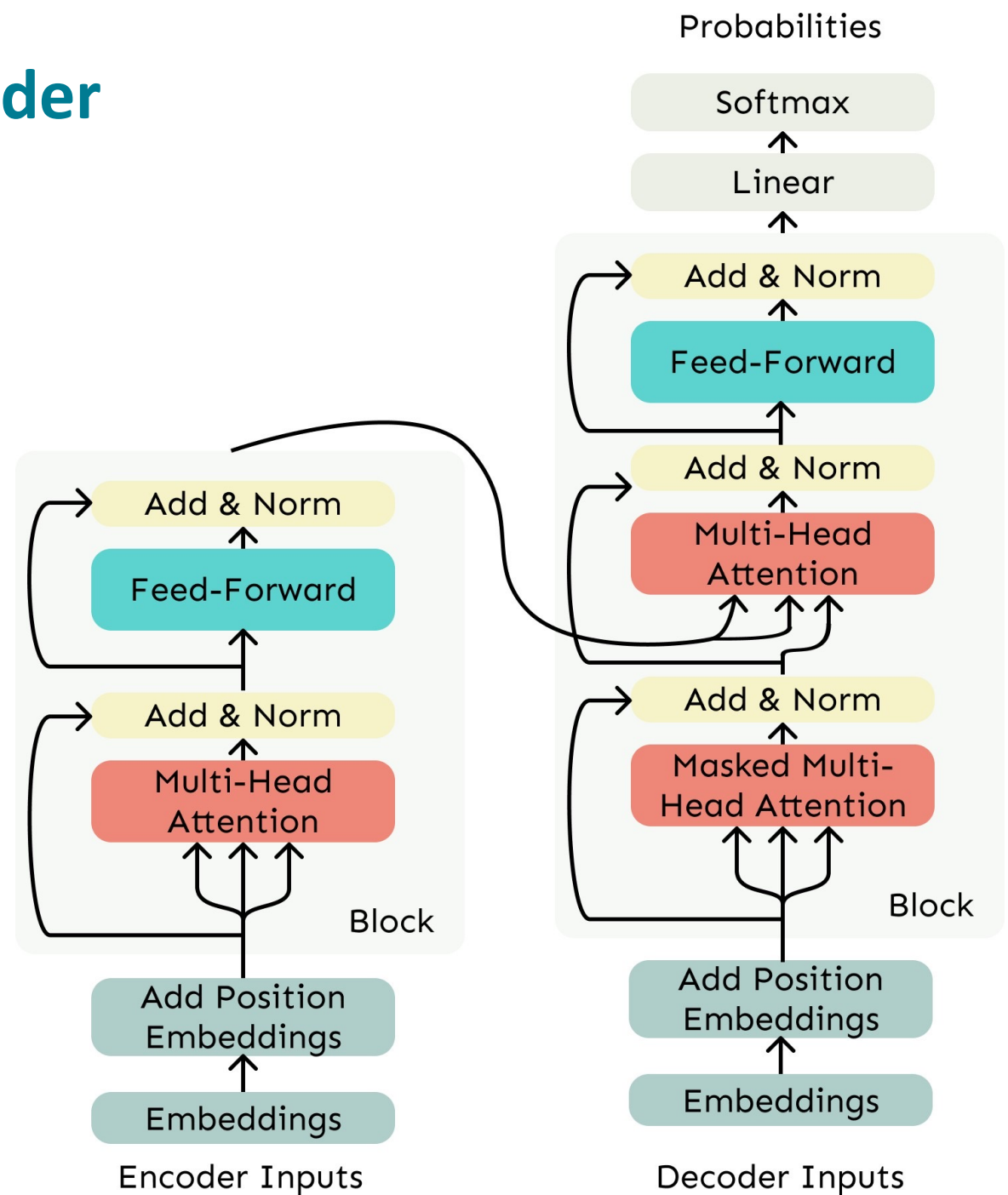
- The Transformer Decoder constrains to **unidirectional context**, as for **language models**.
- What if we want **bidirectional context**, like in a bidirectional RNN?
- This is the Transformer Encoder. The only difference is that we **remove the masking** in the self-attention.

**No Masking!**



# The Transformer Encoder-Decoder

- Recall that in machine translation, we processed the source sentence with a **bidirectional** model and generated the target with a **unidirectional model**.
- For this kind of seq2seq format, we often use a Transformer Encoder-Decoder.
- We use a normal Transformer Encoder.
- Our Transformer Decoder is modified to perform **cross-attention** to the output of the Encoder.



# Outline

1. From recurrence (RNN) to attention-based NLP models
2. Introducing the Transformer model
3. Great results with Transformers

# Great Results with Transformers

First, Machine Translation from the original Transformers paper!

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	<b>41.29</b>	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$

# Great Results with Transformers

Next, document generation!

Model	Test perplexity	ROUGE-L
<i>seq2seq-attention, <math>L = 500</math></i>	5.04952	12.7
<i>Transformer-ED, <math>L = 500</math></i>	2.46645	34.2
<i>Transformer-D, <math>L = 4000</math></i>	2.22216	33.6
<i>Transformer-DMCA, no MoE-layer, <math>L = 11000</math></i>	2.05159	36.2
<i>Transformer-DMCA, MoE-128, <math>L = 11000</math></i>	1.92871	37.9
<i>Transformer-DMCA, MoE-256, <math>L = 7500</math></i>	1.90325	38.8

The old standard



Transformers all the way down.



# Great Results with Transformers





Before too long, most Transformers results also included **pretraining**.

Transformers' parallelizability allows for efficient pretraining, and have made them the de-facto standard.

On this popular aggregate benchmark, for example:



**All** top models are Transformer (and pretraining)-based.

Rank Name		Model	URL	Score
1	DeBERTa Team - Microsoft	DeBERTa / TuringNLRv4		90.8
2	HFL iFLYTEK	MacALBERT + DKM		90.7
+	3	Alibaba DAMO NLP	StructBERT + TAPT	 90.6
+	4	PING-AN Omni-Sinitic	ALBERT + DAAF + NAS	90.6
5	ERNIE Team - Baidu	ERNIE		90.4
6	T5 Team - Google	T5		90.3

**More results Thursday when we discuss pretraining.**

[[Liu et al., 2018](#)]

**Questions?**