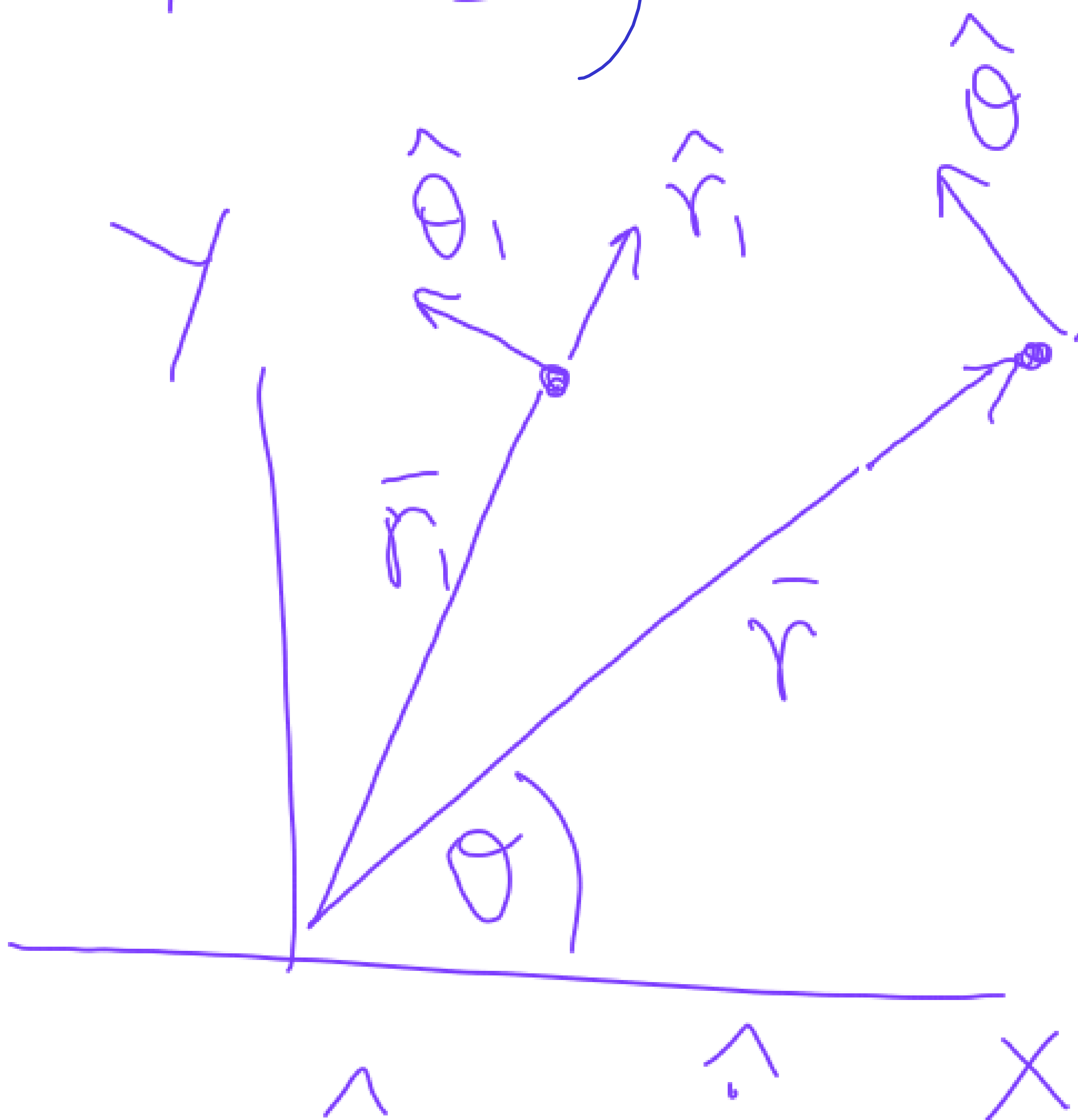
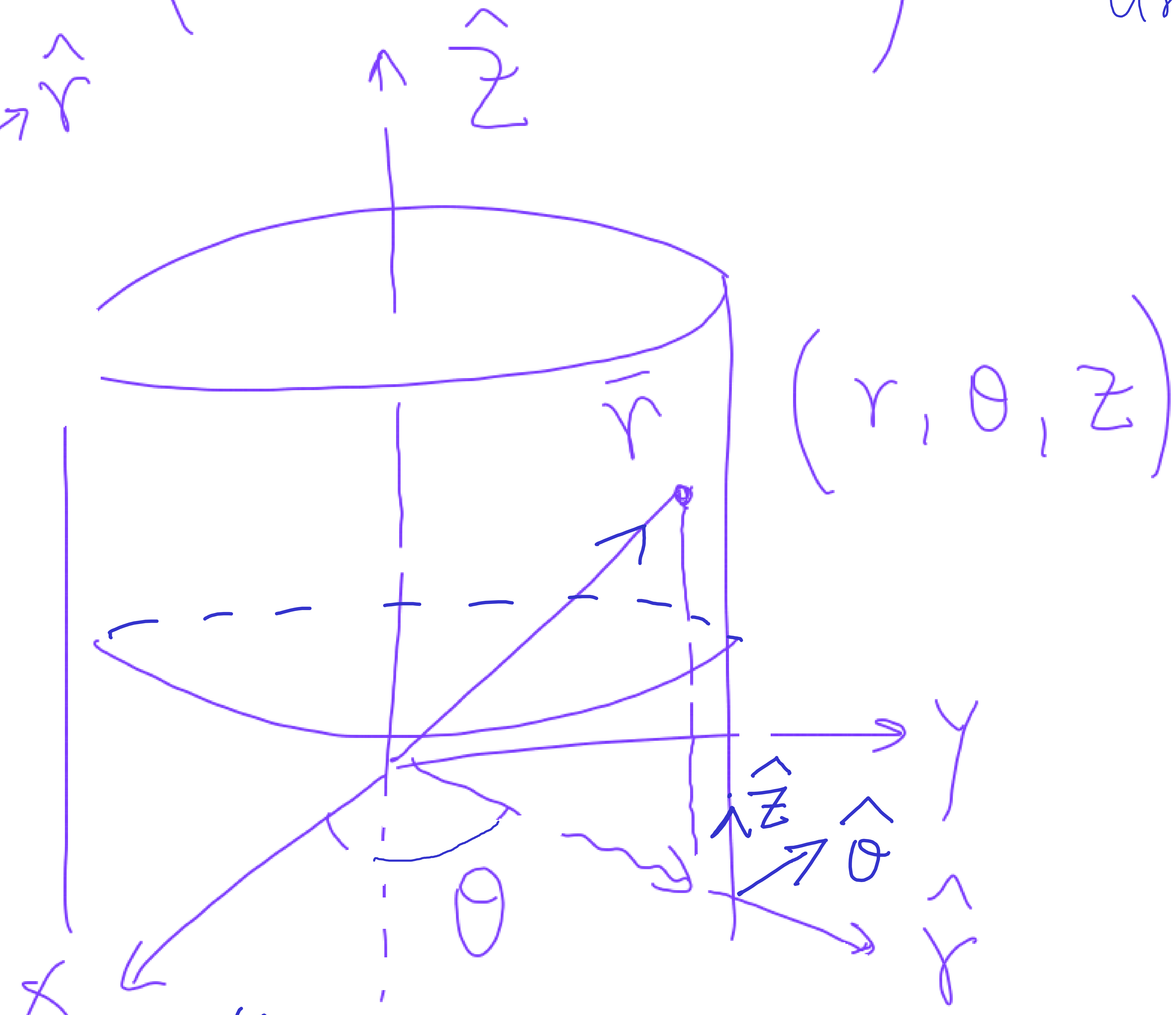
one orthogonal set to another orthogonal set of co-ordinates

$$(\hat{X} \perp \hat{Y} \perp \hat{Z}) \longrightarrow (\hat{r}, \hat{\theta}, \hat{z})$$

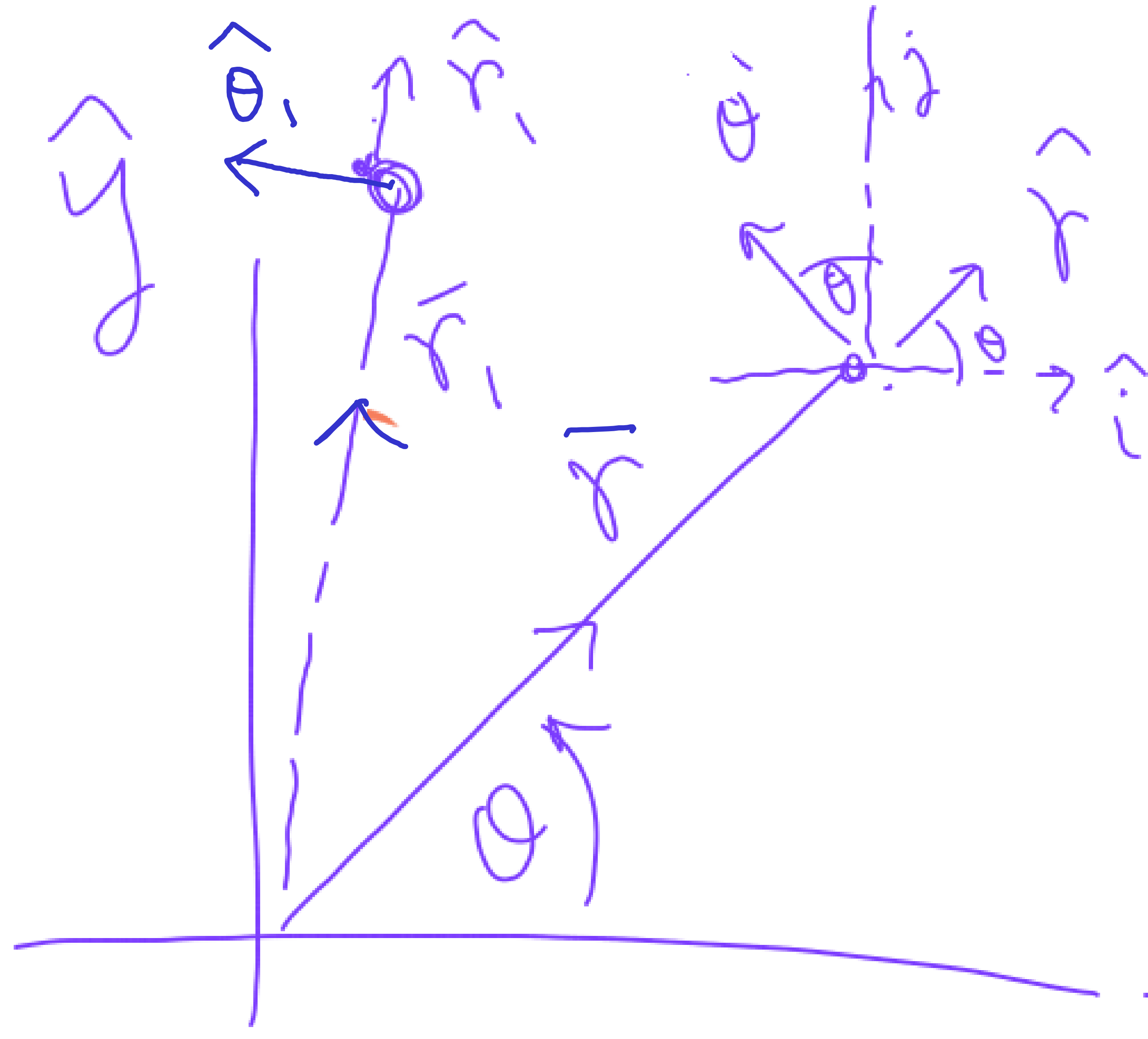
Hat ^ implies unit vector




$$\bar{r} = x\hat{i} + y\hat{j}$$
$$= r\hat{r}$$




While $(\hat{x}, \hat{y}, \hat{z})$ are fixed in space, $(\hat{r}, \hat{\theta})$ keeps changing depending on $\bar{r}$




$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\hat{i} = \hat{x}$$
$$\hat{j} = \hat{y}$$
$$\hat{k} = \hat{z}$$

$$\hat{i} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$
$$\hat{j} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$
$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$
$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\bar{r} = r\,\hat{r}$$

$$\bar{r} = x\,\hat{i} + y\,\hat{j}$$

$$d\bar{r} = dx\,\hat{i} + dy\,\hat{j} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

(prove this)

Relation among $(r,\theta,z)$ & $(\hat{i},\hat{j},\hat{k})$

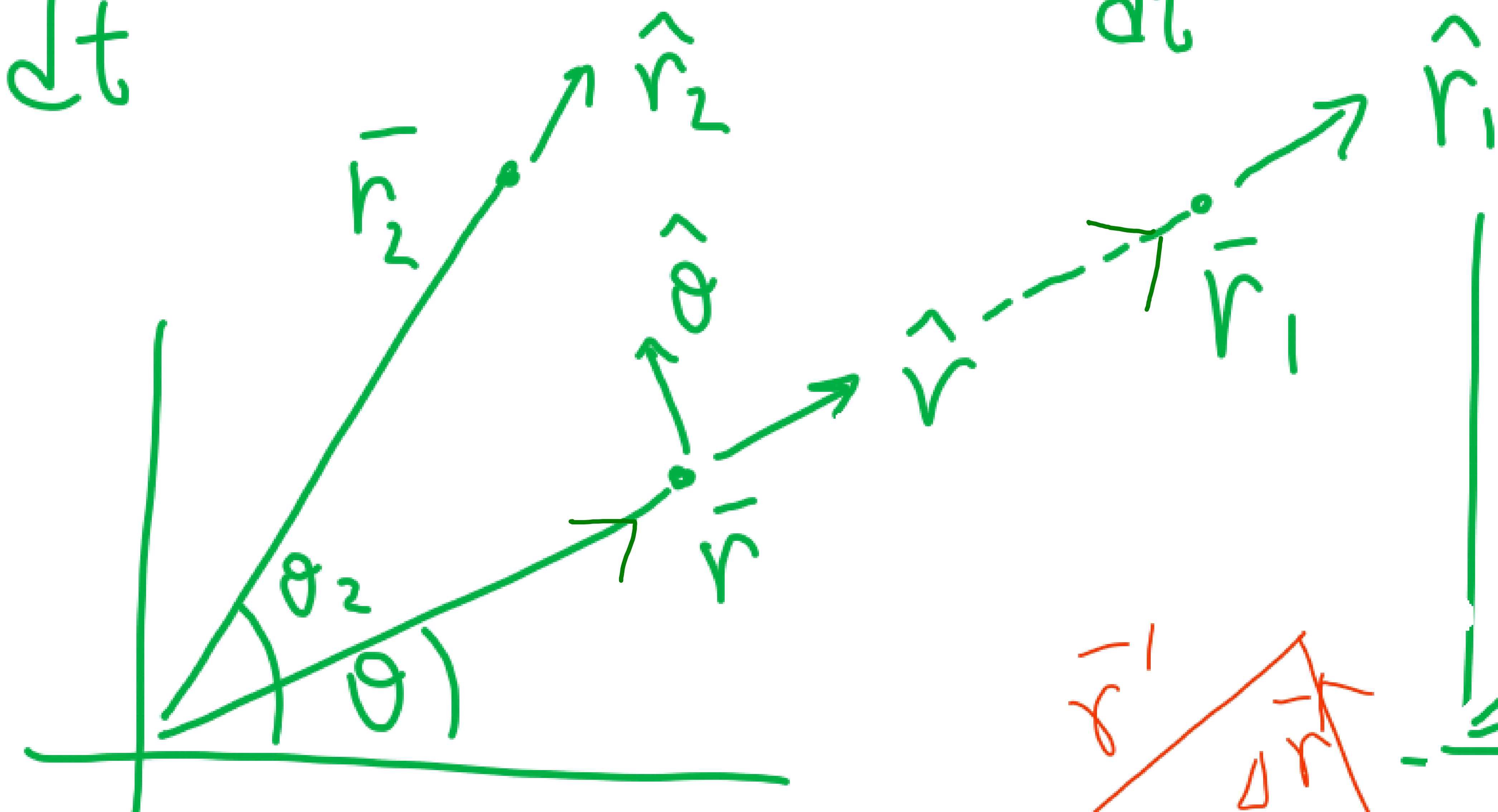
Compute velocity $\frac{d\bar{r}}{dt}$:

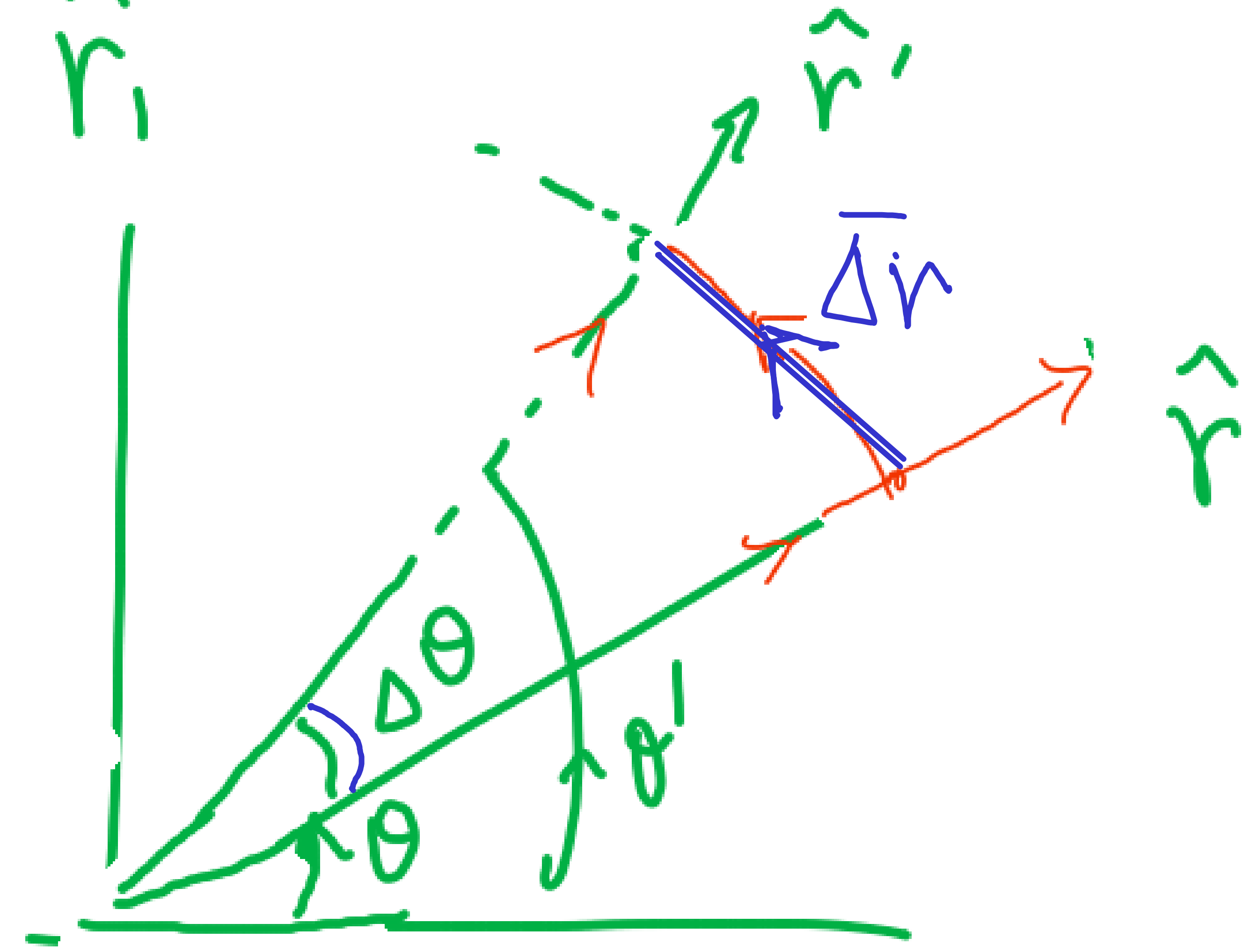
$$\bar{r} = r\hat{r}$$

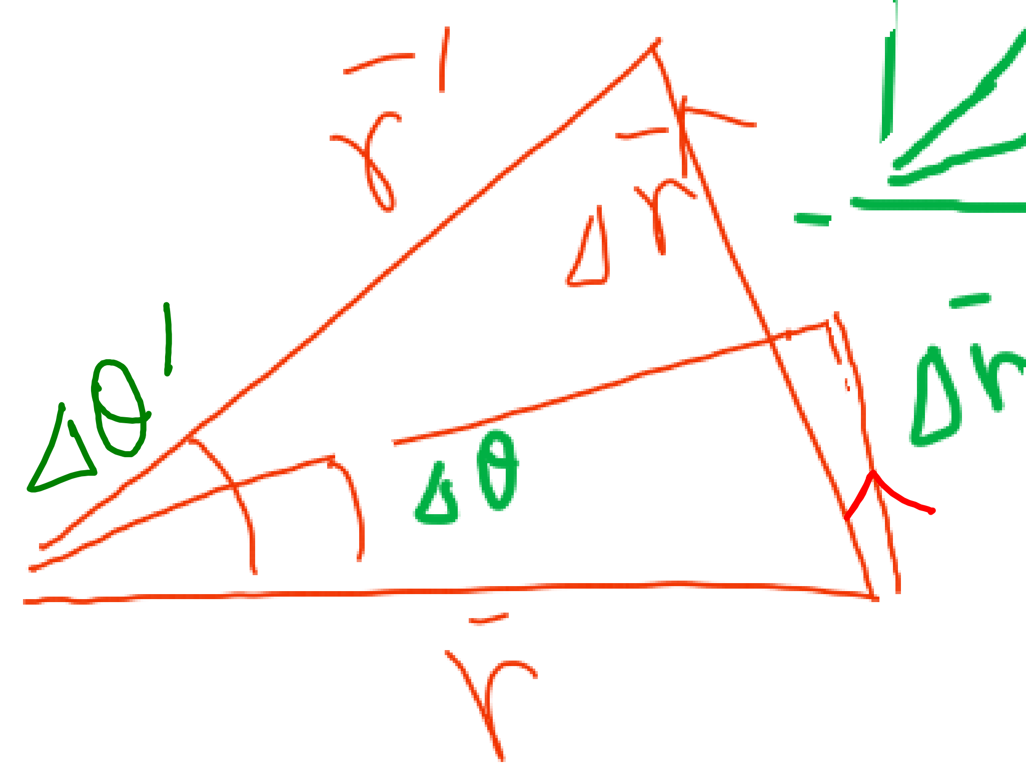
$$\frac{d\bar{r}}{dt} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\frac{d\hat{r}(r,\theta)}{dt} = \cancel{\frac{\partial\hat{r}}{\partial r}}\dot{r} + \frac{\partial\hat{r}}{\partial\theta}\dot{\theta}$$

$$= \dot{r}\hat{r} + r\frac{\partial\hat{r}}{\partial\theta}\dot{\theta} \boxed{= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$




$$\frac{\partial\hat{r}}{\partial\theta} = \lim_{\Delta\theta\to 0}\frac{\hat{r}' - \hat{r}}{\Delta\theta}$$

$$= \left(\frac{\bar{r}'}{r'} - \frac{\bar{r}}{r}\right)\frac{1}{\Delta\theta}$$

$r' = r$

$$= (\bar{r}' - \bar{r})\frac{1}{r\Delta\theta}$$

$$= \cancel{\frac{r\Delta\theta}{r\Delta\theta}}\hat{\theta}$$

because $\partial\hat{r}/\partial\theta$ is partial derivatv

As $\Delta\theta \to 0$, $\Delta\bar{r}$ becomes $\perp$ to $\bar{r}$

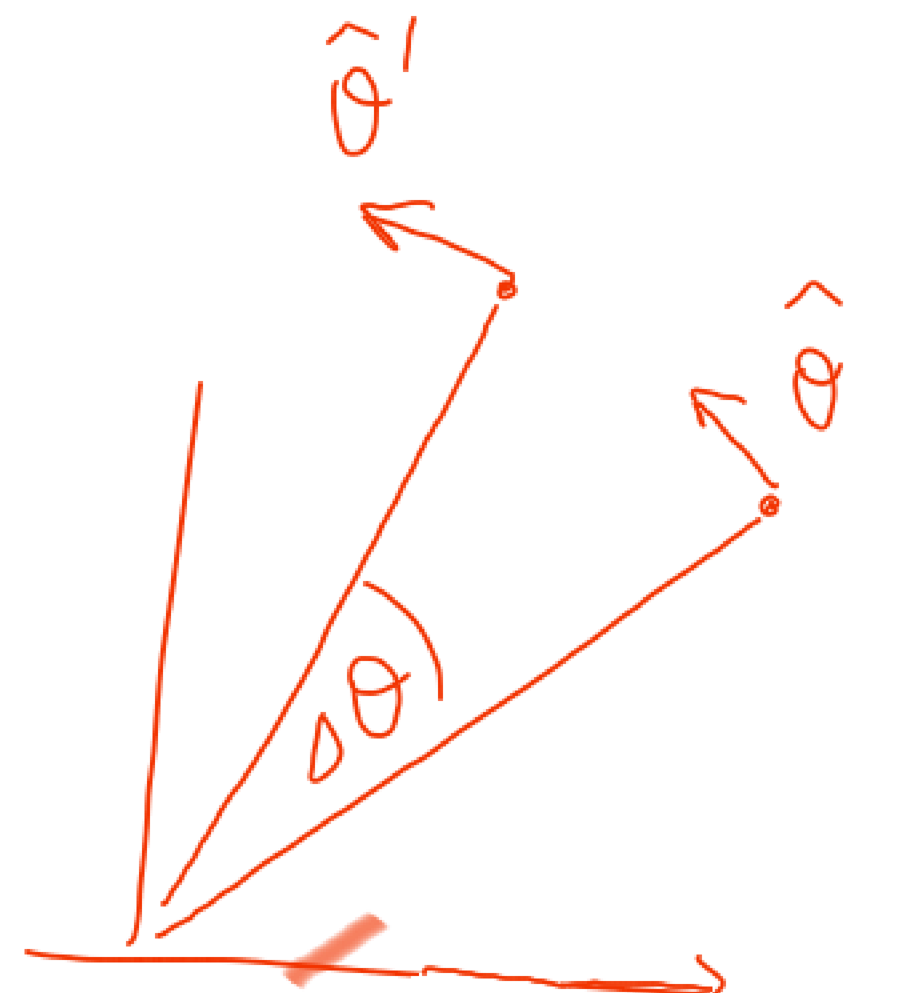
All the partial derivatives of the unit vectors.

$$\frac{\partial \hat{r}}{\partial \theta} \approx \frac{\hat{r}' - \hat{r}}{\Delta\theta} = \frac{\Delta\theta\,\hat{\theta}}{\Delta\theta} = \hat{\theta}$$

$$\frac{\partial \hat{r}}{\partial r} = 0$$

$$\frac{\partial \hat{\theta}}{\partial r} = 0$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}$$



$-\hat{r}\,\Delta\theta =$

Note: as $\Delta\theta \to 0$, $d\bar{r}$ becomes $\perp$ to $\hat{\theta}$

# Calculate Accl$^n$

$$\frac{\partial \hat{r}}{\partial \theta} = \hat{\theta}$$

$$\bar{V} = \frac{d\bar{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = \frac{\partial \hat{\theta}}{\partial \theta}\dot{\theta} = -\hat{r}\dot{\theta}$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}$$

$$\bar{a} = \frac{d^2\bar{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\hat{\theta}}{dt}$$

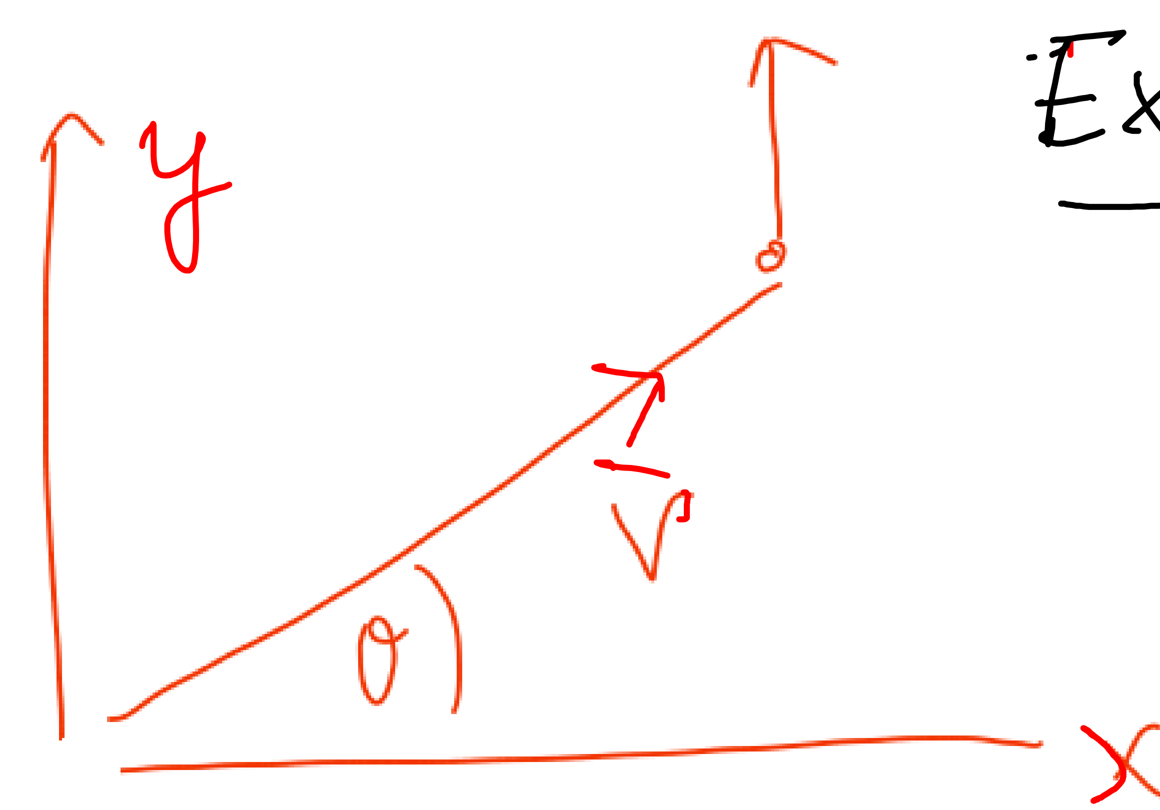
$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$\frac{\partial \hat{r}}{\partial r} = \frac{\partial \hat{\theta}}{\partial r} = 0$$

$$= \hat{r}(\ddot{r} - \underline{r\dot{\theta}^2}) + \hat{\theta}(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$r(t)$

$\theta(t)$




Examples

1) Stdy circular motn $\dot{r}=0$

only $-\hat{r}\, r\dot{\theta}^2 \neq 0$

centripetal accl$^n$

2) $\dot{\theta}=0, \ddot{\theta}=0$

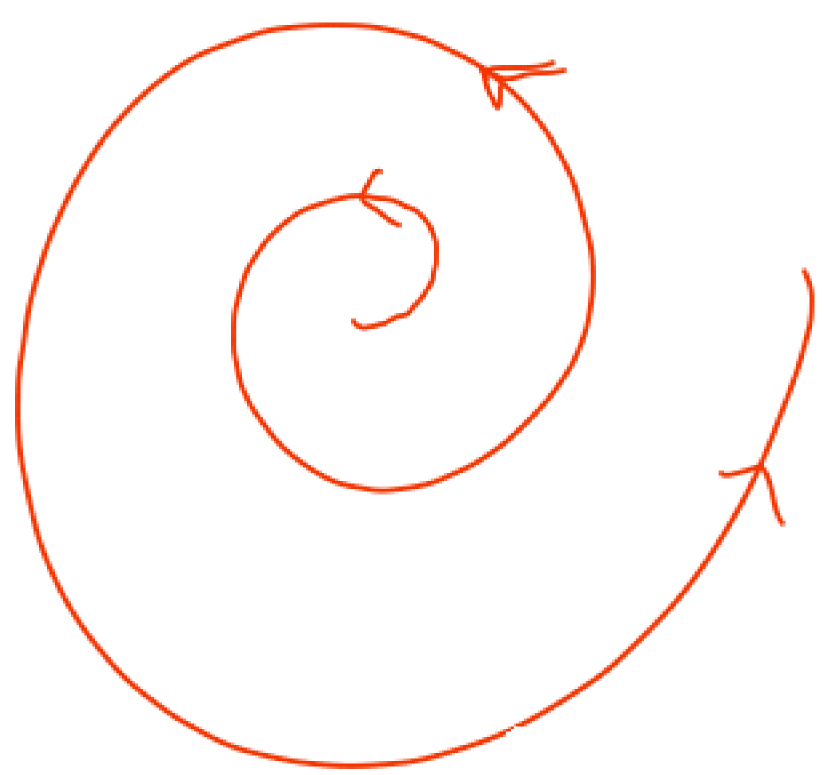
1-D motn along $\hat{r}$

3) $\dot{\theta} = \omega_0$ , $\dot{r} = \alpha$

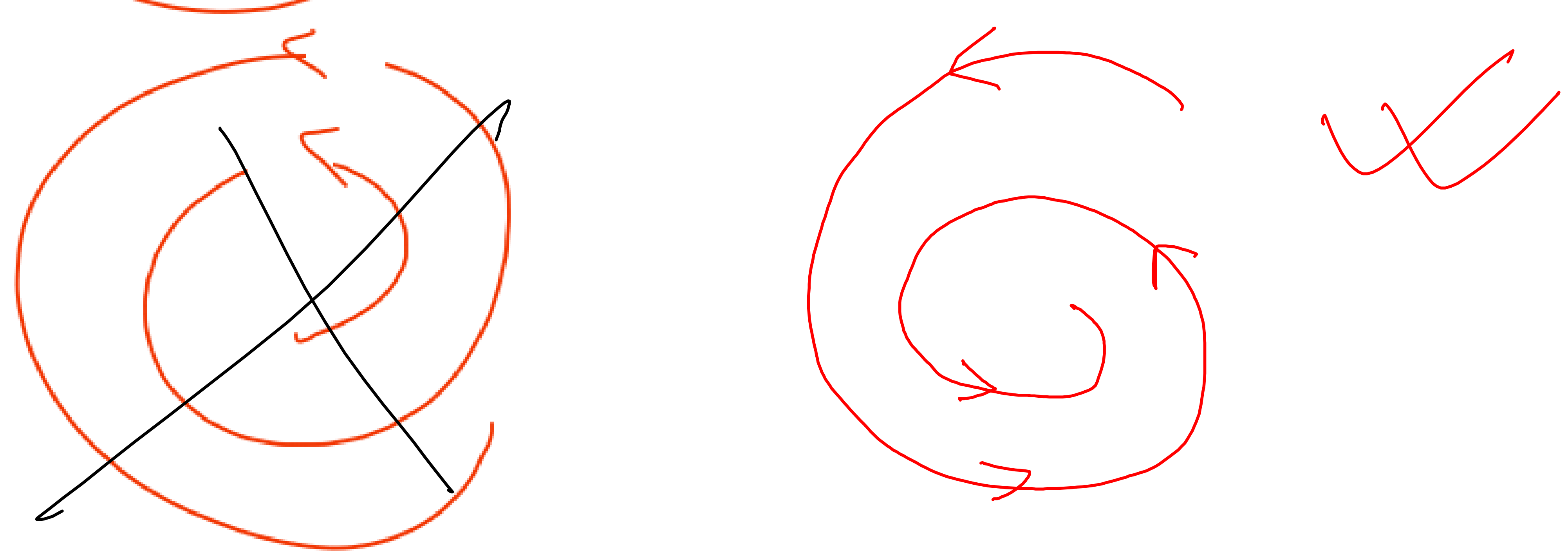
$\downarrow$

$r(t) = \alpha t + \alpha_0$

$\alpha > 0$



$\alpha < 0$

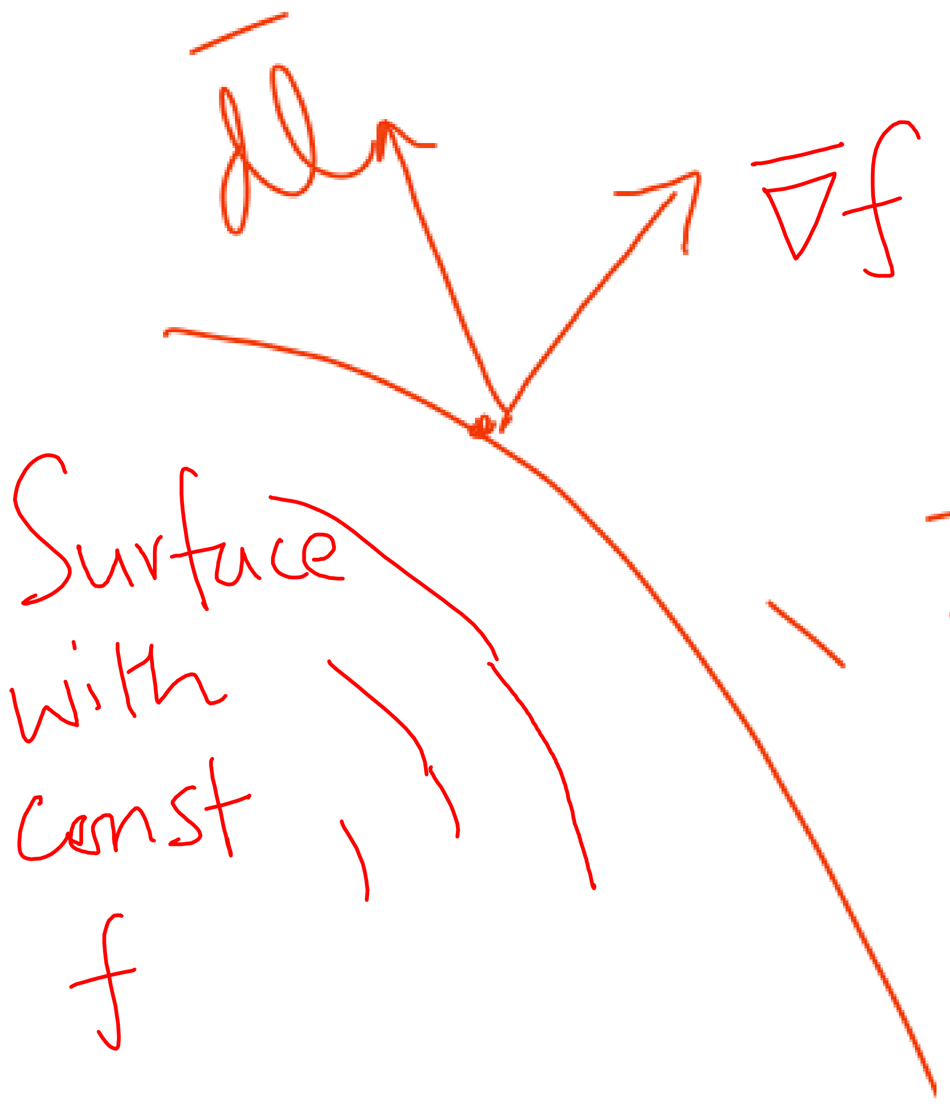


Since $\dot{\theta} = \omega_0 > 0$

$\theta$ keeps increasing

while $r(t)$ decreases

grad, div, curl: all are certain types of space derivatives, but what's the difference?




$\vec{\nabla} f$ is $\perp$ to the surface

$f(x,y,z) = c_1$

$df = \vec{\nabla} f \cdot d\vec{l}$ (change in $f$ along $d\vec{l}$)

$\vec{\nabla}\times\vec{A}(x,y,z)$

Fields with nonzero curl

$\overline{\text{div}}\, \bar{A}(x\,y\,z) = c_0 \hat{r}$